Analysis of magnetic dipole transitions between *sd*-shell states

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(Received 9 May 1988)

Experimental values of magnetic dipole transition strengths between states of *sd*-shell ($A=17-39$) nuclei are analyzed with one-body transition densities from complete-space $0d_{5/2}$-$1s_{1/2}$-$0d_{3/2}$ shell-model wave functions in order to extract empirical values for the higher-order corrections to the model $M1$ operator. These corrections, or renormalizations, are obtained in terms of the values of the effective single-particle matrix elements which, when combined with the model transition densities, yield a minimum deviation of the model predictions for $M1$ strengths from a set of selected experimental values. A total of 250 experimental data are investigated. Confirmation that the particular realizations of shell-model wave functions employed in the analysis of the data yield adequate descriptions of the actual nuclear states, and that the set of experimental data provides a thorough and accurate sampling of the relevant phenomena, are prerequisites to obtaining meaningful results from this procedure. These issues are studied with a variety of tests of the internal consistency of the procedures used.

I. INTRODUCTION

It is currently of considerable interest to understand the response of finite nuclei to simple, spinlike probes such as the Gamow-Teller (GT) and the magnetic dipole ($M1$) operators. The simplicity of these probes allows quantitative investigation of the effects upon the phenomenology of low-energy nuclear spectroscopy which arise from the existence of nuclear isobars, mesonic-exchange currents, and configuration mixing over many oscillator shells. Our fundamental understanding of nuclear structure suggests that the approximations of the standard nuclear shell model, namely, the truncation to configurations spaces of one major shell and the restriction to operators which reflect the properties of free protons and neutrons, should be corrected to take account of higher-order effects such as those mentioned. Ultimately, of course, the introduction of such corrections must be justified also in terms of improved theoretical prediction of experimental values.

Theoretical studies have produced predictions (Refs. 1 and 2) of how the $M1$ and Gamow-Teller operators in the *sd*-shell region should be corrected, or "renormalized," to account for higher-order effects of configuration mixing over many major shells and for the modifications of the properties of free nucleons which occur in the nuclear medium. In complementary studies, values for these renormalizations have been empirically extracted from magnetic moment (Refs. 3 and 4) and GT (Refs. 3 and 5) data by analyzing the experimental values with shell-model predictions of the many-nucleon structure of the states concerned. There is evidence from both the theoretical and the empirical studies that the $M1$ renormalizations and the GT renormalizations differ from each other in significant ways. There is also reason to think that the different aspects of nuclear structure sampled in ground-state moment data on the one hand, and in data from transitions from excited states on the other, might affect the conclusions drawn from the empirical studies. Although the effective $M1$ operator is expected to be the same in both cases, the ground-state moment data emphasize the diagonal single-particle matrix elements which tend to have larger than average orbital contributions, and the data from transitions emphasize the off-diagonal single-particle matrix elements which tend to be dominated by the spin contribution. Hence, it is of interest to contrast the empirical renormalizations of the $M1$ and GT operators drawn from the analyses of magnetic moments and beta decay, respectively, with the renormalizations extracted from an analogous analysis of magnetic dipole transitions. The present study is a step towards such a synthesis (Ref. 4).

II. THEORETICAL FRAMEWORK OF THE ANALYSIS

The key elements in the empirical determination of higher-order corrections to the basic shell-model operators are accurate model descriptions of the many-nucleon structure of the nuclear wave functions $|NJTn\rangle$, where $N=A-16$ is the number of active particles in the model representation, J and T are the total angular momentum and isobaric spins of the state, respectively, and n is the numbering of the state, 1 for the lowest eigenvalue of that

(N,J,T), 2 for the second, etc. The parities of all states of interest in this study are positive. With these wave functions, the coefficients of the model-allowed one-body transition paths by which the operator connects one many-body state to another can be calculated. These terms, the multiparticle transition densities for a transition from an initial state $|i\rangle = |NJTn\rangle$ to a final state $|f\rangle = |NJ'T'n'\rangle$, are defined for a $\Delta J=1$ operator by

$$D(\Delta J=1,\Delta T,j,j',f,i)=[3(2\Delta T+1)]^{-1/2}\langle f|||[a^{\dagger}(j)\otimes\tilde{a}(j')]^{(\Delta J=1,\Delta T)}|||i\rangle , \quad (1)$$

where j and j' represent single-particle states of the model basis, $\tilde{a}$ and $a^{\dagger}$ are annihilation and creation operators, respectively, for these states, and ΔT represents their coupling to isospin values of 0 (isoscalar) or isospin 1 (isovector). The model values of the $M1$ matrix elements, $M(M1)^{\rm sm}$, which correspond to experimental measurements $M(M1)^{\rm exp}$, are obtained by summing the products of these D values with the values of the corresponding matrix elements of the $M1$ operator between the single-particle states of the model space,

$$S(M1,\Delta T=0,j,j')=\langle j|||O(ISM1)|||j'\rangle , \quad (2a)$$

and

$$S(M1,\Delta T=1,j,j')=\langle j|||O(IVM1)|||j'\rangle . \quad (2b)$$

In terms of these transition densities and single-particle matrix elements, the $M1$ matrix elements which we consider in this work are given by

$$M(M1,f,i)=\tfrac{1}{2}(-1)^{T_f-T_Z}\times\sum_{j,j',\Delta T}\begin{pmatrix} T_f & \Delta T & T_i \\ -T_Z & 0 & T_Z\end{pmatrix}\times D(\Delta J=1,\Delta T,j,j',f,i)\times S(M1,\Delta T,j,j') , \quad (3)$$

where T_f, T_i, and T_Z refer to the total isospins and $(N-Z)$ value of the initial and final states. The reduced $M1$ transition probability $B(M1)$ is given by

$$B(M1)=[M(M1,f,i)]^2/(2J_i+1) . \quad (4)$$

The standard, or "free-nucleon," form of the $M1$ operator corresponds to the values obtained for the $S(M1,\Delta T,j,j')$ when the coupling constants of the operator have the values consistent with those for nucleons in free space. The expression for the free-nucleon $M1$ operator is given by

$$O(XM1)^{\rm free}=(3/4\pi)^{1/2}\sum_k[g_s^{\rm free}(XM1)\mathbf{s}^k+g_l^{\rm free}(XM1)\mathbf{l}^k]P^k(X)\mu_N , \quad (5)$$

where μ_N is the nuclear magneton. The "g's" are the spin and orbital g factors for the free proton and neutron. The equation can be applied separately to the proton $(X=p)$ and neutron $(X=n)$ contributions, in which case $P^k(X)$ is the appropriate projection operator, or it can be applied to the isoscalar and isovector forms of the operator required for Eq. (2), in which case $P^k(IS)=1$ and $P^k(IV)=\tau^k$.

