Shell effects in hot isobaric nuclei

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Level densities for hot isobaric nuclei are investigated by incorporating the isospin fluctuations in the statistical theory of nuclei. The single-particle level density parameter a is extracted as a function of temperature for various isospins of the system. At large temperatures, the empirical value of $a \simeq A/8$ is reproduced. A new method of extracting the neutron-proton asymmetry parameter is proposed. The neutron-proton asymmetry parameter is found to depend very strongly on deformation and isospin at low temperatures. The effect of the shell structure on the asymmetry energy is predominant for certain isospins corresponding to stable neutron-proton combinations. Results are presented for $A=42$, 44, 46, and 208. The excitation energy versus isospin plot for constant entropy of the system exhibits pockets similar to yrast traps in high-spin states of highly excited nuclei.

I. INTRODUCTION

The introduction of isospin by Heisenberg[1] to treat neutrons and protons on the same footing, i.e., as nucleons in two different isospin states, has proved to be a convenient way of calculating certain nuclear properties. Wigner and Feenberg[2] have obtained expressions for the masses of isospin multiplets of cold isobars in the usual liquid drop formalism by exploiting the symmetry of the Hamiltonian with respect to the total isospin of the system in the absence of Coulomb interaction. In an approximate way, they have assumed that the $2\tau+1$ states, where τ is the total isospin of the system, have a common energy level with τ_z values ranging from $-\tau$ to τ.

The purpose of the present work is twofold:

(i) to highlight the effects of the shell structure on these multiplets at low temperatures.

(ii) to extract the neutron-proton asymmetry parameter from the statistical model of the nucleus as a function of temperature, deformation, and isospin. This dependence of the asymmetry parameter on the isospin and the deformation of the nucleus has been overlooked in most calculations[3–11] which take recourse to liquid drop assumptions.

The present calculation elucidates the following important points:

(i) The neutron-proton asymmetry parameter is strongly dependent on the shell structure which is grossly different for various deformations of the nucleus. In the liquid drop model (LDM), the single-particle level spacing is assumed to be inversely proportional to the mass number A of the nucleus, irrespective of the isospin τ of the system, and the asymmetry energy is assumed to be proportional to the square of the isospin. Both these assumptions are not justified in view of the nuclear shell structure which causes large fluctuations in the single-particle level density which is a function of deformation, particle number, and temperature as illustrated by Strutinsky,[12] Ramamurthy *et al.*,[13] and the present authors.[14–16]

(ii) Another important aspect of the present calculation is that the asymmetry energy contributions are less significant at large temperatures owing to the fluctuations in the occupation probabilities of the different single-particle states at higher excitations. This leads to a decrease in the neutron-proton asymmetry parameter a_{asy} at large temperatures. It is estimated that the asymmetry parameter takes a value approximately equal to 20 MeV at very low temperatures, in good agreement with earlier works, and less than 5 MeV at temperatures greater than 1.5 MeV. The dependence of the asymmetry parameter on deformation is very strong at low temperatures. The value of the parameter at $T=0.4$ MeV decreases from 20 to 5 MeV as the deformation parameter δ is changed from 0 to $+0.6$.

(iii) The single-particle level density parameter, $a(\tau,T)$, τ as a function of isospin τ and temperature T, is calculated for $A=208$, and it is found that at higher temperatures the level density parameter approaches a constant value which is almost equal to the empirical value $a \simeq A/8$, irrespective of the isospin τ.

