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Random phase approximation for light nuclei based on fully relativistic Hartree-Fock calculations

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The particle-hole spectra of light nuclei are examined in the self-consistent random phase approximation based on fully relativistic Hartree and Hartree-Fock models for the nuclear ground state. The particle-hole interaction is completely prescribed by the ground-state calculation. It includes σ, ω, ρ, and π meson exchanges, with σ and ω parameters adjusted to fit the bulk properties of nuclear matter. Differences between Hartree (no exchange) and Hartree-Fock (with exchange) predictions for the spectra are discussed.

I. INTRODUCTION

In recent years relativistic models[1] for the ground-state properties of nuclei have been pursued with some success. These models are based on a relativistic quantum field theory of baryons and mesons called quantum hadrodynamics (QHD). Calculations for finite nuclei have been performed in the Hartree or mean-field approximation[2] to this theory, with an interaction based on the exchange of σ, ω, and ρ mesons.

The couplings of the σ and ω mesons are determined by fitting the saturation density (1.30 fm^{-1}) and binding energy (15.75 MeV) of nuclear matter, while the mass of the phenomenlogical σ is fixed by the rms charge radius of ^{40}Ca. The model gives a reasonably good description of the bulk properties of finite nuclei, viz. the neutron and proton density distributions, rms radii, energy levels and spin-orbit splittings.

Recently Blunden and Iqbal[3,4] have extended the above model to include exchange terms (i.e., the relativistic Hartree-Fock approximation). In addition to the above mesons a pseudovector pion interaction is included, together with a tensor coupling term for the rho. (The pion does not contribute in the Hartree approximation.) The sigma and omega meson couplings are again determined from nuclear matter, while the rho and pion couplings are taken from experiment. Although the effect of the Fock or exchange terms is large, the final results have gross features similar to those of the Hartree calculations. This is probably not too surprising since both calculations saturate nuclear matter at the same place and give similar results for the bulk properties (at normal nuclear densities).

It is of interest to see how well these relativistic models do in reproducing the excited-state properties of finite nuclei. Since there are no additional parameters introduced, this constitutes a severe test of the interactions as well as the underlying theory.

Another consideration is that the pion will play a more important role in excited states than it does in the ground state. In particular, the unnatural parity $(0^-,1^+,2^-,3^+,\ldots)$ isovector modes are highly sensitive to the nature of the pionic interaction. The Hartree-Fock coupling constants are all, to some extent, constrained by the pion, since the theory incorporates pions consistently from the start. However, this is not true in the Hartree model. Therefore, even though the ground-state properties are similar in the two theories, the excited state spectra may be very different.

Properties of collective states were examined in a relativistic semiclassical approximation by Horowitz and Walecka.[5] They found that the experimental systematics of the collective vibrational modes (giant resonances) could be reproduced in their model. Furnstahl[6] has considered the negative parity states in ^{16}O in a microscopic random phase approximation (RPA) calculation based on the Hartree approximation. He found that a reasonable excitation spectrum could be obtained with a pseudovector pion coupling but not with a pseudoscalar coupling. One problem reported in his results was that the spurious $1^-, T=0$ state did not occur at zero energy, as is expected in a self-consistent model where the wave functions of the ground and excited states are determined from the same underlying Hamiltonian. We shall comment on this further in Sec. III.

The organization of this paper is as follows. In Sec. II we formulate the RPA equations of motion, and give formulae for the particle-hole matrix elements of the relativistic interaction. The meson parameters determined from the Hartree and Hartree-Fock calculations are reproduced.

In Sec. III we present results for the excited-state spectra in both Hartree and Hartree-Fock approximations. We discuss the strength distribution of the giant dipole $(1^-, T=1)$ resonance, as well as the spurious $1^-, T=0$ state. In Sec. IV the unnatural parity states are examined in greater detail. Additional interactions are introduced, in particular a tensor coupling for the ρ, and a contact (or Landau-Migdal) term for the π exchange. Section V contains discussion of the major results and comments on possible modifications to the theory.

II. FORMULATION

Excited states $|\Psi^J\rangle$ of multipolarity J are formed as superpositions of particle-hole states

$$|\Psi^J\rangle=\sum_{p,h}(X^J_{ph}a^\dagger_p a_h - Y^J_{ph}a^\dagger_h a_p)\,|\,0\rangle \ . \tag{1}$$

Here $|\,0\rangle$ is the exact Hartree-Fock ground state, which is itself described as an antisymmetrized Slater determinant

$$|\,0\rangle=\prod_{i=1}^{A} a_i^\dagger\,|\,\mathrm{vac}\rangle \ . \tag{2}$$

$a_p^\dagger$ and a_h are creation and annihilation operators for positive-energy single-particle states above and below the Fermi surface. X^J_{ph} and Y^J_{ph} are the amplitudes for creating a particle-hole pair and for annihilating a particle-hole pair already present in the ground state. Negative energy states are excluded from the present description (i.e., particle-antiparticle states). The implications of this omission are discussed later on in Sec. V.

The transition strength between the ground and excited states is given by

$$\begin{aligned}B(J)&=|\,\langle\Psi^J\|T^J|\,|\,0\rangle^2\\&=\sum_{p,h}|\,\langle h^{-1}p;J\|T^J\|0\rangle X^J_{ph}\\&\qquad+(-1)^J\langle 0\|T^J\|h^{-1}p;J\rangle Y^J_{ph}\,|^2\\&=\sum_{p,h}|\,\langle p\|T^J\|h\rangle[X^J_{ph}+(-1)^kY^J_{ph}]\,|^2 \ .\end{aligned}\tag{3}$$

The phase $(-1)^k$ is related to the hermiticity of the transition operator T^J via

$$\langle p\|T^J\|h\rangle=(-1)^{h-p+k}\langle h\|T^J\|p\rangle^* \ , \tag{4}$$

where the convention of Edmonds[7] has been used for the reduced matrix element. The notation $h-p$ in the exponent is shorthand for j_h-j_p.

