

PHYSICAL REVIEW C

NUCLEAR PHYSICS

THIRD SERIES, VOLUME 38, NUMBER 3

SEPTEMBER 1988

Particle-hole stretched states excited in the $^{26}\text{Mg}(p, n)^{26}\text{Al}$ reaction at 134 MeV

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(Received 18 May 1988)

Strong excitation of both $(\pi d_{5/2}, \nu d_{5/2}^{-1})$ 5^+ , $0 \hbar\omega$ and $(\pi f_{7/2}, \nu d_{5/2}^{-1})$ 6^- , $1 \hbar\omega$ stretched-state strength is observed in the $^{26}\text{Mg}(p, n)^{26}\text{Al}$ reaction at 134.4 MeV. The $T=0$, 5^+ strength is observed to be predominantly in the ground state which has 56% of the strength predicted by a distorted-wave impulse approximation calculation that assumes the extreme single-particle-hole model. Another 17% of this strength is observed in a 5^+ state at 3.4 MeV. The 6^- strength is split into three isospin components: 61% of the extreme single-particle-hole model $T=0$, 6^- strength is observed in three states between 6.9 and 11.0 MeV; 58% of the predicted $T=1$ strength is observed in nine states between 9.3 and 16.5 MeV; 27% of the predicted $T=2$ strength is observed in a single state at 18.2 MeV. The observed 6^- -state excitation energies are in general agreement with the $T=0$ and 1, 6^- states observed in the $^{25}\text{Mg}(\alpha, t)^{26}\text{Al}$ reaction and with the analog $T=1$ and 2, 6^- states in ^{26}Mg observed in elastic electron and proton scattering. The total observed 5^+ strength and its distribution are in good agreement with distorted-wave impulse approximation calculations that use full S - D shell-model wave functions. The total 6^- strength and its distribution are generally indicated by a shell-model calculation that assumes only 2p2h correlations in the ^{26}Mg target nucleus and requires one particle in the $1f_{7/2}$ orbital for the final 6^- states; quantitatively, the calculation predicts less fragmentation than observed.

I. INTRODUCTION

Particle-hole stretched-state excitations, formed by a pair involving the highest-spin orbitals in their respective shells coupled to the maximum angular momentum, are expected to be relatively pure shell-model states. These particle-hole couplings are unique within $2\hbar\omega$ of excitation, and only more complicated configurations can mix with these states. Reduction and spreading of such strength can arise from mixing with the more complicated configurations and from excitation of collective modes. These effects are expected to be minimized near closed-shell nuclei and strongest for deformed nuclei. Indeed, strong excitation of the $(d_{5/2}, p_{3/2}^{-1})$, 4^- and the $(f_{7/2}, d_{5/2}^{-1})$, 6^- “ $1 \hbar\omega$ ” particle-hole stretched states in ^{16}O and ^{28}Si , respectively, are observed¹⁻⁶ primarily in single states, whereas the same stretched configurations are observed to be fragmented in the deformed nuclei ^{14}C and ^{26}Mg .^{7,8} Comparison with structure calculations of the measured strength and fragmentation of these stretched

states provides an important test of such calculations; such comparisons are needed for both spherical and non-spherical nuclei.

Stretched-state excitations have been studied with a variety of reactions, including proton and electron inelastic scattering,⁹ particle-transfer reactions,¹⁰ and charge-exchange reactions.¹¹ These various reactions provide both supplementary and complementary information regarding stretched-state excitations; accordingly, one should consider all of this information simultaneously in order to obtain the fullest understanding. The (p, n) charge-exchange reaction provides important contributions to this systematic approach. First, because the (p, n) reaction is strictly isovector, the absence of isoscalar strength allows unambiguous identification of the isovector strength. Second, in contrast to inelastic scattering, such as (p, p') and (e, e') reactions, the (p, n) reaction can, for a target nucleus with a neutron excess, excite strength in the residual nucleus with isospin one less than the target; because the fragmentation and mixing of particle-hole states is often sensitive to the isospin, this feature can provide important new information. Exam-

ples of the use of the (p,n) reaction to take advantage of the isovector selectivity include the earlier studies of stretched-state excitations on targets of the self-conjugate nuclei ^{16}O , ^{28}Si , and ^{40}Ca , where the strictly isovector nature of the (p,n) reaction provided unambiguous identification of the isovector strength.^{3,6,12} An important example of the second feature of the (p,n) reaction is the excitation of $0 \hbar\omega$ stretched states.^{13,14} These states have a proton particle and a neutron hole in the same orbital and can be excited only in a charge exchange reaction. Because the $0 \hbar\omega$ excitations usually involve orbitals near the Fermi surface, they are typically fragmented less than the $1 \hbar\omega$ excitations and provide simpler cases for study of the reaction mechanisms and the target wave functions. The earlier studies of these excitations were performed on relatively spherical, closed-shell nuclei and revealed the largest concentrations of stretched-state strength yet observed, typically about two-thirds of that expected in the simple shell model, hereafter referred to as the extreme single-particle-hole model (ESPHM).

In this paper, we extend the earlier (p,n) studies of both 1 and $0 \hbar\omega$ stretched states from relatively spherical nuclei to the deformed target nucleus ^{26}Mg . The $(f_{7/2}, d_{5/2}^{-1})$, 6^- , $1 \hbar\omega$ stretched-state strength was studied in the $A=26$ system via inelastic electron and proton scattering^{7,8} on ^{26}Mg and with the $^{25}\text{Mg}(\alpha,t)^{26}\text{Al}$ proton-stripping reaction.¹⁰ Inelastic scattering can excite $T=1$ and 2 strength in the ^{26}Mg nucleus, and the (α,t) reaction can excite $T=0$ and 1 strength in the analog nucleus ^{26}Al . By comparison of the spectra observed in these three reactions, it is possible to identify the three different isospin components. The $T=0$, 6^- strength is observed in the (α,t) reaction to be split into two states near 7 MeV of excitation and has about 30% of the single-particle strength. All three reactions see $T=1$, 6^- strength fragmented into several states between $E_x=9$ and 17 MeV of excitation, which account for about 60% of the single-particle strength. Both (e,e') and (p,p') see a $T=2$, 6^- state near 18 MeV of excitation in ^{26}Mg which has about 30% of the single-particle-hole strength expected for the $T=2$ component.

The (p,n) reaction on ^{26}Mg excites all three isospin components seen in the above reactions, although the isospin geometrical factors are different so that the (p,n) reaction excites the lower isospin components more strongly relative to the inelastic scattering reactions. Additionally, the (p,n) reaction excites the $(\pi d_{5/2}, \nu d_{5/2}^{-1})$, 5^+ , $0 \hbar\omega$ stretched state. [This state can be described also as $(\pi d_{5/2}^{-1}, \nu d_{5/2}^{-1})$, if one assumes only the simple shell model.] It is significant to see if this type of strength, observed to be highly concentrated in spherical nuclei, is fragmented in this more deformed nucleus. The amount of $0 \hbar\omega$ strength observed provides additional information regarding possible quenching of stretched-state strength in the $A=26$ system. Detailed comparisons of the stretched-state excitations observed in the (p,n) reaction with the results from the other reaction studies and with shell-model calculations are presented and discussed. These comparisons are seen to provide a consistent picture of stretched-state strength in the $A=26$ system.

II. EXPERIMENTAL PROCEDURE

The experiment was performed at the Indiana University Cyclotron Facility with the beam-swinger system. The basic experimental arrangement and data-reduction procedures were similar to those described previously.¹⁵ The forward-angle data were analyzed and reported earlier by Madey *et al.*;¹⁶ the analysis described here is an extension of that work to the wide-angle measurements.