II. METHOD

A. The statistical theory

In the statistical formalism, we start with the grand canonical partition function $Q_0(\alpha,\beta,\gamma)$ for a system of A nucleons at a temperature $T=1/\beta$. The Lagrangian multipliers α, β, and γ conserve the total number of particles, the total energy, and the total isospin of the system[17,18]

$$Q_0(\alpha,\beta,\gamma)=\sum_{E_i,N_i,\tau_i}\exp(\alpha N_i-\beta E_i+\gamma\tau_i)\ . \tag{1}$$

The partition function in Eq. (1) does not include the Coulomb interaction which will be added subsequently. The average number of particles, the average total energy, and the total z component of isospin are projected out of the partition function by the following equations:

$$\langle N\rangle = A = \partial \ln Q_0/\partial\alpha \ , \quad (2)$$

$$\langle E\rangle = -\partial \ln Q_0/\partial\beta \ , \quad (3)$$

$$\langle \tau\rangle = \partial \ln Q_0/\partial\gamma \ . \quad (4)$$

The corresponding equations in terms of the single-particle energies ϵ_i are

$$\langle N\rangle = A = \sum n_i^+ + \sum n_i^- \ , \quad (5)$$

$$\langle E\rangle = \sum (n_i^+ + n_i^-)\epsilon_i \ , \quad (6)$$

$$\langle \tau\rangle = \sum n_i^+\tau_z^+ + \sum n_i^-\tau_z^- \ , \quad (7)$$

where n_i^+ and n_i^- are the occupation probabilities at single-particle energies ϵ_i of neutrons and protons with isospin projections $\tau_z^+ = +\frac{1}{2}$ and $\tau_z^- = -\frac{1}{2}$, respectively

$$n_i^+ = [1+\exp(-\alpha+\beta\epsilon_i-\gamma\tau_z^+)]^{-1} \ ,$$
$$n_i^- = [1+\exp(-\alpha+\beta\epsilon_i-\gamma\tau_z^-)]^{-1} \ . \quad (8)$$

The occupation probabilities are displayed in Fig. 1 as a function of ϵ_i for the two states $\tau_z^{\pm}$. The single-particle energies ϵ_i, as a function of the deformation parameter δ, are generated by the Nilsson Hamiltonian for the deformed oscillator diagonalized[19] in the cylindrical basis[20]

$$H = p^2/2m + (m/2)(\omega_x^2 x^2 + \omega_y^2 y^2 + \omega_z^2 Z^2) + Cl\cdot s + Dl^2 \ . \quad (9)$$

The coefficients for the $l\cdot s$ and l^2 terms are taken from Seeger[5] who has fitted them to reproduce the shell corrections[12] to ground-state masses. The deformation parameter δ is varied from -0.6 to $+0.6$.

The coupled nonlinear Eqs. (5) and (7) have to be solved for the Lagrangian multipliers α and γ for a given mass number A, temperature T, and the total z component of the isospin τ [$=(N-Z)/2$] of the system. The energy of the system is then calculated using Eq. (6). The corresponding excitation energy $E^*(\tau,T)$ and the entropy $S(\tau,E^*)$ are obtained using the following expressions:

$$E^*(\tau,T) = \sum (n_i^+ + n_i^-)\epsilon_i - \sum_{i=1}^{A} \epsilon_i \ ,$$
$$S(\tau,E^*) = S^+ + S^- \ , \quad (10)$$

where

$$S^+ = -\sum [n_i^+ \ln n_i^+ + (1-n_i^+)\ln(1-n_i^+)]$$

and

$$S^- = -\sum [n_i^- \ln n_i^- + (1-n_i^-)\ln(1-n_i^-)] \ . \quad (11)$$

The level densities for various excitation energies and isospins of the system are given by[9]

$$\rho(\tau,E^*) = \beta \exp[S(\tau,E^*)]/S_{\max} \ . \quad (12)$$

The normalization factor $S_{\max}$ depends upon the dimensionality of phase space which is the number of eigenstates used.[14]

The total energy E of the system for each temperature T is minimized with respect to the deformation parameter δ. The lines of constant entropy are then drawn in the E^* versus τ plane for $A=42$, 44, and 46, and the results are displayed in Figs. 2–4. Collective deexcitations along the constant entropy lines are possible through the emission of beta particles.

B. The neutron-proton asymmetry parameter

The neutron-proton asymmetry parameter a_{asy} is extracted from the statistical theory using the following approach.

