Unified theory of $\gamma N \rightarrow \pi N, \pi\pi N$, and $\pi N \rightarrow \pi N, \pi\pi N$ reactions

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We present a set of coupled integral equations for the reactions $\gamma N \rightarrow \pi N, \pi\pi N$ and $\pi N \rightarrow \pi N, \pi\pi N$, that satisfy two- and three-body unitarity. These equations are based on a chiral bag Lagrangian in which the coupling to the photon is introduced at the quark level by demanding U(1) local gauge invariance. The final equations include the contribution of both meson exchange and isobar currents.

I. INTRODUCTION

Pion photoproduction from a single nucleon can be a powerful tool for the study of the structure of the low-energy baryon spectrum in terms of quarks and gluons. In particular, it can test the different chiral bag models,[1,2] as the photon couples to both the quarks and pions in the baryon by imposing U(1) gauge invariance on the chiral Lagrangian.[3] The detailed form of this coupling is determined by the chiral Lagrangian for the model. As a result of this, any comparison between theory and experiment may provide an insight into the form of the effective chiral Lagrangian. However, to carry out such a comparison at medium energies, e.g., in the energy region in which the $\Delta(1232)$ and $N(1440)$ are produced, it is necessary to formulate the theory to satisfy the Watson theorem.[4] This requires that two- and three-body unitarity in the pion channel be satisfied. Recently, Araki and Afnan[5] presented such a formulation based on the cloudy bag model (CBM).[2] Their final amplitude for $\pi B \leftarrow \gamma B$ ($B = N, \Delta$), was given as a distorted-wave matrix element of the form

$$\tilde{T}(\pi B \leftarrow \gamma B) = \langle \psi_{\pi B}^{(0)} \mid (T_{B;B}^{(0)} g + 1)\tilde{v}_{B;B} \mid \psi_{\gamma B}^{(0)} \rangle$$
$$= \langle \psi_{\pi B} \mid \tilde{v}_{B;B} \mid \psi_{\gamma B}^{(0)} \rangle , \quad (1.1)$$

where $T_{B;B}^{(0)}$ is the elastic $\pi - B$ amplitude, $\psi_{\pi B}$ is the corresponding distorted-wave function in the $\pi - B$ channel, while g is the $\pi - B$ propagator (i.e., $g = d_B d_\pi$). Here, $\psi_{\gamma B}^{(0)}$ and $\psi_{\pi B}^{(0)}$ are the plane waves in the initial and final states. The effective operator $\tilde{v}_{B;B}$ includes the contribution from meson exchange, isobar current, as well as the effects of pion multiple scattering, to the extent of satisfying two- and three-body unitarity, i.e., it includes the effect of $\gamma N \rightarrow \pi N^*, \pi\Delta, \pi\pi N$. It was also shown that in lowest order, $\tilde{v}_{B;B}$ includes the standard mechanisms that have been included in the past by Blomqvist and Laget[6] and by Tanabe and Ohta[7] with the added feature that all the vertices are written in terms of parameters of the chiral Lagrangian. Unfortunately, the determination of $\tilde{v}_{B;B}$ to all orders involves the solution of the three-body $\pi\pi B$ ($B = N, \Delta, \ldots$) problem, and the determination of the full off-shell three-body amplitudes. This limits the use of Eq. (1.1) to lowest-order calculations (i.e., distorted-wave Born approximation), and to the energy region below the threshold for $N(1440)$ photoproduction, as three-body unitarity is not satisfied.

In the present paper we will show how the results Araki and Afnan,[5] for pion photoproduction, can be extended to include $\gamma N \rightarrow \pi\pi N$, and in this way give a unified description of both pion photoproduction and pion elastic scattering. In other words, we get one set of matrix integral equations that describe the reactions

$$\begin{aligned} \pi + N &\rightarrow \pi + N \\ &\rightarrow \pi + (\pi B)_{N^*} \\ &\rightarrow (\pi\pi)_\rho + B \\ &\rightarrow \pi + \pi + B \end{aligned} \quad (1.2)$$

and

$$\begin{aligned} \gamma + N &\rightarrow \pi + N \\ &\rightarrow \pi + (\pi B)_{N^*} \\ &\rightarrow (\pi\pi)_\rho + B \\ &\rightarrow \pi + \pi + B \ . \end{aligned} \quad (1.3)$$

Here, the baryon B is considered as a three-quark state (i.e., $B = N, \Delta, \ldots$), and the $(\pi B)_{N^*}$ corresponds to the N^* resonances, not included as a three-quark state, and observed in pion elastic scattering or photoproduction. In Eqs. (1.2) and (1.3), the $(\pi\pi)_\rho + B$ corresponds to the production of two-pion resonances such as the ρ meson. With these equations, one can address such questions as to the structure of the N^* resonances in terms of quark excitation in the chiral bag model[8] versus the more traditional mechanisms such as the formation of resonances due to the coupling to the $\pi - \Delta$ threshold[9] or the opening of the $\pi\pi N$ channel.[10,11] The interesting feature of the final equations is the fact that the kernel of the two sets of coupled equations that describe the reactions in Eqs. (1.2) and (1.3) are identical. The only difference between the two sets of coupled integral equations is the inhomogenous term, which specifies the initial boundary condition. In this way, we have included the contribution of $\tilde{v}_{B;B}$ to the amplitude for photoproduction to all orders and avoided the problem of having to evaluate the distorted-wave matrix element given in Eq. (1.1).

In Sec. II we summarize the results of Afnan and Pearce[12] for pion-nucleon scattering above the threshold for pion production and the work of Araki and Afnan[5] for pion photoproduction. Although, intutively, we expect a set of coupled integral equations for the reactions in Eqs. (1.2) and (1.3) to establish this unification, we need to examine the amplitude for $\gamma B \to \pi\pi B$. This is achieved in Sec. III. In Sec. IV we combine the results of Secs. II and III to get our unified coupled integral equations for the reactions on Eqs. (1.2) and (1.3). Finally, in Sec. V we present some concluding remarks.

II. REVIEW OF PREVIOUS RESULTS

Before we can proceed to derive a set of coupled equations for the reactions in Eqs. (1.2) and (1.3), it is important to remind the reader of the origin and form of the underlying Hamiltonian and its relation to the chiral bag Lagrangian. The main aim in constructing the Hamiltonian is to integrate the quark degrees of freedom in favor of the hadronic degrees of freedom. In this way, we hope to maintain some of the information about the quark degrees of freedom in the form factors for the coupling between the baryons, mesons, and photons. At this stage of the game, this procedure involves a number of approximations that are difficult to overcome—the most famous being the center-of-mass motion. The coupling to the electromagnetic field is introduced at the quark level by demanding local U(1) gauge symmetry.[3] Thus, the Lagrangian before projection onto the baryon space is, in fact, gauge invariant. The projection onto the baryon space involves taking matrix elements of the gauge and chiral invariant Lagrangian between baryon eigenstates. These eigenstates, which are three-quark states, are constructed from a solution of the equation of motion resulting from the chiral Lagrangian in some approximation. Thus, in the case of the cloudy bag model,[2] the baryon eigenstates are given in terms of the M.I.T. bag wave function.[13] The resultant matrix elements in the baryon space are given by Araki and Afnan.[5] In this case, the form factors for the coupling between the baryons, mesons, and photon are known analytically. A more elaborate and possibly more realistic procedure is to solve the equations of motion for the chiral Lagrangian in the mean-field approximation. In this case, the form factors for all the coupling are known numerically.

