Application of a cubic barrier in exotic decay studies

G. Shanmugam and B. Kamalaharan
Department of Physics, Presidency College, Madras 600 005, India
(Received 5 February 1987)

In exotic decay studies, the branching ratios for spontaneous emissions of fragments heavier than alpha particle have been found to be very sensitive to the shape of the potential barrier. In order to fix the top of barrier correctly, finite range effects are included in our calculations. Experimental Q values for different decay modes are chosen so as to incorporate the shell effects. The shape of the barrier in the overlapping region is approximated by a third-order polynomial suggested by Nix. The cubic barrier is found to be more suitable near the penetrating region. This model is applied to calculate the branching ratios for the spontaneous emission of heavier fragments. The results obtained compare well with those of other theoretical models and experimental values.

I. INTRODUCTION

In the year 1939 itself, alpha decay processes were recognized to be similar to the spontaneous asymmetric fission. But it is conventional to treat the alpha decay on a quantum-mechanical foundation while fission was studied for a long time classically in terms of the liquid drop model. But after Strutinsky's hybrid model, microscopic methods penetrated fission theory. Thus Poenaru *et al.*[1] proved that the alpha decay or any other particle evaporation could be considered as a very asymmetric fission and the methods used in fission for the computation of Q values and half-lives can be adopted for treating such processes also. A numerical superasymmetric fission model[1] (NSAFM) was derived by them extending the liquid drop model (LDM), finite range of nuclear forces model, and Yukawa-plus exponential model to the systems with charge asymmetry different from the mass asymmetry and by using phenomenological shell corrections. This model was tested for alpha decay, the half-lives being computed with Wentzel-Krammers-Brillouin (WKB) method successfully used in fission. The NSAFM involving manifold numerical quadratures was found to be too slow to be used for a systematic search of new exotic decay modes and hence an analytical relationship for the half-life [analytical superasymmetric fission model[2] (ASAFM)] was derived and used by them.

The stability of a particular nucleus (A,Z) with respect to the split into a heavy (A_1,Z_1) and light (A_2,Z_2) fragments can be studied by using the deformation energy curve $V(R)$ of the system, R being the distance between the centers of the fragments. If the energy of the two nuclei at infinite separation is taken as the origin of the potential, the initial energy is equal to the Q value which can be computed from the experimental masses. For $Q>0$, the nucleus is unstable if $V(R)$ is monotonously decreasing with R or metastable if the fragments are held together by the potential barrier. In the latter case, there is a finite probability per unit time of penetrating this barrier by the quantum-mechanical tunneling effect.

In the two-center spherical parametrization of shapes during the deformation from the parent nucleus with a radius $R_0=r_0A^{1/3}$, to the touching point of the fragments, R is varied between $R_i=R_0-R_2$ and $R_t=R_1+R_2$ where $R_j=r_0A_j^{1/3}$, $(j=1,2)$ and $r_0=1.2249$ fm. In the framework of LDM for separated spherical fragments, $R>R_t$, only the Coulomb interaction energy $Z_1Z_2e^2/R$ has been considered by Poenaru *et al.*[2] and the maximum of the potential energy at $R=R_t$ was $E_i=Z_1Z_2e^2/R_t$, where e is the electron charge. In the overlapping region, a convenient analytical approximation of the potential energy curve $V(R)$ going from $V(R_i)=Q$ to $V(R_t)=E_i$ which allows them to get a closed formula for the half-life T is a second-order polynomial in R. The elegant ASAFM is known to have a disadvantage that, in it, the height of the fission barrier is overestimated.

Shi and Swiatecki[3] have independently developed another barrier penetration model based on the proximity plus Coulomb potential (PPCPM) which has the virtue of no adjustable parameters. The inclusion of the nuclear proximity attraction by them reduces the barrier height closer to the experimental values. The disadvantage of this model is that the zero-point vibration energy E_v cannot be incorporated explicitly.

Our aim is to develop a model in which the height of the barrier is correctly determined and E_v is explicitly taken. In order to fix the top of the barrier correctly, finite-range Yukawa-plus-exponential model with latest constants[4,5] is used in our calculations. Q values for different decay modes are calculated using the latest mass table[6] and used so as to incorporate the shell effects at the ground states. The shape of the barrier in the overlapping region which connects the ground state and the contact point is approximately by a third-order polynomial suggested by Nix.[7] According to Nix, the shape of the fission barrier departs from a parabola at very low energies. Although the true shape of the barrier is not known, there are nevertheless two firm guidelines that one should follow when choosing a shape for it; the barrier should be parabolic near its top, and it should have a

TABLE I. Comparison of interaction barrier heights.

