Coulomb Faddeev calculation of d+p→p+p+n spectra near threshold and three-body effects

P. Doleschall,* H. Kröger, and R. J. Slobodrian
Université Laval, Département de Physique, Laboratoire de Physique Nucléaire, Québec, Canada G1K 7P4
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Coulomb effects are relevant near the breakup threshold and simple corrections of neutral Faddeev calculations are not sufficient. At a center-of-mass energy $E_{c.m.}=0.243$ MeV, an approximate Coulomb Faddeev calculation is shown to be reliable. Comparison to experimental data shows deviations, which remain unexplained in terms of pure two-body nuclear-plus-Coulomb forces.

I. INTRODUCTION

It is well known that two-body nuclear forces are rather dominant in explaining nuclear binding energies. Although it is known that the three-nucleon binding energy is strongly influenced by the three-nucleon force, there are open questions.[1,2] On the one hand, calculations with three-nucleon forces produce an overbound triton;[1,2] on the other hand, a new version of the Bonn potential produces a nearly perfect triton binding energy without a three-nucleon force.[1] Therefore, at present it is not clear whether or not one really has to include a three-nucleon force in order to describe the three-nucleon bound-state properties.

Another possibility for finding an effect of the three-nucleon force via low-energy p-d breakup measurements was suggested in Ref. 3. However, neither the magnitude of effects nor the region of measurable quantities to observe them are known accurately. Earlier calculations[4,5] indicated that near the breakup threshold the breakup differential cross section depends only on the S-wave component of the N-N interaction and is not sensitive to the details of this interaction; although, it has to be noted that this was checked for a certain class of separable interactions whose form factors decrease rather fast in momentum space.

This advantage is overshadowed by the fact that near the breakup threshold the influence of the Coulomb field is expected to be strong, and cannot be handled by final-state Coulomb corrections, as was done previously with some success in Ref. 4 above threshold. If one could take into account properly the Coulomb interaction, a comparison of the calculations using only two-body forces and experimental measurements could yield information on effects due to the three-nucleon force.

II. COULOMB FADDEEV CALCULATION OF CORRELATION SPECTRA

The nuclear-plus-Coulomb interaction in the S-wave channel has a repulsive barrier with a height of about 0.20 MeV around 5 fm. Thus, at the low final-state energy $E_{c.m.}=0.243$ MeV the relative p-p energy is below the barrier for a significant region. Therefore, one should expect that the breakup process is suppressed to some extent. Based on this simple physical picture one can expect that if this barrier is properly included in the calculations the basic effect of the Coulomb field is taken into account.[6] To avoid the troubles caused by the long-range character of the Coulomb interaction, it will be cut off somewhere beyond the radius of the barrier. The model assumptions upon which the calculations are based are as follows.

(1) A cut-off Coulomb interaction with a form factor

$$V^{c}_{\text{cut-off}}=\begin{cases}\dfrac{1}{r} & \text{if } r\le R_{\text{cut}}\\ \dfrac{1}{r}e^{-[a(r-R_{\text{cut}})]^2} & \text{if } r>R_{\text{cut}}\end{cases}$$

($a=0.2$ fm^{-1}) was chosen.

(2) Since this cut-off Coulomb interaction is of short range it will be substituted by a proper separable approximation.

(3) The full p-p interaction is represented as a sum of the separable cut-off Coulomb and a separable N-N interaction.

Based on these approximations the Faddeev equations are solved at a center-of-mass $E_{c.m.}=0.243$ MeV. The numerical method of the solution is described in Ref. 7.

To make sure that the numerical approximations were accurate enough, the number of mesh points for the integration was increased up to a level (30 points) where the off-shell T-matrix elements, necessary for the construction of the breakup T-matrix, became accurate up to 1–2%. In a next step the sensitivity of the breakup differential cross sections relative to the cut-off parameter (R_{cut}) and to the number of separable terms was checked for the investigated angle combinations. It has to be noted that, of course, the necessary number of terms is increasing with the increase of the cut-off parameter. It was found that at energy $E_{c.m.}=0.243$ MeV the cut-off parameter must be at least 20 fm, but some regions of the investigated angle combinations require at least a 30 fm cut off. The criterion was the stability of the differential cross sections.