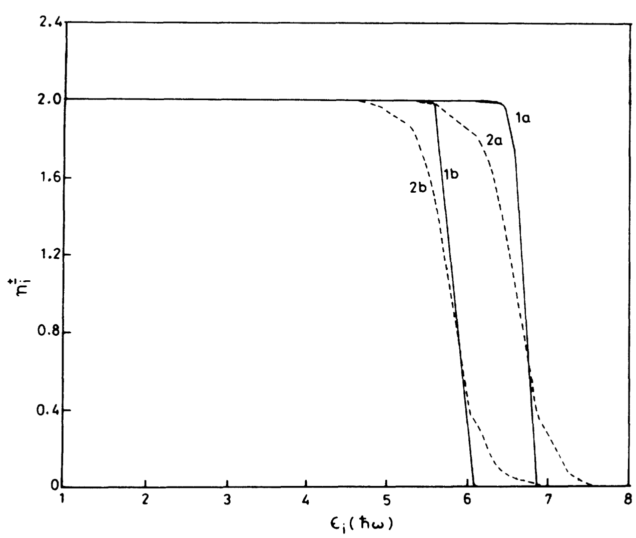

FIG. 1. Occupation probabilities n_i^+ (1a and 2a) for τ_z^+ and n_i^- (1b and 2b) for τ_z^- as a function of the single-particle energies ϵ_i for $A=208$ corresponding to a total isospin of 22. The solid curves are for $T=0.2$ MeV, whereas the dashed curves are for $T=1.2$ MeV.

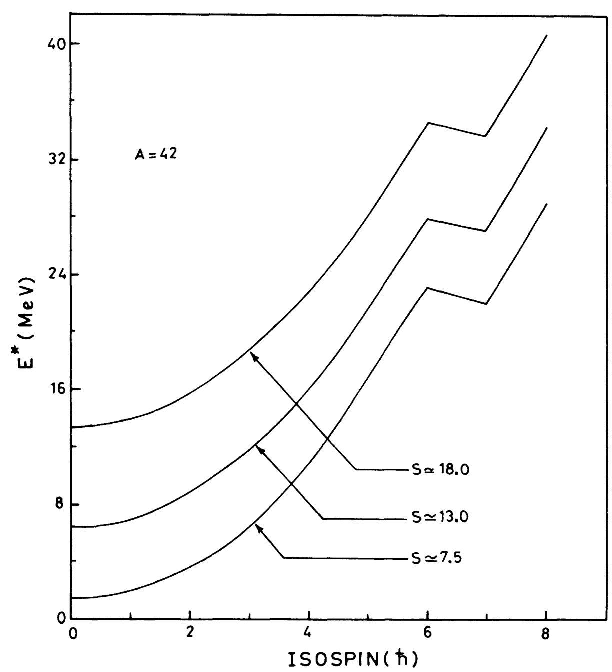

FIG. 2. Constant entropy lines for $A=42$.

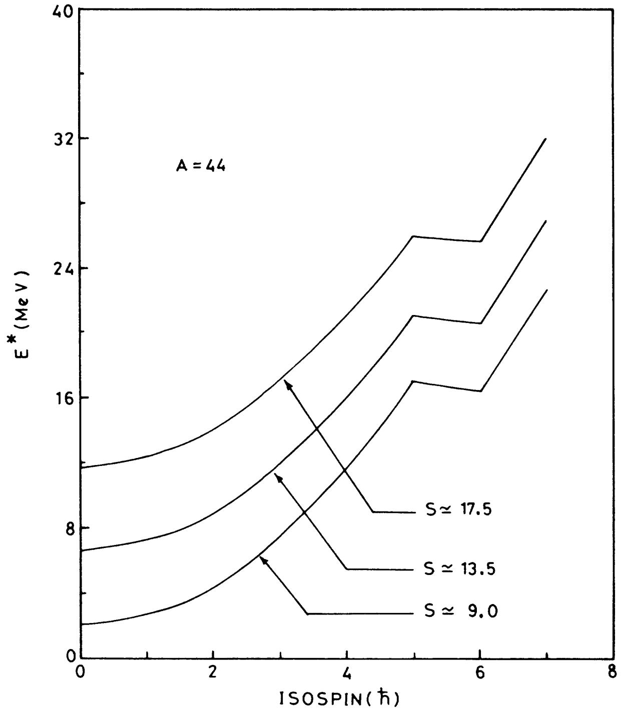

FIG. 3. Constant entropy lines for $A=44$.