Decay mode	Barrier height (MeV) LDM	YEM	Experimental
$^{221}Fr \rightarrow ^{207}Tl + ^{14}C$	37.3439	26.6349	28.4476
$^{221}Ra \rightarrow ^{207}Pb + ^{14}C$	37.0811	26.2740	27.9502
$^{222}Ra \rightarrow ^{208}Pb + ^{14}C$	36.3418	25.5570	27.2902
$^{223}Ra \rightarrow ^{209}Pb + ^{14}C$	37.4630	26.7025	28.4902
$^{224}Ra \rightarrow ^{210}Pb + ^{14}C$	38.7046	27.9625	29.8102
$^{225}Ac \rightarrow ^{211}Bi + ^{14}C$	39.5300	28.7198	30.4828
$^{226}Ra \rightarrow ^{212}Pb + ^{14}C$	38.6091	30.1728	29.8702
$^{231}Pa \rightarrow ^{207}Tl + ^{24}Ne$	47.7879	33.6222	32.3880
$^{232}U \rightarrow ^{208}Pb + ^{24}Ne$	47.1156	32.8608	31.5190
$^{233}U \rightarrow ^{209}Pb + ^{24}Ne$	48.8080	34.5784	33.3290

local minimum corresponding to the ground-state equilibrium configuration. The cubic shape is the simplest form which satisfies these two physical requirements. We therefore employ in the prescission region a cubic barrier whose form was given by Nix.[7] It should be emphasized that there is no evidence that the true shape of the potential is entirely cubic,[8] but for spontaneous fission yielding heavier fragments the assumption that it is cubic in the prescission region is intrinsically more reasonable than the assumption that it is parabolic. In Sec. II we describe the main features of our model. Section III contains the results obtained and conclusion.

II. THE MODEL

If the Q value of the reaction is taken as the origin, then the potential as a function of r (which is the distance of mass centers of the fragments) for the post-scission region is given by

$$V(r)=\frac{Z_1Z_2e^2}{r}+V_n(r), \quad r>r_t , \tag{1}$$

where

$$V_n(r)=-D\left[F+\frac{r-r_t}{a}\right]\frac{r_t}{r}\exp[(r_t-r)/a] ,$$

and $r_t=R_1+R_2$ is the sum of their equivalent sharp-surface radii. The depth constant D is given by

