RAPID COMMUNICATIONS

PHYSICAL REVIEW C VOLUME 38, NUMBER 2 AUGUST 1988

Shape transitions in hot rotating ^{158}Yb nuclei

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(Received 5 May 1988)

The finite-temperature Hartree-Fock-Bogoliubov cranking equation predicts the *most probable* shape for ^{158}Yb nuclei. At a spin-dependent critical temperature, there is a transition from prolate collective rotation to oblate noncollective rotation. For spins above $39\hbar$, there is oblate noncollective rotation at all temperatures. However, inclusion of thermal shape fluctuations produces an *average* shape which is prolate collective at spins 40 and $50\hbar$ and thermal excitation energy $E^* \approx 10$ MeV.

The transitional nucleus ^{158}Yb has an yrast line which displays a shape transition.[1-3] For low spins, this nucleus has a prolate shape which rotates collectively (the rotation axis is perpendicular to the symmetry axis). At high spins the shape is oblate and the rotation is noncollective (the rotation axis coincides with the symmetry axis). Recent experiments at Oak Ridge National Laboratory, Oak Ridge, Tennessee suggest that for spins $38-51\hbar$, the noncollective yrast states give way to collective structures with increasing temperature.[4,5] The purpose of this article is to calculate the shape of ^{158}Yb nuclei as a function of spin and temperature, and to search for an explanation of this Oak Ridge experiment.

Mean-field theories such as the microscopic finite-temperature Hartree-Fock-Bogoliubov cranking (FTHFBC) theory[6-8] and the macroscopic Landau theory[9,10] have been used to study the shapes of hot rotating nuclei. These articles have concentrated on the strongly deformed nuclei ^{164,166}Er, although the Landau theory[9,10] has also been used to determine the universal features of shape transitions.

Consider the FTHFBC equation[11-13]

$$\begin{pmatrix} \mathcal{H} & \Delta \\ -\Delta^* & -\mathcal{H}^* \end{pmatrix} \begin{pmatrix} U_i \\ V_i \end{pmatrix} = E_i \begin{pmatrix} U_i \\ V_i \end{pmatrix}, \quad (1)$$

where the Hartree-Fock Hamiltonian $\mathcal{H}$ includes the cranking term $-\omega J_x$, and Δ is the pair field. The quasiparticle operators are defined by the eigenvectors (U_i, V_i), and the quasiparticle energies are given by the eigenvalues E_i. At finite temperature, the quasiparticles are thermally excited, in accordance with the Fermi-Dirac occupation probability

$$f_i = \frac{1}{1+e^{E_i/T}} . \quad (2)$$

The FTHFBC equation determines the shape and pair gaps as self-consistent microscopic functions of spin and temperature.

The pairing-plus-quadrupole (PPQ) interaction of Kumar and Baranger (KB) is used.[14,15] KB fit the interaction strengths to the ground-state shapes and pair gaps of rare-earth nuclei. We have not introduced any additional parameters to fit the thermal or rotational properties of nuclei, and we are using the same Hamiltonian for ^{158}Yb as we have employed for other rare-earth nuclei.[8,16,17]

The FTHFBC phase diagram for ^{158}Yb is shown in Fig. 1. The ground state is prolate ($\beta = 0.173$) and axially symmetric. At spin zero, raising the excitation energy to $E^* = 23.3$ MeV ($T = 1.04$ MeV) causes the deformation to collapse, and the equilibrium shape becomes spherical. For rotating nuclei there are two phases. If the spin is below $39\hbar$, and if the thermal excitation energy (temperature) is below a critical value, then the shape is approximately prolate and the rotation is collective. However, if the spin is above $39\hbar$, or if the thermal excitation energy exceeds a critical value, then there is oblate noncollective rotation. The critical temperature goes to zero at spin $39\hbar$. Even though nuclei are finite systems, the mean-field approximation creates a shape transition which occurs rapidly when the excitation energy crosses the phase-transition line. The response of this transitional nucleus can be contrasted with the strongly deformed nucleus ^{166}Er, which retains its prolate collective structure at spin $60\hbar$ for all temperatures up to 1.4 MeV.