The EST method as it was suggested by Haidenbauer and Plessas[8] was used to construct the separable approximation of the cut-off Coulomb potential. However, we used the zero and negative energy points as the base of the approximation because the integration in the Faddeev equations is going through this region. The "on-shell

momenta" were chosen as the square roots of the absolute values of the energy points. Table I shows the energy values used (E_i) and the number of terms (N) for the different cut-off parameters (R_{cut}).

The $R_{cut}=10$ fm, $N=5$ separable expansion calculation reproduces the phase shifts of the original potential up to $E_{lab}=50$ MeV. In the cases $R_{cut}=10$ fm, $N=3$ and $R_{cut}=20$ fm, $N=5$ the phase shifts are reproduced up to 20 MeV, and in the other cases the phase shifts are reproduced up to 10 MeV. The breakup results become stable up to a few percent if the separable expansions reproduced the phase shifts up to 10 MeV.

The Haidenbauer-Plessas[8] separable expansion of the Paris potential was chosen for the nuclear part of the interactions. It was checked (together with the S-wave part of the cut-off Coulomb interaction) that neither the choice of a rank-3 single S-wave interaction nor the inclusion of a nuclear P-wave interaction really influences the breakup results at energy $E_{c.m.}=0.243$ MeV. Therefore, the nuclear part of the interaction was fixed as the rank-1 PEST1 for the $^1S_0$ channel, and the rank-1 PEST1 interaction for $^3S_1$-$^3D_1$ channel.[8]

III. RESULTS AND DISCUSSION

Firstly we have studied the influence of the Coulomb cut-off parameter R_{cut} on three-body observables. In Fig. 1 we show the computed p + d breakup differential cross section when the two protons are observed at 13°, −13°, the curves are calculated and plotted as a function of the length of the arc on the kinematical locus of the proton-proton correlation at such angles: E_S.

The results seem to converge for the values $R_{cut}=10,15,20$ fm; however, the effect for the value $R_{cut}=30$ fm is somewhat surprising for small values of the arc E_S. We could not establish the reason for this behavior and, since obtaining a separable expansion becomes more and more difficult with an increasing cut-off parameter, no attempt was made to perform calculations with larger values of R_{cut}.

We have searched values of the parameter R_{cut} for stability of the breakup cross sections over the full kinematic locus, without success. Hence we performed calculations with two sets of separable expansions.

(1) The cut-off parameter was chosen to be 20 fm. The rank-5 S wave and rank-3 P-, D-, and F-wave separable expansions of the cut-off Coulomb potential were used.

(2) The cut-off parameter was chosen to be 30 fm, and the rank-5 S-, P-, and D-wave separable expansions of the cut-off Coulomb potential was used.

The inclusion of the higher partial-wave components of the cut-off Coulomb potential was done one by one to see their respective effect on the breakup differential cross section. In addition, a reference n-d calculation was performed with the same nuclear interactions and the nucleon-deuteron breakup differential cross sections were calculated without (n-d) and with (p-d) final-state Coulomb corrections. Here the final-state Coulomb correction is a simplified version of that suggested by Koike:[9] The p-p two-body pure nuclear propagator is multiplied by a Coulomb penetration factor C_0^2.

The results are shown in Figs. 2–10. Although the purpose of these calculations was to produce reasonably accurate breakup differential cross sections, the elastic differential cross section could still be used as an indicator of a possible error. Since we know the qualitative effect of a repulsive Coulomb interaction on the elastic differential cross section, we can verify how these expected effects are qualitatively reproduced by the present calculation. In Fig. 2 one can see that the increase of the cut-off parameter from 20 fm to 30 fm will lead to an increase of the forward peak which is the consequence of the longer range. Figure 3 shows (this was not an expectation) that the forward Rutherford peak is formed by partial-wave components higher than P wave. Figure 4 gives the details of the dependence of the backward peak on the included components of the cut-off Coulomb interaction. It has to be noted that this backward peak does not seem to be too sensitive to the cut-off parameter: for $R_{cut}=20$ fm and $R_{cut}=30$ fm the same backward peak is produced. Finally we conclude that the elastic results indicate that we have included a long-range repulsive Coulomb-type interaction.

