Exact multistep distorted-wave method

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(Received 6 July 1987)

We show that the coupled channel equations are exactly transformed into the T matrix of the multistep process. The consistency with empirical models is discussed.

Since Gell-Mann and Goldberger[1] have formulated the distorted-wave Born approximation (DWBA), it has been successfully used in analyses of quantum scattering in various fields of physics. This method is based on a perturbation expansion in the residual interaction (the original interaction minus the distorting potential). The DWBA model used conventionally in nuclear reactions is also able to reproduce measured cross sections even with a strong characteristic interaction for which the perturbation expansion does not converge.[2] This is generally understood as follows: The entrance and exit distorting potentials are replaced by empirical optical potentials which contain the contribution of higher-order terms in the interaction. Hence, the perturbation expansion in the DWBA interaction may be diverge because of over counting. This model cannot be viewed as the first-order model in a perturbation expansion. Here we call this method the conventional DWBA (CDWBA).

There have been some exceptions[3] reported where the CDWBA fails in fitting certain kinds of reactions such as ^{48}Ca(^{16}O,^{16}O)^{48}Ca*$(3^-,2^+,5^-)$. Ascuitto *et al.*[4] have shown that these reactions can successfully be described in terms of the so-called asymmetric DWBA (hereafter referred to as ADWBA) proposed by Satchler,[5] which uses for the exit channel the bare potential, i.e., the unperturbed potential without coupling effects (the entrance-channel distorting potential remains the empirical optical potential). Ichimura and Kawai[6] have pointed out that the success of ADWBA does not depend on the strength of the coupling between the entrance and exit channels. Kubo and Hodgson[7] have studied how to determine the bare exit-channel potential from the empirical optical potential.

In Ref. 8 we have shown that CDWBA and ADWBA are valid in alternative coupling schemes: For a system with sequential excitations like a vibration and a rotation, CDWBA reproduces well the coupled-channel result, whereas ADWBA does not. On the other hand, for the case of exit channels coupled only to the entrance channel, ADWBA predicts exactly the coupled channel result, but CDWBA fails.

Under this situation it is quite desirable to understand the two models from a unified viewpoint. In a previous paper[9] we gave a solution to this problem. There we presented the exact first- and second-order distorted-wave Born methods derived by transforming the coupled-channel equations. The first-order method has the very desirable structure which reproduces well the CDWBA result in the sequential coupling scheme, while it coincides with ADWBA in the case where the exit channels are coupled only to the entrance channel. Here we rename the first- and second-order distorted-wave Born methods as the one- and two-step DW methods, respectively.

The advantage of these DW methods is to allow one to assess the validity of CDWBA or ADWBA and to find suitable exit-channel distorting potential and transition interactions by estimating polarization effects reliably. However, there is no consistent formulation for the higher-step method other than the two-step method.

The purpose of this report is to formulate the multistep DW method on the basis of the coupled-channel method in a consistent manner with the optical model, and the one-step and two-step DW methods.

This formalism is different from Feshbach's multistep direct formalism.[10] Each step method of our model is equivalent with the coupled-channel method and hence, the higher-order effects in the original interaction, i.e., the polarization effects, are correctly included in the distorting potentials and the transition interactions in each step. On the other hand, the latter formalism is based on a perturbation expansion in the residual interaction, which does not contain the polarization effects, and thus, has such a structure that an expansion up to a higher-order term comes closer to the exact theory (a coupled-channel method).

First we define the projection operators on channel spaces. Following Ref. 9, we divide a coupled-channel space into the entrance-channel P and the other channels $Q(P+Q=1)$. We specify the channel by $c(=(I_\alpha,I_\beta))$, where $I_\alpha(I_\beta)$ denotes the channel spin of nucleus $\alpha(\beta)$. We divide Q into N groups as

$$Q=Q^1+Q^2+\cdots+Q^N , \tag{1}$$

where Q^n is the projection operator onto the channels reached in the n-step process and/or the less-step processes [explained below Eq. (2)]. Let us decompose Q^n further into the subchannels $i=(c,I,L)$ with the total spin I and the orbital-angular momentum L:

$$Q^n=\sum_i Q_i^n . \tag{2}$$

The summation over i contains those over α and β if one

treats a rearrangement reaction. Here we assume that the channel states are orthogonal to each other and that $R^2=R$ for any projection operator R. The channel state projected by Q_i^n is assumed to be excited via the n-step process, $P\rightarrow\sum'_j Q_j^1\rightarrow\sum'_k Q_k^2\rightarrow\cdots\rightarrow Q_i^n$, and/or the less-step processes, e.g., $P\rightarrow Q_i^n$ where the prime means the restricted summation over the states enhanced in that process.