In the present relativistic calculation, the operators T^J will have the structure of 2×2 matrices constructed from the Lorentz-invariant matrices 1, γ^μ, and γ_5. The two-component relativistic wave functions have the form[2]

$$|\,p\,\rangle\equiv\begin{bmatrix} i\dfrac{G_p(r)}{r}\Phi_{\kappa m}(\hat r)\\ -\dfrac{F_p(r)}{r}\Phi_{-\kappa m}(\hat r)\end{bmatrix}\equiv\begin{bmatrix} i\phi_p(r)\\ -\tilde\phi_p(r)\end{bmatrix} , \tag{5}$$

where $\kappa=(l-j)(2j+1)$. $\Phi_{km}(\hat r)$ is a vector spherical harmonic which couples orbital angular momentum to spin, and a tilde is used to distinguish lower component quantities from those of the upper component.

Using the ansatz of Eq. (1) for the wave functions and linearizing the equations of motion leads (this is strictly true only if one ignores retardation corrections) to the familiar RPA eigenvalue problem[8]

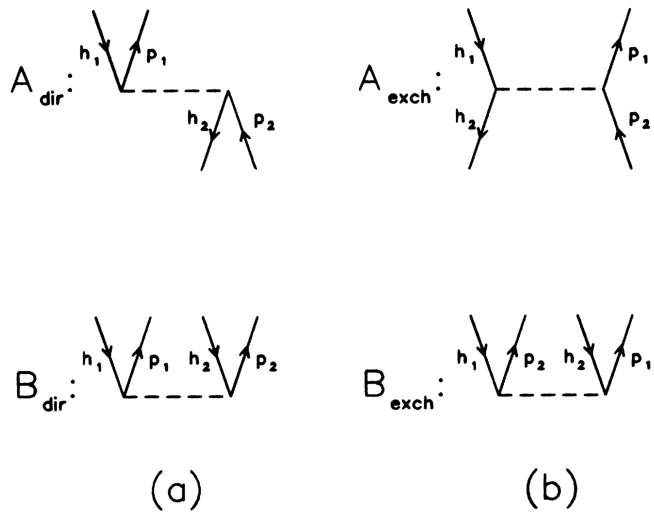

FIG. 1. Direct and exchange particle-hole interactions which appear in the RPA.

$$\begin{bmatrix} A & B\\ -A & -B\end{bmatrix}\begin{bmatrix}X\\Y\end{bmatrix}=E\begin{bmatrix}X\\Y\end{bmatrix} , \tag{6}$$

with

$$A^{J,T}_{1,2}=(\varepsilon_{p_1}-\varepsilon_{h_1})\delta_{h_1,h_2}\delta_{p_1,p_2}+\langle h_1^{-1}p_1;J,T\,|\,V\,|\,h_2^{-1}p_2;J,T\rangle_{\mathrm{dir+exch}} \ , \tag{7}$$

and

$$B^{J,T}_{1,2}=(-1)^{h_2-p_2}\langle h_1^{-1}p_1;J,T\,|\,V\,|\,p_2^{-1}h_2;J,T\rangle_{\mathrm{dir+exch}} \ . \tag{8}$$

The two-body matrix elements are reduced with respect to angular momentum J and isospin T. In this paper we will ignore the Coulomb interaction, and work with states of good isospin.

The direct and exchange terms of $A^{J,T}$ and $B^{J,T}$ are depicted graphically in Fig. 1. The exchange matrix element may be rewritten in terms of direct particle-hole matrix elements by angular momentum recoupling. By writing the interaction V schematically in terms of an isoscalar piece V_0 and an isovector piece $V_1\tau_1\cdot\tau_2$, the isospin dependence may be factored out, so that

$$\begin{aligned}\langle h_1^{-1}p_1;J,T\,|\,V\,|\,h_2^{-1}p_2;J,T\rangle_{\mathrm{dir+exch}}&=(2V_0\delta_{T,0}+2V_1\delta_{T,1})\langle h_1^{-1}p_1;J\,|\,V\,|\,h_2^{-1}p_2;J\rangle_{\mathrm{dir}}\\&\quad+\sum_{J'}(-1)^{h_2-p_1+J'+J}(2J'+1)\begin{Bmatrix}h_1 & h_2 & J'\\ p_2 & p_1 & J\end{Bmatrix}\\&\quad\times[(V_0+3V_1)\delta_{T,0}+(V_0-V_1)\delta_{T,1}]\langle h_1^{-1}h_2;J'\,|\,V\,|\,p_1^{-1}p_2;J'\rangle_{\mathrm{dir}} \ .\end{aligned}\tag{9}$$

The first term corresponds to the direct particle-hole interaction $A_{\rm dir}$ of Fig. 1(a), while the second corresponds to the exchange term $A_{\rm exch}$ of Fig. 1(b). The terms for B may be obtained similarly using Eq. (8).

We will be comparing the results of calculations based on both Hartree and Hartree-Fock models of the nuclear ground state. Consistency demands that the prescription for evaluating two-body matrix elements of the excited states should be the same as that used to construct the ground state. In other words, with the Hartree basis one should only take direct particle-hole matrix elements, while for the Hartree-Fock basis direct and exchange matrix elements should be used. This point is crucial if one wants to get the spurious $1^-, T=0$ state to go to zero energy.

