Unified theory of $\gamma d \to np$, $\pi^0 d$, πNN, and $pp \to pp\gamma$ and the chiral bag model

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A unified theory of photopion reactions in two-nucleon systems ($\gamma d \to pn$, $\pi^0 d$, and πNN) and NN bremsstrahlung ($NN \to NN\gamma$) is presented. By exposing the two-body [BB, where $B = N$ or $\Delta(1232)$] and three-body (πBB and γBB) unitarity, we derive a set of coupled integral equations to determine the amplitudes for these reactions. These equations have the same kernel as the equations one gets for the BB-πBB system. The two-body input amplitudes are the result of a coupled channel unitary theory for $\pi N \to \pi N$ and pion photoproduction on a single baryon, within the framework of a gauge and chirally invariant Lagrangian, which is obtained from the chiral bag model Lagrangian. The renormalization due to the πB interaction is incorporated in a consistent manner.

I. INTRODUCTION

Over the past ten years the NN-πNN equations[1-5] have been developed into a unified theory for the reactions $NN \to NN$, πd and $\pi d \to \pi d$, NN at medium energies.[6-14] This has allowed the analyses of a vast data bank consisting of both the differential cross section and polarization observables. The input to these equations are the low energy π-N and N-N amplitudes. To improve the results for N-N elastic scattering, some groups[10,14] have included heavy meson exchanges. There has been great consistency in the results produced by the different groups, even though the details of the numerical calculations and the treatment of the kinematics are quite different. This has led to the situation where the present discrepancy between theory and experiment will require the introduction of either new physics in the form of higher order pionic effects,[1,15] or new degrees of freedom in the form of quarks.[16] Before we proceed with the introduction of explicit quark degrees of freedom into the theoretical description of these reactions, we need to establish how far we can reduce the discrepancy between theory and experiment within the framework of hadronic degrees of freedom. At the same time we need to guarantee that a theory based on the NN-πNN equations is consistent with quantum chromodynamics (QCD), to the extent that the baryons (i.e., $N, \Delta, \ldots$) are treated on equal footing. This is achieved by the constraint on the coupling of the meson to the baryons, as well as the corresponding form factors, in such a way that both are consistent with QCD.

Recently, Afnan and Blankleider[17] showed how to extend the NN-πNN equations so as to include all the baryons on equal footing, with the coupling constants and form factors determined from the quark structure of the hadrons. This was achieved by considering a chiral bag model[18-20] (e.g., cloudy bag model[19]) Lagrangian in which the quarks are confined to the bag, and are coupled to the pion in order to satisfy chiral symmetry. This Lagrangian, after expansion in the pion field, is projected onto the space of baryons. In this way the quark degrees of freedom are integrated out, in favor of the hadronic degrees of freedom. However, the resultant Lagrangian in terms of the hadronic degrees of freedom, has both the coupling constants and form factors determined by the bag model. In particular, the parameters of the bag model are the only free parameters in the Lagrangian and these can be determined by the π-N scattering data. The inclusion of the electromagnetic coupling is achieved by introducing the minimal coupling at the quark level as was done previously by Kälberman and Eisenberg.[21] In this way the number of parameters in the Lagrangian are not increased, and as we will show, the resultant theory based on this Hamiltonian includes the contribution of the exchange currents.

In a previous publication[22] we extended the work of Afnan and Pearce[23] on the πN-$\pi\pi N$ system, to formulate a multichannel unitary theory of single pion photoproduction from a single baryon B. Here, the baryon B includes not only the nucleon and $\Delta(1232)$, but may also include the Roper and strange baryons. Though, in principle, the theory is applicable to all types of interactions, we took the gauge and chiral Lagrangian which is based on the chiral bag model Lagrangian,[18-20] as described above. In that theory, we treat the baryons (the N, Δ, Roper, . . .) on equal footing as a three-quark state, and ignored the antiquark contribution. In other theories,[21,24] the relation between the quark model and the multiple scattering theories is not clear. In particular, it is not clear whether the coupling constants used in these theories are bare or renormalized. In our previous paper on pion photoproduction from a single baryon,[22] the renormalization was treated in a consistent manner. We found that (a) In the Born term of the multiple scattering series for the amplitude, the s-channel pole diagram has bare vertices and bare coupling constants, while the rest of the diagrams have dressed vertices that correspond to the physical coupling constants. This result was obtained by exposing the three-body ($\pi\pi B$ and $\gamma\pi B$) unitarity cuts. (b) We were provided with an off-shell unitary pion photoproduction amplitude. (c) The theory is valid both below and above the pion production threshold. (d) As previously stated, all the baryons were treated on equal footing as three-quark states. (e) Finally, the renormalization due to the πB interaction was treated self-consistently.

We will employ the methods used previously to formulate pion photoproduction on a single nucleon,[22] to give a unified description of the reactions $\gamma d \rightarrow pn, \pi^0 d, \pi NN$, and $NN \rightarrow NN\gamma$. By exposing the two-body [BB, where $B = N, \Delta(1232), \ldots$] and then the three-body (πBB and γBB) unitarity cuts, we are able to derive a set of coupled integral equations for the reactions under consideration. These equations have the same kernel as the BB-πBB equations,[17] suggesting that we can give a unified description of the above reactions as well as $\pi d \rightarrow \pi d, NN, \pi NN$, and $NN \rightarrow NN, \pi d, \pi NN$. The input to these equations are the two-body amplitudes for πN scattering[23,25] and pion photoproduction on a single nucleon.[22] The new feature of this formulation is the fact that both the renormalization and unitarity are treated consistently within the framework of the Hamiltonian given in Sec. II, and at the same time give a unified description of pion photoproduction, photodisintegration of the deuteron, and NN bremsstrahlung. By including the coupling to the πNN channel, we are able to examine, in a consistent manner, the role of the pion exchange current in these reactions. Finally, by commencing with a Hamiltonian that is based on a chiral Lagrangian at the quark level, the form factors for all vertices are related to each other, and to the basic size of the baryon. What is not included, apart from the antiquark contributions, are the effects of any change in the size of the baryon in a two-baryon system, and the possibility of baryons overlapping to the extent of allowing quarks to penetrate from one baryon to the next. Such effects are most likely to be small in the low to medium energy region, and could be treated at the perturbation level.

Although there have been no previous attempts at such a unification, there is a long history for each of the reactions under consideration. Thus for $\gamma d \rightarrow \pi^0 d$, one of the first such calculations goes back to Zachariasen,[26] who used the classification of the diagrams that contribute to the amplitude for this reaction, according to the number of pions in intermediate states. This classification method was generalized by Taylor,[27] to use the last-cut-lemma to derive integral equations that satisfy unitarity within the framework of field theory. This method has been used by the present authors[22] and others to derive integral equations for the πB-$\pi\pi B$ (Ref. 23), and $BB \rightarrow \pi BB$ (Refs. 4, 5, and 17) systems.

More recently, Laget[28] and his collaborators have calculated the amplitude for $\gamma d \rightarrow pn$ (Ref. 29), $\pi^0 d$ (Ref. 30), and πNN (Ref. 31) in a nonunitary but consistent manner. They examined the multiple scattering series for these reactions and included those diagrams which have singularities close to the physical region. In this way, they hoped that all the reactions were treated on equal footing and that the important contributions were included. Thus for the photodisintegration of the deuteron, Laget[29] also included the meson exchange current contributions which arise due to gauge invariance. He found that meson exchange currents give a substantial contribution to the total cross section. This is in contrast to the work of Arenhövel and collaborators[32] who found that meson exchange currents give a relatively small contribution to the cross section. This difference is due to the fact that Laget[29] included the meson exchange current diagrams by coupling the photon to the pion which is exchanged between the two nucleons in the multiple scattering series. This approach is similar to ours,[22] in that we introduce the minimal electromagnetic coupling at the quark level in the chiral Lagrangian, and after projection onto the baryon states, the meson exchange current contribution is naturally incorporated as a result of the photon coupling to the pion in the πBB part of the Hilbert space. The advantage of our approach is that all form factors are related to each other and to the size of the hadron. A relativistic calculation of the photodisintegration of the deuteron at higher energies has been carried out by Ogawa *et al.*[33] by taking into account the covariant Born amplitudes only, which includes the deuteron pole, the nucleon pole, and the isobar contribution. In the above three calculations,[29,32,33] the deuteron propagator is taken to be elementary, i.e., the dissociation of the deuteron is neglected. In all the above analyses, the agreement with experiment is generally good for the cross section. However, the recent TRIUMF measurements[34] of the analyzing power for $H(\vec{n},\gamma)d$, at medium energies, suggests[35] that one need not include the isobar degrees of freedom. This is surprising and may need further analyses.

Turning to pion photoproduction, i.e., $\gamma d \rightarrow \pi^0 d$, Lazard *et al.*[36] have included the single and double scattering terms under some kinematical assumptions, while Bosted and Laget[30] removed these assumptions and included the leading terms exactly. On the other hand, Hǎndel *et al.*[37] used the isobar model to give a unified treatment of both $\gamma d \rightarrow \pi d$ and $\pi d \rightarrow \pi d$, while Miyachi *et al.*[38] included the dibaryon term in a relativistic calculation. Finally, Laget[31] has carried out the most extensive study of the reaction $\gamma d \rightarrow \pi NN$.

For NN bremsstrahlung ($pp \rightarrow pp\gamma$ and $np \rightarrow np\gamma$) there is an extensive history of calculations based mainly on the N-N potential models, and they are always treated separately from the other reactions considered above. Some of the highlights of these calculations involve the inclusion of the single and double scattering terms, by Drechsel and Maximon,[39] and the effect of the Coulomb interaction on the N-N wave function, by Marker and Signell.[40] More recently, Kamal and Szyjewicz[41] have carried out relativistic calculations based on the one-Boson exchange model for the N-N interaction. This allowed them to include explicitly the contribution of the Δ degrees of freedom and the meson exchange current due to the π, ρ, and ω mesons.

In the present investigation, we have a unified formulation of the reactions $\gamma d \rightarrow np, \pi^0 d, \pi NN$, and $NN \rightarrow NN\gamma$ which is similar in spirit to the work of Laget.[28] However, the following differences should be noted: (i) We will include the multiple scattering of the pions and baryons to all orders in perturbation theory by deriving integral equations for the relevant amplitudes. (ii) To maintain consistency with QCD we commence with a chiral Lagrangian at the quark level and then project it onto the Hilbert space of baryons. This will give us a parameterfree form factor for all the vertices, in terms of the basic size of the hadrons. (iii) We have a consistent description of both (a) the renormalization of the propagators and form factors, and (b) the inclusion of unitarity at the two- and three-body levels. (iv) The integral equation for the reaction under consideration has the same kernel as the integral equation for the BB-πBB system. This will allow us to give a unified description of the reactions,

$$\begin{aligned}\gamma+d&\to n+p\\&\to\pi^0+d\\&\to\pi+N+N\ ,\end{aligned}\tag{1.1a}$$

$$\begin{aligned}\pi+d&\to\pi+d\\&\to N+N\\&\to\pi+N+N\ ,\end{aligned}\tag{1.1b}$$

$$\begin{aligned}\pi+d&\to\gamma+d\\&\to\gamma+N+N\ ,\end{aligned}\tag{1.1c}$$

$$\begin{aligned}N+N&\to N+N\\&\to\pi+d\\&\to\pi+N+N\ ,\end{aligned}\tag{1.1d}$$

and

$$\begin{aligned}N+N&\to\gamma+d\\&\to\gamma+N+N\ .\end{aligned}\tag{1.1e}$$

The reaction $\pi+d\to\gamma+N+N$ will not be discussed in this paper, but can easily be treated in a similar manner. (v) With the advent of medium energy electromagnetic probes, we can, in a consistent manner, include the higher isobars in terms of their quark structure. Here, the inclusion of three-body unitarity might be essential. Finally, the advent of high precision data will require the calculations to go beyond the first few orders in the multiple scattering series. This has not been done in the past for many of the reactions.