We assume that the higher-order corrections to the $M1$ operator can, analogously, be expressed in terms of altered (empirically renormalized, or "effective") values of the single-particle operators

$$O(XM1)^{\rm eff}=(3/4\pi)^{1/2}\sum_k[g_s^{\rm eff}(XM1)\mathbf{s}^k+g_l^{\rm eff}(XM1)\mathbf{l}^k+g_p^{\rm eff}(XM1)\mathbf{p}^k]P^k(X)\mu_N . \quad (6)$$

The p operator is given by

$$\mathbf{p}^k=(8\pi)^{1/2}[Y^{(2)}(\mathbf{r}^k)\otimes\mathbf{s}^k]^{(1)} \quad (7)$$

and will be referred to as the "tensor" term. This term is needed in order to specify the most general form for a $\Delta J=1$ operator (Ref. 2).

As in Ref. 3 we will express $g^{\rm eff}$ in terms of the δ parameters defined by

$$\begin{aligned} g_s^{\rm eff}(XM1)&=g_s^{\rm free}(XM1)+g_s^{\rm free}(XM1)\delta_s(XM1) ,\\ g_l^{\rm eff}(XM1)&=g_l^{\rm free}(XM1)+g_s^{\rm free}(XM1)\delta_l(XM1) ,\\ g_s^{\rm eff}(XM1)&=g_s^{\rm free}(XM1)\delta_p(XM1) . \end{aligned} \quad (8)$$

We will assume that the δ parameters have a mass dependence of the form (Ref. 3)

$$\delta(A)=\delta(A=28)(A/28)^{0.35} .$$

As discussed in detail in Ref. 3, there are in general two δ_s parameters, one associated with the d to d transition $[\delta_s(d-d)]$ and another associated with the s to s transition $[\delta_s(s-s)]$. Likewise there are in general two δ_p parameters associated with the tensor term, one for the d to d transition $[\delta_p(d-d)]$ and another associated with the s to d transition $[\delta_p(s-d)]$. We find that the experimental data and the structure of the D factors are such that the empirical values for these two tensor parameters cannot be determined independently. However, theoretical studies suggest that the higher-order corrections to their values (which are zero in the free-nucleon limit) should be similar (Ref. 3), and, thus guided, we shall couple them together and determine a common correction term $\delta_p(X)=\delta_p(X,d-d)=\delta_p(X,s-d)$. We thereby eliminate one IS and one IV degree of freedom in Eq. (6).

The empirical values of the renormalizations of the $M1$ operator are extracted from a selected set of experimental values $M(M1,f,i)^{\rm exp}$ by solving for the values of the two

$\delta_s(X)$, the $\delta_l(X)$, and the $\delta_p(X)$ terms which yield the rms minimum for the set of differences

$$M(M1,f,i)^{\text{exp}} - M(M1,f,i)^{\text{eff}} , \tag{9}$$

summed over all of the cases considered.

Obviously, for this procedure to yield meaningful results the values of D must accurately reflect the dominant structures of the actual nuclear states which give rise to the $M(M1)^{\text{exp}}$. We now have available shell-model wave functions for every allowed state in every *sd*-shell nucleus, all calculated in the complete $0d_{5/2}$-$1s_{1/2}$-$0d_{3/2}$ basis space from a unified description of the model Hamiltonian (Ref. 6). If they are sufficiently accurate representations of the actual nuclear states, these wave functions offer the possibility of a systematic analysis of *sd*-shell data. Studies with them to date indicate that their accuracy may indeed be sufficient to make this approach useful.

As mentioned, these shell-model wave functions and the procedure implied by the above equations have been used to analyze magnetic moments and analogous GT decays for mirror ground states of *sd*-shell nuclei. They have also been used in a comprehensive analysis of all *sd*-shell GT decay data. The present study deals with data from $M1$ transitions between *sd*-shell states. Since these $M(M1)^{\text{exp}}$ involve the wave functions of many excited states as well as of the ground states, many more model wave functions are involved than were used in the mirror-state magnetic moment study.

The excited states of a nucleus should involve rather different structural features than those dominating the ground states. Hence, the transition data constitute an extended test of the internal consistency of the shell-model components of the analysis. Moreover, these structural differences are reflected in average values of the D factors which are different from those used in the mirror-state study. Hence, some δ values which were poorly determined in the analysis of the magnetic moments can be much better determined from the transition data. For example, the $D(d_{5/2},d_{3/2})$ transition density is never dominant in the ground-state wave functions and hence the value of the corresponding effective single-particle matrix element $S(d_{5/2},d_{3/2})^{\text{eff}}$ is correspondingly poorly determined from data sets consisting only of ground states. On the other hand, the $D(d_{5/2},d_{3/2})$ term is the largest contributor to many transitions and hence these data should provide a good determination of the empirical value for $S(d_{5/2},d_{3/2})^{\text{eff}}$.