The two-body matrix elements

$$\langle h_1^{-1}p_1;J \mid V \mid h_2^{-1}p_2;J\rangle_{\rm dir}$$

are easily evaluated in terms of products of one-body matrix elements. A multipole decomposition of $V(r)$, where $\mathbf{r}=\mathbf{r}_1-\mathbf{r}_2$, can be written in the general form

$$V(r)=\sum_J M^J(r_1,\sigma_1)\cdot M^J(r_2,\sigma_2) , \tag{10}$$

where $M^J(r_1,\sigma_1)$ is a 2×2 matrix whose elements are spherical tensors of rank J formed from the position and spin coordinates of particle 1. Then we can use the relation

$$\langle h_1^{-1}p_1;J \mid V \mid h_2^{-1}p_2;J\rangle_{\rm dir}=\frac{(-1)^J}{2J+1}\langle h_1^{-1}p_1;J\|M^J(r_1,\sigma_1)\|0\rangle\langle 0\|M^J(r_2,\sigma_2)\|h_2^{-1}p_2;J\rangle . \tag{11}$$

For the current problem, the interaction in momentum space is given by the exchange of mesons in the form

$$\begin{aligned}V(q)=&\frac{-g_\sigma^2}{-q_\mu^2+m_\sigma^2}1^{(1)}1^{(2)}\\&+\left[\frac{g_\omega^2}{-q_\mu^2+m_\omega^2}+\frac{g_\rho^2}{-q_\mu^2+m_\rho^2}\tau_1\cdot\tau_2\right]\gamma^{(1)}\cdot\gamma^{(2)}\\&-\frac{g_\pi^2}{4M^2}\frac{\gamma_5^{(1)}\gamma_\mu^{(1)}q^\mu\gamma_5^{(2)}\gamma_\nu^{(2)}q^\nu}{-q_\mu^2+m_\pi^2}\tau_1\cdot\tau_2 ,\end{aligned} \tag{12}$$

where $q_\mu^2=q_0^2-\mathbf{q}^2$. Retardation effects are small for low-lying excitations, so we can set $q_0=0$. We have also used the following conventions:

$$\gamma^0=\begin{bmatrix}1&0\\0&-1\end{bmatrix},\ \gamma^i=\begin{bmatrix}0&\sigma^i\\-\sigma^i&0\end{bmatrix},\ \gamma_5=\begin{bmatrix}0&1\\1&0\end{bmatrix} .$$

There are two important features of this interaction to point out. First, we have not included a tensor coupling for the ρ. Secondly there is no contact (or Landau-Migdal) term for the π exchange. This is commonly added to nonrelativistic calculations in order to simulate the effect of short-range correlations. One of our aims in the present calculation is to see how well the bare QHD interaction does without the addition of phenomenological terms. In Sec. IV we shall examine these additional terms to see their effect on the unnatural parity spectra, for which they play the greatest role.

The coordinate space interaction $V(r)$ is related to $V(q)$ by the Fourier transform

$$V(r)=\int\frac{d^3q}{(2\pi)^3}e^{-i\mathbf{q}\cdot(\mathbf{r}_1-\mathbf{r}_2)}V(q) . \tag{13}$$

The multipole decomposition may be performed in either coordinate space or momentum space. We prefer the latter for the present problem, since the resulting one-body spherical tensors of Eq. (11) turn out to be the same ones needed to evaluate the one-body matrix elements of Eq. (3). Details are given in the Appendix, where the explicit form of the momentum space multipole expansions are given for the various types of interactions.

Table I gives the Hartree and Hartree-Fock coupling constants used in the present calculation. All parameter sets saturate nuclear matter at 1.30 $\rm fm^{-1}$ and 15.75 MeV in their respective approximations, and give reasonable values for the bulk symmetry energy.

The unperturbed basis states are obtained from the numerical Dirac-Hartree-Fock code of Blunden and Iqbal.[4] The continuum has been discretized similarly to Ref. 6 by

TABLE I. Hartree and Hartree-Fock parameters sets which saturate nuclear matter at 1.30 $\rm fm^{-1}$. The second set includes a tensor coupling for the ρ and a "spin-isospin" contact interaction, Eq. (15). The masses are 520, 783, 770, and 138 MeV for the σ, ω, ρ, and π mesons, respectively.

	g_σ	g_ω	g_ρ	g_π	K_ρ	g'
Hartree I	10.47	13.80	2.63	13.45		
Hartree-Fock I	10.00	11.90	2.63	13.45		
Hartree II	10.47	13.80	2.63	13.45	6.6	0.7
Hartree-Fock II	8.11	12.47	2.63	13.45	6.6	0.7

imposing a boundary condition that the lower wave function component $F(r_{max})=0$ at $r_{max}=12$ fm and restricting all integrations to within this sphere. This choice of r_{max} places the $d_{3/2}$ resonance in ^{17}O at about the right energy, yet has a negligible effect on the bound states. Other boundary conditions were found to give similar unperturbed spectra. In order to keep the calculation tractable, the basis has been truncated at three major shell spacings (between 30–50 MeV). Within this model space we were able to obtain reasonable convergence for all but the most highly collective states.

In the following we will somewhat loosely refer to "Hartree" as the result of using direct interactions with the Hartree basis and parameter set, and "Hartree-Fock" as the results of using both direct and exchange interactions with the Hartree-Fock basis and parameter set.

III. RESULTS

One should keep in mind that different multipole excitations are sensitive to different pieces of the residual interaction. Although exchange terms mix the contributions of all the mesons into the particle-hole interaction, the dominant direct terms are highly selective. Therefore, the Hartree based spectra will strongly reflect the features of the various mesons in the interaction.

In general a multipole excitation of a given spin-parity and isospin will receive a direct contribution from those mesons which carry the same spin-parity and isospin quantum numbers. For example, the $\sigma(0^+,T=0)$ meson will only contribute to natural parity $(0^+,1^-,2^+,3^-,\ldots)T=0$ excitations (electric multipoles). The $\omega(1^-,T=0)$ meson has both a "Coulombic" interaction which contributes to natural parity excitations, and a weaker "magnetic" interaction which contributes to both natural parity and unnatural parity $(1^+,2^-,3^+\ldots$ but not $0^-)$ excitations (magnetic multipoles). The $\rho(1^-,T=1)$ behaves similarly to the ω, but for $T=1$ modes. The $\pi(0^-,T=1)$ only affects the unnatural parity $T=1$ modes (including the 0^-).