The above formulation generates the N-N interaction to the extent that it includes one pion exchange, and the box diagram due to the two-pion exchange with the N-Δ and Δ-Δ intermediate states. To fully describe N-N scattering we might need to go beyond the pion exchange, and include the heavy meson exchange. This can be achieved in this formalism as was the case in the NN-πNN equations, where heavy meson exchange was introduced as a static potential.[10,14]

In Sec. II, we introduce the interactions and give a summary of the two-body coupled equations, which were derived in our previous papers[22,23] and are to be used as input to the present theory. To derive the coupled equations for the reactions in Eqs. (1.1a), (1.1c), and (1.1e), we need the integral equations for the BB-πBB system. In Sec. III, we present a summary of the results of Ref. 17 for the BB-πBB system, and the method used to expose three-body unitarity. Sections IV and V are devoted mainly to the derivation of the amplitudes for $\gamma d\to np$, $NN\to NN\gamma$, and $\gamma d\to\pi^0 d$ and πNN, and how they are related to the amplitudes for the reactions in Eqs. (1.1b) and (1.1d). We then proceed in Sec. VI to derive the integral equations for the reactions in Eqs. (1.1a), (1.1c), and (1.1e). Finally, in Sec. VII we discuss our final results.

II. THE TWO-BODY INTERACTIONS

To formulate our theory for the photodisintegration of the deuteron, NN bremsstrahlung, and pion photoproduction off the deuteron, we need to define the Hamiltonian under consideration and set up the input for the resulting equations. Although the derivation of the equations does not depend on the detailed form of the Hamiltonian, any numerical calculations will require the use of a specific form for the interaction. Furthermore, the details of the derivation become more transparent if the reader has a specific Hamiltonian at hand. Since one of the ultimate aims of the present formulation is to maintain consistency with QCD, we will consider, as our starting point, the chiral bag model Lagrangian.[18-20] This Lagrangian has the quarks confined to a finite volume, and to maintain chiral symmetry, a pion field is coupled to the quarks.[19] In general, such Lagrangians are nonlinear in the pion field. To derive integral equations that satisfy two- and three-body unitarity, we need a renormalizable Lagrangian at the baryon level. This can be achieved if we expand the chiral Lagrangian,[19] in powers of the coupling constant, and keep terms up to order $g^2=(2f_\pi)^{-2}$, where f_π is the pion decay constant. The resultant Lagrangian, often referred to as the cloudy bag model (CBM),[19] has been used extensively to describe both s- (Ref. 42) and p-wave[19,24] π-N scattering. We introduce the electromagnetic interaction by employing the minimal coupling demanded by $U(1)$ gauge symmetry (i.e., $\partial_\mu\to\partial_\mu-ieA_\mu$). This was used by Kälbermann and Eisenberg[21] to study the reaction $\gamma N\to\pi N$ within the framework of the chiral bag model. Since we need to include the electromagnetic coupling to first order, we retain only the terms in the Lagrangian of order g, g^2, e, and eg, and discard the higher order terms in e, such as the $e^2, e^2g, e^2g^2, \ldots$ terms, by expecting them to be negligible. We also neglect the higher order terms in g such as $eg^2, g^3, \ldots$ to make the final equations computationally manageable. These higher order terms, however, can be included in perturbation theory. The resulting total Lagrangian may be written as the sum,

$$L=L_{\rm MIT}+L_\pi+L_\gamma+L_I\ ,\tag{2.1}$$

where $L_{\rm MIT}$ is the Lagrangian for the MIT bag, and L_π and L_γ are the Lagrangians for the free pion and photon, respectively. The interaction Lagrangian L_I consists of the following six different couplings:

$$L_I=L_{qq\pi}+L_{qq\pi\pi}+L_{\pi\pi\pi\pi}+L_{qq\gamma}+L_{qq\pi\gamma}+L_{\pi\pi\gamma}\ ,\tag{2.2}$$

where the explicit forms of the different terms in L_I were given previously.[22] We can now project this Lagrangian, after quantization, onto the Hilbert space of the baryon as was carried through for the CBM,[18,19] to get an effective Hamiltonian at the baryon level. We restrict our present analysis for the baryon-equal-to-one system, to the Hilbert space of one-, two-, and three-particle states (i.e., $|B\rangle$, $|\pi B\rangle$, $|\gamma B\rangle$, $|\pi\pi B\rangle$, and $|\gamma\pi B\rangle$, where $|B\rangle$ is the bare baryon which is composed of three quarks). The effective Hamiltonian, $\hat{H}$, can be expressed in terms of the creation and annihilation operators in the Fock space representation. The interaction part of $\hat{H}$ is the sum of the twelve terms:

$$\hat{H}_I=\langle B\mid\hat{H}\mid B\pi\rangle+\langle B\pi\mid\hat{H}\mid B\pi\rangle+\langle\pi\pi\mid\hat{H}\mid\pi\pi\rangle$$
$$+\langle B\mid\hat{H}\mid B\gamma\rangle+\langle B\pi\mid\hat{H}\mid B\gamma\rangle+\langle\pi\pi\mid\hat{H}\mid\gamma\rangle$$
$$+\langle\pi\mid\hat{H}\mid\pi\gamma\rangle$$
$$+\text{five Hermitian conjugate terms}\ . \tag{2.3}$$

For example, $\langle B\pi\mid\hat{H}\mid B\gamma\rangle$ may be related to the Hamiltonian, H, at the quark level by

$$\langle B\pi\mid\hat{H}\mid B\gamma\rangle$$
$$=\sum_{mn}\sum_{\alpha\lambda}\int d^3q\,d^3k\,\langle m,\alpha q\mid H_{qq\pi\gamma}\mid n,\lambda k\rangle$$
$$\times B_m^\dagger B_n a_{\alpha q}^\dagger C_k^\lambda\ , \tag{2.4}$$

where $B_m^\dagger$ and $a_{\alpha q}^\dagger$ are the creation operators for the quark and pion, respectively, while C_k^λ is the annihilation operator for the photon. In writing the above Hamiltonian, we have truncated our interaction to avoid any direct coupling between the single-particle state $|B\rangle$ and the three-particle state $|B\pi\pi\rangle$ or $|B\pi\gamma\rangle$. This was found[23] to be necessary so that our final equations would be computationally viable. Finally, we note that the interaction Hamiltonian $\hat{H}_I$ is not covariant due to the absence of terms such as $\langle B\mid\hat{H}\mid B\pi\gamma\rangle$ from Eq. (2.3).

Having defined our Hamiltonian in the space of baryons and mesons, we now turn to the input for our final three-body equations presented in the next section. This input is the off-shell amplitude for the reactions $B(\pi,\pi)B$ and $B(\gamma,\pi)B$, as predicted by the above Hamiltonian. In our previous paper,[22] we derived integral equations for these amplitudes by exposing the two-body (πB or γB) and three-body ($\pi\pi B$ or $\pi\gamma B$) unitarity cuts. This was accomplished by using the last-cut-lemma, introduced by Taylor,[27] to derive integral equations in field theory, by classifying diagrams according to their irreducibility. The final amplitude for π-N scattering is given by[22,23]

$$t^{(0)}=t^{(1)}+f^{(1)\dagger}d_0f^{(0)} \tag{2.5a}$$
$$=t^{(1)}+f^{(1)\dagger}d_Bf^{(1)}\ , \tag{2.5b}$$

where $t^{(i)}$ and $f^{(i)}$ are the i-particle irreducible amplitudes for $\pi B\leftarrow\pi B$ and $B\leftarrow\pi B$, respectively, while d_B is the dressed baryon propagator and is given in terms of the bare baryon propagator d_0, by the relation

$$d_B^{-1}=d_0^{-1}-\Sigma^{(1)}\equiv E-m_\alpha^{(0)}-\Sigma^{(1)}\ . \tag{2.6}$$

Here $m_\alpha^{(0)}$ is the bare mass of the baryon α, and $\Sigma^{(1)}$ is the self-energy of the baryon and is given by

$$\Sigma^{(1)}=\Sigma^{(2)}+f^{(1)}gf^{(2)\dagger}\ , \tag{2.7}$$

where g is the πB propagator and is given by $g=d_Bd_\pi$. The one-particle irreducible πBB form factors $F^{(1)}$ and πB amplitude $t^{(1)}$, required to evaluate the full πB amplitude $t^{(0)}$ using Eq. (2.5), satisfy the equations,

$$f^{(1)}=f^{(2)}+f^{(2)}gt^{(1)}\ , \tag{2.8}$$

and

$$t^{(1)}=t^{(2)}+t^{(2)}gt^{(1)}\ . \tag{2.9}$$

Here, the structure of the two-particle irreducible amplitudes, in terms of the interaction Hamiltonian $\hat{H}_I$, requires the exposure of the three-body unitarity cuts. This procedure results in having[23] $\Sigma^{(2)}=0$, $f^{(2)}=\langle B\mid\hat{H}\mid B\pi\rangle$, and $t^{(2)}$ given by

$$t^{(2)}=t^{(3)}+\sum_{ij}f^{(1)}(i)d_\pi(i)d_Bu_{ij}^{(2)}d_\pi(j)d_Bf^{(1)\dagger}(j)\ , \tag{2.10}$$

where $u_{ij}^{(2)}$ are the Alt-Grassberger-Sandhas[43] (AGS) amplitudes for the $\pi\pi B$ system. In Eq. (2.10), $f^{(1)}(i)$ is the amplitude for the ith pion being absorbed by the baryon. Finally, $t^{(3)}$ in Eq. (2.10) is the three-particle irreducible amplitude which for the present Hamiltonian is given by $t^{(3)}=\langle\pi B\mid\hat{H}\mid\pi B\rangle$.

Here we would like to point out that there is an alternative way of obtaining the πB amplitude $t^{(0)}$, other than by using Eqs. (2.5)–(2.9). This involves writing the amplitude as a solution of a two-body equation of the form,[44]

$$t^{(0)}=v+vgt^{(0)}\ , \tag{2.11}$$

with the potential v given by

$$v=t^{(2)}+f^{(2)\dagger}d_0f^{(2)}\ . \tag{2.12}$$

The advantage of this latter procedure is its numerical simplicity, since we need not calculate the off-shell amplitude $t^{(1)}$ as an intermediate step in our calculation of $f^{(1)}$.

The potential v in Eq. (2.12) is illustrated diagrammatically in Fig. 1. Here we see that the diagram which has the s-channel pole, in Fig. 1(e) [the second term on the right-hand side (rhs) of Eq. (2.12)], has bare vertices, while the rest of the diagrams that arise from $t^{(2)}$, in Figs. 1(b), 1(c), and 1(d), have dressed vertices. This was first pointed out by Pearce and Afnan.[25] We also note that the contact term $t^{(3)}$, in Fig. 1(a), gets dressed by the diagram in Fig. 1(c).

In the above discussion, we have implicitly assumed that the electromagnetic coupling is included to first order only. As a result of this, we have no radiative corrections to the πB amplitude.

We now turn to the amplitude for pion photoproduction on a single baryon, $\tilde{t}^{(0)}$. This has been considered previously in great detail.[22] Here, we will only present a brief summary of our results.[22] Taking into consideration the fact that $\tilde{t}^{(i)}$ and $\tilde{f}^{(i)}$ are the i-particle irreducible am-

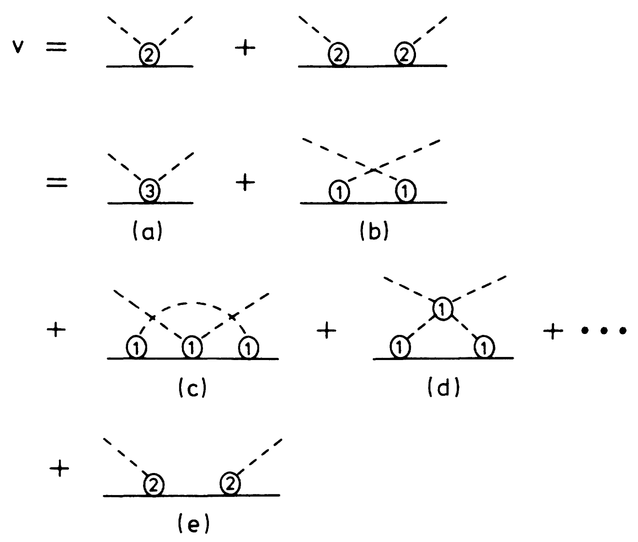

FIG. 1. The diagrammatic representation of the lowest order terms in the πN potential v, given in Eq. (2.12).