Neutron kinetic energies were measured by the time-of-flight (TOF) technique. A beam of 135 -MeV protons was obtained from the cyclotron in narrow beam bursts typically 350 ps long, separated by 128 ns. Neutrons were detected in three detector stations at 0° , 24° , and 45° with respect to the undeflected proton beam. The flight paths were 85.8 , 89.0 , and 62.8 m, respectively. The neutron detectors were rectangular bars of fast plastic scintillator 10.2 cm thick. Three separate detectors each 1.02 m long by 0.51 m high were combined for a total frontal area of 1.55 m² in the 0° station. Two detectors were used in the 24° station, one was 1.02 m long by 1.02 m high and the other was 1.02 m long by 0.51 m high, for a combined frontal area of 1.55 m². Two detectors were used also in the 45° station, both were 1.52 m long by 0.76 m high, for a combined frontal area of 2.31 m². Each neutron detector had tapered plexiglass light pipes attached on the two ends coupled to 12.7 -cm diam phototubes. Timing signals were derived from each end and combined in a mean-timer circuit¹⁷ to provide the timing signal from each detector. Overall time resolutions of about 750 ps were obtained, including contributions from the beam burst width (~ 350 ps) and energy spread (~ 280 ps), energy loss in the target (~ 350 ps), neutron transit times across the 10.2 cm thickness of the detectors (~ 530 ps), and the intrinsic time dispersion of each detector (~ 300 ps). This overall time resolution provided an energy resolution of about 370 keV in the first two detector stations and about 500 keV in the widest-angle station. The large-volume neutron detectors were described in more detail previously.¹⁸ Protons from the target were rejected by anticoincidence detectors in front of each neutron detector array. Cosmic rays were vetoed by anticoincidence detectors on top and at the front of each array.

The target was a 35.8 (± 0.9) mg/cm² self-supporting foil of ^{26}Mg , enriched to 99.45% . Time-of-flight spectra were measured in approximately 4° steps from 0° to 63° . Spectra from each detector were recorded at many pulse-height thresholds ranging from 25 to 90 MeV equivalent-electron energy (MeV EE). Calibration of the pulse-height response of each of the detectors was performed with a ^{232}Th γ -source, which emits a 2.61 -MeV γ ray, and a calibrated fast amplifier. The values of the cross sections extracted for several thresholds (from 40 to 70 MeV EE) were found to be the same within statistics.

III. DATA REDUCTION

Excitation-energy spectra were obtained from the measured TOF spectra using the known flight path and a calibration of the time-to-amplitude converter. At forward angles, the known excitation energy of the strongly

excited 1^+ state at 1.09 MeV was taken to provide an absolute reference point. At wide angles, the strongly excited 5^+ ground-state transition was taken to provide the reference point. All of the excitation energies reported here are estimated to be accurate to ± 0.1 MeV.

Yields for transitions in the $^{26}\text{Mg}(p,n)^{26}\text{Al}$ reaction were obtained by peak fitting of the TOF spectra. The spectra were fitted with an improved version of the peak-fitting code of Bevington.¹⁹ Examples of peak fitting of similar neutron TOF spectra were presented earlier for ^{48}Ca and the forward-angle spectra for ^{26}Mg .^{15,16} The TOF spectra were subdivided into regions where groups of peaks and a polynomial background could be fit simultaneously. Cross sections were obtained by combining the yields with the measured geometrical parameters, the beam integration, and the target thickness. The neutron-detector efficiencies were obtained from a Monte Carlo computer code²⁰ which was tested extensively at these energies.^{21,22} The overall absolute cross sections so obtained were checked by remeasuring the known $^{12}\text{C}(p,n)^{12}\text{N(g.s.)}$ reaction.²¹ The experimental procedure and data reduction is similar to that described in more detail in Refs. 15 and 16. The uncertainty in the overall scale factor is dominated by the uncertainty in the detector efficiencies and is estimated to be $\pm 12\%$.

IV. RESULTS

The excitation-energy plot for the $^{26}\text{Mg}(p,n)^{26}\text{Al}$ reaction at 45° is shown in Fig. 1. High-spin stretched excitations were identified by extraction and analysis of angular distributions and by comparison with known excitation energies in ^{26}Al or analog states identified in ^{26}Mg , as discussed below. We see the excitation of $T=0$, 5^+ and $T=0$, 1 and 2, 6^- stretched-state strength. We consider these excitations individually below.

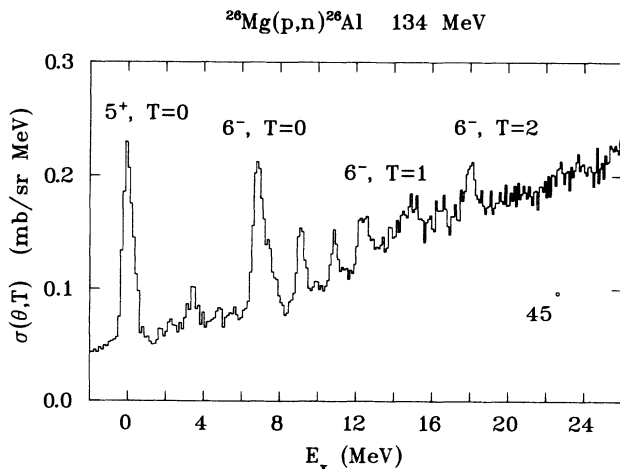


FIG. 1. The differential cross section at 45° vs excitation energy for the $^{26}\text{Mg}(p,n)^{26}\text{Al}$ reaction at 134 MeV.

A. 5^+ , $0 \hbar\omega$ stretched-state strength

The ground state of ^{26}Al is known to be a $T=0$, 5^+ state.²³ We see this state to be excited strongly in the (p,n) reaction (see Fig. 1). The angular distribution, shown in Fig. 2, was extracted by fitting the TOF spectra as discussed above. The 5^+ state cross section is obtained reliably from 24° outward. At smaller angles, interference from other (known) states, too close in excitation energy to be resolved in this experiment, prevented unambiguous extraction of the 5^+ peak area. The experimental angular distribution is compared in Fig. 2 with a “standard” distorted-wave impulse approximation (DWIA) calculation for this transition. The calculation was performed with the distorted-wave code DWBA70;²⁴ the optical-model parameters were taken from Olmer *et al.*²⁵ and were fit to both cross-section and analyzing-power data for the elastic scattering of 135-MeV protons on ^{28}Si . The nucleon-nucleon effective interaction assumed is that of Franey and Love at 140 MeV.²⁶ The structure assumed is simply that of the extreme single-particle-hole model, viz., that this state is a pure $(\pi d_{5/2}, \nu d_{5/2}^{-1})$ particle-hole excitation with six neutrons and four protons in the $d_{5/2}$ orbital for the ^{26}Mg target nucleus. Because both the particle and hole states involved are in the

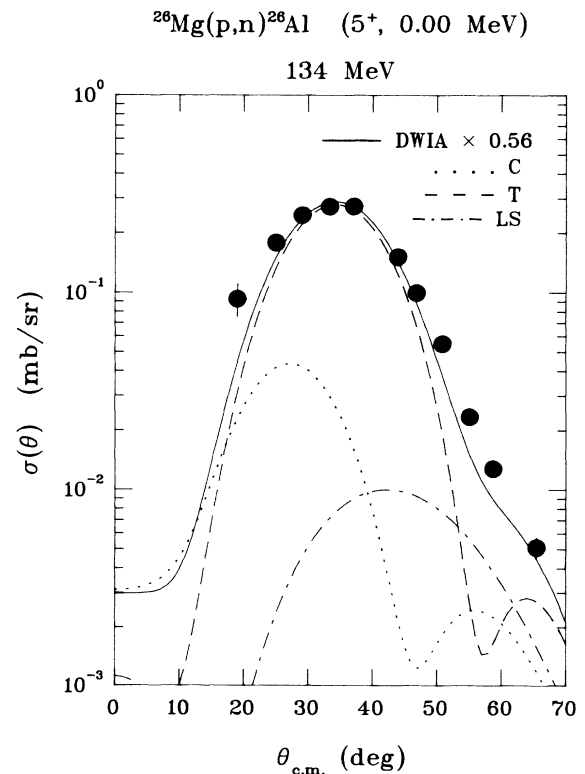


FIG. 2. Angular distribution for the transition to the 5^+ ground state in the $^{26}\text{Mg}(p,n)^{26}\text{Al}$ reaction at 134 MeV. The curves represent DWIA calculations with the normalization factor indicated (see text).

same major shell (in fact, in the same orbital), this is a $0 \hbar\omega$ excitation. The particle and hole states are described by harmonic oscillator wave functions, with an oscillator parameter $b = 1.70$ fm. As seen in Fig. 2, the DWIA calculation reproduces the measured angular distribution well with a normalization factor of 0.56. [Note that this normalization factor includes a reduction by $[(A-1)/A]^{J-1} = 0.85$, for center-of-mass (c.m.) motion not accounted for in the impulse approximation calculation. See Ref. 27.] Shown also are the individual contributions from the central, tensor, and spin-orbit parts of the N - N interaction; similar to other stretched-state excitations (both 0 and $1 \hbar\omega$), the tensor contribution is seen to dominate strongly in the peak region. The good fit of the DWIA calculations to the experimental angular distribution gives us confidence that these calculations can reproduce reliably the shape for stretched-state transitions in this reaction.