III. EXPERIMENTAL FOUNDATIONS OF THE ANALYSIS

We have made a compilation (Ref. 7) of electromagnetic transition data for *sd*-shell nuclei with the aim of identifying experimentally well-characterized pairs of states which appear to have counterparts in the spectra of theoretical states generated from an *sd*-shell model basis. The foundation of our compilation was obtained from the values of spin, isospin, and parity assignments and lifetime, branching-ratio, and mixing-ratio values which are presented in the invaluable collection of Endt and van der Leun (Ref. 8). We have supplemented the "consensus" experimental values of Endt and van der Leun with results from all later experimental results we could find which bear on these same spectroscopic properties and, where appropriate, have rederived our own "consensus" values. A more detailed description of our compilation is in preparation (Ref. 7). From these compiled properties of the experimentally determined electromagnetic transition parameters of *sd*-shell nuclear levels we calculated the corresponding values of $M(E2)^{\text{exp}}$ and $M(M1)^{\text{exp}}$ and their associated uncertainties. The values of $M(M1)^{\text{exp}}$ from this compilation are those used in the present study. The values were matched with the appropriate values of the transition densities D and the set of equations corresponding to Eq. (9) were thus constructed.

The number of entries in Eq. (9) which we use to determine the empirical values of the correction factors δ is limited by the existence of appropriate experimental data. From the shell-model calculations we have available as many states $NJTn$ (all of positive parity, of course) as we desire, and, in turn, as many sets of D values. The first criterion for an experimental matrix element to be an acceptable entry for our data set is that it must be possible to make a unique association between the states involved in the observed transition and states $NJTn$ and $NJ'T'n'$ of the model spectra. This requires that the experimental states have secure assignments of positive parity and definite values of J and, where appropriate, T. Moreover, all lower-lying states in the experimental spectrum must be completely characterized so that the ordering index N is also unambiguously known.

Almost all of the data for such unambiguously characterized transitions is included in our data set. The exceptions are some transitions in the $A=18$, 19, 20, 36, 37, 38 nuclei which involve so-called "intruder" states. These are positive-parity states for which considerations of energy and spectroscopic factor essentially rule out an *sd*-shell origin. It is presumed that their "parent" configurations involve multiparticle, multihole excitations across the *sd*-shell boundaries. The excitation energies of such states move up rapidly as the mass numbers move away from the shell boundaries of $A=16$ and 40. For $A=18$, 19, 20, 36, 37, and 38 we accept into the data set only transitions between the lowest-lying, most securely understood states. For $A=21-35$ we assume that none of the "unambiguously characterized" transitions involve intruder states.

Of course, there are many pairs of unambiguously identified experimental states for which a value of $M(M1)^{\text{exp}}$ is not available. The branching ratios for electromagnetic decay of a level are the most commonly available data beyond the knowledge of its existence and excitation energy. However, the lifetimes or widths of many transitions are unknown, either because they fall outside the province of current measurement techniques or just because the relevant experiment has not been performed. Perhaps more surprising is that many experimental transitions for which lifetime data are available do not yield values of $M(M1)^{\text{exp}}$ because the $E2/M1$ mixing ratio for the transition has not been measured. This factor reduces the potential data set by a factor of 2. Our

acceptance criteria and the available experimental results combine to yield 250 entries in our data set of $M(M1)^{\text{exp}}$ values.

IV. OVERVIEW OF THE POTENTIAL DIFFICULTIES WITH THE ANALYSIS

It is useful to take an overview before discussing the details of the present study. This overview is provided by decades of general study and analysis of $M1$ phenomena and considerable work with detailed shell-model analyses such as we apply here. The primary fact which emerges from these studies is that the standard, free-nucleon form of the $M1$ operator is not a bad first approximation to the empirically optimum prescription. Putting it another way, values of $M(M1)^{\text{sm}}$ calculated with the free-nucleon values of g_s and g_l yield agreement with experimental values good enough that a casual inspection does not always detect any room for improvement, particularly when the inevitably necessary allowances are made for random inadequacies in the model wave functions (values of D). This means that in searching for higher-order corrections to $O(M1)$ we are searching for effects which are small at best and, at worst, invisible. As a consequence, our analyses must be precise and comprehensive, and precautions must be taken to evaluate both random and systematic sources of error.

In particular, the validity of the present procedure rests very heavily on the accuracy with which the shell-model wave functions represent, or "model," nature. Small systematic deviations between the model wave functions and the corresponding aspects of the nuclear states they represent will tend to be transformed, through the D factors in Eq. (3), into spurious contributions to the values of the δ. Of course, the δ corrections we actually seek also are reflections of systematic deficiencies in the wave functions, but these latter are deficiencies which are intrinsic to the basic model assumptions, deficiencies such as the exclusion from the model of nucleon resonances, mesonic-exchange currents, and non-sd-shell configurations.

The deficiencies against which we have to be on guard in our study correspond to nonoptimum utilization of the degrees of freedom which are included in the model space. Examples would be too much occupation of the $d_{5/2}$ orbit coupled with too little of the $d_{3/2}$ orbit, too much configuration mixing between the three sd orbits in general, too little configuration mixing, etc. In addition to such systematic deficiencies, random inaccuracies in the wave functions could also impair our study, because the "noise" thereby introduced into the least-squares solutions would tend to mask the desired correction "signals."

Studies to data have not revealed clear-cut systematic deviations between the current model wave functions and the states they are designed to represent. These studies have treated the densities of energy levels (Ref. 6), single-nucleon transfer (Ref. 6), electric quadrupole matrix elements (Ref. 9), and the previously mentioned $M1$ moments (Ref. 4) and GT transition studies (Ref. 5). Relative spectroscopic factors for single-nucleon transfer yield a direct measure of whether the occupations of the active model orbits in the calculated wave functions are similar to those in the physical states. In the cases of the $E2$, $M1$, and GT matrix elements, internal consistency in the relationships between theoretical and experimental matrix elements over the entire range of sd-shell masses is a primary criterion for whether systematic deficiencies are present. In the present study we have investigated the issue of systematic errors in the shell-model wave functions (and, of course, in the data as well) by carrying out the procedure for extracting the δ correction factors in several different contexts, as will be described in the following discussion.