$$D=\frac{4a^3g_1g_2e^{r_t/a}[C_s(1)\cdot C_s(2)]^{1/2}}{r_0^2r_t} .$$

The constant F is given by

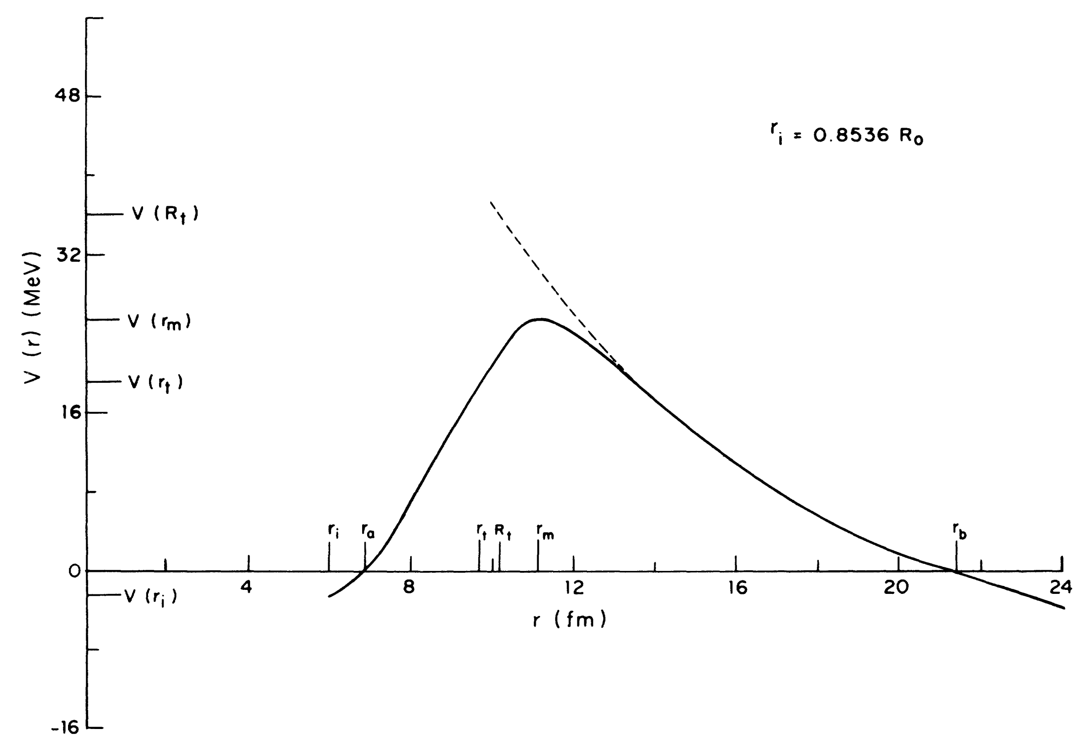

FIG. 1. The shape of the potential barrier used in this work (——) and the liquid drop model barrier in post-scission region (– – –) for ^{14}C emission from ^{222}Ra.

$$F=4+\frac{r_t}{a}-\frac{f_1}{g_1}-\frac{f_2}{g_2} ,$$

where

$$g_j=(R_j/a)\cosh(R_j/a)-\sinh(R_j/a) ,$$

$$f_j=(R_j/a)^2\sinh(R_j/a) ,$$

$$C_s(j)=a_s(1-K_sI_j^2) ,$$

$$I_j=(N_j-Z_j)/A_j, \quad (j=1,2) .$$

Here, $r_0=1.16$ fm; $a=0.68$ fm; $a_s=21.13$ MeV, and $K_s=2.3$.

The interaction barrier of the two fragments can be easily computed in this one-dimensional parametrization as the maximum of the interaction potential energy. For spherical fragments, this maximum $V(R_t)$ occurs at a distance of R_t from the origin in the LDM. But, in the Yukawa-plus-exponential model (YEM), the maximum $V(r_m)$, occurs at a distance of r_m that can be calculated by using the relation

$$\left[\frac{dV(r)}{dr}\right]_{r=r_m}=0 . \tag{2}$$

Q values are computed as

$$Q=[M(A,Z)-M(A_1,Z_1)-M(A_2,Z_2)] \times 931.501 \text{ MeV} , \tag{3}$$

$$A=A_1+A_2, \quad Z=Z_1+Z_2 .$$

In Table I we show the barrier heights calculated by LDM $[V(R_t)]$ and by YEM $[V(r_m)]$ which are compared with the experimental values obtained by the relation[9]

$$[V(r)]^{\max}=10.107+0.1021Z_1Z_2-Q . \tag{4}$$

It is seen that while the LDM overestimates the barrier heights by about 10 MeV or more, the YEM reproduces the experimental values which are uncertain by about 2 Mev.[10]

For the overlapping region, we approximate the barrier by a third-order polynomial in r having the form (see Figs. 1 and 2)

$$V(r)=-E_v+[V(r_t)+E_v]\left[s_1\left[\frac{r-r_i}{r_t-r_i}\right]^2 - s_2\left[\frac{r-r_i}{r_t-r_i}\right]^3\right] , \quad r_i<r<r_t , \tag{5}$$

where r_i is the distance between the centers of mass of two portions of a sphere cut (by a plane) in two pieces, with volume asymmetry of the decay in question. For r_i, we obtain the expression

$$r_i=\frac{3}{4}\left[\frac{h_1^2}{R_0+h_1}+\frac{h_2^2}{R_0+h_2}\right] , \tag{6}$$

where h_1 and h_2 are the heights of the spherical segments. For symmetric case ($h_1=h_2=R_0$), this reduces to $r_i=\frac{3}{4}R_0$.