The division of $Q=Q^1+\cdots+Q^N$ is arbitrary in our formal theory. In practical calculation, one can use the following choice of division: When a particular step of process dominates the problem under consideration, one can use that particular step ($=N$ step) method. On the other hand, when it is not clear which particular step dominates the reaction, one first examines the maximum step of the processes. Then the result should show which step process dominates the reaction or that some different step processes compete in the problem [this point is discussed in detail following Eq. (20) for the case of two-step process]. This fixes the division of the Q space, $Q=Q^1+\cdots Q^N$. For notational convenience we introduce the operator $\bar{Q}_i^n$:

$$\bar{Q}_i^n=\sum_{k=n}^{N} Q^k-Q_i^n \ . \tag{3}$$

We set up the Schrödinger equation of the total system: $(E-H)\Psi=0$. Then we define the polarization operator[9] generated by elimination of a space R:

$$\Delta V(R)=HR(E^{(+)}-RHR)^{-1}RH \ . \tag{4}$$

For simplicity we write

$$\tilde{H}(\bar{Q}_i^n)=H+\Delta V(\bar{Q}_i^n) \ . \tag{5}$$

We start with the optical model (referred to as the zero-step DW method), which has been formulated by Feshbach.[11]

(1.) The zero step: The entrance-channel (elastic) wave function $P\Psi^{(+)}(\equiv\Psi_P^{(+)})$ satisfies[11]

$$(E-P\tilde{H}(Q)P)\Psi_P^{(+)}=0 \ . \tag{6}$$

The elastic T-matrix element is given by

$$T_{PP}=T_P^{(0)}+\langle\Phi_P^{(-)}\mid P\Delta V(Q)P\mid\Psi_P^{(+)}\rangle \ , \tag{7}$$

with

$$(E-PHP)^{\dagger}\Phi_P^{(-)}=0 \ , \tag{8}$$

where $T_P^{(0)}$ is the T-matrix element determined from $\Phi_P^{(-)}$, which represents the unperturbed wave function with the incoming boundary condition. Equation (6) can be regarded as the optical model equation, since $P\tilde{H}(Q)P\simeq T+$ (optical potential).

(2.) The one step: The exit-channel wave function $\Psi_{Q_i^1}^{(+)}$ obeys[9]

$$(E-Q_i^1\tilde{H}(\bar{Q}_i^1)Q_i^1)\Psi_{Q_i^1}^{(+)}=Q_i^1\tilde{H}(\bar{Q}_i^1)P\Psi_P^{(+)} \ . \tag{9}$$

Then we find the one-step T-matrix element as

$$T_{Q_i^1P}=\langle\Phi_{Q_i^1}^{(-)}\mid Q_i^1\tilde{H}(\bar{Q}_i^1)P\mid\Psi_P^{(+)}\rangle \tag{10}$$

with

$$(E-Q_i^1\tilde{H}(\bar{Q}_i^1)Q_i^1)^{\dagger}\Phi_{Q_i^1}^{(-)}=0 \ . \tag{11}$$

(3.) The multistep: We define the Green's function and the n-step transition interaction by

$$G(Q_i^n)=(E^{(+)}-Q_i^n\tilde{H}(\bar{Q}_i^n)Q_i^n)^{-1} \ , \tag{12}$$

and

$$\nu_i^{(n)}=\tilde{H}(\bar{Q}_i^n)+\sum_{k=1}^{n-1}\sum_{j\in Q^k}\tilde{H}(\bar{Q}_i^n)Q_j^kG(Q_j^k)Q_j^k\nu_j^{(k)} \ . \tag{13}$$

Equation (13) gives the recurrence equation for $\nu_i^{(k)}$, which is evaluated in order from $\nu_i^{(1)}=\tilde{H}(\bar{Q}_i^1)$ [the one-step interaction of Eq. (10)].