The Figs. 5–10 show the breakup results. Since the sensitivity to the different effects (R_{cut}, higher partial-wave components of the cut-off Coulomb interaction) are

TABLE I. Parameters for the numerical solutions of Faddeev equations.

R_{cut} (fm)	N	E_i (MeV)	R_{cut} (fm)	N	E_i (MeV)
		0.0			0.0
	3	−2.0		3	−1.0
		−8.0			−4.0
10.0		0.0	20.0		0.0
		−3.0			−1.0
	5	−10.0		5	−3.0
		−20.0			−7.0
		−40.0			−12.0
		0.0			0.0
		−1.5			−0.5
15.0	3	−6.0	30.0	5	−1.5
					−3.0
					−5.0

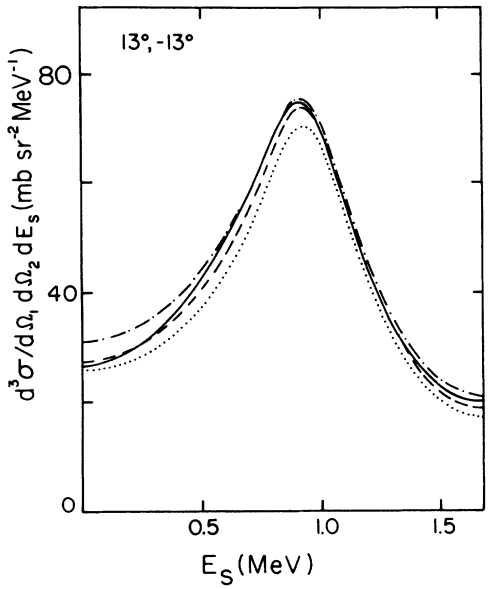

FIG. 1. Proton-proton correlation cross section as a function of the arc of the kinematic locus at 7.4-MeV incident deuteron energy, calculated with cut-off Coulomb S waves. The dotted line corresponds to a cut-off radius $R_{cut}=10$ fm, the dashed line to $R_{cut}=15$ fm, the dashed-dotted line to $R_{cut}=20$ fm, and the solid line to $R_{cut}=30$ fm. Notice that all curves lie within a reasonably well defined band. The $E_S=0$ point corresponds to the $E_{p_1}=E_{p_2}$ proton energies closest to $E_{p_1}=E_{p_2}=0$ in this figure and the following ones, except Fig. 11. E_S measures the energy on the kinematic locus of the E_{p_1}, E_{p_2} plane.

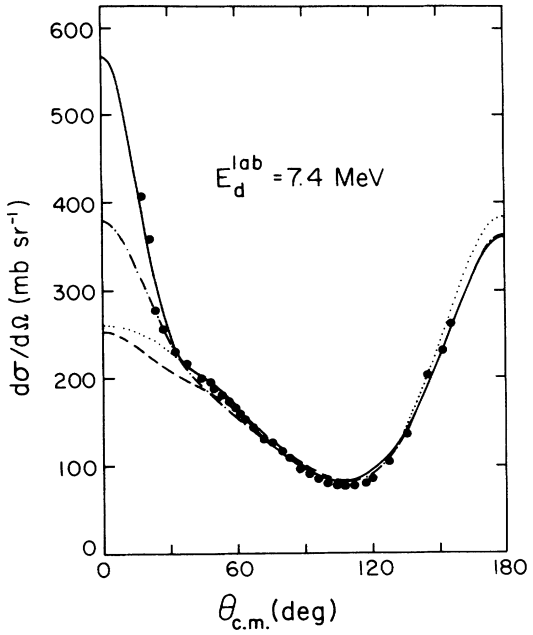

FIG. 3. Comprehensive view of several calculations of p-d scattering cross sections. For comparison the n-d cross section is shown as a dotted line. The experimental data of Lahlou *et al.* [J. Phys. **41**, 485 (1980)] are also shown. The dashed line is the S-P Coulomb calculation and the dashed-dotted line is the S-P-D Coulomb calculation. The solid line is the S-P-D-F Coulomb calculation, it shows that F waves are relevant in the entrance channel.