In the Oak Ridge experiment, the noncollective yrast states at spins $38-51\hbar$ give way to collective structures when the thermal excitation energy is increased from 0 to ≈ 10 MeV. For spins $40-50\hbar$, the FTHFBC equilibrium phase is oblate noncollective at all temperatures. Therefore, the FTHFBC mean-field description does not provide an explanation for the Oak Ridge experiment. How-

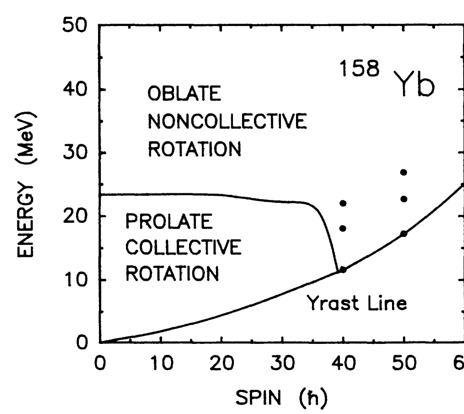

FIG. 1. Phase diagram for ^{158}Yb. The dots at spins 40 and $50\hbar$ correspond to temperatures of 0, 0.6, and 0.8 MeV. Thermal shape fluctuations are not included.

ever, it must be emphasized that the FTHFBC equation is derived by a variational principle, which minimizes the free energy

$$F = E - TS , \tag{3}$$

where S is the entropy. Consequently the FTHFBC self-consistent shape is the *most probable* shape, i.e., the shape which minimizes $F(\beta,\gamma;I,T)$ for given spin and temperature.

For finite temperatures, there are thermal fluctuations which create shapes different from the most probable shape. These shape fluctuations can significantly alter the properties of hot rotating nuclei.[18–23] The probability for a given shape to occur is

$$P(\beta,\gamma;I,T) \propto \exp[-F(\beta,\gamma;I,T)/T] . \tag{4}$$

For given spin and temperature, consider an ensemble of nuclei with this deformation distribution. The ensemble *average* of β is

$$\bar{\beta} = \langle\beta\rangle = \int \beta P(\beta,\gamma)\beta d\beta d\gamma \Big/ \int P(\beta,\gamma)\beta d\beta d\gamma . \tag{5}$$

Averages are also calculated with the volume element $\beta^4 |\sin 3\gamma| d\beta d\gamma$. The fluctuation in β is

$$(\Delta\beta)^2 = \langle\beta^2\rangle - \langle\beta\rangle^2 . \tag{6}$$

There are similar definitions for $\bar{\gamma}$ and $\Delta\gamma$. Equation (4) shows that when the temperature is zero, there are no thermal shape fluctuations. Then the average shape is identical to the most probable shape. However at finite temperature, the average shape may be different from the most probable shape.

Consider nuclei with spin $40\hbar$ and a temperature of 0.8 MeV. The most probable shape has a thermal excitation energy $E^* = 10.4$ MeV, as shown in Fig. 1. The probability distribution $P(\beta,\gamma)$ is given in Fig. 2. The dot indicates the most probable phase, i.e., oblate noncollective rotation, which is assigned a relative probability equal to 1. The contours map the relative probabilities for other shapes. Observe that there are prolate collective states ($\gamma = 0°$) which have a high relative probability of 0.7. Even oblate collective states ($\gamma = 60°$) and prolate noncollective states ($\gamma = 120°$) are populated by the thermal fluctuations. This demonstrates that even though the most probable phase is oblate noncollective rotation, a considerable fraction of the members of the ensemble populate collective states.

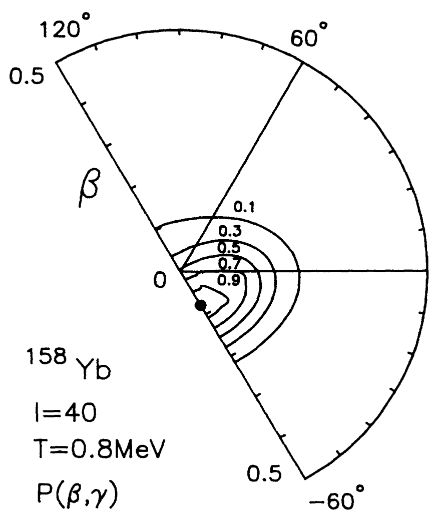

FIG. 2. Contour map of the shape probability distribution in the β,γ plane. The lines have constant values of relative probability. The spin is $40\hbar$ and the temperature is 0.8 MeV.