plitudes for $\pi B \leftrightarrow \gamma B$ and $B \leftarrow \pi B$, respectively, and the fact that (in our classification of diagrams according to their irreducibility) we count the photon on equal footing with the pion, we have, to first order in the electromagnetic coupling,[22]

$$\tilde{t}^{(0)}=\tilde{t}^{(1)}+f^{(1)\dagger}d_B\tilde{f}^{(1)}\ , \tag{2.13}$$

$$\tilde{f}^{(1)}=\tilde{f}^{(2)}+f^{(1)}g\tilde{t}^{(2)}\ , \tag{2.14}$$

$$\tilde{t}^{(1)}=\tilde{t}^{(2)}+t^{(1)}g\tilde{t}^{(2)}\ , \tag{2.15}$$

and

$$d_B\tilde{f}^{(1)}=d_0\tilde{f}^{(0)}\ . \tag{2.16}$$

In Eqs. (2.13)–(2.16), the amplitudes $f^{(1)}$ and $t^{(1)}$ are given in Eqs. (2.8) and (2.9), respectively. An alternative way of writing the pion photoproduction amplitude, $\tilde{t}^{(0)}$, is in terms of the total πB amplitude $t^{(0)}$, as

$$\begin{aligned}\tilde{t}^{(0)}&=\tilde{v}+t^{(0)}g\tilde{v}\\&=(t^{(0)}g+1)\tilde{v}\ ,\end{aligned} \tag{2.17}$$

where

$$\tilde{v}=\tilde{t}^{(2)}+f^{(2)\dagger}d_0\tilde{f}^{(2)}\ . \tag{2.18}$$

In this way, we have illustrated that the photoproduction amplitude can be written as a distorted wave Born amplitude, where $(t^{(0)}g+1)$ is the distortion operator in the πB channel. As before, with the Hamiltonian given in Eq. (2.3), $\tilde{f}^{(2)}=\langle B\,|\,\hat{H}\,|\,B\gamma\rangle$. On the other hand, the determination of $\tilde{t}^{(2)}$, in terms of the interaction Hamiltonian, requires the exposure of three-body unitarity and results in[22]

$$\begin{aligned}\tilde{t}^{(2)}=\tilde{t}^{(3)}&+\tilde{f}_b^{(1)}d_\pi f^{(1)\dagger}+\tilde{f}^{(1)}d_B f^{(1)\dagger}+\sum_i f^{(1)}(i)d_\pi(i)d_B u_{i3}^{(2)}d_\pi(1)d_\pi(2)\tilde{f}_a^{(1)\dagger}\\&+\sum_{ijk} f^{(1)}(i)d_\pi(i)d_B u_{ij}^{(2)}d_\pi(j)d_B\tilde{t}^{(1)}(j)\bar{\delta}_{jk}d_\pi(k)d_B f^{(1)\dagger}(k)\ ,\end{aligned} \tag{2.19}$$

where $\bar{\delta}_{jk}=1-\delta_{jk}$, and $u_{\alpha\beta}^{(2)}$ are the AGS (Ref. 43) amplitudes for the $\pi\pi B$ system, which are two-particle irreducible, and where $f^{(1)}$, $\tilde{f}^{(1)}$, and $\tilde{t}^{(1)}$ are given in Eqs. (2.8), (2.14), and (2.15), respectively. The one-particle irreducible amplitude for $\pi \leftarrow \gamma\pi, \tilde{f}_b^{(1)}$, required to calculate $\tilde{t}^{(2)}$ using Eq. (2.19), is given in terms of the interaction Hamiltonian by

$$\tilde{f}_b^{(1)}=\tilde{f}_b^{(2)}=\langle \pi\,|\,\hat{H}\,|\,\gamma\pi\rangle\ , \tag{2.20}$$

while the amplitude for $\pi\pi \leftarrow \gamma, \tilde{f}_a^{(1)\dagger}$, is given in terms of the one-particle irreducible $\pi\pi$ amplitude $t^{(1)}(3)$, as

$$\tilde{f}_a^{(1)\dagger}=[t^{(1)}(3)d_\pi d_\pi+1]\tilde{f}_a^{(2)\dagger}\ , \tag{2.21}$$

$$\tilde{f}_a^{(2)\dagger}=\langle \pi\pi\,|\,\hat{H}\,|\,\gamma\rangle\ , \tag{2.22}$$

$$t^{(1)}(3)=t^{(2)}(3)[1+d_\pi d_\pi t^{(1)}(3)]\ , \tag{2.23}$$

$$t^{(3)}(3)=\langle \pi\pi\,|\,\hat{H}\,|\,\pi\pi\rangle\ . \tag{2.24}$$

Finally, in Eq. (2.19), $\tilde{t}^{(3)}$ is the three-particle irreducible $\pi B \leftarrow \gamma B$ amplitude and is given in terms of the interaction Hamiltonian by

$$\tilde{t}^{(3)}=\langle \pi B\,|\,\hat{H}\,|\,\gamma B\rangle\ . \tag{2.25}$$

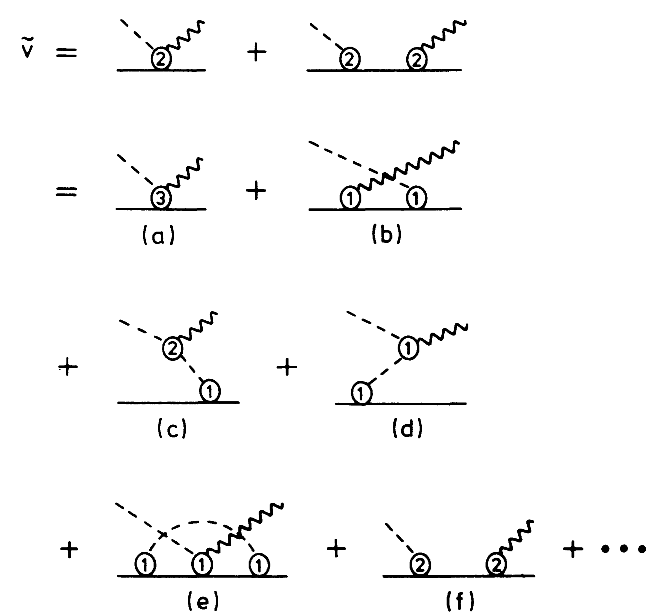

FIG. 2. The diagrammatic representation for the lowest order contribution to the effective operator for pion photoproduction on a single nucleon $\tilde{v}$. See Eqs. (2.18) and (2.19).

In this way, we have determined the potential $\tilde{v}$ required to calculate the pion photoproduction amplitude off-shell. Considering the complexity of Eq. (2.19), it might be appropriate to examine the content of the lowest order contribution to $\tilde{v}$. This is illustrated in Fig. 2, where we observe that the diagram with the s-channel pole, in Fig. 2(f) [the second term on the rhs of Eq. (2.18)], has undressed vertices, while all vertices in $\tilde{t}^{(2)}$ are dressed. the exception is the $\pi \leftarrow \gamma\pi$ vertex, $\tilde{f}_b^{(1)}$, which gets no dressing though it is one-particle irreducible. This is due to the fact that in our analysis the electromagnetic interaction is included to first order only. Of particular interest is the observation that the contact diagram, corresponding to $\tilde{t}^{(3)}$, in Fig. 2(a), gets dressed due to the contributions such as those in Fig. 2(e). Thus, by neglecting such terms, one can take the strength of the contact diagram from experiment. This was done for the π-N elastic scattering in the P_{11} channel with considerable success.[25]

III. THE THREE-BODY EQUATIONS

The Hamiltonian defined in Eq. (2.3) of Sec. II, can be defined, in principle, for baryon-number-greater-than-one systems. This involves writing the Hamiltonian in Eq. (2.3), in the Fock representation, and ignoring the antibaryon part of the spectrum. In this case, the resultant Hamiltonian does not include all the degrees of freedom allowed for in the original quark model Lagrangian. In particular, we have neglected the possibility of six or more quark bags, and the leakage of quarks from one bag to the next. Also not included, is the possibility for a change in the size of hadrons due to direct quark leakage. However, the new feature of the resultant Hamiltonian is that all baryons (i.e., N,Δ,Roper, . . .) are treated on equal footing in terms of their quark structure and the coupling between the baryons and mesons is derived from the underlying chiral Lagrangian in terms of the quark degrees of freedom. In this way, the strength and form of the coupling of the meson to the baryon is predetermined by the quark Lagrangian in a consistent manner. When the electromagnetic coupling is introduced via the minimal coupling postulate at the quark level, the form factors associated with the coupling of the meson and photon to the baryon are related, via the size of the hadrons which is determined by the bag radius. Finally, the photons are coupled to both the mesons and baryons, and thus the meson exchange currents are included in the Hamiltonian in a natural way.

This Hamiltonian, for the baryon number-two system, can in principle describe the reactions involving pion production and absorption, i.e., $BB \rightarrow BB, \pi d, \pi BB$ and $\pi d \rightarrow BB, \pi d, \pi BB$, where $B = N, \Delta$, Roper, ...[17] If the electromagnetic coupling is included to first order, we can also describe the reactions $\gamma d \rightarrow BB$, $\pi^0 d$, and πBB, as well as nucleon-nucleon bremsstrahlung, $BB \rightarrow BB\gamma$. For this Hamiltonian to actually give a good description of NN bremsstrahlung, it is important that it also gives a good description of N-N scattering. This will necessitate the inclusion of other mesons besides the pion. Such a generalization is not contemplated at this stage, particularly since there has been no numerical results for the BB-πBB system using the equations in Ref. 17.

In order to describe the above reactions, satisfy two- and three-body unitarity, and incorporate meson exchange currents, we need to take, for our Hilbert space, the states $|BB\rangle$, $|\pi BB\rangle$, and $|\gamma BB\rangle$. In this basis, the operators which give the amplitudes for the reactions under consideration are the following:

(a) $T^{(n)}$ is the amplitude for $BB \leftarrow BB$,
(b) $F^{(n)}$ is the amplitude for $BB \leftarrow \pi BB$,
(c) $\tilde{F}^{(n)}$ is the amplitude for $BB \leftarrow \gamma BB$,
(d) $M^{(n)}$ is the amplitude for $\pi BB \leftarrow \pi BB$,
(e) $\tilde{M}_A^{(n)}$ is the amplitude for $\pi BB \leftarrow \gamma BB$,
(f) $\tilde{M}_B^{(n)}$ is the amplitude for $\gamma BB \leftarrow \gamma BB$.

In the above, the superscript n stands for the irreducibility of the diagrams that contribute to the corresponding amplitude. Here we note that this notation is slightly different from that used in Ref. 17. To get some of the physical amplitudes under consideration, we may need to take the right or left residue of the matrix element of the above operators in our Hilbert space. For example, to get the amplitude for $np \leftarrow \gamma d$, we take the right-hand residue of $\tilde{F}_c^{(1)}$ at the deuteron pole, where the subscript c denotes the connected part of the amplitude.

Although we will be considering reactions involving photons later, we find it necessary to briefly summarize the equations that describe the reactions in Eqs. (1.1b) and (1.1c) in the BB-πBB system.[17] This particularly is the case, since the amplitudes for the photodisintegration of the deuteron and pion photoproduction will be given, in Secs. IV and V, in terms of the amplitude for the BB-πBB system. For a detailed discussion of the BB-πBB equations, the reader is referred to the work of Afnan and Blankleider, Ref. 17.