We can generalize the one- and two-step DW methods to the higher-step methods. The resulting wave function of the n-step process satisfies

$$(E-Q_i^n\tilde{H}(\bar{Q}_i^n)Q_i^n)\Psi_{Q_i^n}^{(+)}=Q_i^n\nu_i^{(n)}P\Psi_P^{(+)} \ . \tag{14}$$

Then the T-matrix element can be written as

$$T_{Q_i^nP}=\langle\Phi_{Q_i^n}^{(-)}\mid Q_i^n\nu_i^{(n)}P\mid\Psi_P^{(+)}\rangle \tag{15}$$

with

$$(E-Q_i^n\tilde{H}(\bar{Q}_i^n)Q_i^n)^{\dagger}\Phi_{Q_i^n}^{(-)}=0 \ . \tag{16}$$

We prove Eq. (14) by the induction method: We assume that Eq. (14) holds for $k=1,\ldots,n$. This equation is formally solved using Eq. (12) as

$$\Psi_{Q_i^k}^{(+)}=G(Q_i^k)Q_i^k\nu_i^{(k)}P\Psi_P^{(+)}, \quad \text{for } k=1,2,\ldots,n \ . \tag{17}$$

For the $(n+1)$-step process we multiply $(E-H)\Psi=0$ by Q_i^{n+1} on the left-hand side and eliminate the wave function $\Psi_{\bar{Q}_i^{n+1}}^{(+)}$, so that we obtain

$$(E-Q_i^{n+1}\tilde{H}(\bar{Q}_i^{n+1})Q_i^{n+1})\Psi_{Q_i^{n+1}}^{(+)}=Q_i^{n+1}\tilde{H}(\bar{Q}_i^{n+1})P\Psi_P^{(+)}+\sum_{k=1}^{n}\sum_{j\in Q^k}Q_i^{n+1}\tilde{H}(\bar{Q}_i^{n+1})Q_j^k\Psi_{Q_j^k}^{(+)} \ , \tag{18}$$

Substituting Eq. (17) into Eq. (18), we find that the right-hand side becomes

$$Q_i^{n+1}\left[\tilde{H}(\bar{Q}_i^{n+1})+\sum_{k=1}^{n}\sum_{j\in Q^k}\tilde{H}(\bar{Q}_i^{n+1})Q_j^kG(Q_j^k)Q_j^k\nu_j^{(k)}\right]P\Psi_p^{(+)}=Q_i^{n+1}\nu_i^{(n+1)}P\Psi_P^{(+)} \ . \tag{19}$$

Hence, Eq. (18) coincides with Eq. (14) when $n\rightarrow n+1$. It follows that the $(n+1)$-step T-matrix element is given by

Eq. (15) with $n \rightarrow n+1$, where $\Phi^{(-)}_{Q_i^{n+1}}$ satisfies Eq. (16) with $n \rightarrow n+1$. That establishes the proof.

Next we examine the two-step T-matrix element given by

$$T_{Q_i^2 P} = \langle \Phi^{(-)}_{Q_i^2} \mid Q_i^2 \left[\tilde{H}(\bar{Q}_i^2) + \sum_{j \in Q^1} \tilde{H}(\bar{Q}_i^1) Q_j^1 G(Q_j^1) Q_j^1 \tilde{H}(\bar{Q}_j^1) \right] P \mid \Psi^{(+)}_P \rangle \ . \tag{20}$$

The first term represents the direct transition process, $P \rightarrow Q_i^2$, while the second term represents the two-step process, $P \rightarrow \sum_{j \in Q^i} Q_j^1 \rightarrow Q_i^2$. By this formula one can calculate the T-matrix amplitude quite generally: (i) the direct process dominates it, (ii) the two-step process dominates it, and (iii) it is not clear which process is important or several processes compete equally. When the direct process is hindered for a certain reason, it corresponds to the conventional two-step DWBA.[12] We note that the initial and final interactions have different structures in the truncation spaces, i.e., $\bar{Q}_i^1$ and $\bar{Q}_k^2$, respectively. The distorting potentials of the exit, the intermediate, and the entrance channels are also different in truncation spaces. In inelastic scattering, therefore, these interactions and potentials can be different.[13]