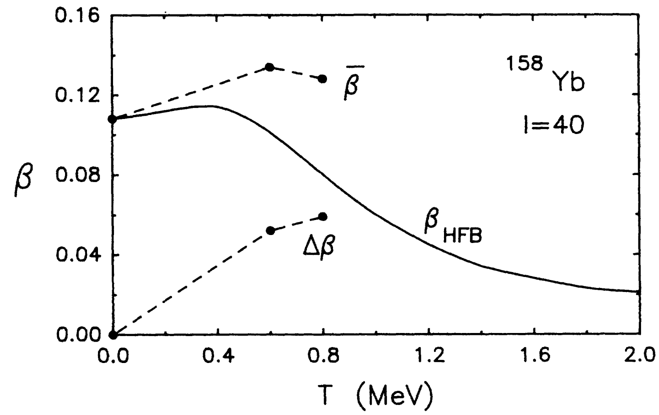

FIG. 3. The quadrupole deformation β vs the temperature T at spin $40\hbar$.

The ensemble average shape is compared to the most probable (HFB) shape in Figs. 3 and 4 for $I = 40$. For $T = 0.8$ MeV, the average $\bar{\beta}$ is considerably larger than β_{HFB}, and the magnitude of the fluctuation in β indicates a broad distribution of shapes. Although $\gamma_{HFB} = -60°$ (oblate noncollective), the average $\bar{\gamma} = -4°$ (nearly prolate collective). If the volume element $\beta^4 |\sin 3\gamma| d\beta d\gamma$ is used, then $\bar{\gamma} = -7°$. The large fluctuation in γ again indicates a wide distribution of shapes. Since the average shape is essentially prolate collective, one may conclude that the ensemble contains a significant proportion of nuclei which display collective structures.

The shape fluctuations are also calculated for $I = 40$ and

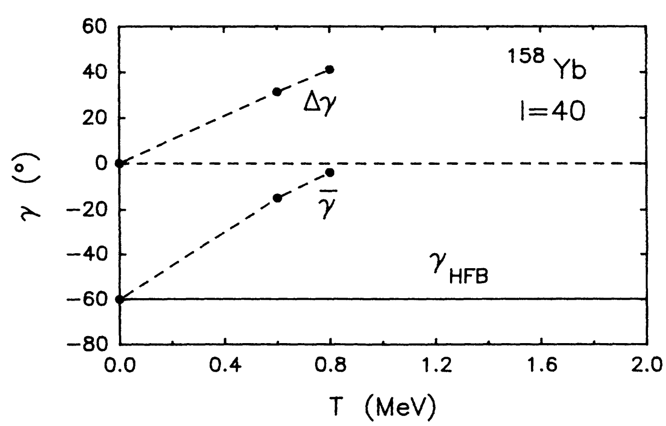

FIG. 4. The quadrupole deformation γ vs the temperature T at spin $40\hbar$.

$T = 0.6$ MeV (see Figs. 3 and 4). Then the most probable shape has $E^* = 6.5$ MeV, as shown in Fig. 1. Figure 4 shows that the importance of collective structures increases with the temperature.

The fluctuations are also determined for $I = 50$ and $T = 0.6$, 0.8 MeV. The results for $I = 50$ are very similar to those for $I = 40$. For $I = 50$ and $T = 0.8$ MeV ($E^* = 9.7$ MeV), the average $\bar{\gamma} = -7°$.

In conclusion, for spins 40 and $50\hbar$ and thermal excitation energies of approximately 10 MeV, there is a significant probability that statistical shape fluctuations will populate prolate collective structures. This mechanism may provide a possible explanation for the Oak Ridge experiment.

This work was supported in part by the National Science Foundation.

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