In addition to the strongly excited ground state, the known 5^+ state at $E_x = 3.40$ MeV is seen also. The extracted angular distribution for this transition is shown in Fig. 3; it is fitted well by a 5^+ , DWIA calculation as shown. This excitation is about 30% of the strength of the ground state, and its angular distribution cannot be extracted reliably at angles less than about 35° because of interference from other lower-spin states. There appears to be even more 5^+ strength near the state at 3.4 MeV

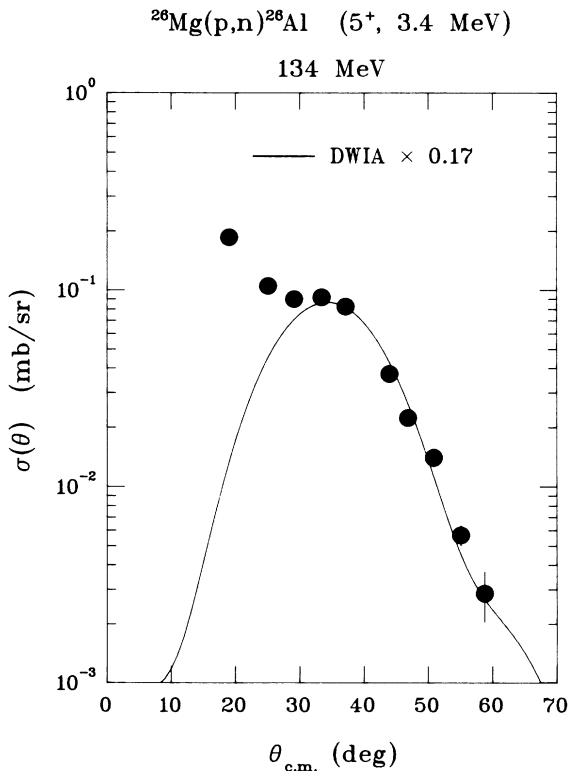


FIG. 3. Angular distribution for the transition to the 5^+ state at 3.4 MeV in the $^{26}\text{Mg}(p,n)^{26}\text{Al}$ reaction at 134 MeV. The curve represents a DWIA calculation with the normalization factor indicated (see text).

(see Fig. 1), but it is too weak to be fit reliably. If we sum the strengths for the two 5^+ states which can be extracted, we see about 73% of the $(\pi d_{5/2}, \nu d_{5/2}^{-1})$ particle-hole strength expected in the ESPHM.

B. $6^-, 1 \hbar\omega$ stretched-state strength

As seen in Fig. 1, several strong excitations are seen at large momentum transfer between about 6 and 20 MeV of excitation in ^{26}Al . The experimental angular distributions for these excitations are shown in Figs. 4–10. These angular distributions are compared with DWIA calculations for the excitation of a $(\pi f_{7/2}, \nu d_{5/2}^{-1})$, 6^- particle-hole state. The calculations are essentially the same as those described above for the 5^+ excitations, except for the different structure, which is again assumed to be given by the ESPHM. The DWIA calculations are seen to describe the angular distributions well at large ($>40^\circ$) angles. The disagreement between the calculations and the measured angular distributions observed at angles less than 40° is due to the presence of unresolved states of lower spin; the level density is known to be high in this region of excitation. As discussed below, these excitations correspond to known 6^- states in ^{26}Al , or to analog 6^- states in ^{26}Mg , identified in other reactions. Because of the presence of the unresolved states of lower spin, the DWIA calculations are normalized to the wide-angle ($>40^\circ$) portions of the distributions only. The earlier study of particle-hole excitations in the $^{48}\text{Ca}(p,n)^{48}\text{Sc}$ reaction,¹⁵ where good agreement was obtained between DWIA calculations and particle-hole excitations of various spins, gives us confidence in this procedure. Our experience in that work and others^{3,6,11} is that only stretched-state excitations will contribute significantly at angles beyond about 40° . As discussed above, the (p,n) reaction can excite $T=0, 1$, and 2 components of the 6^- stretched-state strength. All three of these components are believed to be seen in this region. The relative strengths assumed in the DWIA calculations for the $T=0, 1$, and 2 isospin components are assumed to be given by the isospin geometrical factors²⁸ which are $\frac{1}{3}$, $\frac{1}{2}$, and $\frac{1}{6}$, respectively, of the total calculated 6^- strength.

In a study of 6^- stretched states in ^{26}Al with the $^{25}\text{Mg}(\alpha,t)$ proton-stripping reaction, Peterson *et al.*¹⁰ identified two $T=0, 6^-$ states at $E_x = 6.89$ and 7.53 MeV. We see these two states as part of a complex involving at least three states near 7 MeV of excitation. The first state is seen at 6.9 MeV. This state is the most strongly excited 6^- state observed in the (p,n) reaction. The 7-MeV complex was fitted with three states at angles from 24° to 63° . The angular distribution obtained for the 6.9-MeV state is shown in Fig. 4. The middle peak of the complex is at $E_x = 7.3$ MeV and peaks at a more forward angle by at least several degrees than does the 6.9-MeV state; hence, we conclude that this state is not a 6^- state. The third state of the complex is at about 7.5 MeV of excitation. The angular distribution for this transition is shown in Fig. 5; because it is similar to that for the 6.9-MeV transition, we conclude that this is the 6^- state seen

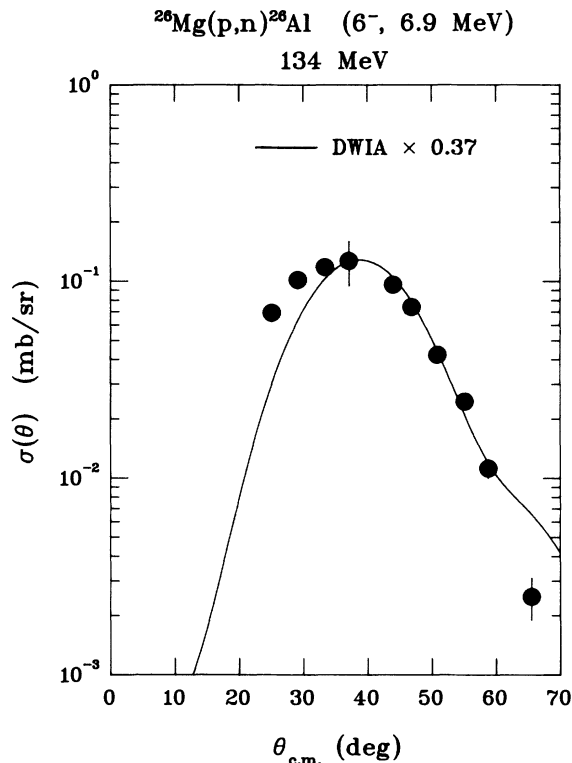


FIG. 4. Angular distribution for the transition to the 6^- state at 6.9 MeV in the $^{26}\text{Mg}(p,n)^{26}\text{Al}$ reaction at 134 MeV. The curve represents a DWIA calculation with the normalization factor indicated (see text).