A. ^{16}O

With these comments in mind, we shall begin by examining the negative parity states of ^{16}O, shown in Fig. 2. In the first column is shown the unperturbed energies of the Hartree basis. The unperturbed Hartree-Fock energies are quite similar and have therefore not been drawn. The next two columns are the Hartree and Hartree-Fock predictions for the spectra. Only those levels which have a non-negligible transition strength to the ground state are shown. The fourth column is a selection from the compilation of Ref. 9 of those experimental levels which show predominantly 1p-1h character. Clearly not all levels have been shown.

There are only minor differences between the Hartree and Hartree-Fock $T=0$ spectra. Both predict a strongly collective 3^- as the first (negative parity) excited state, with the positions of most of the remaining levels being comparable. One exception is the second 0^- level, which lies at is unperturbed level in the Hartree calculation (recall that the ω can contribute to all unnatural parity exci-

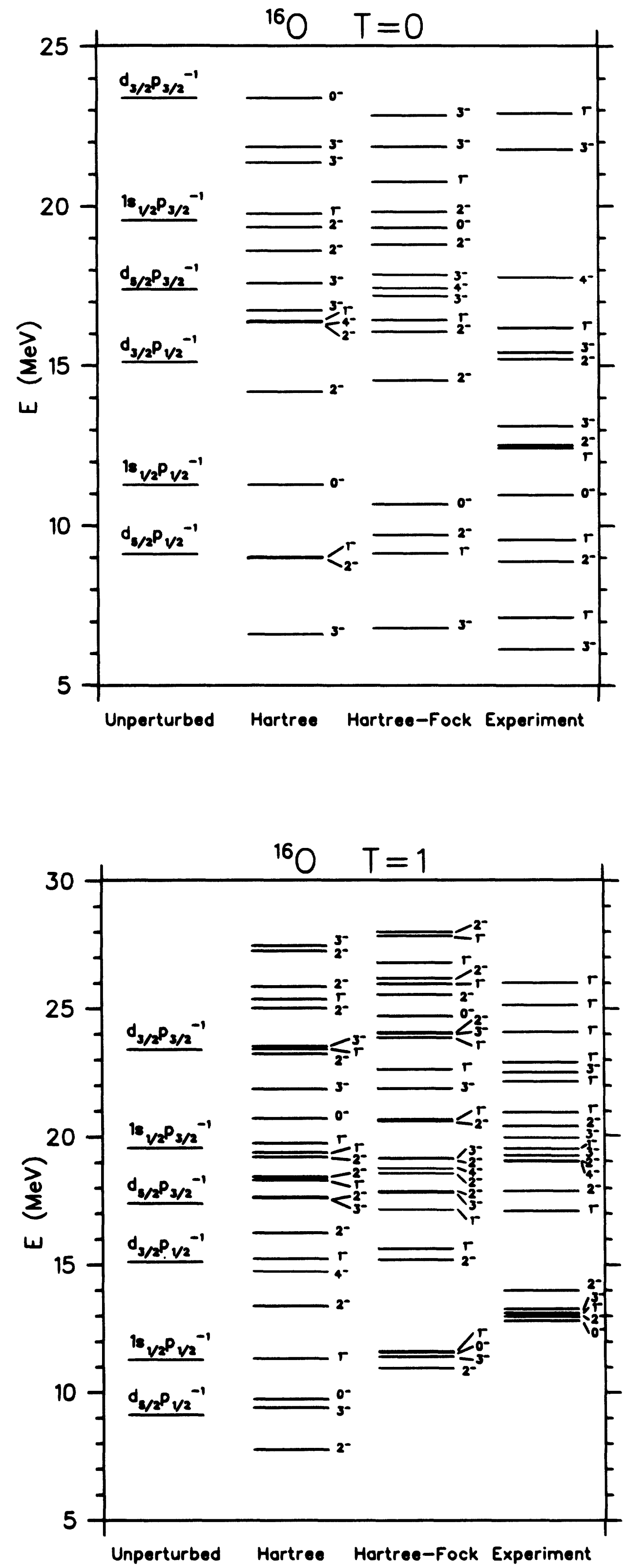

FIG. 2. Isoscalar and isovector negative parity states in ^{16}O. Shown are the unperturbed basis states in a Hartree model, the Hartree-based RPA, Hartree-Fock RPA, and the experimental levels (from Ref. 9). Not all levels are shown.

tations except the 0^-) but lies some 4 MeV lower in the Hartree-Fock calculation. The 4^- level also does somewhat better in this case. The overall agreement with the experimental systematics is modest.

Differences are more marked for the $T=1$ excitations. The Hartree is too attractive, particularly for the unnatural parity states, which are strongly influenced by the pion interaction. Indeed, one notices that for these cases the lowest-lying $T=1$ states are below the corresponding $T=0$ states, in strong disagreement with experiment. For example, the first $2^-, T=0$ occurs at 9.0 MeV while the first $2^-, T=1$ occurs at 7.7 MeV. Similarly, the $0^-, T=0$ lies at 11.3 MeV and the $0^-, T=1$ at 9.7 MeV; the $4^-, T=0$ occurs at 16.4 MeV and the $4^-, T=1$ at 14.7 MeV.

The Hartree-Fock interaction is repulsive, pushing the levels above their unperturbed values. The correct ordering of the $T=0$ unnatural parity levels below the $T=1$ ones is now seen. The low-lying cluster of 0^-, 1^-, 2^-, and 3^- levels is reproduced, although it lies about 2 MeV below experiment. The rest of the spectrum is also in reasonably good agreement.