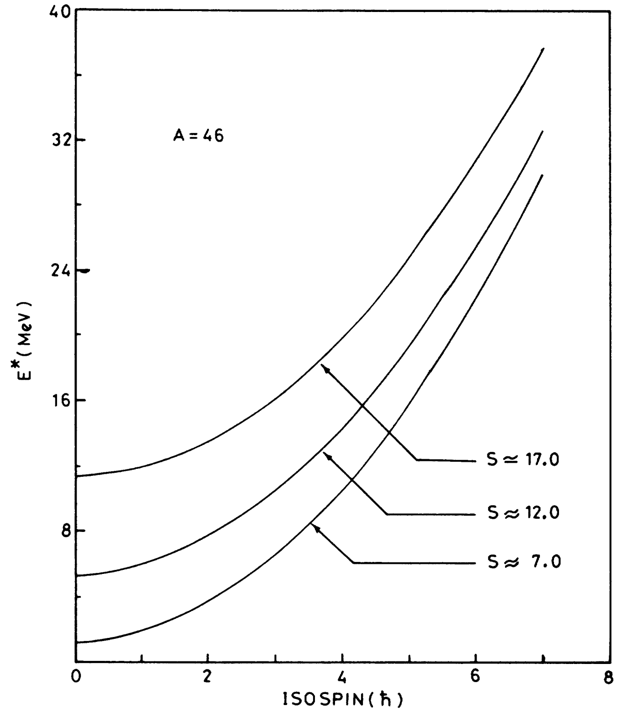

FIG. 4. Constant entropy lines for $A=46$.

The mass of the multiplets of total isospin can be written in the liquid drop model following Wigner and Feenberg[2] who, however, have not included the asymmetry energy

$$E_{\text{asy}}=a_{\text{asy}}(N-Z)^2/A$$

which removes the mass degeneracy between different multiplets. Introducing the net isospin of the nucleus with Z protons and N neutrons as $\tau=(N-Z)/2$, the nuclear mass is given by

$$M=(M_n-M_p)\tau+3[(A/2)-\tau]\times[(A/2)-\tau-1]e^2/5R+\text{const}\ , \tag{13}$$

where the nuclear radius $R=r_0A^{1/3}$, with $r_0=1.2$ fm. The constant term includes all other factors independent of τ and M_n and M_p are the neutron and the proton masses, respectively. Incorporating the asymmetry term in the expression for M, we have

$$M=(M_n-M_p)\tau+a_c[(A/2)-\tau][(A/2)-\tau-1]/A^{1/3}+4a_{\text{asy}}\tau^2/A+\text{const}\ , \tag{14}$$

where the Coulomb constant $a_c=3e^2/5r_0$. In terms of the isospin τ, the variation of binding energy with isospin is given by

$$(\partial B/\partial\tau)_{\text{LDM}}=-\partial M/\partial\tau=-(M_n-M_p)-8a_{\text{asy}}\tau/A-2a_c\tau/A^{1/3}+a_c(A-1)/A^{1/3}\ . \tag{15}$$