Inevitably, some level of random error must exist in the model wave functions. It is important to have an estimate of the magnitudes of these "model" errors relative to the typical experimental errors. The value of the rms deviation between theory and experiment suggests an average model uncertainty in $M(M1)^{\text{sm}}$ of between 0.1 and 0.2 nuclear magnetons. This value is about 5–10% of the magnitudes of the single-particle matrix elements and of the largest multiparticle matrix elements. It is also an order of magnitude larger than the uncertainties in the values of a significant fraction of the experimental matrix elements, while comparable to those of many others. This disparity between the typical model uncertainty and some of the experimental errors has the potential consequence of giving inordinate weight to the D factors associated with very accurate data. To avoid this, and to ensure a more equitable, statistically safer sampling of the model predictions in the set of Eq. (9) we add in quadrature to the experimental errors a "model" uncertainty of 0.100 nuclear magnetons. This specific value is arbitrary, but it produces values of chi (see footnote to Table I) which are close to unity.

There is another probable correlation between relative uncertainties and the role of the data in the shell-model analysis. The transitions which have been more frequently studied and more accurately measured tend to be those involving the ground state and first few excited states. The model wave functions for these same states are probably more reliable than those for higher-lying states, since increasing excitation energy correlates with increasing level density and an intrinsic source of error in the model wave functions is the mixing between wave functions of states of nearly degenerate energies.

Our $M(M1)^{\text{exp}}$ data set contains values from 22 pure isoscalar transitions ($T=0$ to $T=0$) and a few pure isovector transitions ($T=0/1$ to $T=1/0$), but the great preponderance come from transitions for which both isoscalar and isovector components are allowed. As is well known, the net strength of the isovector component of the $M1$ operator is much larger than that of the isoscalar component. In a transition which allows both components, the value of $M(M1)$ is, on the average, strongly dominated by the isovector component. As a corollary, the details of the isoscalar part of the $M1$ operator have very little effect upon the value of $M(M1)^{\text{sm}}$ for such transitions and such details cannot be extracted from a fit of the type of Eq. (9) when the data set consists primarily of such isovector-dominanted terms. As a consequence

of this situation, treatment of the corrections to the isoscalar component of the free-nucleon form of the $M1$ operator must focus on the pure isoscalar transitions. We will discuss this topic in a separate subsection.

In applying the procedure of Eq. (9) to our full data set we have chosen to fix the values of the isoscalar components and obtain the least-squares solution by allowing variation only of the coefficients of the isovector components. Because of the negligible contribution of the isoscalar $M1$ components to the aggregate magnitude of $M1$ matrix elements in our data set, it is inconsequential whether the standard form or the "renormalized" form of the isoscalar part of the $M1$ operator is used to evaluate these fixed isoscalar contributions. The results presented in this paper for isovector corrections are obtained by using the renormalized parametrization of the isoscalar operator obtained in the "$A=18-38$" fit of Ref. 3, namely, $\delta_s(ISM1,d-d)=-0.138$, $\delta_s(ISM1,s-s)=-0.056$, $\delta_l(ISM1)=0.022$, and $\delta_p(ISM1,d-d)=\delta_p(ISM1,s-d)=0.034$. As a result of removing the isoscalar terms from active participation, extraction of higher-order corrections involves determining four unknowns from the set of Eq. (9), these unknowns being $\delta_s(d-d)$, $\delta_s(s-s)$, δ_l, and δ_p for the isovector $M1$ operator.

V. RESULTS OF THE ANALYSIS

A. Dependence of the extracted correlation values upon the relative experimental errors of the data

As a first attempt in studying the reliability of our procedure we solve the set of Eq. (9) for four different (nested) subsets of the 228-element basic set of $M1$ data. These subsets consist of (a) the entire 228-element set, (b) the 192-element subset consisting of data with relative experimental errors smaller than 20%, (c) the 117-element subset consisting of data with relative experimental errors smaller than 10%, and (d) the 41-element subset consisting of data with relative experimental errors smaller than 5%. The results of these solutions are presented in Table I.

If one assumes that the experimental errors are uncorrelated with any underlying aspect of nuclear structure and are in themselves completely accurate, then the quoted errors just provide a mechanism for randomly segmenting the data set. The results shown in Table I indicate that the level of agreement of the shell-model predictions $M(M1)^{\text{free}}$ and $M(M1)^{\text{eff}}$ with the experimental values $M(M1)^{\text{exp}}$ is quantitatively the same for any subset of the data. Quantitatively, the two data sets incorporating the larger errors are less well fitted, either with the free-nucleon or with the effective form of the operator, than are the smaller sets of the more accurate data. This could result from the largest errors still being underestimates of the true experimental errors. Another possibility is that the wave functions for higher-lying states, for which uncertainties are typically larger, are less accurate estimates of the many-nucleon structure of the actual states than are those for the ground and low-lying states. The agreement between experiment and theory is virtually the same for the 5% and 10% data sets. As a compromise between best agreement in terms of rms dev and smallest uncertainties associated with the correction terms, we select the results from the 10% error data set as the optimum solution.

B. Dependence of the extracted correction factors upon the nuclear mass number

The various subsets of data treated in the analyses summarized in Table I are not correlated with mass number. Because of the ordered filling of the orbits of the *sd* shell between mass numbers $A=16$ and 40, the mass numbers are correlated with the dominance of the various *sd* orbits in the D values, the $d_{5/2}$ orbit being most important for the lighter masses, the $s_{1/2}$ orbit for the region around $A=30$, and the $d_{3/2}$ orbit for the heavier masses. To study the dependence of the extracted correction values upon this aspect of the data set and the model wave functions we have also analyzed the total data set in subsets categorized in terms of mass numbers. The first subset consists of the 40 data from the $A=18-24$ nuclei, the second subset consists of the 66 data from the $A=25-27$ nuclei, the third subset of the 62 data from the $A=28-31$ nuclei, and the fourth subset of the 60 data from the $A=32-38$ nuclei. These subsets were

TABLE I. Values of isovector operator corrections as a function of the size/accuracy of the set of experimental data.

Number of elements	$<100\%$ errors 228	$<20\%$ errors 180	$<10\%$ errors 107	$<5\%$ errors 41
χ(free)[a]	2.30	2.39	2.11	2.17
χ(eff)[a]	1.66	1.72	1.42	1.33
$\delta_s(d-d)$	−0.20(2)	−0.20(2)	−0.16(2)	−0.16(3)
$\delta_s(s-s)$	−0.17(3)	−0.15(3)	−0.14(3)	−0.18(4)
δ_l	0.025(4)	0.027(5)	0.027(5)	0.031(7)
δ_p	0.10(1)	0.09(1)	0.09(1)	0.07(2)

[a] $\chi^2=\sum_n^{N_d}\{[M(M1,f,i)_n^{\text{exp}}-M(M1,f,i)_n^{\text{eff}}]/\Delta_n\}^2/(N_d-N_p)$,

where N_d are the number of data, N_p are the number of parameters, and Δ_n is the error for each transition (the experimental error folded in with a theoretical error of $0.1\mu_N$).