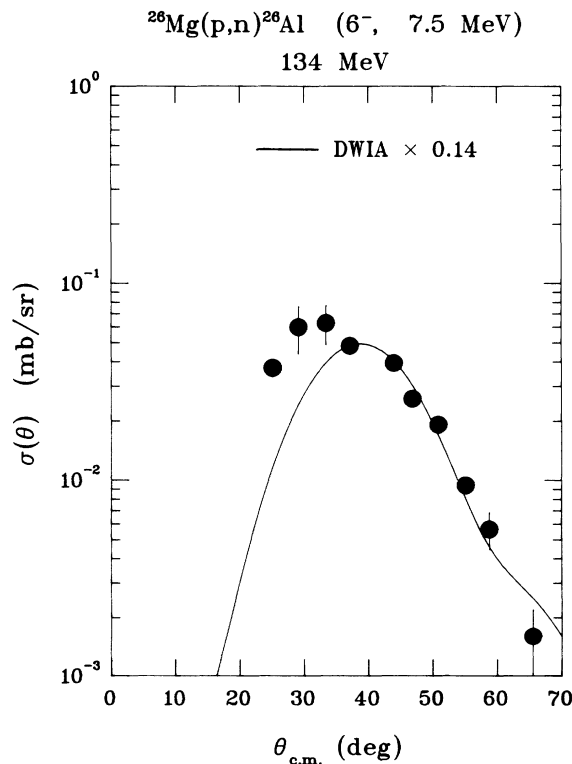


FIG. 5. Angular distribution for the transition to the 6^- state at 7.5 MeV in the $^{26}\text{Mg}(p,n)^{26}\text{Al}$ reaction at 134 MeV. The curve represents a DWIA calculation with the normalization factor indicated (see text).

in the (α, t) measurements at 7.53 MeV. The strength of the 7.5-MeV state is about 50% of that of the strong 6.9-MeV state. As indicated in Figs. 4 and 5, the DWIA normalization factors required to make the calculations agree in magnitude with the experimental angular distributions are 0.37 and 0.14, respectively. [As for the $0 \hbar\omega$ transitions, these normalization factors include a reduction by the factor $[A/(A-1)]^{J-1}$ for the center-of-mass motion, as discussed earlier.] Thus the total strength observed in the two $T=0$, 6^- states is about 51% of the ESPHM expectation for the $T=0$ component.

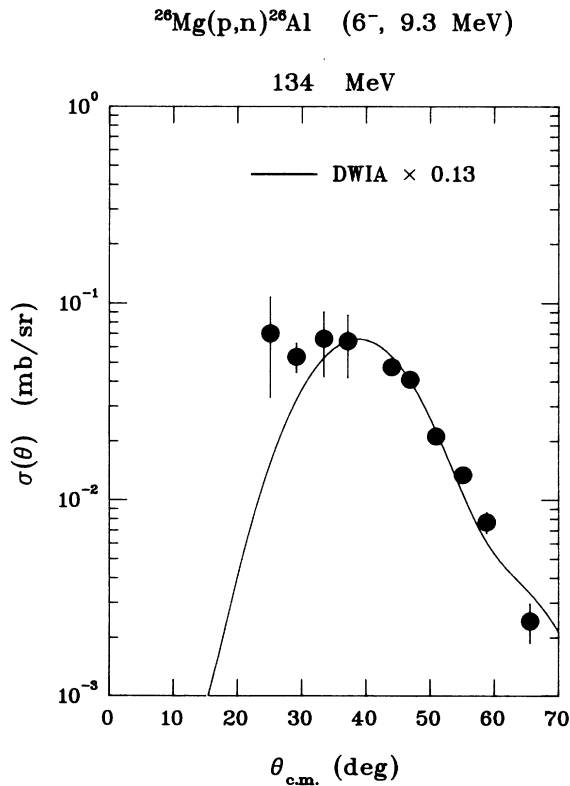
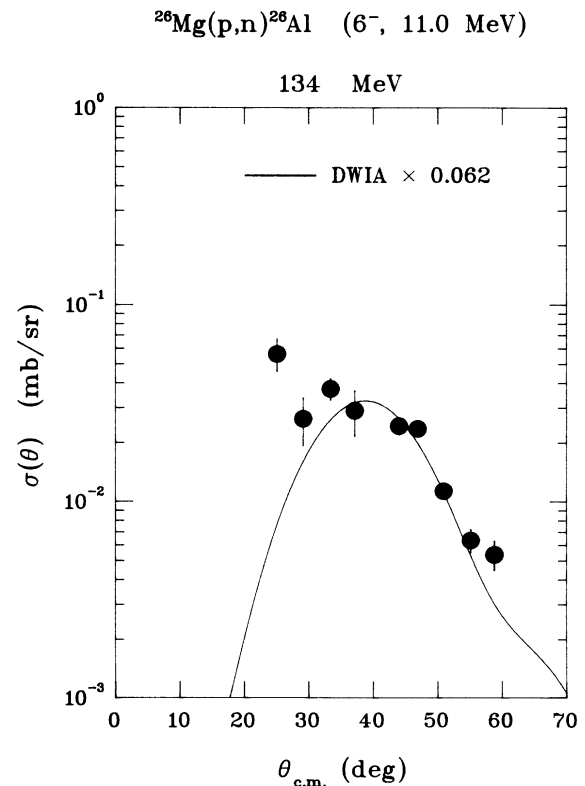
Several candidates for $T=1$ ($f_{7/2}, d_{5/2}^-$), 6^- stretched states were observed between about 9 and 17 MeV of excitation in ^{26}Al or ^{26}Mg . The isobaric analog state (IAS) of the ground state of ^{26}Mg is known to be at 0.23 MeV of excitation in ^{26}Al , so that excitation energies in the two nuclei are expected to be shifted by about this amount; because the actual shift for each analog state depends on the detailed location of the extra charge in ^{26}Al , this difference is sensitive to the nuclear structure and can vary by a few tenths of an MeV from state to state. The (α, t) measurements indicate the clear excitation of a $T=1$, 6^- state at $E_x=9.26$ MeV, and the probable excitation of 6^- states at 11.97, 12.40, 12.55, and 16.83 MeV in ^{26}Al .¹⁰ Inelastic-proton-scattering measurements indicate possible $T=1$, 6^- states at $E_x=9.17$, 11.98, 12.49, 12.90, 14.50, and 15.36 MeV in ^{26}Mg .⁸ Inelastic-electron-scattering measurements indicate $T=1$, 6^- states at 9.17, 12.50, 12.88, 14.50, 15.36, and 15.46 MeV in ^{26}Mg , with good confidence, and at 13.00, 13.97, and 16.5 MeV with less confidence.⁷ The results from inelastic (p, p') and (e, e') scattering reactions are listed in Table I, where they are compared with the (p, n) results discussed below.

In the (p, n) measurements, we see clearly the 6^- excitation at $E_x=9.3$ MeV; its angular distribution is presented in Fig. 6. The angular distribution is fit well beyond about 35° . Inside 35° unresolved strength begins to contribute significantly so that the angular distribution does not fall off as expected for a 6^- state. The DWIA normalization factor required for the excitation at 9.3 MeV is 0.13. The (p, n) measurements show several other states between 11 and 17 MeV of excitation in ^{26}Al which appear to have significant 6^- strength. The first such state, at $E_x=11.0$ MeV, is not observed in the (p, p') and (e, e') inelastic-scattering reactions. Its (p, n) angular distribution is shown in Fig. 7. The shape of the angular distribution at wide angles is consistent with a 6^- excitation. Because this excitation is not seen in the inelastic (p, p') and (e, e') scattering reactions, we conclude that it must be $T=0$.

The angular distributions of eight such transitions are shown in Figs. 8 and 9. In general, these excitations are in complexes and not resolved well in this experiment. These angular distributions correspond to those states seen to be excited above the general background at angles beyond 40° (see Fig. 1) and to fall off with increasing angle in a manner consistent with that observed for the other 6^- excitations near 7 and 9 MeV discussed above. The 6^- states observed in this experiment, along with the DWIA normalizations required for each, are listed in

TABLE I. Stretched-state excitations in ^{26}Mg and ^{26}Al .