Let us consider the amplitude for the reactions $\pi BB \leftarrow BB$ as the sum of all the connected diagrams that are one-particle irreducible, $F_c^{(1)\dagger}$. These diagrams can be divided into two classes: (i) Those with at least one pion in every intermediate state (i.e., two-particle irreducible). These we denote by $F_c^{(2)\dagger}$. (ii) Those diagrams not included in (i), but contributing to the pion production amplitude. These can be written, using the last-cut-lemma,[27] as $F^{(2)\dagger}G^{(1)}T^{(1)}$, where $T^{(1)}$ is the $BB \leftarrow BB$ amplitude, and $G^{(1)}$ the BB propagator (i.e., $G^{(1)} = d_B \otimes d_B$). To keep the particle label and operator structure, we have introduced the direct product $\otimes$.[17,45] We now can write the $\pi BB \leftarrow BB$ amplitude as

$$F_c^{(1)\dagger} = F_c^{(2)\dagger} + F^{(2)\dagger}G^{(1)}T^{(1)} . \tag{3.1}$$

The $BB \leftarrow \pi BB$ amplitude, $F^{(2)}$, can be written as the sum of a connected and disconnected part as

$$F^{(2)} = F_d^{(2)} + F_c^{(2)} , \tag{3.2}$$

where the subscript d labels the class of disconnected diagrams. The connected part of the $BB \leftarrow \pi BB$ amplitude, $F_c^{(2)}$, is related to the AGS amplitudes[43] for the πBB system, $U_{i\alpha}^{(2)}$, by[17]

$$F_c^{(2)} = \sum_{i\alpha} F_d^{(2)}(i) G^{(2)} U_{i\alpha}^{(2)} G^{(2)} M_d^{(2)}(\alpha) , \tag{3.3}$$

with $G^{(2)} = [d_B \otimes d_B] d_\pi$, the πBB propagator. In Eq. (3.3), $F_d^{(2)}(i)$ and $M_d^{(2)}(\alpha)$ are the $B \leftarrow \pi B$ vertex functions and two-body amplitudes in the πBB Hilbert space, respectively. These in turn can be written in terms of the amplitudes given in the last section. Thus, the disconnected amplitude for $BB \leftarrow \pi BB, F_d^{(n)}$, is given by

$$\begin{aligned} F_d^{(n)} &= f^{(n-1)}(1) \otimes d_B^{-1}(2) + d_B^{-1}(1) \otimes f^{(n-1)}(2) \\ &= \sum_{i=1}^{2} F_d^{(n)}(i) . \end{aligned} \tag{3.4}$$

On the other hand, the disconnected amplitude for $\pi BB \leftarrow \pi BB, M_d^{(2)}$, is related to the two-body subamplitudes by

$$\begin{aligned} M_d^{(2)} &= t^{(1)}(1) \otimes d_B^{-1}(2) + d_B^{-1}(1) \otimes t^{(1)}(2) + t^{(1)}(3) d_\pi^{-1} \\ &= \sum_{\alpha=1}^{3} M_d^{(2)}(\alpha) . \end{aligned} \tag{3.5}$$

Here, $t^{(1)}(i)$, $i=1,2$, are the amplitudes for pion scattering off the ith baryon, and $t^{(1)}(3)$ is the BB amplitude in the πBB Hilbert space. In this section and throughout the rest of this paper, we will use the labeling convention where particles 1 and 2 are the baryons, while particle 3 is the pion. Furthermore, labels $i,j,\ldots$ $(=1,2)$ refer to the interactions of baryon $i,j,\ldots$ with the pion, while label 3 refers to the interaction of the two baryons. Finally, $\alpha,\beta,\ldots$ $(=1,2,3)$ in the sums, run over all three particles.

By combining the results of Eqs. (3.1)–(3.3), we can write the one-particle irreducible $\pi BB \leftarrow BB$ amplitude, $F_c^{(1)\dagger}$, as

$$\begin{aligned} F_c^{(1)\dagger} &= F_d^{(2)\dagger} G^{(1)} T^{(1)} + F_c^{(2)\dagger}[1 + G^{(1)} T^{(1)}] \\ &= F_d^{(2)\dagger} G^{(1)} T^{(1)} \\ &\quad + \sum_{\alpha i} M_d^{(2)}(\alpha) G^{(2)} U_{\alpha i}^{(2)} G^{(2)} F_d^{(2)\dagger}(i)[1 + G^{(1)} T^{(1)}] . \end{aligned} \tag{3.6}$$

Taking the residue of the amplitude $F_c^{(1)\dagger}$ at the BB or πB poles, we get

$$T_{\lambda;B} = \sum_i U_{\lambda i}^{(2)} G^{(2)} F_d^{(2)\dagger}(i)(1 + G^{(1)} T^{(1)}) , \tag{3.7}$$

where, $\lambda = 1$ or 2 corresponds to the $B + (\pi B)$ channel, while $\lambda = 3$ corresponds to the $\pi + (BB)$ channel. In taking the left-hand side (lhs) residue of Eq. (3.6), we have neglected the contribution from the first term on the rhs of the equation for $\lambda = 1$ or 2, even though $F^{(2)\dagger}$ has a πB quasiparticle pole. This was motivated by the fact that (i) The π-B quasiparticle poles correspond to resonances other than those described in terms of three quarks. In other words, these quasiparticles do not include the $\Delta(1232)$, and possibly the Roper resonance, $N(1440)$, as these belong to the baryons B. (ii) These resonances are above the threshold for two pion production in N-N scattering, and do not contribute substantially in the energy region of interest. (iii) The amplitudes $T_{i,B}$ $(i=1,2)$ are not physical observables and are required only in constructing the amplitude for reactions with three-body final states (i.e., $\pi BB \leftarrow BB$). In fact, in constructing the amplitude for $\pi BB \leftarrow BB$, we have to include the contribution from the first term on the rhs of Eq. (3.6). (iv) Finally, there is no loss of generality or any approximation in our integral equations, due to the exclusion of the contribution to the residue at the quasiparticle pole from the first term on the rhs of Eq. (3.6). The advantage gained from this definition of $T_{i;B}$ is that the resultant coupled integral equations are written in terms of the dressed form factor $F_d^{(1)}$ and two-body subamplitudes $M_d^{(2)}$. Using the AGS equations[43] for the πBB system, i.e.,

$$U_{\lambda i}^{(2)} = \bar{\delta}_{\lambda i} G^{(2)-1} + \sum_\gamma \bar{\delta}_{\lambda\gamma} M_d^{(2)}(\gamma) G^{(2)} U_{\gamma i}^{(2)} , \tag{3.8}$$

we can rewrite Eq. (3.7), as

$$\begin{aligned} T_{\lambda;B} &= \sum_i \bar{\delta}_{\lambda i} F_d^{(2)\dagger}(i)(1 + G^{(1)} T_{B;B}) \\ &\quad + \sum_\gamma \bar{\delta}_{\lambda\gamma} M_d^{(2)}(\gamma) G^{(2)} T_{\gamma;B} , \end{aligned} \tag{3.9}$$

where $T_{B;B} = T^{(1)}$, is the BB amplitude. This equation can be closed by deriving the equation for $T_{B;B}$, which is given by[17]

$$\begin{aligned} T_{B;B} &= V_{\mathrm{OPE}}(1 + G^{(1)} T_{B;B}) \\ &\quad + \sum_{i\lambda} F_d^{(2)}(i) \bar{\delta}_{i\lambda} G^{(2)} M_d^{(2)}(\lambda) G^{(2)} T_{\lambda;B} , \end{aligned} \tag{3.10}$$

where V_{OPE} is the one-pion-exchange potential and is given by the relation[17]

$$V_{\mathrm{OPE}} = \sum_{ij} F_d^{(2)}(i) \bar{\delta}_{ij} G^{(2)} F_d^{(2)\dagger}(j) . \tag{3.11}$$

In a similar way, we can write the equations for the BB-πBB system with an initial state of πd or $B(\pi B)$.[17] Here, we consider the connected one-particle irreducible amplitude for $\pi BB \leftarrow \pi BB$, $M_c^{(1)}$. By classifying the diagrams that contribute to $M_c^{(1)}$ according to their irreducibility, and making use of the last-cut-lemma, we get

$$\begin{aligned} M_c^{(1)} &= M_c^{(2)} + \{F^{(2)\dagger} G^{(1)} F^{(1)}\}_c \\ &= M_c^{(2)} + \{F^{(1)\dagger} G^{(1)} F^{(2)}\}_c . \end{aligned} \tag{3.12}$$

Making use of Eq. (3.6), the fact that $F_d^{(1)} = F_d^{(2)}$, and that

$$M_c^{(2)} = \sum_{\alpha\beta} M_d^{(2)}(\alpha) G^{(2)} U_{\alpha\beta}^{(2)} G^{(2)} M_d^{(2)}(\beta) , \tag{3.13}$$

allows us to write $M_c^{(1)}$ as

$$M_c^{(1)}=\sum_{ij}F_d^{(2)\dagger}(i)G^{(1)}T^{(1)}F_d^{(2)}(j)+\sum_{\alpha\beta}M_d^{(2)}(\alpha)G^{(2)}U_{\alpha\beta}^{(2)}G^{(2)}M_d^{(2)}(\beta)$$
$$+\sum_{\alpha ij}M_d^{(2)}(\alpha)G^{(2)}U_{\alpha i}^{(2)}G^{(2)}F_d^{(2)\dagger}(i)(G^{(1)}T^{(1)}+1)G^{(1)}F_d^{2}(j)$$
$$+\sum_{ij\beta}F_d^{(2)\dagger}(i)G^{(1)}(T^{(1)}G^{(1)}+1)F_d^{(2)}(j)G^{(2)}U_{j\beta}^{(2)}G^{(2)}M_d^{(2)}(\beta)$$
$$+\sum_{\alpha ij\beta}M_d^{(2)}(\alpha)G^{(2)}U_{\alpha i}^{(2)}G^{(2)}F_d^{(2)\dagger}(i)G^{(1)}(T^{(1)}G^{(1)}+1)F_d^{2}(j)G^{(2)}U_{j\beta}^{(2)}G^{(2)}M_d^{(2)}(\beta)\ . \tag{3.14}$$

Taking the lhs and rhs residues of Eq. (3.14), and neglecting the contribution from the quasiparticle pole in $F_d^{(2)}$, we find

$$T_{\alpha;\beta}=U_{\alpha\beta}^{(2)}+\sum_i U_{\alpha i}^{(2)}G^{(2)}F_d^{(2)\dagger}(i)G^{(1)}T_{B;\beta}\ . \tag{3.15}$$

Here again we can justify neglecting the contribution to the residue, from the pole in $F_d^{(2)}$ and its adjoint, on the ground that the corresponding amplitudes are not observables, and the above definition of $T_{i,j}$ involves no loss of generality. Making use of the AGS equations for the πBB system, we can rewrite the above expression as

$$T_{\alpha;\beta}=\bar{\delta}_{\alpha\beta}G^{(2)-1}+\sum_i\bar{\delta}_{\alpha i}F_d^{(2)\dagger}(i)G^{(1)}T_{B;\beta}$$
$$+\sum_\gamma\bar{\delta}_{\alpha\gamma}M_d^{(2)}(\gamma)G^{(2)}T_{\gamma;\beta}\ . \tag{3.16}$$

To close this equation we need to write an equation for $T_{B;\beta}$. This can be achieved by considering the connected one-particle irreducible amplitude for $BB\leftarrow\pi BB, F_c^{(1)}$, and taking the rhs residue at either the BB pole or the πB quasiparticle pole. The resultant equation is[17]

$$T_{B;\beta}=V_{\rm OPE}G^{(1)}T_{B;\beta}+\sum_i F_d^{(2)}(i)\bar{\delta}_{i\beta}$$
$$+\sum_{i\gamma}F_d^{(2)}(i)\bar{\delta}_{i\gamma}G^{(2)}M_d^{(2)}(\gamma)G^{(2)}T_{\gamma\beta}\ . \tag{3.17}$$

In Eqs. (3.9), (3.10), (3.16), and (3.17), we have a set of coupled integral equations, first derived in Ref. 17, for the BB-πBB system. These equations satisfy two- and three-body unitarity, and give a good description of the reactions in Eqs. (1.1b) and (1.1d) for the case where $B=N$.[12,14] We expect the extension to $B=N,\Delta$ will improve the agreement with experiment.

IV. PHOTODISINTEGRATION OF THE DEUTERON

We would now like to consider all reactions that result from the interaction of a real or virtual photon with the deuteron, i.e.,

$$\gamma+d\to N+N \tag{4.1a}$$
$$\to\pi+(NN) \tag{4.1b}$$
$$\to N+(N\pi) \tag{4.1c}$$
$$\to\pi+N+N\ . \tag{4.1d}$$

The first two of these reactions involve two-body final states, while the third reaction involves the production of an isobar, and thus has a quasi-two-body final state. Here we should note that under the substitution of $N\to B$ in Eq. (4.1), where $B=N,\Delta,R\ldots$, the isobars $(B\pi)$ in the reaction (4.1c) will not include those baryons already included in the set B. The fourth reaction, Eq. (4.1d), involves three-body final states, and its amplitude could be written in terms of the amplitude for the reactions in Eqs. (4.1b) and (4.1c).[46]

Because all these reactions are initiated by a photon incident on the deuteron, we expect to get a coupled set of equations for the corresponding amplitudes (in particular for the first three of these reactions). The advantage of a coupled channel approach, in momentum space, is the fact that we need not calculate the off-shell amplitudes for these reactions. On the other hand, in a distorted-wave Born approximation, we need the distorted-wave functions in the initial and final states. These are given in terms of the off-shell amplitude for the corresponding reactions. However, before we proceed to the derivation of the coupled equations, we need to examine the contribution of three-body unitarity to the amplitude for each of these reactions. We will then proceed in a later section to derive coupled equations for the reactions in Eqs. (4.1a)–(4.1c).