Using the above procedure, we can write the Hamiltonian, projected onto the Hilbert space of baryons, mesons, and photons, as[5]

$$H=\sum_n \epsilon_n B_n^\dagger B_n+\sum_\alpha \int d^3q\, \omega_q a_{\alpha q}^\dagger a_{\alpha q} + \sum_\lambda \int d^3k\, k_0 C_k^{\lambda\dagger} C_k^\lambda + H_I\ , \tag{2.1}$$

where B_n, $a_{\alpha q}$, and C_k^λ are the annihilation operators for a baryon in state n, a meson in state α and momentum q, and a photon with polarization λ and momentum k. The interaction Hamiltonian H_I is given by

$$\begin{aligned} H_I = {}& \langle B \mid \hat H \mid B\pi\rangle + \langle B\pi \mid \hat H \mid B\pi\rangle \\ &+\langle \pi\pi \mid \hat H \mid \pi\pi\rangle + \langle B \mid \hat H \mid B\gamma\rangle \\ &+\langle B\pi \mid \hat H \mid B\gamma\rangle + \langle \pi\pi \mid \hat H \mid \gamma\rangle + \langle \pi \mid \hat H \mid \pi\gamma\rangle \\ &+ \text{five Hermitian conjugate terms}\ . \end{aligned} \tag{2.2}$$

In writing the above interaction Hamiltonian, we have excluded terms that couple a single-baryon state $|B\rangle$ and the three-body state $|B\pi\pi\rangle$ and $|B\pi\gamma\rangle$. This truncation was introduced to facilitate the truncation of the equation at the level of including only two- and three-body unitarity. The effect of this truncation can be investigated at a later stage using perturbation theory.

Given the above Hamiltonian, we can proceed to derive equations for the amplitude corresponding to the reactions in Eqs. (1.2) and (1.3). The method used to derive the equations relies on the classification of the diagrams that contribute to a given amplitude according to their irreducibility. This classification scheme does not depend on the detailed form of the Hamiltonian, but on the kind of vertices included. To that extent, we do not have to specify the interaction Hamiltonian beyond the form given in Eq. (2.2), and our final equations are valid irrespective of the form chosen for each of the terms in the interaction Hamiltonian H_I. This approach, which relies on the last-cut lemma, was first introduced by Taylor[14] to derive integral equations for the scattering amplitude in quantum field theory. The method has also been used to derive equations for the $NN-\pi NN$ system[15–17] and the $\pi N-\pi\pi N$ system.[12]

To derive equations for the amplitude for a given process, we need to classify the diagrams that contribute, in perturbation theory, to this amplitude according to their irreducibility using the last-cut lemma. To achieve this we need to first define a k cut as an arc that separates the initial state from the final state, in a given diagram, and cuts k-particle lines with at least one line being internal. Second, an amplitude is r-particle irreducible if all diagrams that contribute to this amplitude will not admit any k cut with $k \le r$. With these two definitions, we can introduce the last-cut lemma which states that for a given amplitude that is $(r-1)$-particle irreducible, there is a unique way of obtaining an internal r-particle cut closest to the final (initial) state for all diagrams that contribute to the amplitude. By virtue of this lemma, we can expose one-, two-, and three-particle intermediate states and the corresponding unitarity cuts and in this way derive equations for the amplitude that satisfy unitarity. From the above statement of the lemma, it is clear that we need to expose the n-particle unitarity cut before the $(n+1)$-particle unitarity cut.

Making use of the last-cut lemma, we can expose the one-, two-, and three-particle intermediate states in our analysis of the $\pi-N$ elastic amplitude. After some algebra, we get[12]

$$T_{B;B}^{(0)} = T_{B;B}^{(1)} + f^{(1)\dagger} d_B f^{(1)} \tag{2.3}$$

$$= v_{B;B}(1+gT_{B;B}^{(0)})\ , \tag{2.4}$$

where $g=d_\pi d_B$ is the πB propagator, and d_π and d_B are the pion and dressed baryon propagators. The effective potential $v_{B;B}$ is given by[12]

$$v_{B;B}=T^{(3)}_{B;B}+f^{(2)\dagger}d_0f^{(2)}+\sum_{ij}F^{(2)}_d(i)GU_{ij}GF^{(2)\dagger}_d(j)\ , \tag{2.5}$$

where $G=d_\pi d_\pi d_B$ is the $\pi\pi B$ propagator, and $F^{(2)}_d(i)$ is the disconnected two-particle irreducible amplitude for $\pi B\leftarrow\pi\pi B$, given by

$$F^{(2)}_d(i)=\sum_j \bar\delta_{ij}d_\pi^{-1}(j)f^{(1)}(i)\ , \tag{2.6}$$

and $\bar\delta_{ij}=1-\delta_{ij}$. In the above, $T^{(n)}_{B;B}$ is the n-particle irreducible amplitude for $\pi B\rightarrow\pi B$, while $f^{(n)}$ is the n-particle irreducible amplitude for $B\leftarrow\pi B$. Here, and throughout this paper we take particles 1 and 2 to be the pions, while particle 3 is the baryon. Thus, we have that $i,j,\ldots=1,2$, and $\alpha,\beta,\ldots=1,2,3$. In this notation, $f^{(1)}(i)$ is the one-particle irreducible amplitude for the absorption of the ith pion on the baryon (i.e., $B\leftarrow\pi B$). The contribution of three-body unitarity to the $\pi\pi$ potential $v_{B;B}$ comes through the three-body amplitude for $\pi\pi B\leftarrow\pi\pi B, U_{ij}$, which satisfies the Alt, Grassberger, and Sandhas (AGS) equations[18]

$$\begin{aligned}U_{\alpha\beta}&=\bar\delta_{\alpha\beta}G^{-1}+\sum_\gamma\bar\delta_{\alpha\gamma}M^{(2)}_d(\gamma)GU_{\gamma\beta}\\&=\bar\delta_{\alpha\beta}G^{-1}+\sum_\gamma U_{\alpha\gamma}GM^{(2)}_d(\gamma)\bar\delta_{\gamma\beta}\ ,\end{aligned} \tag{2.7}$$

with $M^{(2)}_d$, the disconnected part of the two-particle irreducible amplitude for $\pi\pi B\leftarrow\pi\pi B$, given in terms of the two-body input amplitudes by

$$\begin{aligned}M^{(2)}_d&=\sum_\gamma M^{(2)}_d(\gamma)\\&=\sum_{ij}\bar\delta_{ij}d_\pi^{-1}(i)T^{(1)}_{B;B}(j)+d_B^{-1}t^{(1)}(3)\ ,\end{aligned} \tag{2.8}$$

where $t^{(1)}(3)$ is the one-particle irreducible amplitude for $\pi-\pi$ scattering.