TABLE II. Values of the isovector operator corrections as a function of the mass number.

Number of elements	$A=18-24$ 40	$A=25-27$ 66	$A=28-31$ 62	$A=32-38$ 60
χ(free)[a]	2.44	2.29	2.06	2.69
χ(eff)[a]	1.64	1.52	1.74	1.66
$\delta_s(d-d)$	−0.17(4)	−0.24(3)	−0.19(5)	−0.28(4)
$\delta_s(s-s)$	0.52(20)	−0.20(5)	−0.19(5)	−0.17(4)
δ_l	0.03(3)	0.03(1)	0.03(1)	0.01(1)
δ_p	0.12(13)	0.13(3)	0.08(2)	0.08(2)

[a]See footnote to Table I.

chosen in part to have approximately equal numbers of elements and in part to correspond in terms of nuclear structure to, first, the region of strong prolate deformation dominated by strongly mixed $d_{5/2}$ and $s_{1/2}$ configurations; second, the region of quasispherical $d_{5/2}$ hole systems below ^{28}Si; third, the region in which the $s_{1/2}$ orbit is being fitted; and fourth, the region in which the $d_{3/2}$ orbit is being filled. The results are summarized in Table II.

There is less consistency between the values of the corrections extracted for these different mass regions than was obtained for the different groupings of experimental accuracy. This is not surprising given that the mass regions incorporate significant fundamental differences in the underlying nuclear structure while the different subsets classified on the basis of accuracy are distributed uniformly over the entire range of nuclei considered. The most notable excursion from the "typical" values of the corrections occurs for the $\delta_s(s-s)$ term in the 18–24 data set. All the other data sets yield a value of this term of around -0.20 ± 0.05, while the 18–24 value is $+0.5\pm0.2$ The much larger uncertainty in this value is indicative that the 18–24 data set does not provide a good fix for this component of the operator, but this error is not large enough to remove the inconsistency. The only other appreciable inconsistency occurs for the 32–38 data set, for which the parameter $\delta_s(d-d)$ is larger and, perhaps in correlation, the parameter δ_l smaller, than the typical values.

C. Isoscalar results

As noted, roughly 10% of experimentally characterized $M1$ transitions occur between states of nominally pure $T=0$ nature. Thus, to a first approximation, these data should yield information on the corrections to the isoscalar aspects of the $M1$ operator. The data set used in this work encompasses 22 such transitions, as listed in Table III.

TABLE III. Experimental and theoretical matrix elements for *sd*-shell $M1$ transitions between $T=0$ states. The transitions are labeled by the values of A and Z of the nucleus, the values of $2J$, $2T$, n, and the excitation energies for the initial and final states. The values of the free-nucleon and empirically corrected (effective) versions of the shell-model predictions are given along with the experimental values and their uncertainties.

		Initial state				Final state				$M(M1)$ (μ_N)		
A	Z	$2J$	$2T$	n	E_x (keV)	$2J$	$2T$	n	E_x (keV)	Free	Eff.	Experiment
22	11	8	0	1	890.9	6	0	1	0.0	0.126	0.085	0.064±0.004
22	11	10	0	1	1528.1	8	0	1	890.9	0.169	0.120	0.076±0.005
22	11	4	0	1	3059.6	2	0	1	583.0	0.215	0.154	0.277±0.018
22	11	12	0	1	3706.6	10	0	1	1528.1	0.175	0.126	0.075±0.037
24	12	4	0	2	4238.4	4	0	1	1368.6	0.001	0.005	0.008±0.003
24	12	6	0	1	5236.1	4	0	1	1368.6	0.000	0.007	0.016±0.004
24	12	2	0	1	7747.2	0	0	1	0.0	0.015	0.005	0.065±0.008
24	12	10	0	1	7812.0	8	0	1	4122.8	0.007	0.021	0.102±0.046
26	13	6	0	3	2545.2	6	0	1	416.8	0.053	0.032	0.063±0.014
28	14	6	0	1	6276.3	4	0	1	1778.8	0.045	0.049	0.058±0.003
28	14	2	0	1	8328.3	0	0	1	0.0	0.089	0.066	0.029±0.006
28	14	2	0	2	9497.3	0	0	1	0.0	0.175	0.121	0.156±0.034
30	15	4	0	1	1454.7	2	0	1	0.0	0.141	0.132	0.114±0.007
30	15	4	0	2	2724.0	2	0	1	0.0	0.132	0.090	0.093±0.012
32	16	2	0	1	4695.4	0	0	1	0.0	0.056	0.045	0.043±0.003
32	16	2	0	1	4695.4	4	0	1	2230.3	0.101	0.098	0.115±0.023
34	17	4	0	1	1230.2	6	0	1	146.4	0.088	0.078	0.029±0.016
34	17	4	0	1	1230.2	2	0	1	461.0	0.112	0.106	0.062±0.018
34	17	4	0	1	1230.2	2	0	2	665.6	0.189	0.158	0.161±0.007
34	17	4	0	2	1887.3	6	0	1	146.4	0.103	0.088	0.076±0.028
34	17	4	0	2	1887.3	2	0	1	461.0	0.107	0.085	0.090±0.020
34	17	8	0	1	2376.1	6	0	1	146.4	0.114	0.103	0.073±0.023

It is impossible to determine all four of the isoscalar correction terms as was done in Sec. V A and V B for the isovector $M1$ operator, since for an isoscalar $M1$ transition the δ_l and δ_s corrections are fundamentally linearly dependent. This can be seen from the fact that the expectation value of $\langle J_z \rangle$ is zero for these transitions, which implies that $\langle \sum_k s^k \rangle = -\langle \sum_k l^k \rangle$ and thus that only the linear combination $g_s^{\text{eff}} - g_l^{\text{eff}}$ enters in Eq. (6). Hence, in determining the empirical isoscalar corrections from a fit to these data, the parameters $\delta_s(d-d)$, $\delta_s(s-s)$, and δ_p

TABLE IV. Experimental and theoretical matrix elements for *sd*-shell $M1$ transitions with explicit isovector components. The transitions are labeled by the values of A and Z of the nucleus, the values of $2J$, $2T$, n, and the excitation energies for the initial and final states. The values of the free-nucleon and empirically corrected (effective) versions of the shell-model predictions are given along with the experimental values and their uncertainties.