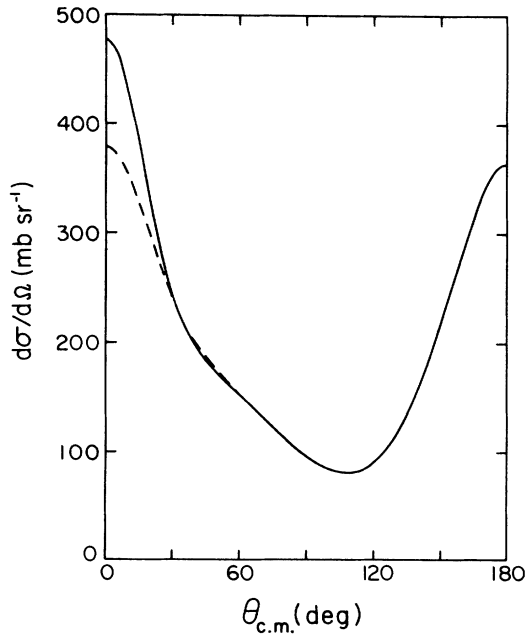

FIG. 2. Elastic p-d scattering cross section, including S-P-D Coulomb partial waves; the dashed line corresponds to $R_{cut}=20$ fm and the solid line to $R_{cut}=30$ fm. These curves correspond to $E_d=7.4$ MeV.

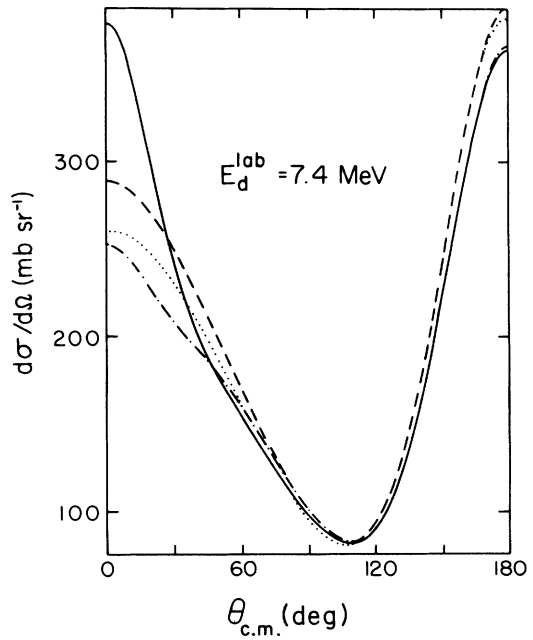

FIG. 4. The n-d cross section is shown by the dotted line. The dashed, dashed-dotted and solid lines show, respectively, the S Coulomb, S-D Coulomb and S-P-D Coulomb calculations. The effect of adding higher partial waves is far from dramatic except at "forward" angles.

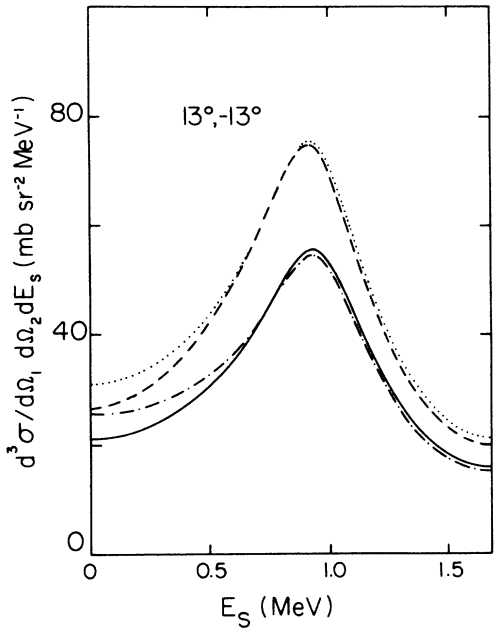

FIG. 5. Comparison of the theoretical proton-proton correlations from d + p→p + p + n at 7.4 MeV for S Coulomb with $R_{cut}=20$ fm (dotted line) and $R_{cut}=30$ fm (dashed line) with S-P-D Coulomb, $R_{cut}=20$ fm (dashed-dotted line) and $R_{cut}=30$ fm (solid line).