To establish the connection between the input amplitudes and the interaction Hamiltonian H_I as given in Eq. (2.2), we make use of the last-cut lemma to relate a given n-particle irreducible amplitude to the corresponding $(n+1)$-particle irreducible amplitude. At some stage in this analysis, the $n,(n+1),\ldots$-particle amplitudes become equal, because the interaction Hamiltonian, by definition, does not couple states that differ by two or more bosons (i.e., mesons or photons). Then the corresponding amplitude is written in terms of the interaction Hamiltonian. Thus for the $B\leftarrow\pi B$ amplitude we have, using the last-cut lemma,

$$f^{(1)}=f^{(2)}+f^{(2)}gT^{(1)}_{B;B}\ , \tag{2.9}$$

and

$$f^{(2)}=f^{(3)}=\cdots=\langle B\mid\hat H\mid\pi B\rangle\ . \tag{2.10}$$

Similarly, we have that

$$T^{(3)}_{B;B}=\langle\pi B\mid\hat H\mid\pi B\rangle\ , \tag{2.11}$$

and for the $\pi\pi$ amplitude $t^{(1)}(3)$ we have

$$t^{(1)}(3)=t^{(2)}(3)[1+d_\pi d_\pi t^{(1)}(3)] \tag{2.12}$$

with the $\pi\pi$ potential given in terms of the interaction Hamiltonian by

$$t^{(2)}(3)=\langle\pi\pi\mid\hat H\mid\pi\pi\rangle\ . \tag{2.13}$$

In this way, we have determined the input required to calculate the effective potential $v_{B;B}$ and the amplitude $T^{(0)}_{B;B}$. For a more detailed derivation of the above results, the reader is referred to the work of Afnan and Pearce.[12]

We now turn to the pion photoproduction amplitude (i.e., $\pi B\leftarrow\gamma B$). Here again we use the last-cut lemma to classify the diagrams that contribute to this amplitude according to their irreducibility. This procedure gives us the amplitude for $\pi B\leftarrow\gamma B$ in the form[5]

$$\tilde T^{(0)}_{B;B}=\tilde T^{(1)}_{B;B}+f^{(1)\dagger}d_B\tilde f^{(1)} \tag{2.14}$$

$$=(T^{(0)}_{B;B}g+1)\tilde v_{B;B}\ , \tag{2.15}$$

where $\tilde T^{(n)}_{B;B}$ is the n-particle irreducible amplitude for $\pi B\leftarrow\gamma B$, and the effective photoproduction potential $\tilde v_{B;B}$ is given by[5]

$$\begin{aligned}\tilde v_{B;B}=\tilde T^{(3)}_{B;B}&+f^{(2)\dagger}d_0\tilde f^{(2)}+\tilde f^{(1)}_b d_\pi f^{(1)\dagger}\\&+\tilde f^{(1)}d_Bf^{(1)\dagger}+\sum_i F^{(2)}_d(i)GU_{i3}G\tilde F^{(2)\dagger}_{1;d}\\&+\sum_{ijk}F^{(2)}_d(i)GU_{ij}G\tilde M^{(2)}_{A;d}(j)\bar\delta_{jk}\tilde G\tilde F^{(2)\dagger}_{2;d}(k)\ ,\end{aligned} \tag{2.16}$$

where d_0 is the bare baryon propagator, and $\tilde G=d_\pi d_\gamma d_B$ is the $\pi\gamma B$ propagator. The disconnected two-particle irreducible amplitudes for $\pi\pi B\leftarrow\pi\gamma B, \tilde M^{(2)}_{A;d}(i)$, $\pi\pi B\leftarrow\gamma B, \tilde F^{(2)\dagger}_{1;d}$, and $\pi\gamma B\leftarrow\gamma B, \tilde F^{(2)\dagger}_{2;d}(i)$ are given by

$$\tilde M^{(2)}_{A;d}(i)=\sum_j\bar\delta_{ij}d_\pi^{-1}(j)\tilde T^{(1)}_{B;B}(i)\ , \tag{2.17}$$

$$\tilde F^{(2)\dagger}_{1;d}=d_B^{-1}\tilde f^{(1)\dagger}_a\ , \tag{2.18}$$

$$\tilde F^{(2)\dagger}_{2;d}(i)=d_\gamma^{-1}f^{(1)\dagger}(i)\ . \tag{2.19}$$

The one-particle irreducible amplitude for $B\leftarrow\gamma B,\tilde f^{(1)}$ is given by

$$\tilde f^{(1)}=\tilde f^{(2)}+f^{(2)}d_\pi d_B\tilde T^{(1)}_{B;B}\ , \tag{2.20}$$

with

$$\tilde f^{(2)}=\langle B\mid\hat H\mid\gamma B\rangle\ . \tag{2.21}$$

In Eq. (2.16), the one-particle irreducible amplitude $\tilde f^{(1)}_b$, for $\pi\leftarrow\gamma\pi$, is given by

$$\tilde f^{(1)}_b=\tilde f^{(2)}_b=\langle\pi\mid\hat H\mid\gamma\pi\rangle\ , \tag{2.22}$$

while the amplitude $\tilde f^{(1)\dagger}_a$, for $\pi\pi\leftarrow\gamma$, in Eq. (2.18) is given by the equations,[5]

$$\tilde f^{(1)\dagger}_a=[t^{(1)}(3)d_\pi d_\pi+1]\tilde f^{(2)\dagger}_a\ , \tag{2.23a}$$

$$\tilde f^{(2)\dagger}_a=\langle\pi\pi\mid\hat H\mid\gamma\rangle\ . \tag{2.23b}$$

Finally, to complete the definition of our input amplitude in terms of the interaction Hamiltonian, we have

$$\tilde{T}^{(3)}_{B;B}=\langle \pi B\mid \hat{H}\mid \gamma B\rangle\ . \tag{2.24}$$

A careful examination of the above equations reveals that we need the amplitudes $T^{(1)}_{B;B}$ and $\tilde{T}^{(1)}_{B;B}$ under two distinct circumstances. In the one case [Eqs. (2.3) and (2.14)] we need to calculate these amplitudes as the non-pole part of the full amplitude. In the second case [Eqs. (2.8) and (2.17)] where the amplitude is labeled by the pion that interacts with the baryon, the amplitude is an input into the calculations. Because in this second case, the $\pi-B$ interaction takes place in the presence of a spectator pion, the energy at which we need the amplitude is at least m_π less than the energy at which the amplitude is required in the first case. To that extent we do not have a self-consistancy problem on our hand. To calculate the amplitude in the second case, in terms of the interaction Hamiltonian, we require that unitarity be satisfied to one order lower than in the first case. Thus to satisfy three-body unitarity for the amplitude in the first case, the amplitude in the second case needs to satisfy two-body unitarity only, i.e., it is a solution of a two-body problem. In practice, we can parametrize the amplitudes in this second case while the amplitudes in the first case are the output of the calculation.