$J^\pi; T$	^{26}Mg		^{26}Al		Strengths	
	(p, p') E_x (MeV)	(e, e')	(p, n)	(p, p') ^a	(e, e') ^b FESPHM	(p, n) ^c
$5^+; 0$			0.00 3.4			0.56 0.17 $\Sigma(5^+, T=0)=0.73$
$6^-; 0$			6.9 7.5			0.37 0.14
$6^-; 1$	9.18	9.17		0.074	0.072	
$6^-; 0+1$			9.3			0.13
$6^-; 0$			11.0			0.062 $\Sigma(6^-, T=0)=0.61$
$6^-; 1$	11.98		12.0	0.103		0.048
	12.49	12.50	12.5	0.095	0.095	0.081
	12.90	12.88	13.1	0.056	0.042	0.052
		13.00			0.028	
		13.97	14.0		0.039	0.060
	14.50	14.50	14.6	0.051	0.052	0.065
			15.0			0.084
	15.36	15.36	15.5	0.056	0.057	0.062
	(15.46)	15.46			0.104	
	(16.5)	16.5	16.5	<0.020	0.141	0.048
						$\Sigma(6^-, T=1)=0.58$
$6^-; 2$	18.05	18.05	18.2	0.27	0.43	0.27

^aReference 8. See discussion in text.^bReference 7. See discussion in text.^cThis work.FIG. 6. Angular distribution for the transition to the 6^- state at 9.3 MeV in the $^{26}\text{Mg}(p, n)^{26}\text{Al}$ reaction at 134 MeV. The curve represents a DWIA calculation with the normalization factor indicated (see text).FIG. 7. Angular distribution for the transition to the 6^- state at 11.0 MeV in the $^{26}\text{Mg}(p, n)^{26}\text{Al}$ reaction at 134 MeV. The curve represents a DWIA calculation with the normalization factor indicated (see text).

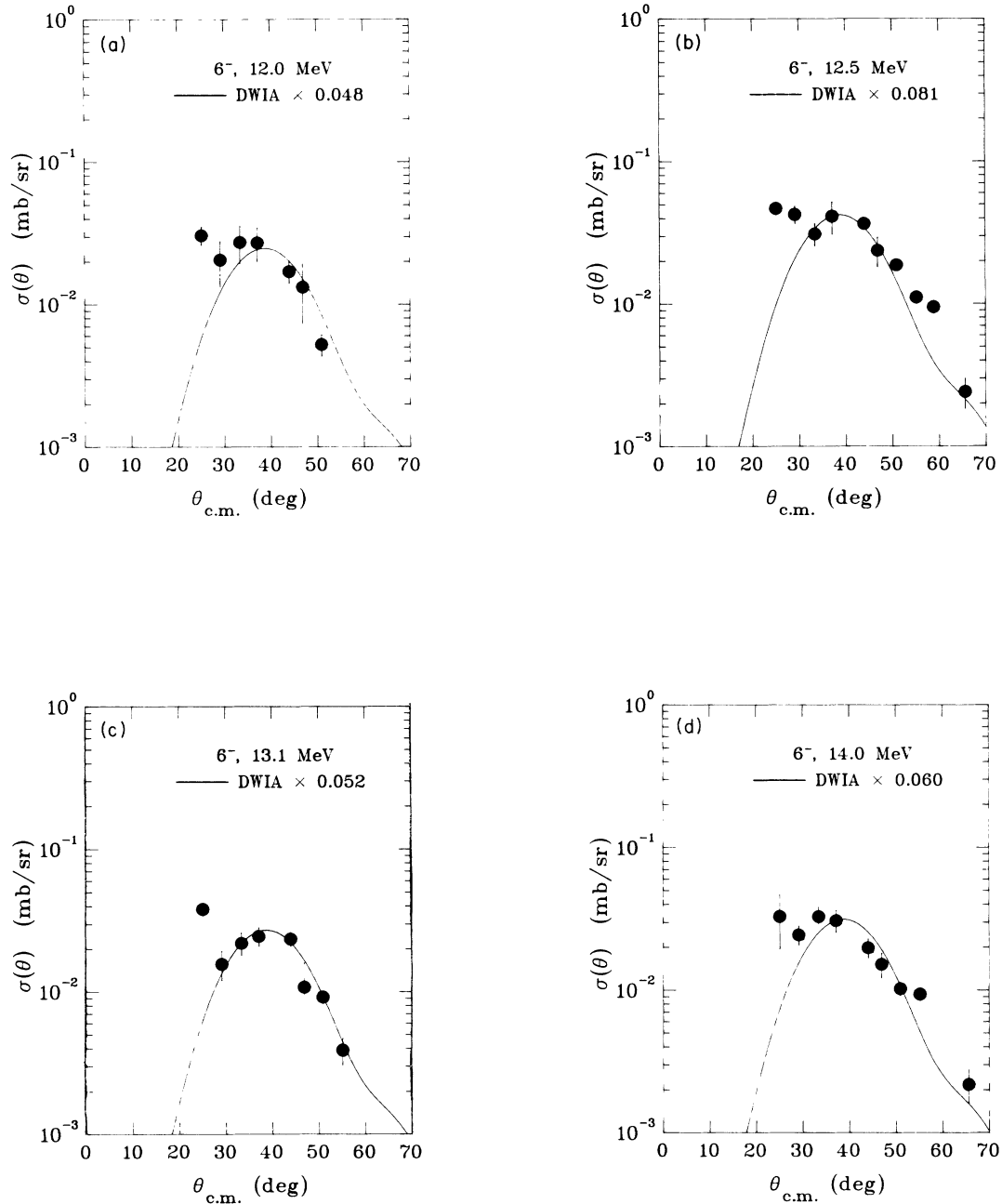


FIG. 8. Angular distributions for the transitions to four 6^- states between 12 and 14 MeV in the $^{26}\text{Mg}(p,n)^{26}\text{Al}$ reaction at 134 MeV. The curves represent DWIA calculations with the normalization factors indicated (see text).

Table I where they can be compared with the 6^- states identified in the (p,p') and (e,e') reactions. These comparisons are discussed more fully in the next section. The total DWIA normalization for the (p,n) results, summed over the nine $T=1$, 6^- states in Table I, is 0.58. It is possible that some of the weaker transitions could be 5^- states, based on the same $1p1h$ configuration as the 6^-

states. Such states will also have $\Delta l=5$ angular distributions and be similar to those for the 6^- states; however, as discussed previously,¹⁵ the medium-energy (p,n) reaction suppresses these normal-parity transitions by more than an order of magnitude relative to the stretched-state transitions, so that we expect most of the observed strength to be 6^- .