The negative sign indicates that as M increases B decreases. The corresponding expressions in the statistical theory can be obtained using the partition function which includes the Coulomb interaction

$$Q=\exp(-\beta\Omega)\ , \tag{16}$$

where Ω is the thermodynamical potential of the system. After introducing the Coulomb interaction, the logarithm of the partition function is expressed as a sum of two terms

$$\ln Q=\ln Q_0-\beta E_c\ , \tag{17}$$

where the second term on the right-hand side corresponds to the Coulomb energy of the system and Q_0 is the same as in Eq. (1). Using this partition function and the definition of the thermodynamical potential Ω (free energy), which is the negative of the binding energy, we can obtain the change in binding energy as a function of isospin as

$$(\partial B/\partial\tau)_{\text{shell}}=-\partial\Omega/\partial\tau=-T\tau\left[\sum n_i^+(1-n_i^+)(\tau_z^+)^2+\sum n_i^-(1-n_i^-)(\tau_z^-)^2\right]^{-1}-(2a_c\tau/A^{1/3})+a_c(A-1)/A^{1/3}\ . \tag{18}$$

The first term on the right-hand side of Eq. (18) has been obtained from the partition function as

$$\partial(T\ln Q_0)/\partial\tau=T(\partial\ln Q_0/\partial\gamma)(\partial\gamma/\partial\tau)\ . \tag{19}$$

Equation (18) gives the variation of the binding energy of the system with the isospin τ and can be identified with the liquid drop model neutron-proton asymmetry term $8a_{\text{asy}}\tau/A$. The first differential on the right-hand side is the net isospin as can be seen from Eq. (4). Using Eqs. (2)–(8), we have

$$\partial\gamma/\partial\tau = -\left[\sum n_i^+(1-n_i^+)(\tau_z^+)^2 + \sum n_i^-(1-n_i^-)(\tau_z^-)^2\right]^{-1} . \quad (20)$$

The other terms which are due to the classical Coulomb energy of the nucleus are the same as in the liquid drop model. From Eqs. (15) and (18), the neutron-proton asymmetry parameter is extracted as a function of temperature, deformation, and isospin. Calculations are performed for $A = 208$, 42, 44, and 46. The results are displayed in Figs. 5–8.

C. The single-particle level density parameter

It is commonly observed that the energies calculated using the shell model are usually an order of magnitude larger than the liquid drop value. However, the excitation energies of the system with respect to the ground-state energies of the shell model can be calculated fairly accurately. This fact has been effectively used in the formulation of the shell correction method of Ramamurthy *et al.*[13] which reproduces very well the experimental values of the ground-state masses of the nucleus. The excitation energy E^* of the system is obtained using the equation

$$E^*(\tau,T) = E(\tau,T) - E_0 , \quad (21)$$

where E_0 is the ground-state energy of the nucleus. The single-particle level density parameter $a(\tau,T)$, as a function of isospin τ and temperature T, is extracted using the equation

$$a(\tau,T) = S^2(\tau,E^*)/4E^*(\tau,T) . \quad (22)$$

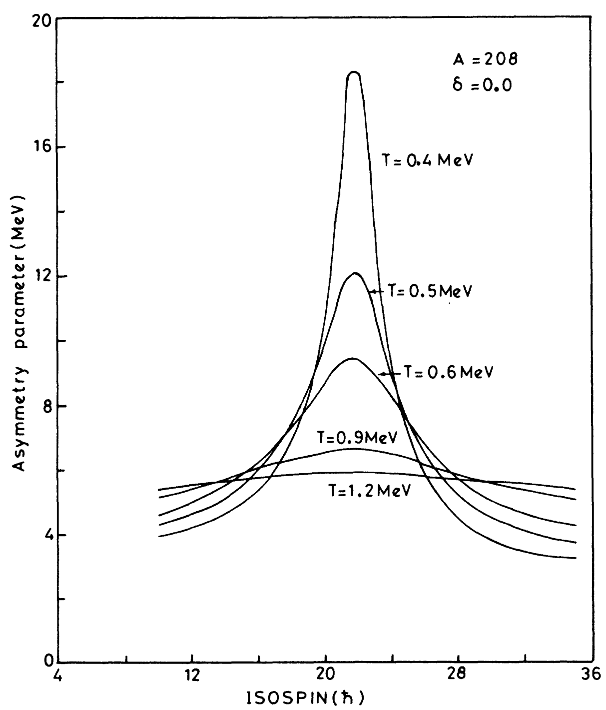

FIG. 5. Variation of the asymmetry parameter with isospin for various temperatures in the case of $A = 208$.