In the above equations, we have presented the amplitude for pion elastic scattering and photoproduction as a distorted-wave Born approximation integral for an effective potential which is by no means simple to calculate, as it requires the full off-shell AGS three-body amplitudes $U_{\alpha\beta}$. For $\pi-N$ elastic scattering, we overcome this problem by examining[12] the amplitude for $\pi B\leftarrow\pi\pi B, F^{(0)}$. This allows us to write a set of coupled equations for the reactions in Eq. (1.2) in the form[12]

$$T_{B;B}=V_{B;B}+V_{B;B}gT_{B;B}+\sum_{i\alpha}F^{(2)}_d(i)G\bar{\delta}_{i\alpha}M^{(2)}_d(\alpha)GT_{\alpha;B}\ , \tag{2.25}$$

$$T_{\alpha;B}=V_{\alpha;B}+V_{\alpha;B}gT_{B;B}+\sum_{\beta}\bar{\delta}_{\alpha\beta}M^{(2)}_d(\beta)GT_{\beta;B}\ , \tag{2.26}$$

where $T_{B;B}\equiv T^{(0)}_{B;B}$. The $\pi B\to\pi B$ potential $V_{B;B}$ is given by

$$V_{B;B}=T^{(3)}_{B;B}+f^{(2)\dagger}d_0f^{(2)}+\sum_{ij}F^{(2)}_d(i)G\bar{\delta}_{ij}F^{(2)\dagger}_d(j)\ , \tag{2.27}$$

while the $\pi\pi B\leftarrow\pi B$ potential reduces to

$$V_{\alpha;B}=\sum_i\bar{\delta}_{\alpha i}F^{(2)\dagger}_d(i)\ . \tag{2.28}$$

In this way, we have a set of coupled equations that will give us, not only the amplitude for pion elastic scattering, but the pion production amplitude as well. More important is the fact that these amplitudes satisfy two- and three-body unitarity. In the next section, we will carry out an analysis of the $\pi\pi B\leftarrow\gamma B$ amplitude that will allow us to write a similar set of equations to those of Eqs. (2.25) and (2.26) for the reactions in Eq. (1.3).

III. THE $\pi\pi B\leftarrow\gamma B$ AMPLITUDE

To get a set of coupled integral equations for pion photoproduction we need to examine the amplitude for $\pi\pi B\leftarrow\gamma B, \tilde{F}^{(0)\dagger}_{1;c}$ where the subscript c refers to the fact that all the diagrams that contribute to this amplitude, in perturbation theory, are connected diagrams. These diagrams can be divided into two classes. (i) Those that are one-particle irreducible, which we denote by $\tilde{F}^{(1)\dagger}_{1;c}$. (ii) The one-particle reducible diagrams can be written, using the last-cut lemma, as

$$\Gamma^{(1)\dagger}d_B\tilde{f}^{(1)}\ , \tag{3.1}$$

where $\Gamma^{(1)\dagger}$ is the one-particle irreducible amplitude for the process $\pi\pi B\leftarrow B$. We, therefore, can write

$$\tilde{F}^{(0)\dagger}_{1;c}=\tilde{F}^{(1)\dagger}_{1;c}+\Gamma^{(1)\dagger}d_B\tilde{f}^{(1)}\ . \tag{3.2}$$

In a similar manner we can write, using the last-cut lemma, the one-particle irreducible amplitude $\Gamma^{(1)\dagger}$ as,

$$\Gamma^{(1)\dagger}=\Gamma^{(2)\dagger}+F^{(2)\dagger}gf^{(1)\dagger} \tag{3.3}$$

with

$$\Gamma^{(2)\dagger}=\Gamma^{(3)\dagger}+M^{(2)}G\Gamma^{(3)\dagger}\ . \tag{3.4}$$

Here, $\Gamma^{(3)\dagger}$ is the three-particle irreducible amplitude for the reaction $\pi\pi B\leftarrow B$. This means it gets contributions only from diagrams with four- or more-particle intermediate states or no intermediate state at all. Since the number of baryons is conserved at one, four-particle intermediate states can only be attained by a term in the interaction Hamiltonian that couples the state with one baryon, and states of one baryon and three bosons (mesons or photon). Since no such term exists in the Hamiltonian given in Eq. (2.2), $\Gamma^{(3)\dagger}$ has a contribution only from the diagram with no intermediate states, i.e., $\Gamma^{(3)\dagger}=\langle \pi\pi B\mid\hat{H}\mid B\rangle=0$, because no such term is included in our Hamiltonian, by definition. We can at a later stage include the contribution due to such a coupling in perturbation theory. This result will allow us to write Eq. (3.2) as

$$\tilde{F}^{(0)\dagger}_{1;c}=\tilde{F}^{(1)\dagger}_{1;c}+F^{(2)\dagger}gf^{(1)\dagger}d_B\tilde{f}^{(1)}\ . \tag{3.5}$$

We now turn to the one-particle irreducible amplitude $\tilde{F}^{(1)\dagger}_{1;c}$. Here again using the last-cut lemma in classifying the diagrams that contribute to this amplitude, we can write

$$\tilde{F}^{(1)\dagger}_{1;c}=\tilde{F}^{(2)\dagger}_{1;c}+F^{(2)\dagger}g\tilde{T}^{(1)}_{B;B}+\tilde{F}^{(2)\dagger}_1\tilde{g}\tilde{\tilde{T}}^{(1)}_{B;B}\ , \tag{3.6}$$

where $\tilde{g}=d_Bd_\gamma$ is the $B\gamma$ propagator, and $\tilde{F}^{(2)\dagger}_1$ is the two-particle irreducible amplitude for $\pi\pi B\leftarrow\gamma B$. Here, $\tilde{\tilde{T}}^{(1)}_{B;B}$ is the one-particle irreducible Compton scattering amplitude $\gamma B\leftarrow\gamma B$. Because we are including only diagrams to first order in the electromagnetic interaction, this amplitude is taken to be zero. For a detailed discussion of the coupling to the Compton scattering channel in this formulation, the reader is referred to the work of Araki *et al.*[19] We now can write Eq. (3.5) as

$$\tilde{F}^{(0)\dagger}_{1;c}=\tilde{F}^{(2)\dagger}_{1;c}+F^{(2)\dagger}g(\tilde{T}^{(1)}_{B;B}+f^{(1)\dagger}d_B\tilde{f}^{(1)}) \tag{3.7a}$$

$$=\tilde{F}^{(2)\dagger}_{1;c}+F^{(2)\dagger}g\tilde{T}^{(0)}_{B;B} . \tag{3.7b}$$

An analysis, using the last-cut lemma of the two-particle irreducible amplitude for $\pi\pi B\leftarrow\pi B, F^{(2)\dagger}$, will give the result that[12]

$$F^{(2)\dagger}=F^{(2)\dagger}_{d}+F^{(2)\dagger}_{c} \tag{3.8}$$

with

$$F^{(2)\dagger}_{c}=\sum_{\alpha i} M^{(2)}_{d}(\alpha)GU_{\alpha i}GF^{(2)\dagger}_{d}(i) , \tag{3.9a}$$

$$F^{(2)\dagger}_{d}=\sum_{i} F^{(2)\dagger}_{d}(i) , \tag{3.9b}$$

and $F^{(2)\dagger}_{d}(i)$ given by Eq. (2.6).