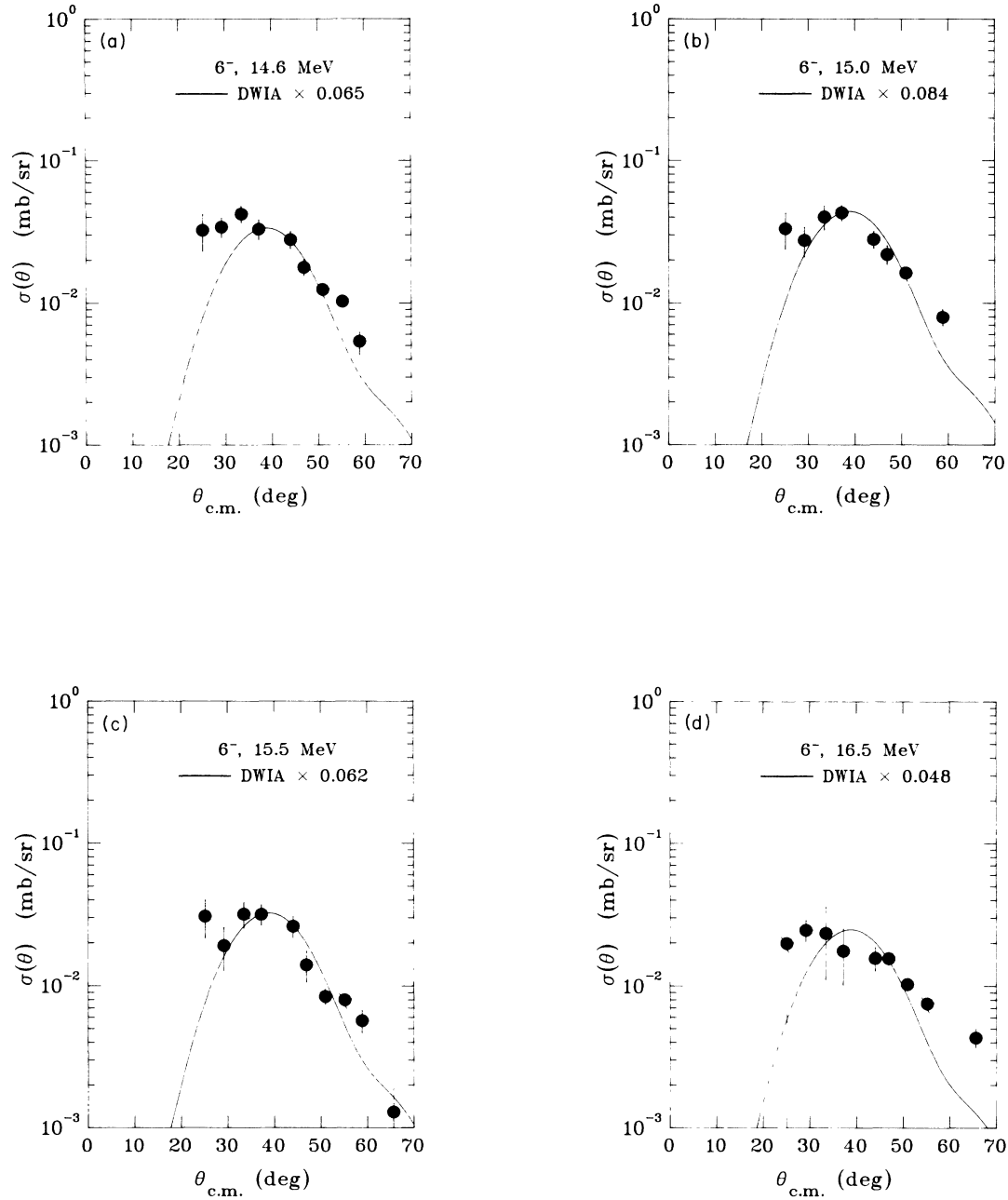


FIG. 9. Angular distributions for the transitions to four 6^- states between 14.6 and 16.5 MeV in the $^{26}\text{Mg}(p,n)^{26}\text{Al}$ reaction at 134 MeV. The curves represent DWIA calculations with the normalization factors indicated (see text).

Near 18 MeV of excitation in ^{26}Mg , a 6^- state is observed which, as discussed below, is identified as $T=2$. We see this excitation in the $^{26}\text{Mg}(p,n)^{26}\text{Al}$ reaction at 18.2 MeV of excitation. In Fig. 10, the angular distribution for this transition is compared with a DWIA calculation. Similar to the other 6^- excitations, contributions from other states are clearly seen at angles less than

about 35° , but the agreement between the experimental and theoretical angular distributions at large angles is good. Based on the normalization factor of 0.27 required for the DWIA calculation, this transition accounts for 27% of the $T=2$ strength expected in the ESPHM. No other state can be identified above the continuum background which might be another candidate for a 6^- state.

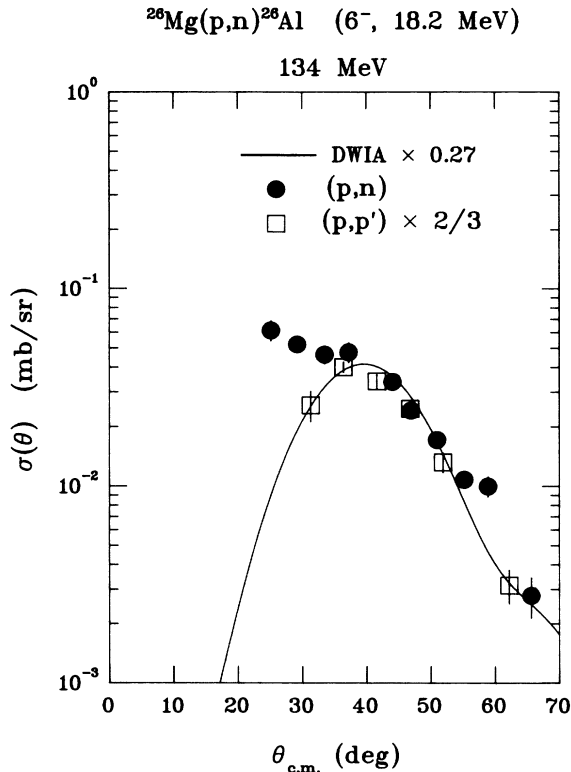


FIG. 10. Comparison of the (p,n) and (p,p') angular distributions for the excitation of the $T=2$, 6^- state at 18.2 and 18.05 MeV in ^{26}Al and ^{26}Mg , respectively. The (p,p') cross sections are reduced by the factor $\frac{2}{3}$, the ratio of the (p,n) to (p,p') isospin geometrical factors. The curve represents a DWIA calculation with the normalization factor indicated (see text).

V. DISCUSSION

The (p,n) reaction at intermediate energies is an excellent probe of $1p1h$ stretched-state strength. The impulsive nature of the reaction causes it to excite predominantly strength that can be reached from the target nucleus in a single-step process; for even-even target nuclei, this strength will be dominated by $1p1h$ configurations. The strong suppression of normal-parity states at large momentum transfer insures that it is the stretched $1p1h$ strength which is excited predominantly at large angles. The net result is that the wide-angle (p,n) spectra provide a relatively clean mapping of the isovector stretched-state strength.

In the $^{26}\text{Mg}(p,n)^{26}\text{Al}$ reaction, two kinds of $1p1h$ stretched-state strengths are excited, viz., the $(\pi d_{5/2}, \nu d_{5/2}^{-1})$, 5^+ , $0 \hbar\omega$ strength and the $(\pi f_{7/2}, \nu d_{5/2}^{-1})$, 6^- , $1 \hbar\omega$ strength. The strength observed with the (p,n) reaction at 135 MeV is compared here with "standard" DWIA calculations that assume only ESPHM excitations of single-particle-hole configurations from the simple shell model. The validity of these calculations has been checked by comparison with inelastic-electron-scattering analyses for analog transitions in the $A=16$ and 28 mass systems;¹¹ these comparisons indicate that the DWIA

analyses are accurate to about 20% relative to the electron-scattering results. This conclusion is supported further by the good agreement observed between DWIA calculations and measured analyzing powers for stretched-state excitations observed in the (p,n) reaction on ^{16}O , ^{28}Si , and ^{48}Ca .¹¹ (In fact, the DWIA shows good agreement with measured analyzing powers for more than 15 strong transitions studied in these reactions.)

A. Comparison with other reactions

Excitation energies and strengths for stretched-state excitations in ^{26}Mg obtained from the (p,p') and (e,e') reactions and in ^{26}Al from this (p,n) study are compared in Table I. The different reactions provide both complementary and supplementary information. Proton and electron inelastic scattering excite $T=1$ and 2 strength in ^{26}Mg . The (p,n) reaction excites $T=0$, 1, and 2 strength in ^{26}Al . The differences can be used to help identify the isospin of the various excitations. The (p,p') reaction has good energy resolution, but excites also isoscalar strength, which can make the identification of isovector stretched-state strength difficult in regions of high-level density. The (e,e') reaction suppresses strongly isoscalar strength and has good energy resolution; however, as indicated above, it cannot excite $T=0$ strength. Finally, the (p,n) reaction can excite all three isospin components, but it does not have the high resolution of the charged-particle reactions.