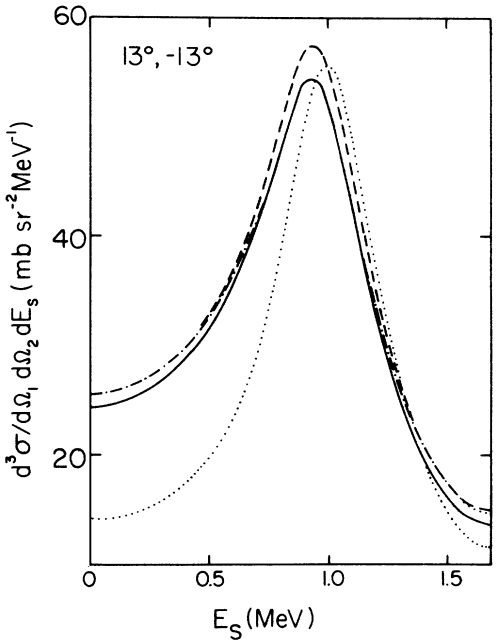

FIG. 6. Comparison of S-P, S-P-D, and S-P-D-F Coulomb calculations, shown, respectively, by dashed, dashed-dotted, and solid lines, with the n-d calculations using final-state Coulomb corrections (dotted). It is clear that the latter calculations are inadequate.

the same for all of our angle combinations, we show only the (13°, −13°) case in detail. In Fig. 5 we plotted the sensitivity of the breakup differential cross section to the cut-off parameter in the presence of the higher partial-wave components of the cut-off Coulomb potentials. It is evident that there is a significant change in the $E_S=0$–0.5 MeV arc-length region (as it was found before), where the relative p-p energy varies between 0.14 and 0.2 MeV, which seems to come basically from the S-wave part of the Coulomb interaction.

The effect of the inclusion of the different partial-wave components of the cut-off Coulomb interaction is shown in Figs. 6 and 7. It is clear that while the inclusion of the P-wave part of the cut-off Coulomb interaction produces a 20–25 % decrease of the differential cross section, the overall effect of the D- and F-wave components is around 5%. Therefore, it does not seem to be a far-fetched assumption that the overall effect of the partial-wave components higher than F wave could be below 5%. Based on this assumption we estimate that the accuracy of our results is about 5% in the 0.6–1.7 MeV region. In the $E_S=0$–0.5 MeV region, due to the effect of the cut-off parameter (see Fig. 5) we have much less accuracy. However, it has to be noted that the shape of the breakup cross section for this $E_S=0$–0.5 MeV region is the same