On the other hand, the connected two-particle irreducible amplitude for $\pi\pi B\leftarrow\gamma B, \tilde{F}^{(2)\dagger}_{1;c}$, can be written using the classification of diagram, and the last-cut lemma, as

$$\tilde{F}^{(2)\dagger}_{1;c}=\tilde{F}^{(3)\dagger}_{1;c}+(\tilde{M}^{(2)}_{A}\tilde{G}\tilde{F}^{(3)\dagger}_{2})_c+(M^{(2)}G\tilde{F}^{(3)\dagger}_{1})_c , \tag{3.10}$$

where the subscript c indicates that we are including only connected diagrams. For the Hamiltonian under consideration, the three-particle irreducible amplitude for $\pi\pi B\leftarrow\gamma B, \tilde{F}^{(3)\dagger}_{1;c}$ is zero, because there is no direct coupling between the γB state and states with three bosons (i.e., pions or photons). In addition, the three-to-three amplitude for $\pi\pi B\leftarrow\pi\pi B, M^{(2)}$ can be written using the Faddeev[20,21] decomposition in terms of the AGS amplitudes for the $\pi\pi B$ system, as

$$M^{(2)}=\sum_{\alpha\beta}[M^{(2)}_{d}(\alpha)\delta_{\alpha\beta}+M^{(2)}_{d}(\alpha)GU_{\alpha\beta}GM^{(2)}_{d}(\beta)] . \tag{3.11}$$

Finally, the two-particle irreducible amplitude for $\pi\pi B\leftarrow\gamma\pi B, \tilde{M}^{(2)}_{A}$ has a connected and a disconnected part with the connected amplitude $\tilde{M}^{(2)}_{A;c}$ given by[5]

$$\tilde{M}^{(2)}_{A;c}=\sum_{ij}\tilde{M}^{(2)}_{A;d}(i)\delta_{ij}\tilde{G}\tilde{M}^{(2)}_{B;d}(j) + \sum_{\alpha ij} M^{(2)}_{d}(\alpha)GU_{\alpha i}G\tilde{M}^{(2)}_{A;d}(i)\bar{\delta}_{ij} \times[1+\tilde{G}\tilde{M}^{(2)}_{B;d}(j)] , \tag{3.12}$$

where the disconnected amplitude for $\pi\pi B \leftarrow\gamma\pi B$, $\tilde{M}^{(2)}_{A;d}(i)$ is defined in Eq. (2.17). On the other hand, the disconnected amplitude for $\pi\gamma B \leftarrow\pi\gamma B, \tilde{M}^{(2)}_{B;d}(i)$ is given in terms of the $\pi\pi$ amplitude by

$$\tilde{M}^{(2)}_{B;d}(i)=d^{-1}_{\gamma}T^{(1)}_{B;B}(i) . \tag{3.13}$$

Making use of the above results, including Eqs. (2.17) and (3.11)–(3.13), and the AGS equations [Eq. (2.7)], we can rewrite the two-particle irreducible amplitude for $\pi\pi B\leftarrow\gamma B$ as

$$\tilde{F}^{(2)\dagger}_{1;c}=\sum_{i}\left[1+\sum_{\alpha}M^{(2)}_{d}(\alpha)GU_{\alpha i}G\right]\times M^{(2)}_{d}(i)G\tilde{F}^{(2)\dagger}_{1;d} + \sum_{ij}\left[1+\sum_{\alpha}M^{(2)}_{d}(\alpha)GU_{\alpha i}G\right]\times M^{(2)}_{A,d}(i)\bar{\delta}_{ij}\tilde{G}\tilde{F}^{(2)\dagger}_{2,d}(j) , \tag{3.14}$$

with $\tilde{F}^{(2)\dagger}_{1;d}$ and $\tilde{F}^{(2)\dagger}_{2;d}$ given in Eqs. (2.18) and (2.19), respectively. In a similar manner, we can write[12] the two-particle irreducible amplitude for $\pi\pi B\leftarrow\pi B, F^{(2)\dagger}$, as

$$F^{(2)\dagger}=\sum_{i}\left[1+\sum_{\alpha}M^{(2)}_{d}(\alpha)GU_{\alpha i}G\right]F^{(2)\dagger}_{d}(i) . \tag{3.15}$$

Making use of Eqs. (3.14) and (3.15) in Eq. (3.7b), we get

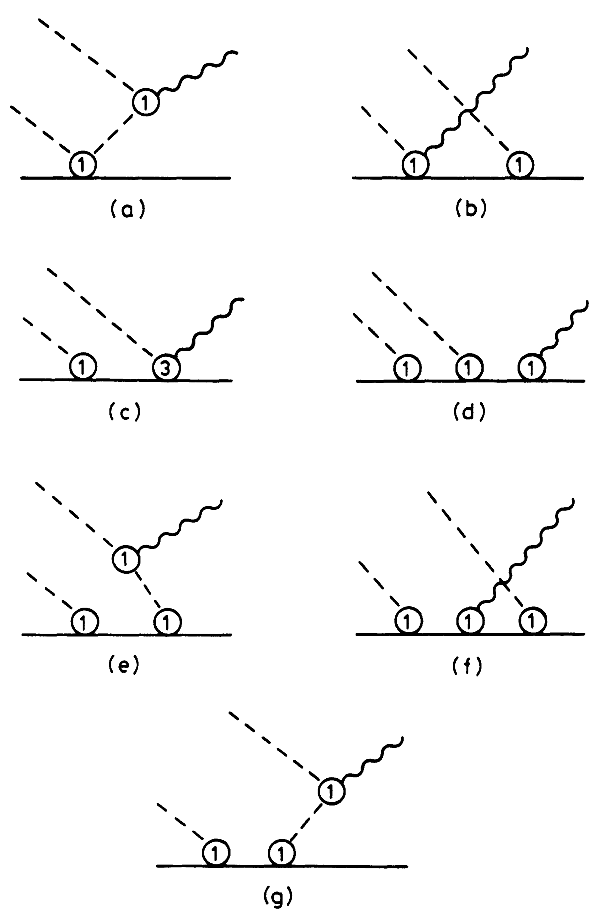

FIG. 1. The lowest-order diagrams that contribute to $\tilde{T}^{(0)}_{B;B}$. They correspond to the terms in Eq. (3.18c).

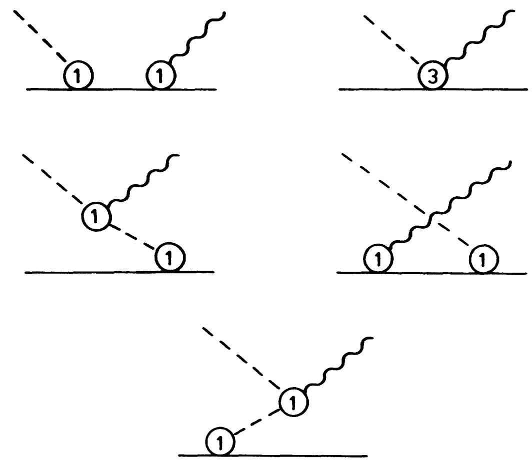

FIG. 2. The lowest-order diagrams that contribute to $\tilde{F}^{(0)}_{1;c}$. They correspond to the terms in Eq. (3.19).

$$\tilde F^{(0)\dagger}_{1;c}=\sum_i\left[1+\sum_\alpha M_d^{(2)}(\alpha)GU_{\alpha i}G\right]\left[M_d^{(2)}(i)G\tilde F^{(2)\dagger}_{1;d}+\sum_{ij}\tilde M^{(2)}_{A;d}(i)\bar\delta_{ij}\tilde G\tilde F^{(2)\dagger}_{2;d}(j)\right]$$
$$+\sum_i\left[1+\sum_\alpha M_d^{(2)}(\alpha)GU_{\alpha i}G\right]F_d^{(2)\dagger}(i)g\tilde T^{(0)}_{B;B}\ . \tag{3.16}$$

Here, we have the amplitude for two-pion photoproduction that satisfies two- and three-body unitarity. As it stands, it is not the most convenient form for computation, because it is expressed in terms of the fully off-shell AGS amplitudes $U_{\alpha\beta}$, and the half-off-shell amplitude for $\pi B\leftarrow\gamma B,\tilde T^{(0)}_{B;B}$. In the next section we will show how this amplitude can be written in terms of solutions of coupled integral equations similar to those given in Eqs. (2.25) and (2.26).