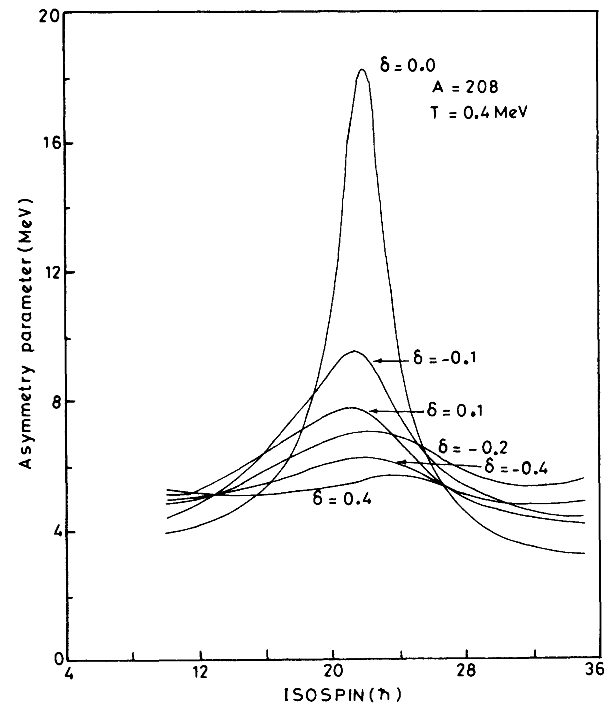

FIG. 6. Variation of the asymmetry parameter with isospin for various deformations in the case of $A = 208$.

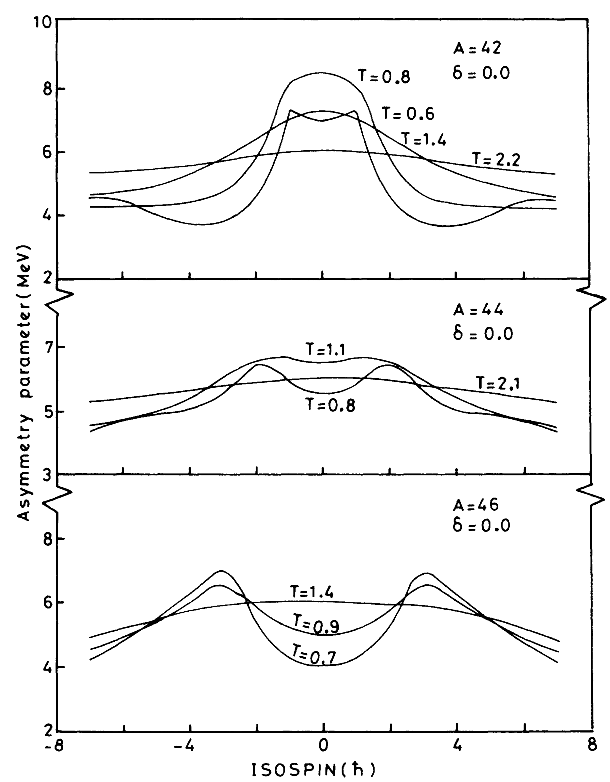

FIG. 7. Variation of the asymmetry parameter with isospin for various temperatures in the case of $A = 42$, 44, and 46.

Calculations are performed for the system $A=208$ and the results are displayed in Fig. 9.