The same observation as in the two-step DW method holds also for the higher-step DW method. The n-step interaction $v_i^{(n)}$ in Eq. (13) contains $G(Q_j^k)$ $[=(E^{(+)}-Q_j^k \tilde{H}(\bar{Q}_j^k) Q_j^k)^{-1}]$ for the propagator in the intermediate state. The Hamiltonian of this propagator is obeyed by the exit-channel wave function in the k-step method: $(E - Q_i^k \tilde{H}(\bar{Q}_i^k) Q_i^k)^{\dagger} \Phi^{(-)}_{Q_i^k} = 0$. Therefore, the multistep DW method is consistent in the sense that the exit distorting potential fixed in each step is also the distorting potential for the intermediate state in the higher-step method.

Let us investigate the inelastic scattering $^{40}\mathrm{Ca}(^{16}\mathrm{O},^{16}\mathrm{O})^{40}\mathrm{Ca}(3^-,5^-)$ on the basis of the one-step DW method given by Eq. (7). The 5^- and 3^- states are weakly and strongly coupled to the ground states, respectively, but both are weakly coupled to the other nonelastic channels. In this case we set all the inelastic states to belong to Q^1: $Q \equiv Q^1 = \sum_{i \neq \mathrm{g.s.}} \mid i \rangle \langle i \mid$. The entrance-channel distorting potential is the optical potential, which contains the polarization term $P \Delta V(Q) P$. For the exit channel, on the other hand, the polarization term

$$Q_i^1 \Delta V(\bar{Q}_i^1) Q_i^1 = Q_i^1 H \bar{Q}_i^1 (E^{(+)} - \bar{Q}_i^1 H \bar{Q}_i^1)^{-1} \bar{Q}_i^1 H Q_i^1$$

can be neglected because it is proportional to the square of the weak interaction $Q_i^1 H \bar{Q}_i^1$ (e.g., $\langle 3^- \mid V \mid 5^- \rangle$). Hence, the exit-channel distorting potentials are the bare potentials. This confirms the success of ADWBA in Ref. 4.

In the case of the single and mutual excitations of the 3^- (6.13 MeV) state in $^{16}\mathrm{O}+^{16}\mathrm{O}$, we set Q^1 and Q^2 to be the single and mutual 3^- channels, respectively. The exit distorting potential of the single 3^- channel is similar to the entrance channel:

$$Q_i^1 H \bar{Q}_i^1 (E^{(+)} - \bar{Q}_i^1 H \bar{Q}_i^1)^{-1} \bar{Q}_i^1 H Q_i^1 \simeq PHQ(E^{(+)} - QHQ)^{-1} QHP \ .$$

It is because the matrix element $\langle (3^-,0^+) \mid V \mid (3^-,3^-) \rangle (\sim Q_i^1 H \bar{Q}_i^1)$ is nearly the same as $\langle (0^+,0^+) \mid V \mid (3^-,0^+) \rangle (\sim PHQ)$ and $(E^{(+)} - \bar{Q}_i^1 H \bar{Q}_i^1)^{-1} \simeq (E^{(+)} - QHQ)^{-1}$ for $E_{\mathrm{c.m.}} \gg 6.13$ MeV. This suggests the validity of CDWBA in this situation.[8]

We finally note that our formalism can treat such coupled-channel effects as the quadrupole reorientation that appears within any particular channel with nonzero spin, say, Q_i^n. These effects are already contained in $Q_i^n H Q_i^n$, which is part of the polarized Hamiltonian $Q_i^n \tilde{H}(\bar{Q}_i^n) Q_i^n$ in Eq. (14).

To summarize, we have formulated the multistep DW method by truncating the coupled-channel space starting with Feshbach's formulation of the optical model. In each step the exit distorting potential and the transition interaction are consistent. This formulation leads to new physical interpretations for the distorting potentials of the exit and intermediate channels, and the transition interactions. For practical applications it relies on reliable approximations to the polarization operator.

The author would like to thank U. Mosel, R. Kaps, and R. Wolf in Universitaet Giessen for valuable discussions. This work was supported by the Deutsche Forschungsgemeinschaft.

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