To examine the physical content of the above result and at the same time be able to compare our amplitudes to similar formulations, we consider the lowest-order contribution to this amplitude. This corresponds to neglecting all multiple scattering in the final state and can be achieved by dropping terms which have the AGS amplitudes. Thus, to lowest order we have

$$\tilde F^{(0)\dagger}_{1;c}\approx\sum_i M_d^{(2)}(i)G\tilde F^{(2)\dagger}_{1;d}+\sum_{ij}\tilde M^{(2)}_{A,d}(i)\bar\delta_{ij}\tilde G\tilde F^{(2)\dagger}_{2;d}(j)+\sum_i F_d^{(2)\dagger}(i)g\tilde T^{(0)}_{B;B}\ . \tag{3.17}$$

If furthermore, we take the lowest-order approximation to $\tilde T^{(0)}_{B;B}$, we get

$$\tilde T^{(0)}_{B;B}=f^{(1)\dagger}d_B\tilde f^{(1)}+\tilde T^{(1)}_{B;B} \tag{3.18a}$$
$$\approx f^{(1)\dagger}d_B\tilde f^{(1)}+\tilde T^{(2)}_{B;B} \tag{3.18b}$$
$$\approx f^{(1)\dagger}d_B\tilde f^{(1)}+\tilde T^{(3)}_{B;B}+\tilde f^{(1)}_b d_\pi f^{(1)\dagger}+\tilde f^{(1)}d_B f^{(1)\dagger}+f^{(1)}d_\pi\tilde f^{(1)\dagger}_a\ . \tag{3.18c}$$

Using this result, for which the right-hand side (RHS) of the equation is given diagramatically in Fig. 1, we can write the lowest-order contribution to the $\pi\pi B\leftarrow\gamma B$ amplitude. The result is

$$\tilde F^{(0)\dagger}_{1;c}\approx\sum_i T^{(1)}_{B;B}(i)d_\pi(i)\tilde f^{(1)\dagger}_a+\sum_{ij}\tilde T^{(1)}_{B;B}(i)\bar\delta_{ij}d_Bf^{(1)\dagger}(j)$$
$$+\sum_i f^{(1)\dagger}(i)d_B\tilde T^{(3)}_{B;B}+\sum_{ij}f^{(1)\dagger}(i)d_B\bar\delta_{ij}f^{(1)\dagger}(j)d_B\tilde f^{(1)}$$
$$+\sum_{ij}f^{(1)\dagger}(i)d_B\bar\delta_{ij}\tilde f^{(1)}_b(j)d_\pi f^{(1)\dagger}(j)+\sum_{ij}f^{(1)\dagger}(i)d_B\bar\delta_{ij}\tilde f^{(1)}d_Bf^{(1)\dagger}(j)+\sum_i f^{(1)\dagger}(i)d_Bf^{(1)}d_\pi\tilde f^{(1)}_a\ . \tag{3.19}$$

This lowest-order result for $\pi\pi B\leftarrow\gamma B$ is illustrated diagramatically in Fig. 2. Comparing the diagrams in Fig. 2 with those used by Laget,[22] we find that he has basically included all the diagrams we get, with the exception of the first two [i.e., Figs. 2(a) and 2(b)]. The diagram in Fig. 2(b) is a different time ordering but the same process as the diagram in Fig. 2(c) and should be included in any calculation, particularly as the diagram in Fig. 2(c) gives the major contribution to the cross section.[22,23] The fact that in the diagram in Fig. 2(b) we have a one-particle irreducible amplitude for $\pi B\leftarrow\gamma B$, while the corresponding amplitude in the diagram in Fig. 2(c) is three-particle irreducible is only a consequence of the fact that in Fig. 2(b) we have not taken the lowest-order expansion. The diagram in Fig. 2(a) may be considered as a higher-order diagram since the pion scatters off the nucleon in the final state. Finally, all the vertices in our analysis are dressed, and are related to the basic chiral Lagrangian we started with. On the other hand, Laget[22] makes use of a local effective Lagrangian, in which the form factors and coupling constants could be adjusted to reproduce the data.

IV. THE COUPLED INTEGRAL EQUATIONS

We now turn to the derivation of the coupled integral equations for the reactions in Eq. (1.3). For this we need to write an expression for the amplitudes describing the reactions $\pi(\pi B)_{B^*}\leftarrow\gamma B$, and $(\pi\pi)_\rho B\leftarrow\gamma B$. This can be achieved by taking the left-hand-side residue of the $\pi\pi B\leftarrow\gamma B$ amplitude $\tilde F^{(0)\dagger}_{1;c}$ at the B^* and ρ poles. To take the residue at B^* pole, we need to expose this pole in the amplitudes $M_d^{(2)}(i)$ and $\tilde M^{(2)}_{A;d}(i)$. Since the B^* is a resonance observed in $\pi-B$ scattering and does not correspond to a three-quark state in the present formulation, the corresponding one-particle irreducible amplitude $T^{(1)}_{B;B}$ will have a pole in the complex energy plane at an energy ε_{B^*} corresponding to this resonance. Exposing this pole allows us to write this amplitude as

$$T^{(1)}_{B;B}=\left[\frac{|\phi_{B^*}\rangle\langle\phi_{B^*}|}{E-\varepsilon_{B^*}}+\cdots\right], \tag{4.1}$$

where ϕ_{B^*} is the form factor for the vertex $B^*\to B\pi$. Making use of this result, we can write the disconnected $\pi\pi B\leftarrow\pi\pi B$ amplitude $M_d^{(2)}(i)$ as

$$M_d^{(2)}(i)=\sum_j \bar{\delta}_{ij} d_\pi^{-1}(j)\left[\frac{|\phi_{B^*}(i)\rangle\langle\phi_{B^*}(i)|}{E-\varepsilon_{B^*}}+\cdots\right] . \tag{4.2}$$

In a similar manner, we can expose the pole in the $\pi B\leftarrow\gamma B$ amplitude and write, using Eq. (2.17), the amplitude $\tilde{M}\,^{(2)}_{A;d}(i)$ as

$$\tilde{M}\,^{(2)}_{A;d}(i)=\sum_j \bar{\delta}_{ij} d_\pi^{-1}(j)\left[\tilde{T}\,^{(2)}_{B;B}(i)+\left[\frac{|\phi_{B^*}(i)\rangle\langle\phi_{B^*}(i)|}{E-\varepsilon_{B^*}}+\cdots\right] g\tilde{T}\,^{(2)}_{B;B}(i)\right] . \tag{4.3}$$