III. RESULTS AND DISCUSSION

The isospin of the system is projected out of the grand canonical partition function using Eq. (4). It is important to note that the occupation probabilities n_i^+ and n_i^- for the positive and the negative projections of the single-particle isospin states in each single-particle level ϵ_i are different. The removal of degeneracy of the $\tau_z^{\pm}$ states is due to the rotation in isospin space. Figure 1 shows these occupation probabilities as a function of ϵ_i at two different temperatures. Curves 1a for n_i^+ and 1b for n_i^- correspond to a total isospin $\tau=22$ at a temperature 0.2 MeV for $A=208$. Curves 2a and 2b correspond to a temperature of 1.2 MeV for the same isospin. These curves help in comprehending the way of generating the net isospin of the system which can be obtained from the graph as

$$\tau=\int dn_i^+\tau_z^+ + \int dn_i^-\tau_z^- \ . \qquad (23)$$

We have drawn constant entropy lines in Figs. 2–4, in the excitation energy E^* versus τ plane. These curves are drawn after minimizing the total energy of the system with respect to deformation for each temperature. In the event of a collective deexcitation along the constant entropy line during isospin fluctuations, the system may be trapped in one of the isobaric states which have relatively lower energies than the neighboring states with a certain net isospin. These traps are similar to the yrast traps[15,21] observed in the collective deexcitation of higher angular momentum states. In the case of $A=46$, no such traps are seen in our calculations as shown in Fig. 7. For $A=42$, we find from Fig. 2 that the minimum occurs at $\tau=7$, corresponding to the proton number 14 and the neutron number 28, which are closed subshells. In Fig. 3, we see that in the case of $A=44$, the stable isobar corresponds to the proton number 16 and the neutron number 28, indicating closed subshells. These relatively stable states can be populated only at very high isospin excitations and do not occur normally, as nature tends towards highly symmetrical states. These changes are mainly due to shell structures which play a major role in the determination of nuclear stability.

From Figs. 5 and 6, we find that the asymmetry parameter extracted from the statistical calculation for $A=208$ shows a maximum only for $\tau=22$, which corresponds to a neutron number 126 and proton number 82. This indicates the relatively higher stability of the $\tau=22$ state for $A=208$. Since single-particle level density at the fermi energy is very low, the level spacing is large. The asymmetry energy is large, since it is directly proportional to the level spacing. In simple terms it means that a small change in the net isospin requires a large energy for nuclei with relatively larger level spacing at the fermi energy. In Fig. 5 it is seen that the variation of the asymmetry parameter with isospin is flattened out for large temperatures. In Fig. 6, the smooth behavior is accomplished by increasing the deformation of the nucleus. In the latter case, the smoothening is due to the decrease in the level spacing of the single-particle levels with increasing deformations. This aspect has not been studied in

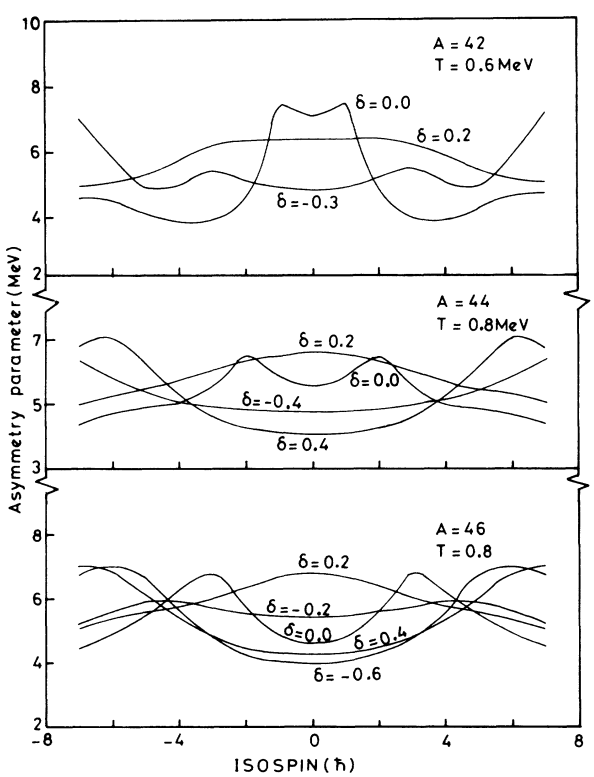

FIG. 8. Variation of the asymmetry parameter with isospin for various deformations in the case of $A=42$, 44, and 46.