Note that in Table I the strengths for the (p,p') , (e,e') , and (p,n) reactions are given in terms of the fraction of the extreme single-particle-hole model (FESPHM), as discussed earlier for the (p,n) reaction. The strengths for the (e,e') reaction are taken directly from the analysis of Plum and Lindgren.⁷ The strengths for the (p,p') reaction were obtained by a renormalization of the analysis by Geesaman *et al.*⁸ In Ref. 8 the (p,p') results are normalized to yield the same FESPHM as obtained in the (e,e') analysis of Ref. 7 for the $T=2$, 6^- state at 18.1 MeV. Instead of using this normalization, we take the (p,p') experimental results and renormalize the extracted DWIA normalizations (i.e., the FESPHM's) to correspond to a DWIA analysis identical to the one described above for the (p,n) analysis. The only differences in the original (p,p') analysis and our (p,n) analysis are (1) the original (p,p') analysis was not corrected by the $[(A-1)/A]^{J-1}$ factor for the c.m. motion (see above), and (2) the original (p,p') analysis was performed with an oscillator parameter value of 1.74 fm^{-1} , compared to a value of 1.70 fm^{-1} for the (p,n) analyses. The first difference makes the (p,p') DWIA normalization factors 18% larger relative to the (p,n) factors, and the second difference makes them 5% smaller. Thus we simply reduced the original (p,p') DWIA normalization factors by 13%. The appropriateness of this renormalization for comparing the (p,n) and (p,p') analyses can be seen by comparing the absolute cross sections for the two reactions for the excitation of the $T=2$, 6^- state near 18 MeV of excitation. This comparison is shown in Fig. 10. The (p,p') experimental cross sections from Ref. 8 were

reduced by the ratio of the isospin geometrical factors for the (p,p') and (p,n) excitation of this state. The absolute cross sections are seen to be in good agreement; both reactions are fitted well by one DWIA calculation with a normalization factor of 0.27. Without the renormalization of the (p,p') DWIA normalization factors, these two reactions would appear to yield different results for this transition, even though the data are in excellent agreement.

Starting with the lowest-energy excitations, only the (p,n) and (α,t) reactions can excite $T=0$ strength. The spectrometer momentum bite for the (α,t) measurements of Ref. 10 did not include the low-excitation-energy region, so that the 5^+ strength could not be observed. The excitation of this strength by the (p,n) reaction was discussed in detail above. Both the (α,t) and (p,n) reactions strongly excite states at 6.9 and 7.5 MeV with angular distributions consistent with J^π assignments of 6^- . The (α,t) reaction excites these two states with approximately equal strength; whereas the (p,n) reaction excites the 6.9-MeV state approximately twice as strongly as the 7.5-MeV state. It must be noted that the (α,t) reaction is a single-particle transfer process and does not sample the same nuclear matrix elements as the (p,n) reaction.

All four reactions considered here can excite the $T=1$ component of the 6^- stretched-state strength. This strength is observed to be highly fragmented in each case. The lowest $T=1$ state is observed at 9.2 MeV in ^{26}Mg by inelastic scattering; its analog is observed at 9.3 MeV in ^{26}Al with the (α,t) and (p,n) reactions. The FESPHM obtained from this (p,n) study is seen to be approximately 50% greater than that extracted from the (p,p') and (e,e') reactions. This extra strength in the (p,n) reaction probably indicates the presence of unresolved $T=0$ strength in this complex; and in Table I, we have assumed that the (p,n) strength is $\frac{2}{3} T=1$, $\frac{1}{3} T=0$.

All four reactions excite strength near 11 MeV. This strength is not identified as 6^- strength in the (p,p') or (e,e') reactions; however, it appears to have an angular distribution at wide angles in the (p,n) reaction consistent with the other 6^- excitations (see Fig. 7). We identify this state as $T=0$, 6^- .

A complex of 6^- strength is observed from 12 to 13 MeV in all four reactions. The (p,n) reaction excites this bump as a single, broad complex (see Fig. 1). The (α,t) reaction shows this complex to be at least three states, at 11.97, 12.40, and 12.55 MeV. The (p,p') and (e,e') studies also find three states in this complex in ^{26}Mg ; they both agree on states at 12.5 and 12.9 MeV, but the (p,p') analysis reports a third one at 12.0 MeV, while the (e,e') analysis reports a third one at 13.0 MeV. We fit this complex with the minimum number of states required, viz., three states, at 12.0, 12.5, and 13.1 MeV. The total strength for the complex obtained from the (p,n) analysis is 71% of that obtained from the (p,p') studies and 110% of that obtained in the (e,e') analysis.

Another complex is observed in the (p,n) reaction from 14 to 15.5 MeV of excitation with an angular distribution consistent with 6^- strength. The centroid and strongest excitation is observed at 15.0 MeV. This complex was fitted also with the minimum number of peaks

required. The result was four "states" at 14.0, 14.6, 15.0, and 15.5 MeV. Unfortunately, the (α,t) experiment did not obtain spectra in the 13-to 16-MeV region. The (p,p') analysis identifies only two 6^- states in this region, while the (e,e') analysis identifies four states [although only three of the four appear to be analogs of the four seen in the (p,n) analysis]. The total FESPHM strength observed in the (p,n) and (e,e') reactions for this region agree to better than 10% (viz., 0.27 vs 0.25), while the (p,p') reaction analysis indicates less than one-half as much strength. The (p,p') reaction does excite more strength in this region with cross-section angular distributions consistent with 6^- strength; however, because the workers performing the (p,p') analysis feel that the analyzing-power angular distributions for this strength are not entirely consistent with those observed for the other 6^- transitions, they do not accept this as 6^- strength. Based on the shapes of the cross-section angular distributions and the general agreement between the (e,e') and (p,n) results, we feel that this strength is in fact $T=1$, 6^- strength. Of course, it is possible that the (p,p') excitations differ from those observed in (e,e') and (p,n) because of possible interference in the (p,p') reaction between isoscalar and isovector strength.

The highest $T=1$, 6^- state observed in the (p,n) reaction is at 16.5 MeV. This excitation is reported to be at 16.8 MeV in the (α,t) studies and its analog is reported to be at 16.5 MeV in ^{26}Mg in the (e,e') studies. The strength observed in the (p,n) reaction is less than one-half of that extracted in the (e,e') analysis. Once again, the (p,p') reaction does excite a state at this energy, but the (p,p') researchers feel that this state is mostly not 6^- strength, based on the (p,p') analyzing-power angular distribution.

Finally, we see a relatively narrow excitation at 18.2 MeV, in good agreement with a $T=2$, 6^- state observed in the (p,p') and (e,e') reactions at 18.0 MeV in ^{26}Al . The (p,n) angular distribution is shown in Fig. 10 and is consistent with a 6^- assignment. The fact that this state is not seen in the (α,t) experiment confirms its $T=2$ assignment. The FESPHM strength extracted from the (p,p') and (p,n) reactions is consistent and is about 0.27; the (e,e') reaction analysis finds significantly more strength in this transition.

These comparisons indicate the value of considering the different reaction studies simultaneously. Although there is general agreement concerning excitation energies and sometimes strengths, one also sees differences. These differences help to provide isospin assignments and to separate isovector and isoscalar contributions. The agreements and differences help also to give one a better feeling for the reliability of any one experiment.

B. Comparison with shell-model calculations

It is important to compare the experimental results with more realistic models of the nuclear structure that proceed beyond the simple shell model to see if the observed strengths can be understood quantitatively. In this regard, the 5^+ , $0 \hbar\omega$ excitation is important because it provides a test of available S - D shell-model wave func-

tions, which are known to describe many observables in the S - D shell.²⁹ The $A = 26$ system has 10 out of a possible 24 nucleons in the S - D shell; positive-parity excitations will be dominated by the degrees of freedom within this shell. Wildenthal performed³⁰ calculations for the ground state of ^{26}Mg and for the 5^+ states in ^{26}Al , using the full S - D shell degrees of freedom and S - D universal matrix elements.²⁹ These matrix elements describe accurately hundreds of observables for S - D shell nuclei, including excited-state energies and electromagnetic moments. One-body transition densities (OBTD) were calculated for the 0^+ to 5^+ transitions. The calculations predict that the total 5^+ strength will be 80% of the simple shell-model expectation and that the ground state will have 80% of this total; thus, the ground state is predicted to have 64% of the simple shell-model limit. We observe this transition to have 56% of the simple shell-model limit, or about 88% of that expected from the full S - D shell-model predictions. The shell-model calculations predict that the majority of the remaining 5^+ strength would reside in two states at 3.4 and 4.5 MeV of excitation, which carry 7% and 4% of the simple shell-model limit, respectively. This result is in reasonable agreement with our experimental measurements that show a few states near this excitation energy which could possibly be 5^+ states (see Fig. 1), although a single state at 3.4 MeV is clearly the strongest of these and has 17% of the ESPHM. The comparison between the observed and predicted 5^+ spectra is shown in Fig. 11. Both the measured and calculat-

ed strengths are shown as fractions of the simple shell-model expectation for this strength. The full S - D calculations and the experimental results agree to first order, and considering the overall uncertainty in absolute strengths ($\pm 12\%$), there is no significant amount of missing strength. The total 5^+ strength predicted is within 10% of that observed.