Let us first consider the photodistintegration of the deuteron [Eq. (4.1a)]. To get the amplitude for this reaction we need to consider the one-particle irreducible amplitude for the reaction $BB\leftarrow\gamma BB$, i.e., $\tilde{F}_c^{(1)}$. The diagrams that contribute to this amplitude can be divided into two classes: (i) Those diagrams that are two-particle irreducible. The sum of these diagrams we denote by $\tilde{F}_c^{(2)}$. (ii) The diagrams that do not belong to (i). These are two-particle reducible and can be written using the last-cut-lemma as

$$\{T^{(1)}G^{(1)}\tilde{F}^{(2)}\}_c=\{T^{(2)}G^{(1)}\tilde{F}^{(1)}\}_c\ .$$

Making use of the fact that $\tilde{F}^{(n)}$, the n-particle irreducible amplitude for $BB\leftarrow\gamma BB$, has a connected and disconnected part, we can write $\tilde{F}_c^{(1)}$ as

$$\tilde{F}_c^{(1)}=\tilde{F}_c^{(2)}+\{T^{(2)}G^{(1)}\tilde{F}_d^{(1)}\}+\{T^{(2)}G^{(1)}\tilde{F}_c^{(1)}\} \tag{4.2a}$$
$$=\tilde{F}_c^{(2)}+\{T^{(1)}G^{(1)}\tilde{F}_d^{(2)}\}+\{T^{(1)}G^{(1)}\tilde{F}_c^{(2)}\}\ , \tag{4.2b}$$

where the disconnected amplitude $\tilde{F}_d^{(n)}$ is given by

$$\tilde F_d^{(n)}=\tilde f^{(n-1)}(1)\otimes d_B^{-1}(2)+d_B^{-1}(1)\otimes\tilde f^{(n-1)}(2)=\sum_{i=1}^{2}\tilde F_d^{(n)}(i)\ . \tag{4.3}$$

In Eq. (4.2), $T^{(n)}$ is the n-particle irreducible amplitude for $BB\leftarrow BB$. Having dressed our baryon propagators using the procedure in Ref. 17, the amplitude $T^{(n)}$ is connected, and we therefore have dropped the subscript c in Eq. (4.2). To further reduce Eq. (4.2) in terms of known amplitudes, we need to examine the two-particle irreducible amplitude for $BB\leftarrow\gamma BB,\tilde F_c^{(2)}$. If we restrict our analysis to the Hamiltonian in Eq. (2.3), then the absence of terms that change the number of bosons (i.e., pions plus photons) by two or more, implies that there is no direct coupling of the BB channel to either the $\gamma\pi BB$ or $\pi\pi BB$ channels. This means that $F_c^{(3)}=F_c^{(4)}=\cdots=\langle BB\mid\hat H\mid\pi BB\rangle$, and $\tilde F_c^{(3)}=\tilde F_c^{(4)}=\cdots=\langle BB\mid\hat H\mid\gamma BB\rangle$. These matrix elements can only be defined for the two-baryon system, and cannot be written in terms of the matrix elements of H_I between the single baryon states. Since our starting Hamiltonian in Eq. (2.3) does not include such matrix elements, we have the result that the three-particle irreducible amplitudes $F_c^{(3)}$ and $\tilde F_c^{(3)}$ are both zero. However, if we admit six-quark bag states into our basis, then it is possible to couple both the photon and pion to the quarks in this six-quark bag. In this case we can calculate a nonzero value for both $\langle BB\mid\hat H\mid\pi BB\rangle$ and $\langle BB\mid\hat H\mid\gamma BB\rangle$ using the chiral Lagrangian under consideration. For the present investigation we will neglect the six-quark states and take the three-particle irreducible amplitudes, $F_c^{(3)}$ and $\tilde F_c^{(3)}$, to be zero. We should stress at this stage that the formalism presented here does admit the introduction of six-quark states, but we have chosen to neglect these states. One can, at a later stage, include these six-quark states in perturbation theory. If we now include the electromagnetic coupling to first order only, then $\tilde M_B^{(n)}=\tilde M_{B,d}^{(n)}$. With these two restrictions and the application of the last-cut-lemma, we can write the connected two-particle irreducible $BB\leftarrow\gamma BB$ amplitude, $\tilde F_c^{(2)}$, as

$$\tilde F_c^{(2)}=\{F_d^{(3)}G^{(2)}\tilde M_A^{(2)}\}_c+\{\tilde F_d^{(3)}\tilde G^{(2)}\tilde M_{B,d}^{(2)}\}_c\ , \tag{4.4}$$

where

$$\tilde M_{B,d}^{(n)}=d_\gamma^{-1}t^{(n-1)}(3)\ , \tag{4.5}$$

and $\tilde G^{(2)}=d_\gamma d_B d_B$, is the γBB propagator. The $\pi BB\leftarrow\gamma BB$ amplitude, $\tilde M_A^{(2)}$, has a connected ($\tilde M_{A,c}^{(2)}$) and disconnected ($\tilde M_{A,d}^{(2)}$) part, with the latter given by

$$\tilde M_{A,d}^{(n)}=\tilde t^{(n-1)}(1)\otimes d_B^{-1}(2)+d_B^{-1}(1)\otimes\tilde t^{(n-1)}(2)=\sum_{i=1}^{2}\tilde M_{A,d}^{(n)}(i)\ . \tag{4.6}$$

To get the structure of the connected two-particle irreducible $\pi BB\leftarrow\gamma BB$ amplitude, $\tilde M_{A,c}^{(2)}$, we classify the diagrams that contribute to this amplitude into two classes: (i) Those that are three-particle irreducible, which we denote by $\tilde M_{A,c}^{(3)}$. (ii) The diagrams not belonging to (i). These are three-particle reducible and can be written using the last-cut-lemma as

$$\{M^{(2)}G^{(2)}\tilde M_A^{(3)}\}_c+\{\tilde M_A^{(2)}\tilde G^{(2)}\tilde M_B^{(3)}\}_c\ , \tag{4.7a}$$

or

$$\{M^{(3)}G^{(2)}\tilde M_A^{(2)}\}_c+\{\tilde M_A^{(3)}\tilde G^{(2)}\tilde M_B^{(2)}\}_c\ . \tag{4.7b}$$

We now can write the two-particle irreducible amplitude for $\pi BB\leftarrow\gamma BB$ as

$$\tilde M_{A,c}^{(2)}=\tilde M_{A,c}^{(3)}+\{M^{(2)}G^{(2)}\tilde M_A^{(3)}\}_c+\{\tilde M_A^{(2)}\tilde G^{(2)}\tilde M_B^{(3)}\}_c \tag{4.8a}$$

$$=\tilde M_{A,c}^{(3)}+\{M^{(3)}G^{(2)}\tilde M_A^{(2)}\}_c+\{\tilde M_A^{(3)}\tilde G^{(2)}\tilde M_B^{(2)}\}_c\ . \tag{4.8b}$$

To proceed further, we need to examine the three-particle irreducible amplitude $\tilde M_{A,c}^{(3)}$. A classification of the Feynman diagrams that contribute to this amplitude according to their irreducibility, using the last-cut-lemma, shows that this amplitude is related to the four-particle irreducible amplitudes for $\pi BB\leftarrow\pi\pi BB$ and $\pi BB\leftarrow\gamma\pi BB$. This coupling to the four-particle intermediate state will introduce four-body unitarity, which we do not want to include at this stage. In the absence of four-body unitarity, we can consider $\tilde M_{A,c}^{(3)}$ as a three-body type force, as was the case with $M_c^{(3)}$ in the BB-πBB equations.[17] In fact, one contribution to this effective three-body interaction comes from the formation of a six-quark bag. Although the amplitude can be included in the formalism, we have chosen at this stage to consistently neglect any contribution from either three-body forces or the formation of six-quark bags.

To further simplify Eq. (4.8), we make use of the fact that both $M^{(2)}$ and $\tilde M_A^{(2)}$ have a connected and disconnected part, with the connected part of $M^{(2)}$ given in terms of the AGS (Ref. 43) amplitudes $U_{\alpha\beta}^{(2)}$ by Eq. (3.13).[17] This allows us to write Eq. (4.8a) as

$$\tilde M_{A,c}^{(2)}=\tilde M_{A,c}^{(2)}\tilde G^{(2)}t^{(2)}(3)d_\gamma^{-1}+\sum_i\tilde M_{A,d}^{(2)}(i)\tilde G^{(2)}t^{(2)}(3)d_\gamma^{-1}+\sum_{\alpha i}\{M_d^{(2)}(\alpha)G^{(2)}\tilde M_{A,d}^{(3)}(i)\}_c$$
$$+\sum_{\alpha\beta i}M_d^{(2)}(\alpha)G^{(2)}U_{\alpha\beta}^{(2)}G^{(2)}M_d^{(2)}(\beta)G^{(2)}\tilde M_{A,d}^{(3)}(i)\ . \tag{4.9}$$

To get an expression for $\tilde M_{A,c}^{(2)}$ in terms of known quantities, we need to formally solve this equation. This is achieved by moving the first term on the right-hand side of the equation to the left-hand side of the equation, and multiplying from the right by

$$[1-\tilde{G}^{(2)}t^{(2)}(3)d_\gamma^{-1}]^{-1}=1+\tilde{G}^{(2)}\tilde{M}_{B,d}^{(2)} .$$

We then make use of the fact that

$$\tilde{M}_{B,d}^{(3)}(1+\tilde{G}^{(2)}\tilde{M}_{B,d}^{(2)})=\tilde{M}_{B,d}^{(2)} ,$$

to write Eq. (4.9) as

$$\tilde{M}_{A,c}^{(2)}=\sum_i \tilde{M}_{A,d}^{(2)}(i)\tilde{G}^{(2)}\tilde{M}_{B,d}^{(2)}+\sum_{\alpha i} M_d^{(2)}(\alpha)G^{(2)}\overline{\delta}_{\alpha i}\tilde{M}_{A,d}^{(3)}(i)(1+\tilde{G}^{(2)}\tilde{M}_{B,d}^{(2)})$$
$$+\sum_{\alpha\beta i} M_d^{(2)}(\alpha)G^{(2)}U_{\alpha\beta}^{(2)}G^{(2)}M_d^{(2)}(\beta)G^{(2)}\tilde{M}_{A,d}^{(3)}(i)(1+\tilde{G}^{(2)}\tilde{M}_{B,d}^{(2)}) . \tag{4.10}$$

We now have to rearrange the multiple scattering series using the AGS equations for $U_{\alpha\beta}^{(2)}$ in order to replace $\tilde{M}_{A,d}^{(3)}$ by $\tilde{M}_{A,d}^{(2)}$ in Eq. (4.10). In this way we have replaced the Born amplitude for pion photoproduction on a single nucleon, by the full amplitude for that process. This gives, after some algebra, an expression for the connected two-particle irreducible amplitude for $\pi BB \leftarrow \gamma BB, \tilde{M}_{A,c}^{(2)}$, in terms of known two-body subamplitudes as

$$\tilde{M}_{A,c}^{(2)}=\sum_i \tilde{M}_{A,d}^{(2)}(i)\tilde{G}^{(2)}\tilde{M}_{B,d}^{(2)}+\sum_{\alpha i} M_d^{(2)}(\alpha)G^{(2)}U_{\alpha i}^{(2)}G^{(2)}\tilde{M}_{A,d}^{(2)}(i)(1+\tilde{G}^{(2)}\tilde{M}_{B,d}^{(2)}) . \tag{4.11}$$

Making use of the above result in Eq. (4.4), we get for the $BB \leftarrow \gamma BB$ amplitude, $\tilde{F}_c^{(2)}$, the result that

$$\tilde{F}_c^{(2)}=\sum_{ij} F_d^{(3)}(i)G^{(2)}\overline{\delta}_{ij}\tilde{M}_{A,d}^{(2)}(j)+\sum_i \tilde{F}_d^{(3)}(i)\tilde{G}^{(2)}\tilde{M}_{B,d}^{(2)}+\sum_{ij} F_d^{(3)}(i)G^{(2)}\tilde{M}_{A,d}^{(2)}(j)\tilde{G}^{(2)}\tilde{M}_{B,d}^{2}$$
$$+\sum_{i\alpha j} F_d^{(3)}(i)G^{(2)}M_d^{(2)}(\alpha)G^{(2)}U_{\alpha j}^{(2)}G^{(2)}\tilde{M}_{A,d}^{(2)}(j)(1+\tilde{G}^{(2)}\tilde{M}_{B,d}^{(2)}) . \tag{4.12}$$