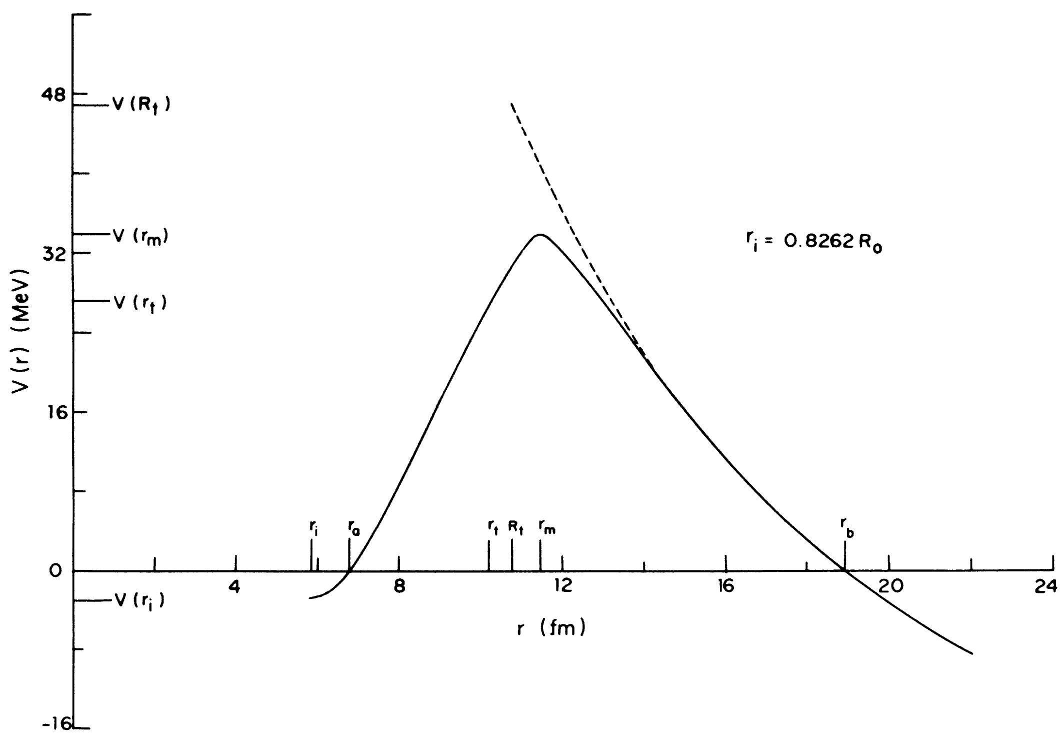

FIG. 2. Same as Fig. 1 for the case of ^{24}Ne emission from ^{232}U.

Table II. Comparison of theoretical and experimental values of $\mathrm{Log}_{10}(T/T_\alpha)$.

Decay mode	Theoretical Ref. 11	Ref. 3	this work	Experimental Ref. 15
$^{221}Fr \to {}^{207}Tl + {}^{14}C$	12.5	11.1	12.0	> 13.1
$^{221}Ra \to {}^{207}Pb + {}^{14}C$	11.9	11.1	11.7	> 12.9
$^{222}Ra \to {}^{208}Pb + {}^{14}C$	11.0	8.8	10.2	9.43
$^{223}Ra \to {}^{209}Pb + {}^{14}C$	8.5	8.2	8.2	9.21
$^{224}Ra \to {}^{210}Pb + {}^{14}C$	11.8	10.2	11.6	10.37
$^{225}Ac \to {}^{211}Bi + {}^{14}C$	12.2	11.8	12.4	> 12.4
$^{226}Ra \to {}^{212}Pb + {}^{14}C$	11.7	10.5	11.8	10.5
$^{231}Pa \to {}^{207}Tl + {}^{24}Ne$	10.0	11.0	10.5	11.22
$^{232}U \to {}^{208}Pb + {}^{24}Ne$	10.9	10.3	11.5	11.7
$^{233}U \to {}^{209}Pb + {}^{24}Ne$	10.3	10.4	11.2	12.12

The constants s_1 and s_2 appearing in Eq. (5) are determined by requiring that the value of the potential $V(r)$ and its first derivative be continuous at the contact point $r = r_t$.

When one wants to include E_v in the calculation of the lifetimes, one has to be careful to see that the conservation of energy is preserved. In order to accomplish this, we follow the consistent procedure to fit the cubic part of the barrier not to zero at $r = r_i$ but to $-E_v$.