Unfortunately, it is not possible to perform a shell-model calculation for the $(f_{7/2}, d_{5/2}^{-1})$, 6^- stretched-state excitations that is as reliable as the S - D calculations for 5^+ strength. Because the 6^- excitations involve orbitals in two major shells, full shell-model calculations including all degrees of freedom in two shells become hopelessly large. We performed a truncated shell-model calculation to compare with the measured isovector 6^- strength spectrum in ^{26}Al . The calculation was performed with the code OXBASH.³¹ The basis for the ^{26}Mg ground state allowed only for $2p2h$ excitations from the $d_{5/2}$ orbital into the $2s_{1/2}$, $d_{3/2}$, and $f_{7/2}$ orbitals. The 6^- final states were required to have nine particles in the $d_{5/2}$ orbital and one particle in the $f_{7/2}$ orbital. The two-body matrix elements were taken from the $SDPF$ interaction of Millener and Kurath (MK).³² The calculated 6^- spectrum is shown compared to the measured spectrum in Fig. 12, again in terms of the fraction of the simple shell-model expectation. The calculated 6^- strength is split into its three isospin components and spread out from about 9 to 16 MeV. All three isospin components are predicted to be fragmented, although the lowest-energy

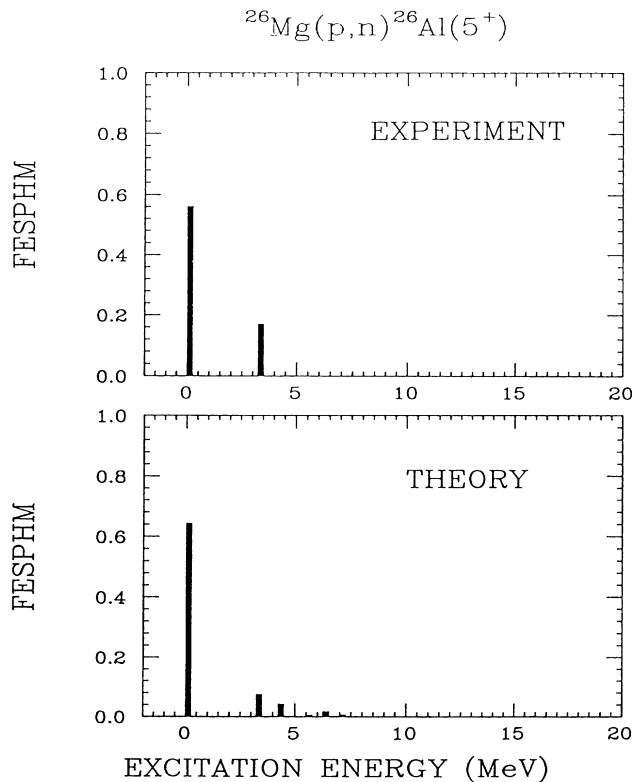


FIG. 11. Comparison between the measured and predicted 5^+ spectra for the $^{26}\text{Mg}(p,n)^{26}\text{Al}$ reaction at 134 MeV. The strengths are in terms of the fraction of the extreme single-particle-hole model strength.

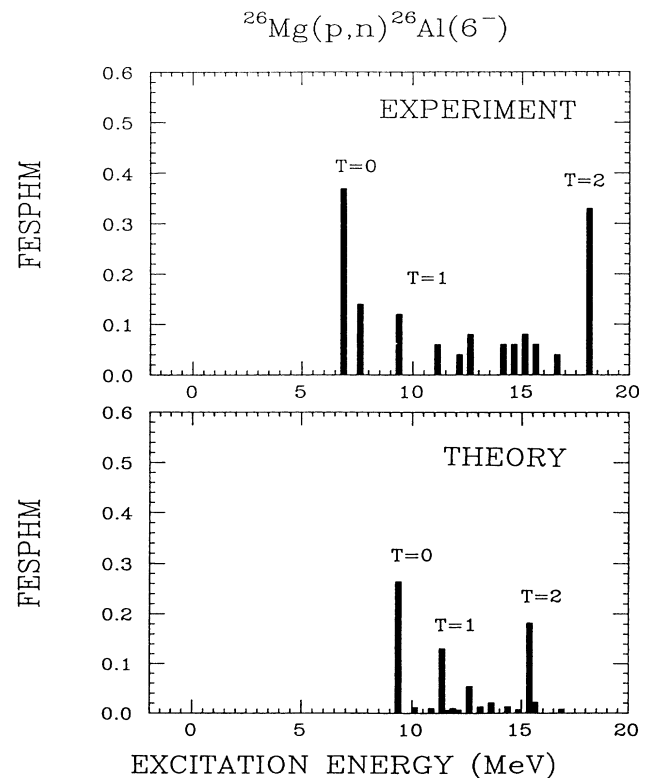


FIG. 12. Comparison between the measured and predicted 6^- spectra for the $^{26}\text{Mg}(p,n)^{26}\text{Al}$ reaction at 134 MeV. The strengths are in terms of the fraction of the extreme single-particle-hole model strength.

state of each component is predicted to have the majority of strength. Experimentally, the 6^- strength is observed to be spread out over a wider range of excitation energy from about 7 to 18 MeV. The observed fragmentation is greater than that predicted; although the lowest-energy state is the largest one observed for each component, other states are seen to have significant strength. Finally, the observed strength is actually greater than that predicted. Clearly, although the shell-model calculations indicate the general spectrum of 6^- strength, they do not describe the results quantitatively nearly as well as do the 5^+ calculations. This quantitative failure is due probably to the severe truncation required for the shell-model calculations and to the fact that the MK interaction was adjusted to reproduce experimental results at or near the boundary between the S - D and F - P shells, and is not expected to be as good for a nucleus near the middle of the S - D shell. Hopefully, more realistic shell-model calculations will be available in the future.

VI. CONCLUSIONS

The $^{26}\text{Mg}(p,n)^{26}\text{Al}$ reaction at 135 MeV shows clearly the excitation of both $(d_{5/2}, d_{5/2}^{-1})$, 5^+ , $0 \hbar\omega$ and $(f_{7/2}, d_{5/2}^{-1})$, 6^- , $1 \hbar\omega$ stretched-state strength. The 6^- strength is observed in $T=0, 1$, and 2 components, in general agreement with inelastic-scattering and transfer-reaction studies. Both the 5^+ and 6^- strength is fragmented, especially the $T=1$ component of the 6^-

strength which is observed in nine states between 9 and 17 MeV of excitation. Comparison of the different reactions, which excite the separate isospin components differently, allows identification of these components in ^{26}Al . The (p,n) experimental results are compared with distorted-wave impulse approximation calculations that assume the extreme single-particle-hole model; 73% of the $T=0$, 5^+ ESPHM strength is observed, and 61%, 58%, and 27% of the $T=0, 1$, and 2 , 6^- , ESPHM strengths, respectively. The magnitude and distribution of the 5^+ strength is described well by full S - D shell-model calculations. The general distribution of the 6^- strength is indicated by a truncated SD - FP shell-model calculation, although quantitative agreement is poor. The (p,n) reaction is shown to complement other reaction studies in significant ways, because of the absence of isoscalar strength and the ability to excite strength with isospin one less than the target.

ACKNOWLEDGMENTS

We wish to thank R. A. Lindgren and D. A. Geesaman for allowing us to use some of the (e,e') and (p,p') results prior to their publication, and for discussions regarding these results. We thank B. A. Brown for help in performing some of the shell-model calculations presented here. We also thank M. Mostajabodavati for preparing the figures. This work was supported in part by the National Science Foundation.

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