We now can make use of Eqs. (4.2) and (4.3) to take the left-hand-side (LHS) residue of Eq. (3.16) at the B^* pole. This gives us the amplitude for $\pi B^*\leftarrow\gamma B$ as

$$\tilde{T}(\pi B^*\leftarrow\gamma B)=\langle\psi^{(0)}_{\pi B^*}|\tilde{T}\,'_{i;B}|\psi^{(0)}_{\gamma B}\rangle , \tag{4.4}$$

where the amplitude $\tilde{T}\,'_{i;B}$ is given by

$$\tilde{T}\,'_{i;B}=\tilde{F}\,^{(2)\dagger}_{1;d}+\sum_j \tilde{M}\,^{(3)}_{A;d}(i)\bar{\delta}_{ij}\tilde{G}\tilde{F}\,^{(2)\dagger}_{2;d}(j)$$
$$+\sum_j U_{ij}G\left[M_d^{(2)}(j)G\tilde{F}\,^{(2)\dagger}_{1;d}+\sum_l \tilde{M}\,^{(2)}_{A;d}(j)\bar{\delta}_{jl}\tilde{G}\tilde{F}\,^{(2)\dagger}_{2;d}(l)\right]+F_d^{(3)\dagger}(i)g\tilde{T}\,^{(0)}_{B;B}+\sum_j U_{ij}GF_d^{(2)\dagger}(j)g\tilde{T}\,^{(0)}_{B;B} . \tag{4.5}$$

In the above result we have the bare amplitudes for $\pi B\leftarrow\gamma B,\tilde{M}\,^{(3)}_{A;d}(i)$ and $\pi B\leftarrow B,F_d^{(3)\dagger}(i)$, appearing in the second and fourth term on the RHS of the above equation. To avoid these bare amplitudes in our integral equations we need to redefine the amplitude for $\pi B^*\leftarrow\gamma B,\tilde{T}_{i;B}$ that will appear in our integral equation. Since the pole corresponding to the B^* is in the complex energy plane, the amplitude for $\pi B^*\leftarrow\gamma B$ is not a physical observable to the extent that we cannot measure it directly. We can only determine its contribution by examining the reaction $\pi\pi B\leftarrow\gamma B$ in certain kinematic regions where this mechanism dominates. In that case, we have to use the amplitude $\tilde{F}\,^{(0)\dagger}_{1;c}$ to calculate the cross section. Thus, with no loss of generality, to close the coupled integral equations and have the input in terms of dressed vertices, we define an amplitude for $\pi B^*\leftarrow\gamma B$ such that it does not include the second and fourth term on the right-hand side of Eq. (4.5), i.e.,

$$\tilde{T}_{i;B}=\tilde{F}\,^{(2)\dagger}_{1;d}+\sum_j U_{ij}GF_d^{(2)\dagger}(j)g\tilde{T}\,^{(0)}_{B;B}+\sum_j U_{ij}G\left[M_d^{(2)}(j)G\tilde{F}\,^{(2)\dagger}_{1;d}+\sum_l \tilde{M}\,^{(2)}_{A;d}(j)\bar{\delta}_{jl}\tilde{G}\tilde{F}\,^{(2)\dagger}_{2;d}(l)\right] . \tag{4.6}$$

In a similar manner, we get the amplitude for the reaction $(\pi\pi)_\rho B\leftarrow\gamma B$ by taking the LHS residue at the ρ pole of the amplitude $\tilde{F}\,^{(0)\dagger}_{1;c}$ as given in Eq. (3.16). Since this pole is present in the amplitude for $\pi-\pi$ scattering, $M_d^{(2)}(3)$, the resultant expression is

$$\tilde{T}(\rho B\leftarrow\gamma B)=\langle\psi^{(0)}_{\rho B}|\tilde{T}_{3;B}|\psi^{(0)}_{\gamma B}\rangle , \tag{4.7}$$

where the amplitude $\tilde{T}_{3;B}$ is given by

$$\tilde{T}_{3;B}=\sum_j U_{3j}GF_d^{(2)\dagger}(j)g\tilde{T}_{B;B}+\sum_j U_{3j}G\left[M_d^{(2)}(j)G\tilde{F}\,^{(2)\dagger}_{1;d}+\sum_l \tilde{M}\,^{(2)}_{A;d}(j)\bar{\delta}_{jl}\tilde{G}\tilde{F}\,^{(2)\dagger}_{2;d}(l)\right] . \tag{4.8}$$

Here, we have redefined the amplitude for $\pi B\leftarrow\gamma B$ by dropping any reference to its reducibility, i.e., $\tilde{T}_{B;B}\equiv\tilde{T}\,^{(0)}_{B;B}$. We now can combine Eqs. (4.6) and (4.8) to write

$$\tilde{T}_{\alpha;B}=\bar{\delta}_{\alpha 3}\tilde{F}\,^{(2)\dagger}_{1;d}+\sum_j U_{\alpha j}G\left[F_d^{(2)\dagger}(j)g\tilde{T}_{B;B}+M_d^{(2)}(j)G\tilde{F}\,^{(2)\dagger}_{1;d}+\sum_l \tilde{M}\,^{(2)}_{A;d}(j)\bar{\delta}_{jl}\tilde{G}\tilde{F}\,^{(2)\dagger}_{2;d}(l)\right] . \tag{4.9}$$

To eliminate the AGS amplitude $U_{\alpha j}$ from the above equation, we first make use of the AGS equations [Eq. (2.7)] to iterate the RHS once, then make use of the definition of $\tilde{T}_{\alpha;B}$, Eq. (4.9), to get

$$\tilde{T}_{\alpha;B}=\tilde{V}_{\alpha;B}+V_{\alpha;B}g\tilde{T}_{B;B}+\sum_\beta \bar{\delta}_{\alpha\beta}M_d^{(2)}(\beta)G\tilde{T}_{\beta;B} , \tag{4.10}$$

where $V_{\alpha;B}$ is given in Eq. (2.28), and

$$\tilde{V}_{\alpha;B}=\bar{\delta}_{\alpha 3}\tilde{F}\,^{(2)\dagger}_{1;d}+\sum_{jl}\bar{\delta}_{\alpha j}\tilde{M}\,^{(2)}_{A;d}(j)\bar{\delta}_{jl}\tilde{G}\tilde{F}\,^{(2)\dagger}_{2;d}(l) . \tag{4.11}$$

This result gives us the amplitudes for $\pi B^*\leftarrow\gamma B$ and $\rho B\leftarrow\gamma B,\tilde{T}_{\alpha;B}$ in terms of the amplitude $\tilde{T}_{\alpha;B}$ and the amplitude for $\pi B\leftarrow\gamma B,\tilde{T}_{B;B}$. To close the set of coupled integral equations, we need to derive an equation for the $\pi B\leftarrow\gamma B$ amplitude $\tilde{T}_{B;B}$ in terms of the amplitude $\tilde{T}_{B;B}$ and the amplitude $\tilde{T}_{\alpha;B}$. To achieve this result, we make use of Eqs. (2.4) and (2.5) to write Eq. (2.15) as

$$\tilde{T}_{B;B}=\tilde{v}_{B;B}+V_{B;B}g(1+T_{B;B}g)\tilde{v}_{B;B}+\sum_{i\alpha j}F_d^{(2)}(i)G\bar{\delta}_{i\alpha}M_d^{(2)}(\alpha)GU_{\alpha j}GF_d^{(2)\dagger}(j)g(1+T_{B;B}g)\tilde{v}_{B;B}\ , \tag{4.12}$$

where $V_{B;B}$ is defined in Eq. (2.27). If we now make use of Eq. (2.15) to replace $(1+T_{B;B}g)\tilde{v}_{B;B}$ by $\tilde{T}_{B;B}$ and substitute for $\tilde{v}_{B;B}$ in the first term on the RHS, using Eq. (2.16), we get

$$\begin{aligned}\tilde{T}_{B;B}=\tilde{T}^{(3)}_{B;B}+f^{(2)\dagger}d_0\tilde{f}^{(2)}+\tilde{f}^{(1)}_{b}d_\pi f^{(1)\dagger}+\tilde{f}^{(1)}d_B f^{(1)\dagger}+V_{B;B}g\tilde{T}_{B;B}\\+\sum_i F_d^{(2)}(i)GU_{i3}G\tilde{F}^{(2)\dagger}_{1,d}+\sum_{ijl}F_d^{(2)}(i)GU_{ij}G\tilde{M}^{(2)}_{A;d}(j)\tilde{G}\bar{\delta}_{jl}\tilde{F}^{(2)\dagger}_{2;d}(l)+\sum_{i\alpha j}F_d^{(2)}(i)G\bar{\delta}_{i\alpha}M_d^{(2)}(\alpha)GU_{\alpha j}GF_d^{(2)\dagger}(j)g\tilde{T}_{B;B}\ .\end{aligned} \tag{4.13}$$