		Initial state				Final state				$M(M1)$ (μ_N)		
A	Z	$2J$	$2T$	n	E_x (keV)	$2J$	$2T$	n	E_x (keV)	Free	Eff.	Experiment
18	9	0	2	1	1041.6	2	0	1	0.0	4.047	3.617	4.320±0.320
19	8	3	3	1	96.0	5	3	1	0.0	0.257	0.204	0.359±0.006
20	9	6	2	1	655.9	4	2	1	0.0	1.541	1.555	1.894±0.073
21	10	5	1	1	350.7	3	1	1	0.0	0.983	0.891	0.878±0.009
21	10	7	1	1	1747.2	5	1	1	350.7	1.582	1.528	1.405±0.045
21	10	9	1	1	2867.2	7	1	1	1745.6	2.079	2.003	1.987±0.085
21	11	5	1	1	311.9	3	1	1	0.0	1.125	0.982	0.953±0.006
21	11	7	1	1	1716.0	5	1	1	331.9	1.764	1.646	1.603±0.081
22	10	2	2	1	5336.0	0	2	1	0.0	0.521	0.647	0.666±0.064
22	11	0	2	1	657.0	2	0	1	583.0	2.745	2.564	2.228±0.031
22	11	2	0	2	1936.9	0	2	1	657.0	1.942	1.509	2.257±0.212
22	11	4	0	1	3059.6	4	2	1	1951.8	2.794	2.225	2.224±0.124
23	11	5	1	1	439.8	3	1	1	0.0	1.557	1.442	1.582±0.044
23	11	7	1	1	2076.4	5	1	1	439.8	1.615	1.396	1.465±0.020
23	11	9	1	1	2703.7	7	1	1	2076.4	2.580	2.455	2.656±0.088
23	11	3	1	2	2982.4	3	1	1	0.0	1.356	1.035	1.008±0.052
23	11	3	1	2	2982.4	5	1	1	439.8	1.735	1.280	1.085±0.057
23	11	5	1	2	3914.7	3	1	1	0.0	0.626	0.443	0.540±0.048
23	11	7	1	2	4775.5	5	1	1	439.8	1.305	1.053	1.277±0.032
23	11	7	1	2	4775.6	7	1	1	2076.4	2.010	1.604	1.767±0.066
23	12	5	1	1	450.7	3	1	1	0.0	1.390	1.335	1.437±0.080
24	11	4	2	1	563.3	2	2	1	472.3	2.816	2.915	2.662±0.205
24	11	4	2	3	1846.1	2	2	1	472.3	0.001	0.005	0.338±0.024
24	11	6	2	2	1885.4	8	2	1	0.0	0.467	0.597	0.738±0.069
24	11	6	2	2	1885.4	4	2	1	563.3	1.891	1.741	1.717±0.159
25	11	3	3	2	2202.0	5	3	1	0.0	1.262	0.821	0.560±0.048
25	11	3	3	2	2202.0	1	3	1	1069.3	0.920	1.044	1.083±0.099
25	12	3	1	1	974.8	5	1	1	0.0	0.179	0.135	0.082±0.002
25	12	3	1	1	974.8	1	1	1	585.1	0.375	0.371	0.336±0.010
25	12	7	1	1	1611.8	5	1	1	0.0	2.099	2.221	2.236±0.160
25	12	9	1	1	3405.2	7	1	1	1611.8	2.582	2.718	2.773±0.084
25	13	5	1	2	1789.6	3	1	1	944.8	0.174	0.286	0.634±0.050
26	12	4	2	2	2938.4	4	2	1	1808.7	0.831	0.964	1.143±0.107
26	12	6	2	1	3940.5	4	2	1	1808.7	0.083	0.081	0.127±0.011
26	12	6	2	1	3940.5	4	2	2	2938.4	0.370	0.369	0.503±0.044
26	12	4	2	4	4834.0	4	2	2	2938.4	0.408	0.489	0.908±0.089
26	13	2	0	1	1057.7	0	2	1	228.4	3.121	3.145	2.884±0.280
26	13	2	0	2	1850.3	0	2	1	228.4	1.395	0.995	0.943±0.042
26	13	2	0	3	2071.5	0	2	1	228.4	0.435	0.271	0.227±0.009
26	13	2	0	4	2739.2	0	2	1	228.4	0.404	0.232	0.490±0.027
27	12	3	3	1	984.7	1	3	1	0.0	0.354	0.414	0.402±0.014
27	12	5	3	2	1940.0	3	3	1	984.7	0.343	0.405	0.483±0.044
27	13	3	1	1	1014.5	5	1	1	0.0	0.016	0.190	0.298±0.006
27	13	3	1	1	1014.5	1	1	1	843.8	0.569	0.765	0.791±0.020
27	13	7	1	1	2211.1	5	1	1	0.0	1.023	0.863	0.940±0.012
27	13	5	1	2	2734.9	5	1	1	0.0	0.640	0.447	0.524±0.042
27	13	5	1	2	2734.9	3	1	1	1014.5	2.216	1.964	1.963±0.152
27	13	3	1	2	2981.3	5	1	1	0.0	1.751	1.296	1.224±0.032
27	13	9	1	1	3004.2	7	1	1	2211.1	1.111	1.302	1.252±0.068

TABLE IV. (*Continued*).