In Fig. 3 we have shown the Hartree and Hartree-Fock predictions for the position and strength of the giant dipole $(1^-, T=1)$ resonance. The two-component dipole transition operator to be used in Eq. (3) has the form $T^{1M}(r)=\frac{1}{2}\tau r Y_{1M}(\hat{r})\gamma^0$. Also shown for comparison is the experimental $^{15}N(p,\gamma)^{16}O$ cross section, taken from Ref. 10, which is predominantly dipole in character. The Hartree levels occur at essentially their unperturbed positions, while the Hartree-Fock pushes the centroid up by about 4 MeV. The position of the dominant configuration is then about right, although the splitting with the next strongest configuration is somewhat too large.

A few remarks about the spurious $1^-, T=0$ state are in order. In any self-consistent model using the same underlying Hamiltonian to generate the ground state and the excited states, there should occur a spurious excitation mode corresponding to a translation of the whole system. In the RPA, this means there will be a $1^-, T=0$ excitation which occurs at zero energy and which carries 100% of the dipole transition strength.[8] For both Hartree and Hartree-Fock we found a collective state at about 3 MeV. It carries over 99% of the total strength, and can therefore be clearly identified with the spurious state. To get convergence toward zero energy would require enlarging the basis considerably, including the introduction of negative energy states.

To illustrate the effect of self-consistency, we also took the Hartree coupling constants and looked at the effect of including both direct and exchange interactions. We then obtained similar results to Ref. 6, namely that there are two states near 10 MeV which carry about 10 and

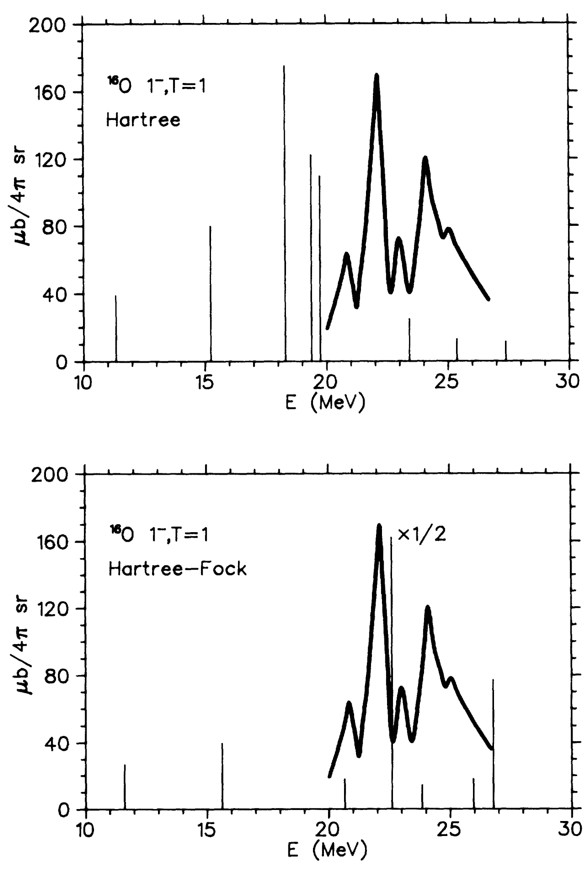

FIG. 3. Hartree and Hartree-Fock predictions for the strength distribution of the giant dipole mode. Also shown is the dipole-dominated $^{15}N(p,\gamma)$ ^{16}O cross section, taken from Ref. 10.

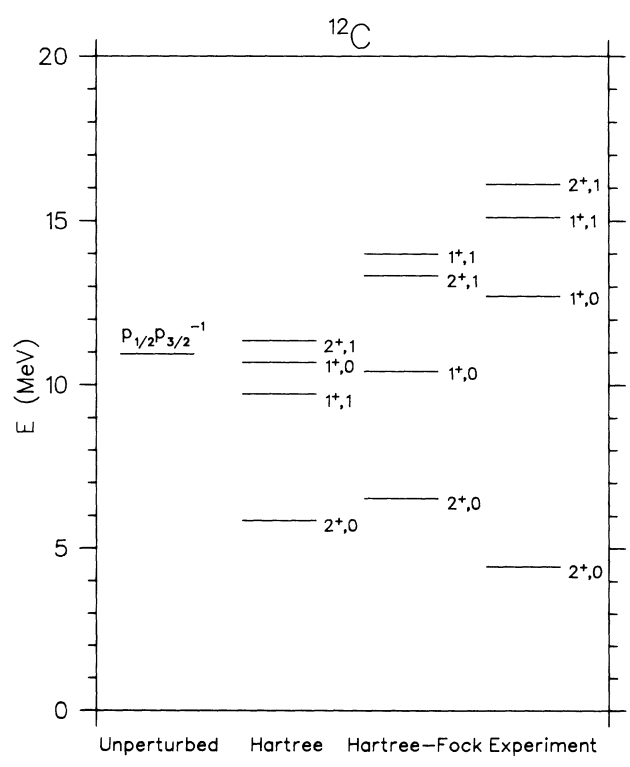

FIG. 4. Positive-parity states of ^{12}C, labeled by spin and isospin. We have only shown those states which occur within a $p_{1/2}-p_{3/2}^{-1}$ model space.

75 % of the strength, with the second excited state being the stronger one. The concerns about the spurious state reported in Ref. 6 are therefore resolved if the correct self-consistent couplings are used for each model.

B. ^{12}C

Positive parity states are usually poorly reproduced in 1p-1h RPA calculations. This is because the low-lying positive parity levels in closed shell nuclei have a significant 2p-2h or even 4p-4h character. In order to examine positive-parity states we therefore consider ^{12}C, for which the simplest basis consists of the $p_{3/2}$ and $p_{1/2}$ levels. Figure 4 shows the results of our calculations, assuming a fully occupied $p_{3/2}$ level for the ground state. The unperturbed energy splitting is somewhat too large here compared with the empirical one deduced from neighboring nuclei. As in ^{16}O, the ordering of the unnatural parity 1^+ states is wrong for the Hartree case and of about the right magnitude for the Hartree-Fock. The natural parity 2^+ states have the correct ordering in the Hartree calculation, but the splitting is too small. The splitting is a little larger for the Hartree-Fock, but it still falls short of experiment.