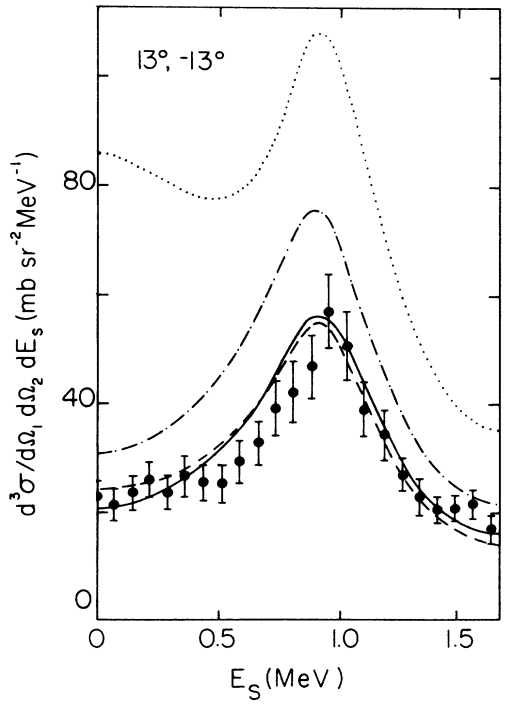

FIG. 7. Neutron-neutron and proton-proton correlation cross sections compared with experimental data. The dotted line corresponds to the n-n correlation from n + d→n + n + p. The dashed-dotted line is the S Coulomb calculation for the p-p correlation from d + p→p + p + n; the dashed line corresponds to the S-P-D-F Coulomb ($R_{cut}=20$ fm) calculation; and the solid line is from the S-P-D Coulomb ($R_{cut}=30$ fm) calculation. An anomaly is seen both at $E_S=0$ and $E_S=$max (corresponding to the symmetric energy partitions $E_{p_1}\simeq E_{p_2}$). The data are scaled up by a factor of 2.

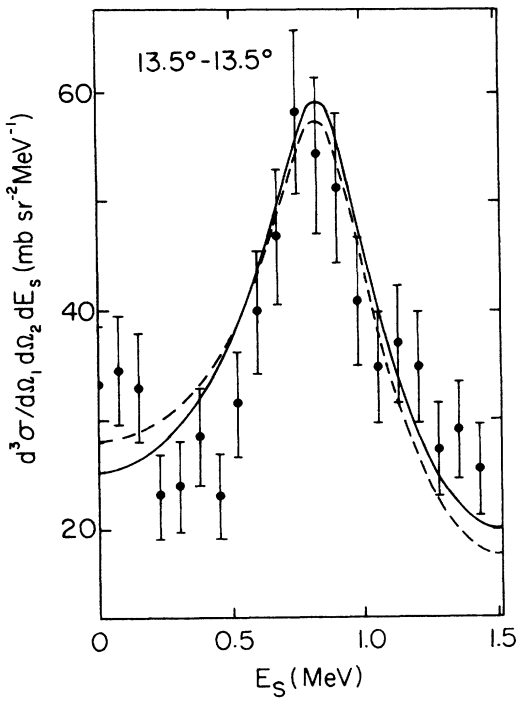

FIG. 8. Comparison of S-P-D Coulomb using $R_{cut}=30$ fm (solid line) and S-P-D-F Coulomb using $R_{cut}=20$ fm (dashed line) with the experimental data at 13.5°, −13.5°, $E_{c.m.}=0.243$ MeV, which have been scaled by a factor of 2.25. However, there is a clear deviation near $E_S=0$.

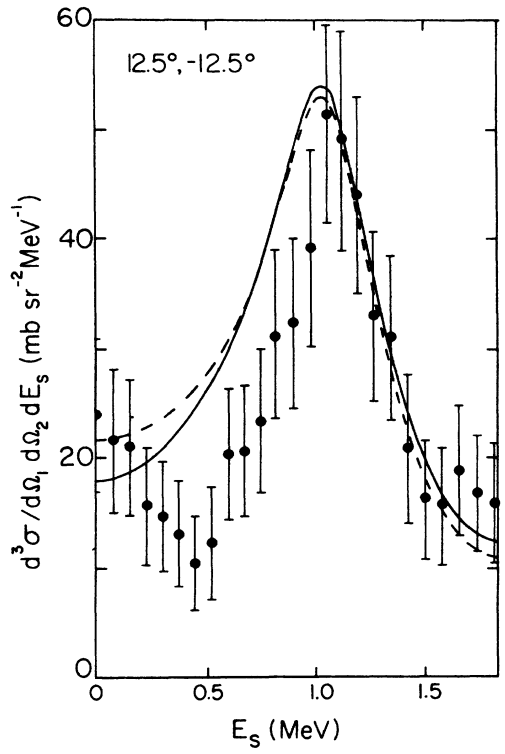

FIG. 9. Correlation angles 12.5°, −12.5°, parameters, and lines as in Fig. 8. The anomaly near $E_S=0$ is even more prominent. The experimental data were also scaled by a factor 2.25.

for all calculations: it increases monotonically.