		Initial state				Final state				$M(M1)$ (μ_N)		
A	Z	$2J$	$2T$	n	E_x (keV)	$2J$	$2T$	n	E_x (keV)	Free	Eff.	Experiment
27	13	5	1	3	4410.2	5	1	1	0.0	1.333	1.104	1.211±0.082
27	13	5	1	3	4410.2	3	1	1	1014.5	1.439	1.294	1.362±0.104
27	13	11	1	1	4510.3	9	1	1	3004.2	0.416	0.262	0.311±0.022
27	14	3	1	1	957.1	5	1	1	0.0	0.088	0.212	0.339±0.010
27	14	7	1	1	2163.7	5	1	1	0.0	0.847	0.745	0.782±0.034
27	14	5	1	2	2647.0	5	1	1	0.0	0.515	0.354	0.364±0.036
27	14	5	1	2	2647.0	3	1	1	957.1	1.873	1.712	1.502±0.098
28	13	4	2	1	30.6	6	2	1	0.0	1.800	1.940	1.820±0.021
28	13	2	2	1	1372.8	4	2	1	30.6	0.461	0.280	0.345±0.027
28	13	2	2	1	1372.8	0	2	1	972.2	1.574	1.455	1.822±0.144
29	12	1	5	1	54.6	3	5	1	0.0	0.486	0.604	0.618±0.017
29	14	3	1	1	1273.3	1	1	1	0.0	0.219	0.359	0.508±0.009
29	14	5	1	1	2028.2	3	1	1	1273.3	0.542	0.311	0.331±0.015
29	14	3	1	2	2425.6	1	1	1	0.0	0.997	0.859	0.710±0.032
29	14	3	1	2	2425.6	3	1	1	1273.3	0.851	0.784	0.876±0.049
29	15	3	1	1	1383.6	1	1	1	0.0	0.312	0.440	0.646±0.049
29	15	5	1	1	1953.9	3	1	1	1383.6	0.732	0.398	0.639±0.053
30	14	4	2	2	3498.7	4	2	1	2235.4	0.846	0.964	0.955±0.043
30	15	0	2	1	677.3	2	0	1	0.0	1.070	1.008	1.161±0.051
30	15	4	2	1	2937.9	4	0	1	1454.7	1.056	1.132	0.663±0.024
31	15	3	1	1	1266.2	1	1	1	0.0	0.250	0.328	0.370±0.012
31	15	1	1	2	3134.1	1	1	1	0.0	0.746	0.601	0.609±0.022
31	15	5	1	2	3295.0	3	1	1	1266.2	0.617	0.567	0.490±0.043
31	15	5	1	2	3295.0	5	1	1	2233.7	0.754	0.834	0.666±0.061
31	15	3	1	2	3505.8	1	1	1	0.0	0.770	0.533	0.466±0.038
31	15	7	1	2	4633.8	5	1	1	2233.7	0.077	0.144	0.256±0.019
31	15	7	1	2	4633.8	5	1	2	3295.0	0.645	0.621	0.721±0.052
31	15	7	1	2	4633.8	7	1	1	3414.6	1.069	1.071	0.864±0.062
31	15	1	1	4	5256.1	1	1	1	0.0	0.703	0.580	0.642±0.051
31	15	1	3	1	7140.6	1	1	1	0.0	0.965	0.898	0.892±0.046
32	15	2	2	2	1149.7	0	2	1	513.0	1.324	1.250	1.196±0.093
32	15	6	2	1	1754.5	4	2	1	78.1	0.152	0.127	0.313±0.021
32	15	6	2	1	1754.5	4	2	2	1323.2	0.546	0.591	0.438±0.027
32	15	6	2	2	2177.8	4	2	1	78.1	0.753	0.767	0.787±0.070
32	15	8	2	1	3149.4	6	2	2	2177.8	0.397	0.343	0.463±0.027
33	15	3	3	1	1431.6	1	3	1	0.0	0.120	0.282	0.296±0.022
33	15	7	3	1	3627.6	5	3	1	1847.6	0.027	0.319	0.347±0.033
33	16	1	1	1	840.9	3	1	1	0.0	0.248	0.363	0.333±0.004
33	16	5	1	1	1966.3	3	1	1	0.0	0.215	0.281	0.478±0.032
33	16	3	1	2	2312.5	1	1	1	840.9	0.498	0.430	0.525±0.043
33	16	9	1	1	4047.8	7	1	1	2968.6	0.189	0.322	0.365±0.035
33	16	7	1	2	4094.0	5	1	1	1966.3	1.180	0.912	0.931±0.073
33	17	5	1	1	1986.5	3	1	1	0.0	0.320	0.374	0.681±0.068
34	16	4	2	2	3303.2	4	2	1	2127.3	0.657	0.807	0.698±0.024
34	16	4	2	3	4114.1	4	2	1	2127.3	0.330	0.307	0.362±0.032
34	17	2	0	1	461.0	0	2	1	0.0	0.573	0.325	0.482±0.016
34	17	2	0	2	665.6	0	2	1	0.0	0.231	0.346	0.209±0.006
34	17	4	2	1	2158.2	6	0	1	146.4	0.272	0.413	0.299±0.017
34	17	4	2	1	2158.2	2	0	1	461.0	0.785	0.782	0.892±0.047
34	17	4	2	1	2158.2	4	0	1	1230.2	0.678	0.441	0.600±0.047
34	18	4	2	2	3287.5	4	2	1	2090.9	0.597	0.754	0.722±0.066
35	17	1	1	1	1219.3	3	1	1	0.0	0.420	0.475	0.531±0.036
35	17	5	1	1	1763.2	3	1	1	0.0	0.059	0.074	0.107±0.005
35	17	7	1	1	2645.3	5	1	1	1763.2	0.305	0.394	0.462±0.042
36	17	2	2	1	1164.8	4	2	1	0.0	0.251	0.072	0.103±0.004
37	17	1	3	1	1726.6	3	3	1	0.0	0.234	0.374	0.335±0.027
37	18	5	1	2	3171.3	3	1	1	0.0	0.934	0.748	0.331±0.023
38	19	2	0	1	458.7	0	2	1	130.4	0.940	0.578	0.688±0.031

were varied and the δ_l value was held fixed at 0.022. The values thus obtained for these corrections are $\delta_s(ISM1,d-d)=-0.11\pm0.04$, $\delta_s(ISM1,s-s)=-0.04\pm0.07$, and $\delta_p(ISM1)=0.03\pm0.04$. These empirical corrections produce about a 30% improvement in the chi value for the 22-element data set. The predictions obtained with the free-nucleon operator and with this transition-based, empirically corrected operator are shown in Table III in comparison with the experimental values.