We now need to replace the three-particle irreducible amplitudes, $F_d^{(3)}$ and $\tilde{F}_d^{(3)}$, by the corresponding two-particle irreducible amplitudes, and in this way dress the basic vertices for photon and pion absorption. This again is achieved by regrouping the expression on the right-hand side of Eq. (4.12) using the AGS equation, and making use of Eqs. (2.8) and (2.14). The resultant expression for the connected two-particle irreducible $BB \leftarrow \gamma BB$ amplitude, $\tilde{F}_c^{(2)}$, is given in terms of the AGS amplitudes $U_{\alpha\beta}^{(2)}$ as

$$\tilde{F}_c^{(2)}=\sum_i \tilde{F}_d^{(2)}(i)\tilde{G}^{(2)}\tilde{M}_{B,d}^{(2)}+\sum_{ij} F_d^{(2)}(i)G^{(2)}U_{ij}^{(2)}G^{(2)}\tilde{M}_{A,d}^{(2)}(j)(1+\tilde{G}^{(2)}\tilde{M}_{B,d}^{(2)}) . \tag{4.13}$$

With this result we now can write the connected one-particle irreducible amplitude for $BB \leftarrow \gamma BB$ given in Eq. (4.2) as

$$\tilde{F}_c^{(1)}=\sum_i T^{(1)}G^{(1)}\tilde{F}_d^{(2)}(i)+\sum_i (T^{(1)}G^{(1)}+1)\tilde{F}_d^{(2)}(i)\tilde{G}^{(2)}\tilde{M}_{B,d}^{(2)}$$
$$+\sum_{ij}(T^{(1)}G^{(1)}+1)F_d^{(2)}(i)G^{(2)}U_{ij}^{(2)}G^{(2)}\tilde{M}_{A,d}^{(2)}(j)(1+\tilde{G}^{(2)}\tilde{M}_{B,d}^{(2)}) . \tag{4.14}$$

To write the physical amplitude for the photodisintegration of the deuteron (i.e., $np \leftarrow \gamma d$), we need to take the rhs residue of the above equation at the deuteron pole. To determine the rhs residue of $\tilde{F}_c^{(1)}$ at the deuteron pole, we need to expose this pole explicitly. This can be achieved by noting that $\tilde{M}_{B,d}^{(n)}=d_\gamma^{-1}t^{(n-1)}(3)$, and that $t^{(1)}(3)$ has the deuteron pole. In other words, we can write $\tilde{M}_{B,d}^{(2)}$ schematically as

$$\tilde{M}_{B,d}^{(2)}=d_\gamma^{-1}t^{(1)}(3)$$
$$=d_\gamma^{-1}\left[\frac{|\phi_d\rangle\langle\phi_d|}{E+\epsilon_d}+\cdots\right] , \tag{4.15}$$

where ϵ_d is the binding energy of the deuteron. Here, $|\phi_d\rangle$ is the deuteron form factor and is related to the deuteron wave function $|\psi_d\rangle$ by the relation

$$|\psi_d\rangle=d_B(1)d_B(2)|\phi_d\rangle . \tag{4.16}$$

The amplitude for $np \leftarrow \gamma d$ can now be written, by taking the rhs residue at the deuteron pole, as

$$T(np \leftarrow \gamma d)\equiv\langle\chi_{np}|\tilde{T}_{B;d}|\psi_d\rangle , \tag{4.17}$$

with

$$\tilde{T}_{B;d}\equiv(T^{(1)}G^{(1)}+1)$$
$$\times\left[\tilde{F}_d^{(2)}+\sum_{ij} F_d^{(2)}(i)G^{(2)}U_{ij}^{(2)}G^{(2)}\tilde{M}_{A,d}^{(2)}(j)\right] . \tag{4.18}$$

In Eq. (4.17), χ_{np} is the spin-isospin wave function for the final two baryons.

To get the multiple scattering series for the photodisintegration amplitude, all we need to do is to use the AGS equations [Eq. (3.8)] for $U_{ij}^{(2)}$ in Eq. (4.18). In fact, the lowest order contribution to $\tilde{T}_{B;d}$ results from taking $U_{ij}^{(2)} = \bar{\delta}_{ij} G^{(2)^{-1}}$ in Eq. (4.18), which gives

$$\tilde{T}_{B;d} \approx (T^{(1)}G^{(1)}+1) \times \left[\tilde{F}_d^{(2)} + \sum_{ij} F_d^{(2)}(i) G^{(2)} \bar{\delta}_{ij} \tilde{M}^{(2)}_{A,d}(j) \right] . \quad (4.19)$$

This expression is presented diagrammatically in Fig. 3. Comparing the contributions to our amplitude for $np \leftarrow \gamma d$ with those of Laget,[29] we find that the diagrams in Fig. 3 are basically the same as those included in Laget's work with one difference being the diagram corresponding to the deuteron current, coupling directly to the photon. This diagram is not included in our theory, since the deuteron is treated as a composite particle. Here, we also note that the diagrams in Fig. 3 do not include any direct coupling of the photon to the pion. This coupling is included in the amplitude for $\pi N \leftarrow \gamma N$. In other words, if the t-channel meson pole terms are included in the pion photoproduction amplitude on a single nucleon, then the meson exchange currents are automatically included in the deuteron photodisintegration amplitude. As the philosophy of these calculations involves the determination of the amplitudes on a single nucleon first, then the inclusion of a t-channel meson pole term in pion photoproduction on a single nucleon should predict the contribution of the meson exchange current in the deuteron photodisintegration. Also, the vertices for pion production, $F_d^{(2)}(i)$ in Eq. (4.19), are dressed and their form is predetermined by π-N scattering (e.g., in the P_{11} channel for $B = N$). These in turn are determined by the chiral bag model Lagrangian: in particular, the bag (or the nucleon) size. A careful comparison of our results with those of Laget reveals that our form factors for both pion and photon absorption include all of the pionic dressing required to satisfy two- and three-body unitarity. In other words, we have a quantum field theory, while Laget is using an effective Lagrangian at the tree level. This in effect means that in Fig. 3(a) we include the process whereby the photon produces a pion via the pion photoproduction amplitude, and this pion is absorbed by that same nucleon. [See Fig. 4(b).] The amplitude for this process, illustrated in Fig. 4(b), should be compared to the amplitude resulting from the diagram in Fig. 5(a), where the pion is absorbed by the other nucleon and is a

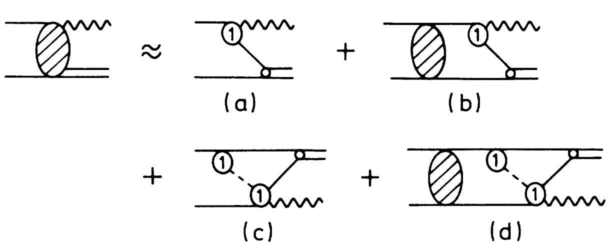

FIG. 3. The diagrammatic representation of Eq. (4.19). Here, we have included the lowest order contribution to the amplitude for $BB \leftarrow \gamma d$.

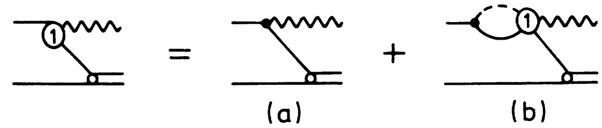

FIG. 4. The decomposition of the dressed γBB vertex in the lowest order contribution to $BB \leftarrow \gamma d$ as given in Fig. 3(a).

part of the contribution to the total amplitude from the diagram in Fig. 3(c). More important is the fact that we include all of the higher order multiple scattering effects. In this way we satisfy unitarity at the two- and three-body levels. Finally, our starting point being the chiral bag model, allows us to treat the nucleon, $\Delta(1232)$, and higher mass isobars, on equal footing, thus maintaining some consistency with QCD. More interesting, is the fact that at a later stage we can examine a nonstandard mechanism such as the six-quark formation within the framework of our theory.

A proper treatment of NN bremsstrahlung will require a theory that gives a good description of the N-N interaction, and therefore may require the inclusion of the heavy meson exchange. The present Hamiltonian does not generate such heavy meson exchanges because we have not included the coupling of such mesons to the quarks. However, we can include the exchange of heavy mesons in the form introduced in the NN-πNN equations,[14] which gives a good description of N-N scattering at medium energies. In our present formulation we have not attempted to include the Coulomb interaction for proton-proton bremsstrahlung because we feel that the inclusion of the heavy meson exchange is more important. (We also note that Marker and Signell[40] estimated the Coulomb effect on the cross section 5–10 %.) Here, we consider NN bremsstrahlung because the amplitude for this reaction is related to the amplitude for the photodisintegration of the deuteron. To illustrate this, we note that the NN bremsstrahlung amplitude ($\gamma BB \leftarrow BB$), $\tilde{F}_c^{(1)}$, is given by the adjoint of Eq. (4.14), i.e.,

$$\begin{aligned} \tilde{F}_c^{(1)\dagger} = & \sum_i \tilde{F}_d^{(2)\dagger}(i) G^{(1)} T^{(1)} \\ & + \sum_i \tilde{M}^{(2)}_{B,d} \tilde{G}^{(2)} \tilde{F}_d^{(2)\dagger}(i)(1+G^{(1)}T^{(1)}) \\ & + \sum_{i,j} (\tilde{M}^{(2)}_{B,d} \tilde{G}^{(2)} + 1) \tilde{M}^{(2)\dagger}_{A,d}(i) G^{(2)} U_{ij}^{(2)} G^{(2)} F_d^{(2)\dagger}(j) \\ & \times (1+G^{(1)}T^{(1)}) . \end{aligned} \quad (4.20)$$

A careful examination of the first two terms on the rhs of

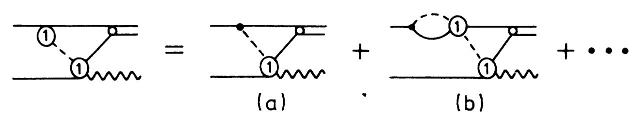

FIG. 5. The decomposition of the dressed γBB vertex in the diagram given in Fig. 3(c).

Eq. (4.20) reveals that (i) the initial and final N-N interactions are not the same, to the extent that the initial N-N interaction $T^{(1)}$ is generated by the BB-πBB equations and includes coupling to the $\Delta(1232)$ as well as including the one-pion exchange diagram explicitly. On the other hand, the final N-N interaction comes via $\tilde{M}^{(2)}_{B,d}=d_{\gamma}^{-1}t^{(1)}(3)$ as an input amplitude. To make the two consistent will be a constraint to be imposed on the equations. (ii) The coupling of the photon to the nucleon involves the dressed vertex $\tilde{F}^{(2)\dagger}_{d}$, which includes the coupling of the photon to the pion as illustrated in Fig. 6. These pionic corrections are normally not included in NN bremsstrahlung calculations. (iii) We can write the amplitudes for $\gamma NN \leftarrow NN$ and $np \leftarrow \gamma d$ in terms of the amplitude for the reaction $BB \leftarrow \pi BB$.

Finally, we need to eliminate the AGS amplitudes, $U^{(2)}_{\alpha\beta}$, from the expressions for the photodistintegration amplitude in Eq. (4.18) and for the NN bremsstrahlung amplitude in Eq. (4.20). At the same time we would like to illustrate the relation between these two amplitudes. Making use of the fact that the disconnected one-particle irreducible amplitude for $\gamma BB \leftarrow BB, \tilde{F}^{(1)\dagger}_{d}$, is equal to the corresponding two-particle irreducible amplitude $\tilde{F}^{(2)\dagger}_{d}$, we can write

$$
\begin{aligned}
\tilde{F}^{(1)\dagger} &= \tilde{F}^{(1)\dagger}_{d} + \tilde{F}^{(1)\dagger}_{c} \\
&= (\tilde{M}^{(2)}_{B,d}\tilde{G}^{(2)}+1) \\
&\quad \times \left[\tilde{F}^{(2)\dagger}_{d} + \sum_{ij} \tilde{M}^{(2)}_{A,d}(i) G^{(2)} U^{(2)}_{ij} F^{(2)\dagger}_{d}\right](1+G^{(1)}T^{(1)}) \\
&= (\tilde{M}^{(2)}_{B,d}\tilde{G}^{(2)}+1)\tilde{T}_{d;B} \ .
\end{aligned} \tag{4.21}
$$

On the other hand, the photodistintegration amplitude in Eq. (4.18) can be written in terms of the amplitudes $T_{B;\lambda}$, given by the adjoint of Eq. (3.7), and $T_{B;B}$ as

$$
\tilde{T}_{B;d} = (T_{B;B}G^{(1)}+1)\tilde{F}^{(2)}_{d} + \sum_{i} T_{B;i} G^{(2)} \tilde{M}^{(2)}_{A,d}(i) \ . \tag{4.22}
$$

Needless to say, the physical amplitude for both reactions are the connected parts of the amplitudes in Eqs. (4.21) and (4.22). In these two equations we have written both the photodisintegration and NN bremsstrahlung amplitudes in terms of the amplitudes that we get from the

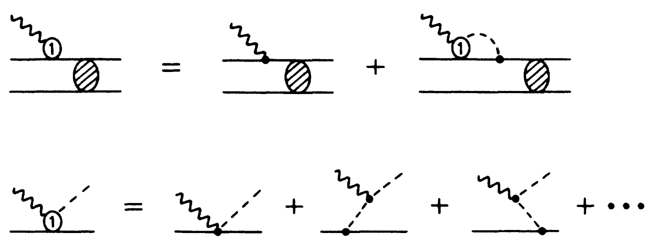

FIG. 6. The decomposition of the dressed γBB vertex as present in NN bremsstrahlung.