For calculating the half-life of the system, we use the formula[11]

$$T = \frac{1.4333 \times 10^{-21}}{E_v}[1 + \exp(K)] , \tag{7}$$

where

$$K = \frac{2}{\hbar}\int_{r_a}^{r_t}[2B_r(r)V(r)]^{1/2}dr + \frac{2}{\hbar}\int_{r_t}^{r_b}[2B_r(r)V(r)]^{1/2}dr . \tag{8}$$

The limits of integration r_a and r_b are the two appropriate zeros of the integrand. In Eq. (8) it is to be noted that the effective mass $B_r(r)$ is taken to be deformation dependent[12] as

$$B_r(r) = \mu + fk(B_r^i - \mu) , \tag{9}$$

where

$$f = \begin{cases} \left[\dfrac{r_t - r}{r_t - r_i}\right]^4, & r \le r_t \\ 0, & r \ge r_t , \end{cases}$$

$$k = 16 ,$$

$$B_r^i = \mu + \tfrac{17}{15}\mu \exp\left[-\frac{128}{51}\left[\frac{r - r_i}{R_0}\right]\right] ,$$

and μ is the reduced mass. For E_v we choose[13]

$$E_v = \frac{\pi\hbar}{2}\frac{(2Q/\mu)^{1/2}}{(C_1 + C_2)} . \tag{10}$$

Here, the "central" radii C_1 and C_2 of the fragments are given by

$$C_j = R_j - (b^2/R_j) \quad (j = 1,2) ,$$

the nuclear "surface width" $b = 1.0$ fm, R_j's are equivalent sharp-surface radii of the fragments.

III. RESULTS AND CONCLUSION

Our model is applied to calculate the half-lives (T) for the spontaneous emission of heavier fragments from certain actinide nuclei. The branching ratios are then obtained by using the experimental half-lives[14] of the respective alpha disintegration (T_α). The results are tabulated in Table II and compared with the values of ASAFM, PPCPM, and the experimental values reported by Barwick *et al.*[15] Our results are found to compare well with those of other theoretical models and experimental values.

To compare our model with the other models, we can say that ASAFM is completely analytical which can be easily extended to numerous cases, but in it the barrier height is overestimated. The PPCPM has the virtue of no adjustable parameters and includes proximity effects, but E_v cannot be included explicitly. In our model, finite-range effects are included which brings down the barrier heights closer to the experimental values, and E_v appears explicitly. Our model involves very short computation time, but numerical.

Considering the main limitation of all these models, namely the fact that only one-dimensional barrier is used, it can be concluded that our model can be used for studying exotic decay modes with confidence.

ACKNOWLEDGMENT

We acknowledge with thanks Professor J. R. Nix for his critical comments on this work.

[1]D. N. Poenaru, M. Ivascu and A. Sandulescu J. Phys. G **5**, L169 (1979).
[2]D. N. Poenaru and M. Ivascu, J. Phys. (Paris) **45**, 1099 (1984).
[3]Y. J. Shi and W. J. Swiatecki, Nucl. Phys. **A438**, 450 (1985).
[4]H. J. Krappe, J. R. Nix, and A. J. Sierk, Phys. Rev. Lett. **42**, 215 (1979); Phys. Rev. C **20**, 992 (1979).
[5]P. Moller and J. R. Nix, Nucl. Phys. **A361**, 117 (1981).
[6]A. H. Wapstra and G. Audi, Nucl. Phys. **A432**, 1 (1985).
[7]J. R. Nix, Ann. Phys. **41**, 52 (1967).
[8]G. Shanmugam, B. Kamalaharan, and K. Ramamurthi, in *Proceedings of the Nuclear Physics Symposium, Waltair, India 1986* (B. A. R. C., Bombay, 1986), Vo. 29B, p. 255.
[9]D. N. Poenaru, D. Mazilu, and M. Ivascu, J. Phys. G **5**, 1093 (1979).
[10]A. J. Sierk, Phys. Rev. Lett. **55**, 582 (1985).
[11]D. N. Poenaru, W. Greiner, M. Ivascu, and A. Sandulescu Phys. Rev. C **32**, 2198 (1985).
[12]P. Moller, J. R. Nix, and W. J. Swiatecki, Nucl. Phys. **A469**, 1 (1987).
[13]H. G. de Carvalho, J. B. Martins, and O. A. P. Tavares, Phys. Rev. C **34**, 2261 (1986).
[14]A. Rytz. At. Data Nucl. Data Tables **12**, 4791 (1973).
[15]S. W. Barwick, P. B. Price, H. L. Ravn, E. Hourani, and M. Hussonnois, Phys. Rev. C **34**, 362 (1986).