Inspection of Table III reveals that the experimental values of the isoscalar $M1$ matrix elements range in magnitude from about 0.15 to $0.30\mu_N$ down to many smaller values at the level of $0.05\mu_N$, with a few being as small as the order of $0.01\mu_N$. The free-nucleon and empirically corrected predictions each have a range similar to the experimental magnitudes. In only two of the 22 cases does the corrected operator yield predictions in significantly worse agreement with experiment than does the free-nucleon operator. In most cases the empirically corrected operator yields quite a significant improvement via à vis experiment, and, in most cases, but not all instances, this improvement corresponds to a reduction of the theoretical magnitude. That is, the empirically corrected isoscalar $M1$ operator yields a "quenched" transition strength on the average. (The complexities of the multiparameter correction and cancellations within the matrix element summation sometimes yield enhancements instead of reductions, of course.)

There are several reasons for maintaining some reservations about these isoscalar results. The data are few in number and the relative experimental uncertainties typically are large. Beyond these simple problems, however, lies the issue of isospin mixing. Since, in terms of $M1$ strength, the IV operator is 2 orders of magnitude stronger than the IS, isospin mixing at the level of only 1% could randomize the experimental data insofar as a meaningful analysis with the pure $ISM1$ theory is concerned. The data and theory certainly seem to have more correspondence with each other than would be implied by such a 1% IV/IS wave-function mixing. This ultimately may yield interesting insight into the actual average magnitude of such mixing. However, the possibility remains that one or two cases of significant isospin mixing could seriously bias the overall conclusions drawn from a small data set such as Table III.

D. Isovector results

In Table IV the matrix elements of the 10%-error data set are presented in comparison with the free-nucleon empirically corrected predictions. The empirical corrections used are those listed in Table I for the 10%-error fit. In summary, these corrections amount to a 15% quenching of the spin operator, a 20% enhancement of the orbital operator, and a positive $[Y^{(2)}\otimes s]^{(1)}$ correction. From Table IV it can be seen that these (typically) IV-dominated $M1$ matrix elements range in magnitude from the order $2-3\mu_N$ down to $0.1-0.4\mu_N$.

Furthermore, it can be seen that the changes in the theoretical matrix element magnitudes between the free-nucleon and empirical correction formulation often amount to as much as $0.3-0.4\mu_N$. Hence, the predictions for even the largest matrix elements can change by 20–30% between the two formulations, while smaller matrix elements can change even more in relative magnitude. The combination of quenching and enhancement in the operator corrections yields a complex profile of changes in the net values of the total matrix elements. Of the 10% cases listed in Table IV, 63 are reduced in magnitude by the empirically corrected operator and 44 are increased.

It is this fluctuating cancellation between the spin and the orbital and tensor terms that creates the original impression that there is little evidence for need of a correction for the $IVM1$ operator, in contrast to the case for the GT operator. Actually, as we have seen from Table I, the spin operator is significantly quenched for the $M1$ case, but the compensating orbital enhancement masks this effect. The net consequence is that the magnitudes of the free-nucleon results are approximately equal to the experimental values on the average, but that there is a significant scatter. The empirical corrections yield a significant reduction of this scatter. Of the 10% cases, the four-parameter adjustment moves 102 theoretical matrix elements in the direction of the experimental value and 25 away from the experimental value. The experimental and theoretical matrix elements for these cases are compared graphically in Fig. 1 of Ref. 4.

VI. CONCLUSIONS

The present study of $M1$ transitions in the sd shell yields corrections to the free-nucleon form of the $M1$ operator which are consistent with the analogous results obtained from the analysis of ground-state magnetic dipole moments. This is true in quantitative detail for the isovector component of the operator and at a qualitative level for the isoscalar operator. The corrections are shown to be stable as a function of the size and accuracy of the data set. When the data are segmented according to A value, the corrections show a few fluctuations from set to set, but are constant for the most part in this context also.

This constancy of the empirical corrections to the $M1$ operator extracted from different sets of nuclear states (ground states versus excited states, measurements of high precision versus those of low precision, lighter mass versus medium mass versus heavier mass) suggests that they reflect a fundamental attribute of the sd-shell model space. Hence, a theoretical understanding of these values would constitute an important advance in understanding the effects of excluded shell-model configurations and nucleonic-mesonic degrees of freedom upon the observables of nuclear physics. We refer the reader to Refs. 1–4 for detailed discussions of these issues.

This research was supported in part by the National Science Foundation under Grant Nos. PHY-87-18772 and PHY-87-14432.

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[1]I. S. Towner and F. C. Khanna, Nucl. Phys. **A399**, 334 (1983); I. S. Towner, Phys. Rep. **155**, 264 (1987).

[2]A. Arima, K. Shimizu, W. Bentz, and H. Hyuga, Adv. Nucl. Phys. **18**, 1 (1987).

[3]B. A. Brown and B. H. Wildenthal, Phys. Rev. C **28**, 2397 (1983).

[4]B. A. Brown and B. H. Wildenthal, Nucl. Phys. **A474**, 290 (1987).

[5]B. A. Brown and B. H. Wildenthal, At. Data Nucl. Data Tables **33**, 347 (1985).

[6]B. H. Wildenthal, in *Progress in Particle and Nuclear Physics,* edited by D. H. Wilkinson (Pergamon Press, Oxford, 1984), Vol. 11, p. 5.

[7]B. H. Wildenthal and J. Keinonen (unpublished).

[8]P. M. Endt and C. van der Leun, Nucl. Phys. **A310**, 1 (1978).

[9]M. Carchidi, B. H. Wildenthal, and B. A. Brown, Phys. Rev. C **34**, 2280 (1986).