C. ^{40}Ca

The ^{40}Ca results concur with our previous observations about ^{16}O and ^{12}C. The $T=0$ spectra, shown in Fig. 5, are very similar for Hartree and Hartree-Fock. The highly collective 3^- states are somewhat too low in energy, but the lowest 5^-, 4^-, 2^-, and 3^- are in good agreement with experiment. Once again, the ordering of the 0^-, 2^-, 4^-, and 6^- $T=0$ and $T=1$ levels is inverted for Hartree. The low-lying cluster of 4^-, 3^-, 2^-, and 5^- $T=1$ levels is well reproduced by the Hartree-Fock results.

The $1^-, T=1$ modes in the giant resonance region have an energy-weighted centroid of 16.8 MeV in the Hartree case compared with 20.0 MeV for the Hartree-Fock. Experimentally the giant resonance occurs at 20.0 MeV.[12] The spurious $1^-, T=0$ excitation occurred at a small imaginary energy in both calculations.

IV. UNNATURAL PARITY STATES AND SHORT-RANGE CORRELATIONS

The most striking feature of the present results is the difference between Hartree and Hartree-Fock based calculations for the $T=1$ spectra. The differences are most apparent for the unnatural parity modes, where the pion plays an essential role. The exchange terms considerably weaken the strongly attractive direct pion interaction. Indeed, without some mechanism to damp this interaction, pion condensates would likely occur at or near nuclear matter densities.[1]

Let us now consider the effect of including a tensor term for the rho in the interaction (12), as well as the effect of "short-range correlations." Typical nonrelativistic calculations take an isovector interaction of the form

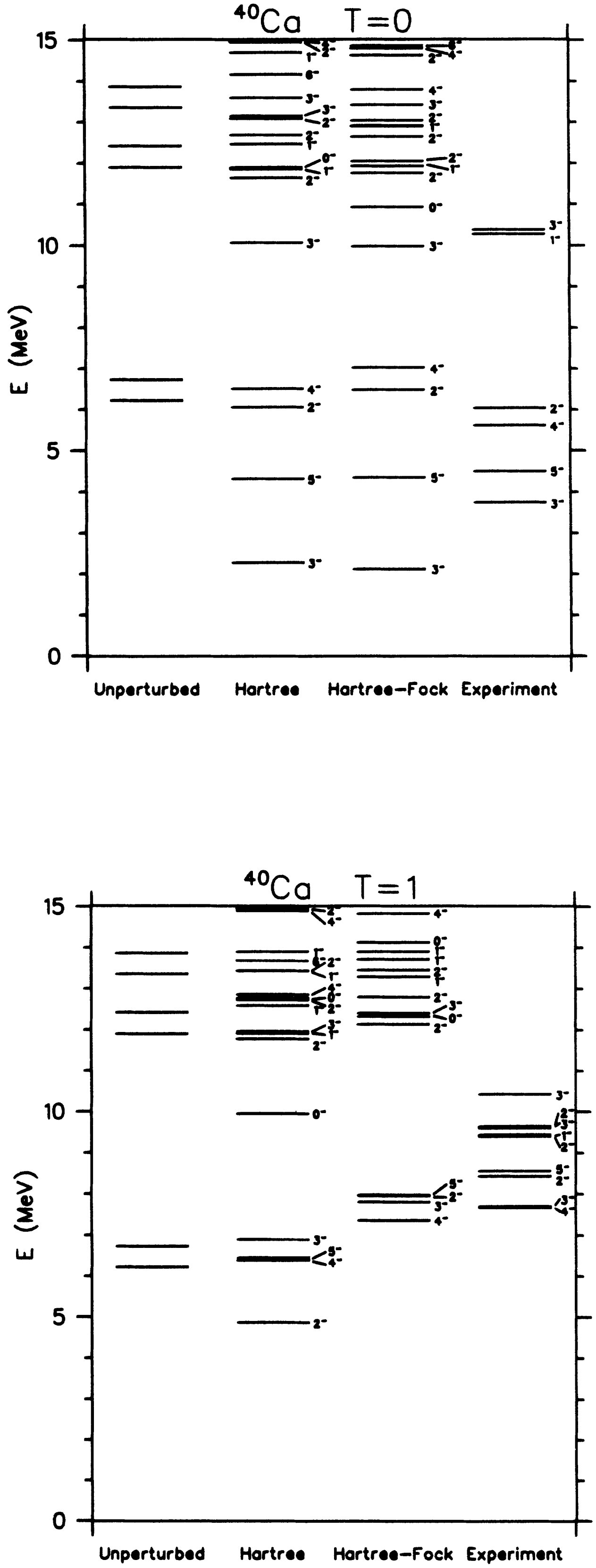

FIG. 5. Isoscalar and isovector negative-parity states in ^{40}Ca. Experimental levels are from Ref. 11. Not all levels are shown.

$$\frac{g_\pi^2}{4M^2}\tau_1\cdot\tau_2\left[g'\sigma_1\cdot\sigma_2-\frac{\sigma_1\cdot\mathbf{q}\sigma_2\cdot\mathbf{q}}{q^2+m_\pi^2}-C_\rho\frac{\sigma_1\times\mathbf{q}\cdot\sigma_2\times\mathbf{q}}{q^2+m_\rho^2}\right], \tag{14}$$

where

$$C_\rho=\frac{g_\rho^2(1+K_\rho)^2}{g_\pi^2}\approx 2.2 ,$$

assuming a tensor coupling of $K_\rho=6.6$ for the rho.