We now make use of the AGS equations to iterate the sixth and seventh term on the RHS of Eq. (4.13). This allows us, with the help of Eq. (4.9), to eliminate the AGS amplitude in favor of the physical amplitude $\tilde{T}_{\alpha;B}$, which we want. After some algebra, we get the second equation in our coupled channel description of the reaction in Eq. (1.3) to be

$$\tilde{T}_{B;B}=\tilde{V}_{B;B}+V_{B;B}g\tilde{T}_{B;B}+\sum_{i\alpha}F_d^{(2)}(i)G\bar{\delta}_{i\alpha}M_d^{(2)}(\alpha)G\tilde{T}_{\alpha;B}\ , \tag{4.14}$$

where $V_{B;B}$ is given in Eq. (2.27), and

$$\tilde{V}_{B;B}=\tilde{T}^{(3)}_{B;B}+f^{(2)\dagger}d_0\tilde{f}^{(2)}+\tilde{f}^{(1)}_{b}d_\pi f^{(1)\dagger}+\tilde{f}^{(1)}d_B f^{(1)\dagger}+\sum_i F_d^{(2)}(i)G\tilde{F}^{(2)\dagger}_{1;d}+\sum_{ijl}F_d^{(2)}(i)G\bar{\delta}_{ij}\tilde{M}^{(2)}_{A;d}(j)\tilde{G}\bar{\delta}_{jl}\tilde{F}^{(2)\dagger}_{2;d}(l)\ . \tag{4.15}$$

If we now compare the coupled integral equation given in Eqs. (4.10) and (4.14) for the reactions in Eq. (1.3) with the coupled integral equations for the reactions in Eq. (1.2) as given by Eqs. (2.25) and (2.26), we find that the kernel of the two sets of equations are identical. In fact, the only difference between them is the inhomogenous term which is governed by the initial state. This observation allows us to write the two sets of coupled equations as a matrix equation by introducing the matrix amplitude

$$\mathbf{T}=\begin{bmatrix}T_{B;B} & \tilde{T}_{B;B}\\ T_{\alpha;B} & \tilde{T}_{\alpha;B}\end{bmatrix} \tag{4.16}$$

and the potential matrix

$$\mathbf{V}=\begin{bmatrix}V_{B;B} & \tilde{V}_{B;B}\\ V_{\alpha;B} & \tilde{V}_{\alpha;B}\end{bmatrix}\ . \tag{4.17}$$

Then the integral equation for the reaction in Eqs. (1.2) and (1.3) can be combined into one matrix equation written as

$$\mathbf{T}=\mathbf{V}+\mathbf{K}\mathbf{T}\ , \tag{4.18}$$

where the kernel is given by

$$\mathbf{K}=\begin{bmatrix}V_{B;B}g & \sum_\alpha V_{B;\alpha}GM_d^{(2)}(\alpha)G\\ V_{\alpha;B}g & \sum_\beta \bar{\delta}_{\alpha\beta}M_d^{(2)}(\beta)G\end{bmatrix}\ , \tag{4.19}$$

and

$$V_{B;\alpha}=\sum_i F_d^{(2)}(i)\bar{\delta}_{i\alpha}\ . \tag{4.20}$$

This result indicates very clearly that we can solve for the amplitudes for pion photoproduction with very little additional work over the effort required to calculate the elastic amplitude. In fact, the only additional input we need are the Born amplitudes $V_{B;B}$ and $V_{\alpha;B}$. However, the new coupled set of integral equations puts a strong constraint on the model to the extent that the form factors and two-body potentials that are required for calculating the elastic amplitudes will also determine the photoproduction amplitude. There are no additional parameters to be adjusted. Also, by introducing the coupling to the electromagnetic field at the quark level and taking the baryon B to be the nucleon or $\Delta(1232)$, we have included both meson exchange current and the isobar current in our final amplitudes.

V. CONCLUSION

In the present paper we have derived a set of coupled integral equations that describe the reactions given in Eqs. (1.2) and (1.3). These equations, which are of the Faddeev form, satisfy two- and three-body unitarity and thus can be used to calculate the amplitude above the threshold for three-body final states. In particular, they can be used to study the resonances just above the $\pi\pi N$ threshold, such as the Roper resonance. Although our starting Hamiltonian is based on the chiral Lagrangian, our derivation does not depend on the detailed form of the interaction Hamiltonian H_I. This will allow us to compare the results of the reactions in Eqs. (1.2) and (1.3) for different chiral Lagrangians and relativistic and nonrelativistic quark models in which the mesons are coupled to the quarks. Furthermore, since the $\pi-N$ resonances can be treated as either a three-quark state B or a purely dynamical effect, we can address such questions as the quark content of a given $\pi-N$ resonance. For example, we have considered[11] in detail the question of the Roper resonance, as observed in $\pi-N$ elastic scattering, and the matter of how important it is to include a three-quark Roper state in the [56] or [70] representation. We can also examine the question of how important a role the mixing of the [56] and [70] representation plays in both pion elastic scattering and photoproduction. The advantage of this formulation over previous analyses[8] is that the $\pi-N$ resonances are treated as resonances corresponding to poles of the S matrix in the complex energy plane. The width for the decay of these resonances into πN, $\pi\Delta$, ρN, and $\pi\pi N$ is determined using the same Hamiltonian and integral equations that give the mass spectrum for the baryon and the cross section for pion elastic scattering and photoproduction.

Because we have introduced the coupling to the electromagnetic interaction into our Lagrangian at the quark level, the model includes the contribution of both meson exchange current and isobar current. Furthermore, the Lagrangian, before projection, satisfies both gauge and chiral symmetry. Any loss of such symmetry is the result of our projection procedure, or the imposition of unitarity by summing only certain classes of diagrams in perturbation theory. In this way, we may be able to investigate the violation of these symmetries in a systematic way.

Finally, the solution of the above equation will give us the input into the $BB-\pi BB-\gamma BB$ equations[24] that describe $N-N$ scattering, pion production, and pion-deuteron scattering, as well as the process involving the interaction of the photon with the $A=2$ system. Since, the parameters of the chiral Lagrangian are determined from pion elastic scattering, the results for both pion photoproduction on a single nucleon and all results for the $A=2$ system are determined with no further parameter adjustment. In other words, we have a parameter-free theory. Most interesting is the fact that all the contributions from the meson exchange current and isobar current in the $A=2$ system are predetermined by the single-nucleon results.

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