BB-πBB equations.[17] In this way we have reduced the evaluation of these amplitudes to integrals over the half-off-shell amplitudes which we get from the BB-πBB equations.

V. PHOTOPRODUCTION OF PIONS OFF THE DEUTERON

Having completed our analysis of the photodisintegration of the deuteron and NN bremsstrahlung, we turn our attention to pion photoproduction [i.e., the reactions given in Eq. (4.1b)–(4.1d)]. In this way we will have the appropriate expressions to derive a set of coupled equations for the reactions given in Eq. (4.1).

To get the amplitudes for pion photoproduction, we need to examine the one-particle irreducible amplitude for the reaction $\pi BB \leftarrow \gamma BB, \tilde{M}^{(1)}_{A,c}$. The diagrams that contribute to this amplitude can be divided into two groups: (i) Those that are two-particle irreducible, which we denote by $\tilde{M}^{(2)}_{A,c}$. (ii) The diagrams that are two-particle reducible. These can be written, using the last-cut-lemma, as

$$
\{F^{(1)\dagger}G^{(1)}\tilde{F}^{(2)}\}_c = \{F^{(2)\dagger}G^{(1)}\tilde{F}^{(1)}\}_c \ .
$$

We now can write

$$
\tilde{M}^{(1)}_{A,c} = \tilde{M}^{(2)}_{A,c} + \{F^{(1)\dagger}G^{(1)}\tilde{F}^{(2)}\}_c \tag{5.1a}
$$

$$
= \tilde{M}^{(2)}_{A,c} + \{F^{(2)\dagger}G^{(1)}\tilde{F}^{(1)}\}_c \ . \tag{5.1b}
$$

Making use of Eqs. (3.3), (4.11), and (4.21), we can write the connected part of the one-particle irreducible amplitude for $\pi BB \leftarrow \gamma BB, \tilde{M}^{(1)}_{A,c}$, as

$$
\begin{aligned}
\tilde{M}^{(1)}_{A,c} = &\sum_{i} \left[1 + \sum_{\alpha} M^{(2)}_{d}(\alpha) G^{(2)} U^{(2)}_{\alpha i} G^{(2)}\right] \{\tilde{M}^{(2)}_{A,d}(i) + F^{(2)\dagger}_{d}(i) G^{(1)} \tilde{T}_{B;d}\} \tilde{G}^{(2)} \tilde{M}^{(2)}_{B,d} \\
&+ \sum_{\alpha i} M^{(2)}_{d}(\alpha) G^{(2)} U^{(2)}_{\alpha i} G^{(2)} \{\tilde{M}^{(2)}_{A,d}(i) + F^{(2)\dagger}_{d}(i) G^{(1)} \tilde{T}_{B;d}\} + \{F^{(2)\dagger}_{d} G^{(1)} \tilde{T}_{B;d}\}_c \ .
\end{aligned} \tag{5.2}
$$

Since our initial state is $\gamma + d$, we need to take the rhs residue of $\tilde{M}^{(1)}_{A,c}$ at the deuteron pole. Making use of Eq. (4.15) for $\tilde{M}^{(2)}_{B,d}$ and the fact that $\tilde{M}^{(2)}_{A,d}$ does not have a deuteron pole, we find that the only nonzero contribution to the $\pi BB \leftarrow \gamma d$ amplitude, comes from the first term on the rhs of Eq. (5.2), i.e.,

$$
\tilde{T}(\pi BB \leftarrow \gamma d) = \sum_{i} \left[1 + \sum_{\alpha} M^{(2)}_{d}(\alpha) G^{(2)} U^{(2)}_{\alpha i} G^{(2)}\right] \{\tilde{M}^{(2)}_{A,d}(i) + F^{(2)\dagger}_{d}(i) G^{(1)} \tilde{T}_{B;d}\} \mid \psi_d \rangle \ . \tag{5.3}
$$

To get the amplitude for the reaction $\pi^0 d \leftarrow \gamma d$, we have to take the lhs residue of $\tilde{T}(\pi NN \leftarrow \gamma d)$ at the deuteron pole.

The only nonzero contribution to this residue comes from $M_d^{(2)}(3)$, which can be written schematically as

$$M_d^{(2)}(3)=d_\pi^{-1}t^{(1)}(3)$$
$$=d_\pi^{-1}\left[\frac{|\phi_d\rangle\langle\phi_d|}{E+\epsilon_d}+\cdots\right] . \tag{5.4}$$

Making use of the definition of the deuteron wave function as given in Eq. (4.16), we can write the amplitude for $\pi^0 d \leftarrow \gamma d$, as

$$\tilde{T}(\pi^0 d\leftarrow\gamma d)=\langle\psi_d|\sum_i U_{3i}^{(2)}G^{(2)}\{\tilde{M}_{A,d}^{(2)}(i)+F_d^{(2)\dagger}(i)G^{(1)}\tilde{T}_{B;d}\}|\psi_d\rangle$$
$$\equiv\langle\psi_d|\tilde{T}_{3;d}|\psi_d\rangle \tag{5.5}$$

with

$$\tilde{T}_{3;d}\equiv\sum_i U_{3i}^{(2)}G^{(2)}\{\tilde{M}_{A,d}^{(2)}(i)+F_d^{(2)\dagger}(i)G^{(1)}\tilde{T}_{B;d}\} \tag{5.6a}$$
$$=\sum_j\left[U_{3i}^{(2)}+\sum_i U_{3,i}^{(2)}G^{(2)}F_d^{(2)\dagger}(i)G^{(1)}T_{B;j}\right]G^{(2)}\tilde{M}_{A,d}^{(2)}(j)+\sum_i U_{3i}^{(2)}G^{(2)}F_d^{(2)\dagger}(i)(1+G^{(1)}T_{B;B})G^{(1)}\tilde{F}_d^{(2)} . \tag{5.6b}$$

To get the result of Eq. (5.6b) from Eq. (5.6a), we have made use of Eq. (4.22) for $\tilde{T}_{B;d}$. To further simplify this result, we make use of Eqs. (3.7) and (3.15), for $T_{\alpha;B}$ and $T_{\alpha;\beta}$, respectively. This gives us an expression for the amplitude for the photoproduction of π^0, in terms of the amplitudes we get from the BB-πBB equations, $T_{\alpha;\beta}$ and $T_{\alpha;\beta}$, i.e.,

$$\tilde{T}_{3;d}=\sum_j T_{3;j}G^{(2)}\tilde{M}_{A,d}^{(2)}(j)+T_{3;B}G^{(1)}\tilde{F}_d^{(2)} . \tag{5.7}$$

This result gives us the π^0 photoproduction amplitude in a form that includes the multiple scattering of the pion, off the two nucleons, to all orders. It also includes the contribution due to true absorption (i.e., $\gamma d \rightarrow BB \rightarrow \pi^0 d$). Making use of the BB-πBB equations in Eq. (5.7), we can derive a multiple scattering series for the amplitude $\tilde{T}_{3;d}$. This series is of the form,

$$\tilde{T}_{3,d}=\sum_i\tilde{M}_{A,d}^{(2)}+\sum_{ij}F_d^{(2)}(i)G^{(1)}\tilde{F}_d^{(2)}(j)$$
$$+\sum_{ij}M_d^{(2)}(i)G^{(2)}\overline{\delta}_{ij}\tilde{M}_{A,d}^{(2)}(j)+\cdots . \tag{5.8}$$

This result can be compared with the work of Laget,[28] who considered the lowest terms in the multiple scattering series. Here, the two-body amplitudes are consistent with pion elastic scattering and photoproduction on a single nucleon, while the propagators and vertices are dressed to give the physical coupling constants and masses. Furthermore, the fact that the photon is coupled to both the baryon and meson at the single baryon level, implies that meson exchange currents are included. This is illustrated in Fig. 7, where we have given a diagrammatic representation of the amplitude for pion photoproduction in Fig. 7(a), and the decomposition of the first term on the rhs of Eq. (5.8), in Fig. 7(b). A comparison of this result with those of Laget indicates that we have included all of the diagrams that he included, with the added constraint that our results must be consistent with the corresponding results on a single nucleon. In addition, we can treat the nucleon and higher mass isobars on equal footing. Finally, all our vertices are dressed and give the correct coupling constant.

We now turn to the reaction $N(\pi N)\leftarrow\gamma d$. To get this amplitude, we need to take the lhs residue of Eq. (5.3) at the quasiparticle (B^*) pole in the πB amplitude. Since this pole is at a complex energy, the corresponding amplitude is not an observable. However, we need to consider this amplitude in order to get a closed set of coupled equations for the reactions given in Eq. (4.1). Assuming the πB amplitude has a quasiparticle pole, we can write $M_d^{(2)}(i)$ as

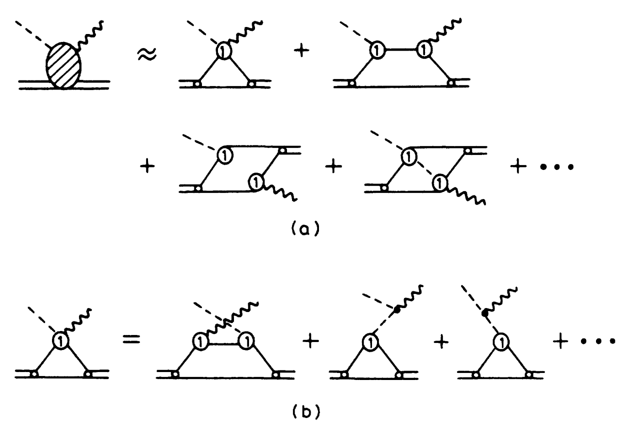

FIG. 7. (a) The lowest order contribution to the amplitude for pion photoproduction. (b) The lowest order diagrams that contribute to the amplitude for pion photoproduction as a result of including the coupling of the photon to the pion in the amplitude for photoproduction at the one nucleon level.

$$M_d^{(2)}(i)=\sum_j \bar{\delta}_{ij} d_B^{-1}(j) t^{(1)}(i)$$
$$=\sum_j \bar{\delta}_{ij} d_B^{-1}(j) \left[\frac{|\phi_{B^*}(i)\rangle\langle\phi_{B^*}(i)|}{E-\epsilon_{B^*}} + \cdots \right] , \quad (5.9)$$

where ϵ_{B^*} is the complex energy at which the πB amplitude $t^{(1)}(i)$ has a pole corresponding to the quasi-particle B^*. This quasi-particle pole is also present in the amplitudes $\tilde{M}^{(2)}_{A,d}(i)$ and $F_d^{(2)\dagger}(i)$ since

$$\tilde{M}^{(2)}_{A,d}(i)=\tilde{M}^{(3)}_{A,d}(i)+M_d^{(2)}(i)G^{(2)}\tilde{M}^{(3)}_{A,d}(i) , \quad (5.10)$$

and

$$F_d^{(2)\dagger}(i)=F_d^{(3)\dagger}(i)+M_d^{(2)}(i)G^{(2)}F_d^{(3)\dagger}(i) . \quad (5.11)$$

With the above results, we can now take the lhs residue of Eq. (5.3), to get the amplitude for $B(\pi B)\leftarrow\gamma d$ to be

$$\tilde{T}[B(\pi B)\leftarrow\gamma d]\equiv\langle\psi_{B^*}(i)|\tilde{T}'_{i;d}|\psi_d\rangle , \quad (5.12)$$

with

$$\tilde{T}'_{i;d}\equiv\tilde{M}^{(3)}_{A,d}(i)+F_d^{(3)\dagger}(i)G^{(1)}\tilde{T}_{B;d} + \sum_j U_{ij}^{(2)}G^{(2)}\{\tilde{M}^{(2)}_{A,d}(j)+F_d^{(2)\dagger}(j)G^{(1)}\tilde{T}_{B;d}\} . \quad (5.13)$$