The π and ρ interactions contain delta function contributions, which are thought to be suppressed by the short-range correlations which keep nucleons apart. The proper way to include these correlations is, of course, to calculate the Brückner particle-hole G matrix. What is usually done instead is to simulate this effect by introducing a repulsive interaction (Landau-Migdal term) of the form $g'\sigma_1\cdot\sigma_2\tau_1\cdot\tau_2$. Removing all the delta-function contributions would require $g'=\frac{1}{3}+\frac{2}{3}C_\rho\approx 1.8$, however most calculations remove less than half, typically giving $g'=0.7$.

The actual results of RPA calculations using this interaction are quite sensitive to the chosen value of g'. This sensitivity arises for two reasons. First, these calculations do not use a self-consistent basis, i.e., the interaction is not also used to determine the single-particle energies and wave functions used as input to the calculation. A more important reason is that the interaction may or may not be consistent with the saturation properties of nuclear matter. One of the virtues of the present approach is that both of these points are addressed. We can therefore examine in a meaningful way the sensitivity of our results to short-range behavior.

A relativistic counterpart to the interaction (14) is problematic since it is not obvious how correlations should be put in, and what their Lorentz structure will be. Furthermore, delta functions only appear for explicitly momentum-dependent interactions—there is no such contribution arising for the vector rho interaction, as there is nonrelativistically.

A simple guess is to include a contact interaction which, to leading relativistic order, would cancel the delta function contribution of the pion. We therefore take

$$-\frac{g_\pi^2}{4M^2}\tau_1\cdot\tau_2\left[g'\gamma_5^{(1)}\gamma_\mu^{(1)}\gamma_5^{(2)}\gamma^{\mu(2)}+\frac{\gamma_5^{(1)}\gamma_\mu^{(1)}q^\mu\gamma_5^{(2)}\gamma^{\nu(2)}q_\nu}{-q_\mu^2+m_\pi^2}\right]+g_\rho^2\tau_1\cdot\tau_2\left[\gamma_\mu^{(1)}-i\frac{K_\rho}{2M}\sigma_{\mu\nu}^{(1)}q^\nu\right]\left[\gamma^{\mu(2)}+i\frac{K_\rho}{2M}\sigma^{\mu\nu(2)}q_\nu\right] \tag{15}$$

for the isovector interaction. We want to reiterate that this is simply a prescription for putting in correlations, and that a proper treatment will require calculating a relativistic G matrix.

Putting $g'=0.7$ and $K_\rho=6.6$, we refit the sigma and omega coupling constants to saturate nuclear matter, giving parameter set II of Table I. The Hartree parameters are the same since isovector mesons do not contribute for symmetric nuclear matter. The Hartree-Fock parameters show a decrease in the scalar coupling and an increase in the vector coupling. The overall central potential (recall that the scalar interaction is attractive and the vector repulsive) will therefore be considerably reduced compared with parameter set I. The reason is that the isovector interaction now is attractive in nuclear matter thus requiring less attraction from the central potential.

The effect of these changes on the unnatural parity states in ^{16}O is demonstrated in Fig. 6, where results using both parameter sets are compared for Hartree and Hartree-Fock. For the Hartree case the short-range interaction pushes these states up considerably, so that they now lie above their $T=0$ counterparts (not shown) and in reasonable agreement with experiment. These results are sensitive to the exact value of g'.

What is interesting, however, is that the Hartree-Fock spectra are virtually unchanged from the results without short-range correlations put in. The strongly repulsive exchange contributions arising from sigma and omega interactions, which are what counteracts the pion interaction for set I, are now much weaker. Instead the repulsion is provided by the g' term of (15).

These results indicate the importance of using an in-

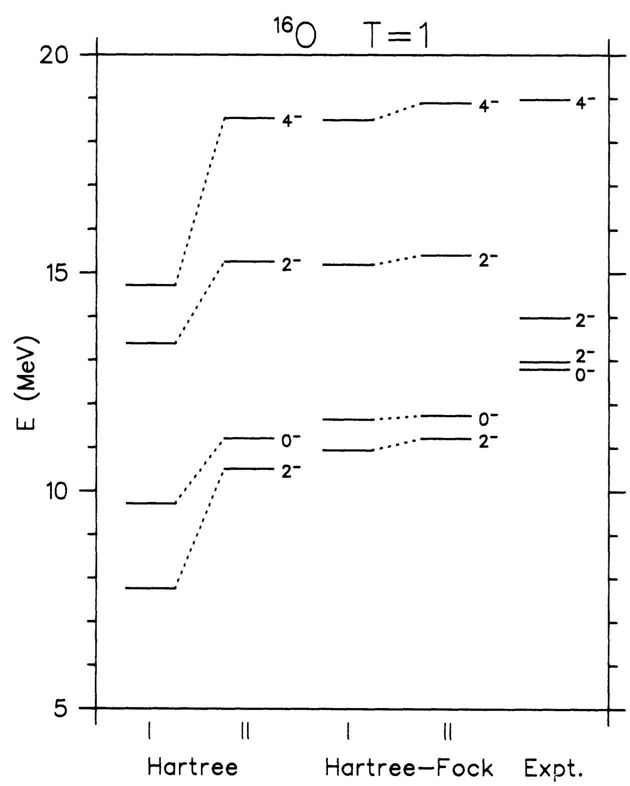

FIG. 6. Isovector unnatural parity states in ^{16}O. Results are compared for parameter sets I and II for Hartree and Hartree-Fock. Parameter set II includes short-range correlations and a tensor coupled rho.

teraction which is consistent with and constrained by the properties of nuclear matter. Since the Hartree-Fock theory incorporates pions consistently form the start, the final results are rather insensitive to the short range behavior of the interaction.