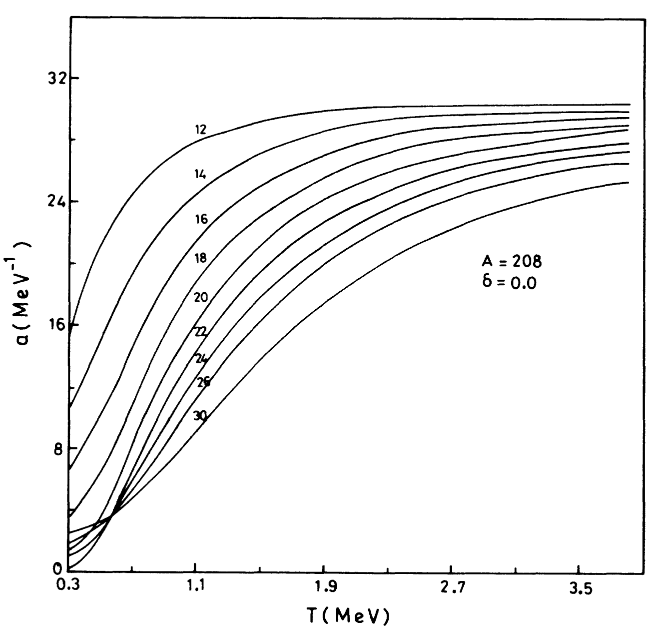

FIG. 9. Variation of the single-particle level density parameter with temperature for various isospins of the isobar $A=208$. The numbers on the curve refer to the net isospin of the system.

earlier calculations. In the case of $A=42$, 44, and 46, symmetric double humps on either side of the $\tau=0$ plane are seen at low temperatures as shown in Figs. 7 and 8. However, at higher temperatures these features are absent. The asymmetry parameter at low temperatures for $A=42$, 44, and 46 shows maxima at $\tau=\pm1$, ±2, and ±3, respectively, corresponding to the proton number 20 in all the cases, implying closed-shell nuclei.

The fact that at large temperatures, the asymmetry energy is very small compared to its value at low temperatures, is important in the study of reactions involving isospin fluctuations. Hot nuclei formed in heavy-ion collisions subsequently experiences rapid isospin fluctuations[22] at high temperatures; since the asymmetry energy is small at large temperatures, the fluctuations may not give rise to large energy changes in the system. The situation at low temperatures is totally different. Since the asymmetry energy is very large at low temperatures, large energy changes may follow isospin fluctuations.

In Fig. 9 we show the variation of the single-particle level density parameter $a(\tau,T)$ temperature, for various isospins of the isobar $A=208$. The value of the level density parameter corresponding to $\tau=22$ is very small at low temperatures, indicating the higher stability of the $\tau=22$ state compared to other isospin values. At higher temperatures, however, the effect of the isospin is less significant, and the curves converge to the value predicted by experimental observation[23,24] which is given by the empirical relation $a\simeq A/8$.

We conclude that by using the statistical theory which involves the shell structure of the nuclei, the most stable isobar can be predicted by treating the isospin as a dynamical variable. A new method of extracting the neutron-proton asymmetry parameter is proposed. The extracted value of the neutron-proton asymmetry parameter from the present theory agrees well with the value fitted in the mass formula of earlier works at low temperatures. The effects of temperature, deformation, and the isospin degree of freedom on the asymmetry parameter value, which have been overlooked in earlier calculations, have been investigated here. In view of the dependence of the asymmetry parameter on the isospin of the system, the asymmetry energy is no longer a parabolic function of τ as assumed in the liquid drop model. The strong shell effects will, however, be slightly altered with the introduction of pairing correlations, but the main results of the paper will remain unchanged. Calculations are underway for isospin fluctuations in fast rotating superfluid nuclei.

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