To get the final equations in terms of two-particle irreducible amplitudes, (i.e., renormalized vertices, and two-body scattering amplitudes), we need to redefine $\tilde{T}_{i;d}$ to include only the last term on the rhs of Eq. (5.13), i.e., replace $\tilde{T}'_{i;d}$ by $\tilde{T}_{i;d}$, where

$$\tilde{T}_{i;d}\equiv\sum_j U_{ij}^{(2)}G^{(2)}\{\tilde{M}^{(2)}_{A,d}(j)+F_d^{(2)\dagger}(j)G^{(1)}\tilde{T}_{B;d}\} . \quad (5.14)$$

Since the amplitude for $B(\pi B)\leftarrow\gamma d$ is not an observable, we can take advantage of working with $\tilde{T}_{i;d}$ rather than $\tilde{T}'_{i;d}$ to simplify the final set of coupled equations. Here, we should stress the point that working with $\tilde{T}_{i;d}$ instead of $\tilde{T}'_{i;d}$ is not an approximation but a matter of convenience. Making use of Eqs. (3.7), (3.15), and (4.22), we can rewrite Eq. (5.14) in terms of the amplitudes for the BB-πBB system, as

$$\tilde{T}_{i;d}=\sum_j T_{i;j}G^{(2)}\tilde{M}^{(2)}_{A,d}(j)+T_{i;B}G^{(1)}\tilde{F}^{(2)}_d . \quad (5.15)$$

We now can combine Eqs. (5.7) and (5.15) into a single equation of the form

$$\tilde{T}_{\alpha;d}=\sum_j T_{\alpha;j}G^{(2)}\tilde{M}^{(2)}_{A,d}(j)+T_{\alpha;B}G^{(1)}\tilde{F}^{(2)}_d . \quad (5.16)$$

This equation describes the reaction for pion photoproduction as a distorted-wave matrix element, which includes the effect of the pion multiple scattering, through the amplitudes one gets from the BB-πBB equations. The above form for the pion photoproduction amplitude is convenient for calculating the lowest order contribution, in the multiple scattering series to the amplitude. In Sec. VI, we show how one can calculate these amplitudes from a set of coupled integral equations that satisfy two- and three-body unitarity. The three-body final-state photoproduction amplitude (i.e., $\pi BB\leftarrow\gamma d$) is given in Eq. (5.3), and can be written in terms of the amplitudes given in Eqs. (5.7) and (5.13).

VI. COUPLED BB-πBB-γBB EQUATIONS

In the above discussion, we concentrated our effort on deriving explicit expressions for the amplitudes corresponding to the reactions in Eq. (4.1). Although the forms of these expressions, as given in Eqs. (4.22) and (5.16), are convenient for expansion as a multiple scattering series or as distorted-wave matrix elements, neither of these forms are the most convenient for numerical computation. In this section we derive a set of coupled equations for these amplitudes. Here, we find that the resultant integral equations, which satisfy two- and three-body unitarity and have the form of the Faddeev equations, have the same kernel as the BB-πBB equation derived in Sec. III. This implies that one can get the cross section for pion-deuteron elastic scattering, photoproduction of pions, and photodisintegration of the deuteron, from the same set of equations. The success of such a program would be a major unification in our understanding of these reactions.

Making use of Eqs. (3.10) and (3.17), for $T_{B;B}$ and $T_{B;i}$, respectively, in Eq. (4.22), we get

$$\tilde{T}_{B;d}=\tilde{F}^{(2)}_d+\sum_{ij}F_d^{(2)}(i)\bar{\delta}_{ij}G^{(2)}\tilde{M}^{(2)}_{A,d}(j)+V_{\text{OPE}}G^{(1)}\left[(T_{B;B}G^{(1)}+1)\tilde{F}^{(2)}_d+\sum_i T_{B,i}G^{(2)}\tilde{M}^{(2)}_{A,d}(i)\right] + \sum_{i\alpha}F_d^{(2)}(i)\bar{\delta}_{i\alpha}G^{(2)}M_d^{(2)}(\alpha)G^{(2)}\left[T_{\alpha;\beta}G^{(1)}\tilde{F}^{(2)}_d+\sum_j T_{\alpha;j}G^{(2)}\tilde{M}^{(2)}_{A,d}(j)\right] . \quad (6.1)$$

Using Eqs. (4.22) and (5.16), we can write the above as

$$\tilde{T}_{B;d}=\tilde{F}^{(2)}_d+\sum_{ij}F_d^{(2)}(i)\bar{\delta}_{ij}G^{(2)}\tilde{M}^{(2)}_{A,d}(j)+V_{\text{OPE}}G^{(1)}\tilde{T}_{B;d}+\sum_{i\alpha}F_d^{(2)}(i)\bar{\delta}_{i\alpha}G^{(2)}M_d^{(2)}(\alpha)G^{(2)}\tilde{T}_{\alpha;d} . \quad (6.2)$$

To close the equations we need to get an equation for $\tilde{T}_{\alpha;d}$. This is achieved by using Eqs. (3.16) and (3.9) for the amplitudes $T_{\alpha;j}$ and $T_{\alpha;B}$, respectively, in Eq. (5.16) to get

$$\tilde{T}_{\alpha;d} = \sum_j \bar{\delta}_{\alpha j} \tilde{M}^{(2)}_{A,d}(j) + \sum_j \bar{\delta}_{\alpha j} F_d^{(2)\dagger}(j) G^{(1)} \left[(T_{B;B} G^{(1)} + 1) \tilde{F}_d^{(2)} + \sum_i T_{B;i} G^{(2)} \tilde{M}^{(2)}_{A,d}(i) \right]$$
$$+ \sum_\gamma \bar{\delta}_{\alpha\gamma} M_d^{(2)}(\gamma) G^{(2)} \left[\sum_j T_{\gamma;j} G^{(2)} \tilde{M}^{(2)}_{A,d}(j) + T_{\gamma;B} G^{(1)} \tilde{F}_d^{(2)} \right] . \tag{6.3}$$

Making use of the definition of the amplitudes $\tilde{T}_{B;d}$ and $\tilde{T}_{\gamma;d}$ as given in Eqs. (4.22) and (5.16), respectively, we can write the second integral equation as

$$\tilde{T}_{\alpha;d} = \sum_i \bar{\delta}_{\alpha i} \tilde{M}^{(2)}_{A,d}(i) + \sum_i \bar{\delta}_{\alpha i} F_d^{(2)\dagger}(i) G^{(1)} \tilde{T}_{B;d} + \sum_\gamma \bar{\delta}_{\alpha\gamma} M_d^{(2)}(\gamma) G^{(2)} \tilde{T}_{\gamma;d} . \tag{6.4}$$

In Eqs. (6.2) and (6.4) we have a set of coupled integral equations for the amplitudes for the photodisintegration of the deuteron, and pion photoproduction. These equations which satisfy two- and three-body unitarity (as far as the strong interactions are concerned) include the threshold for pion production, and can give a good description of these reactions at medium energies, where the $\Delta(1232)$ dominates. The input to these equations are the dressed πBB and γBB vertices as well as the nonpole $\pi B \rightarrow \pi B$ and $\gamma B \rightarrow \pi B$ amplitudes. These amplitudes can be constructed to fit the data for these reactions. If we make use of the chiral Lagrangian given in Eq. (2.1), in conjunction with our previous results for the $\pi B \rightarrow \pi B$ (Refs. 23 and 25) and $\gamma B \rightarrow \pi B$,[22] then the only free parameters in the theory are those associated with the chiral Lagrangian (e.g., the bag radius, the bare masses, and the bare coupling constants). In this case, a comparison of the results of a calculation based on these equations with experiment, could be used to justify the introduction of explicit quark degrees of freedom into the theory. This could partially be accomplished within the present formulation by taking into account, in perturbation theory, the three-particle irreducible amplitudes $F_c^{(3)}$, $\tilde{F}_c^{(3)}$, $M_c^{(3)}$, and $\tilde{M}^{(3)}_{A,c}$, which are to be determined by our model for the six-quark bag.

If we compare Eqs. (6.2) and (6.4) with the BB-πBB equations [e.g., Eqs. (3.16) and (3.17) for $\pi d \rightarrow \pi d$ and $\pi d \rightarrow NN$], we find that the kernel of the two sets of equations are identical. This means that we can give a unified description of all the reactions in Eq. (1). In fact, we can get the cross section for all of the above reactions with two-body final states, from a single set of integral equations. At present we have a detailed analysis of the reactions in Eqs. (1.1b) and (1.1d) within the framework of the NN-πNN equations. This analysis has had considerable success in describing a large set of data covering both differential cross section and polarization observables.[12,14] There are also indications that the inclusion of some of the mechanisms included in the BB-πBB equations, by treating the N and Δ on equal footing,[13,47] improves the agreement between theory and experiment. Thus, an extension of the present analysis based on the NN-πNN equation, to the BB-πBB equations, with the inclusion of the reactions in Eq. (1.1a), could be very promising.

VII. CONCLUSION

In the present paper, we have presented a unified description of the reactions, $\gamma d \rightarrow np, \pi^0 d, \pi NN$ and $NN \rightarrow \gamma NN$. The formulation is based on a Hamiltonian resulting from the projection of the chiral bag model Lagrangian (e.g., cloudy bag model[19]) onto the space of baryons. Although the derivation of the integral equations does not depend on a detailed form of the Lagrangian we have employed, there are four basic assumptions used in formulating the problem: (i) The chiral bag model Lagrangian is expanded in powers of the pion coupling constant, and only the terms up to order $g^2 = (2f_\pi)^{-1}$ are included. This is basically the approximation used in deriving the cloudy bag model.[19] This leads to a Lagrangian that has only the terms that are linear and quadratic in the pion field. This truncated Lagrangian has been used with great success for S- and P-wave π-N scattering. (ii) The coupling to the electromagnetic field is introduced at the quark level by requiring the Lagrangian to have U(1) gauge symmetry (i.e., $\partial_\mu \rightarrow \partial_\mu - ieA_\mu$). This coupling of the electromagnetic field to the quark and pion is included to first order only. (iii) The resultant Lagrangian is projected onto the space of baryons. In this way, our Hamiltonian has only baryons, mesons, and photons with the vertices for the coupling of the baryons to the mesons and photons, determined by the chiral bag model. The corresponding form factors are then related to the size of the baryons. This procedure removes the possibility of having two overlapping bags or the formation of six-quark bags. The contribution of six-quark bags can, in principle, be included in the present formulation by expanding our Hilbert space to include such six-quark states, then using perturbation theory to determine their contribution as discussed in Secs. IV and V. We have also excluded the possible change in the size or shape of the bags corresponding to the two baryons. (iv) To satisfy two- and three-body unitarity, we have included in our Hilbert space, the states $|BB\rangle$, $|\pi BB\rangle$, and $|\gamma BB\rangle$. To truncate our coupled integral equations to the above Hilbert space, we have neglected the coupling between the states that differ by more than two bosons, where the boson can be either a pion or photon.

Within the above framework we have derived a set of coupled integral equations that give a unified formulation for all of the reactions in Eq. (1). In addition, the final equations have the following distinctive features: (a) There are no free parameters, to the extent that all of the parameters of the chiral bag model Lagrangian are fixed at the single baryon level. This has been discussed previously for the π-N system[23,25] and for pion photoproduction on a single nucleon.[22] (b) The renormalization of both the propagators and vertices is handled in a consistent manner with unitarity. In other words, the renormalization of the coupling constants and the calculation of the amplitudes are carried out to the same order. (c) The nucleon and $\Delta(1232)$ are treated on equal footing as three-quark states. Therefore, the backward-going pion contribution to $BB \rightarrow BB$ (where $B = N, \Delta$), which is lacking in the NN-πNN theories,[1-5] is now included. In addition, one can extend the equations to include higher mass isobars [e.g., the Roper $N(1440)$] in terms of their quark structure. This has already been accomplished for the π-N system.[25,48] (d) The present theory can be applied above the threshold for pion production, since three-body unitarity is satisfied. (e) By treating the N and Δ on equal footing and introducing the electromagnetic coupling at the quark level, we have included both the isobar current and the pion exchange current without any ambiguity. This is particularly relevant for the reactions $\gamma d \rightleftarrows pn$, for which these currents have been the subject of recent controversy. (f) The question of dibaryon resonances is still being investigated in both $NN \rightarrow NN$ and $\gamma d \rightarrow pn$ reactions. The present formulation considers both of these reactions in a unified manner. (g) Finally, a consistent discrepancy between experiment and the results of calculations based on the equations presented above, could be considered as possible evidence for the need to introduce explicit quark degrees of freedom into the description of these reactions.

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