V. DISCUSSION

We return now to the implications of omitting negative energy states from the present theory. Although direct transitions to a particle-antiparticle state are ruled out by energy considerations, these states can affect quantities such as the transition strength through virtual polarization. For example, in order to correctly describe magnetic moments in a relativistic model one has to incorporate polarization corrections coming from these negative energy states (so-called backflow).[13] This polarization correction will also screen pieces of the interaction, such as the magnetic interaction of the ω. Therefore the transition strengths, and to a lesser extent the positions, of the magnetic multipoles in our calculation are affected.

A direct extension of the basis to include these negative-energy states appears unwieldy because of the enormous increase in the number of configurations it would entail. Recent work[14] on nonspectral RPA calculations, which treat both the continuum and negative-energy states exactly, has shown that Hartree based calculations are feasible without truncating the full Hilbert space. However, prospects for Hartree-Fock based calculations in this approach are much poorer. Part of the problem is that the energy dependence in the meson propagators is no longer negligible, and must be considered if realistic results are to be obtained.

A more tractable calculational scheme might be to divide the full Hilbert space into a small model space, such as the one we have used, and a larger residual space which includes both the high-lying particle-hole states and the antiparticle states. The RPA can then make use of the small space in the usual way, but the operators will be renormalized by the residual space. Renormalizations of magnetic moment operators along these lines in a local density approximation, based on nuclear matter results, have been shown to give results comparable to those of calculations which explicitly evaluate the antiparticle contribution.[15]

To summarize, we have examined the RPA spectra in a self-consistent treatment based on a relativistic Hartree-Fock model for the nuclear ground state. With no additional parameters one is able to reproduce reasonably well the spectra of light nuclei. Hartree based calculations fail to describe the $T=1$ spectrum without introducing short-range repulsion in the spin-isospin interaction. On the other hand, Hartree-Fock based calculations agree quite well whether such interactions are included or not. This lack of sensitivity to short range behavior is due to the fact that the interaction incorporates pions consistently in a manner constrained by the saturation properties of nuclear matter.

One could argue that adjusting the sigma and omega meson parameters to fit nuclear matter saturation properties, as we have done here, already incorporates to some extent the effect of short range correlations. The fact that properties of both the Hartree-Fock ground state and excited states are not significantly altered by introducing an explicit correlation effect argues in favor of this interpretation. Clearly these calculations are still in their infancy, and more work on the residual particle-hole interaction in relativistic theories, including the effect of correlations, remains to be done.

APPENDIX

Here we present the details of the multipole decomposition (10) of the residual interaction (12). The particle-hole matrix elements can then be obtained via Eqs. (11) and (9).

Consider a 2×2 matrix M^J whose elements are spherical tensors of rank J:

$$M^J=\begin{bmatrix} M^J_{11} & M^J_{12} \\ M^J_{21} & M^J_{22} \end{bmatrix} . \tag{A1}$$

Then

$$\langle h^{-1}p;J\|M^J\|0\rangle=(-1)^{h+p-J}\langle p\|M^J\|h\rangle=(-1)^{h+p-J}(\langle \phi_p\|M^J_{11}\|\phi_h\rangle-\langle \tilde{\phi}_p\|M^J_{22}\|\tilde{\phi}_h\rangle + i\langle \phi_p\|M^J_{12}\|\tilde{\phi}_h\rangle+i\langle \tilde{\phi}_p\|M^J_{21}\|\phi_h\rangle) , \tag{A2}$$

where we have used Eq. (5) to reduce the two-component wave function to its one-component elements.

It should be clear that the interaction (12) can also be reduced to a sum of one-component interactions of spin-independent, spin-dependent $(\sigma_1\cdot\sigma_2)$, and pionic $(\sigma_1\cdot\mathbf{q}\sigma_2\cdot\mathbf{q})$ form. We therefore need only give the multipole expansions of these interactions, starting from their form in momentum space.

Expanding (13) gives

$$V(r)=\frac{2}{\pi}\int_0^\infty dq\, q^2 v(q)\sum_J F_J(q;r_1,r_2) , \tag{A3}$$

where $v(q)$ generally is of the form $\pm g^2/(q^2+m^2)$. We introduce the notation

$$M_J(qr)=j_J(qr)Y_J(\hat{r})$$
$$M_{JL}(qr)=j_L(qr)[Y_L(\hat{r})\otimes\sigma]^J;L=J-1,J,J+1 . \tag{A4}$$

(a) Spin-independent interactions:

$$F_J(q;r_1,r_2)=M_J(qr_1)\cdot M_J(qr_2) . \tag{A5}$$

(b) Spin-dependent interactions $\sigma_1\cdot\sigma_2$:

$$F_J(q;r_1,r_2)=\sum_L(-1)^{J+1-L}M_{JL}(qr_1)\cdot M_{JL}(qr_2)\ . \qquad \text{(A6)}$$

(c) Pionic interactions $\sigma_1\cdot\mathbf{q}\sigma_2\cdot\mathbf{q}$:

$$F_J(q;r_1,r_2)=q^2\sum_{L,L'} i^{L-L'}\alpha_{JL}\alpha_{JL'}M_{JL}(qr_1)M_{JL'}(qr_2)\ , \qquad \text{(A7)}$$

with

$$\alpha_{JL}=\begin{cases}\left[\dfrac{J}{2J+1}\right]^{1/2}, & \text{for } L=J-1,\\ -\left[\dfrac{J+1}{2J+1}\right]^{1/2}, & \text{for } L=J+1.\end{cases}$$

(d) Tensor (ρ) interactions $\sigma_1\times\mathbf{q}\cdot\sigma_2\times\mathbf{q}$:

$$\sigma_1\times\mathbf{q}\cdot\sigma_2\times\mathbf{q}=q^2\sigma_1\cdot\sigma_2-\sigma_1\cdot\mathbf{q}\sigma_2\cdot\mathbf{q}\ . \qquad \text{(A8)}$$

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