This indicates that even within the relatively large error of the theoretical calculations, the measurements,[10] plotted in Figs. 7–10 are not reproduced in the $E_S=0$–0.7 MeV arc-length region. This discrepancy seems to be rather characteristic for the angles (13.5°, −13.5°) and (12.5°, −12.5°) (Figs. 8–9). None of the calculations produce the shape given by the experimental values in the $E_S=0$–0.5 MeV arc-length interval.

IV. CONCLUSIONS

Here we summarize the established effects shown by the calculations.

(1) The n-d calculations give both in magnitude and in shape a poor description of the p-d data (Fig. 7).

(2) The final-state Coulomb correction seems to be reasonable around the value $E_S=1$ MeV, but it is too low around the $E_S=0$ meV region (Fig. 6).

(3) The inclusion of the S-wave part of the cut-off Coulomb interaction is not sufficient. There is a significant effect of the P-wave part and a smaller effect of the D- and F-wave parts (Figs. 6 and 7).

Although the problem of the proper cut-off parameter is unresolved, the overall results seem to be reasonably stable. However, if one reconsiders the assumptions made, it becomes clear that the results themselves somewhat contradict the basic assumption of the barrier effect. The sensitivity to the higher partial-wave components of the cut-off Coulomb interaction indicates that the barrier effect is not very strong; there is a considerable rescattering occurring that will involve the higher partial-wave components of the p-p subsystem. This could be seen in a calculation performed at a center-of-mass energy

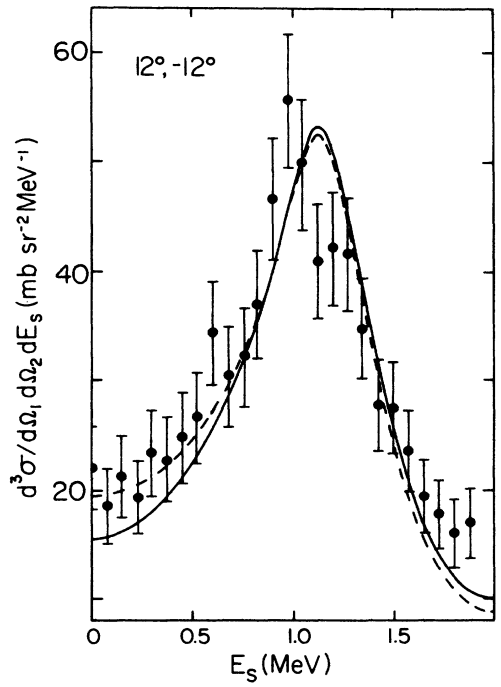

FIG. 10. Correlation angles 12, −12°. The experimental data are scaled by a factor 1.5. The lines are as in Fig. 8.

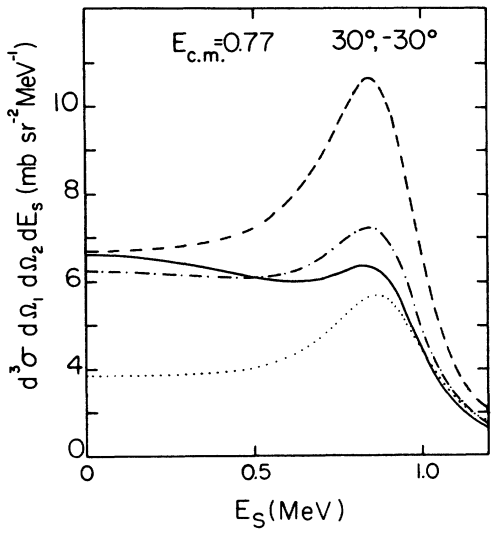

FIG. 11. Correlation cross sections at $E_{\text{c.m.}}=0.77$ MeV, 30°, −30°, calculated with S Coulomb (dashed line), S-P Coulomb (dashed-dotted line), S-P-D Coulomb (solid line), compared with the n-d plus final-state Coulomb corrected calculations. Here again there is evidence of the inadequacy of the latter. However, there is not much evidence of convergence of the Coulomb calculations. Here $S=0$ corresponds to proton energies $E_{p_1}=E_{p_2}$ farthest from $E_{p_1}=E_{p_2}=0$.

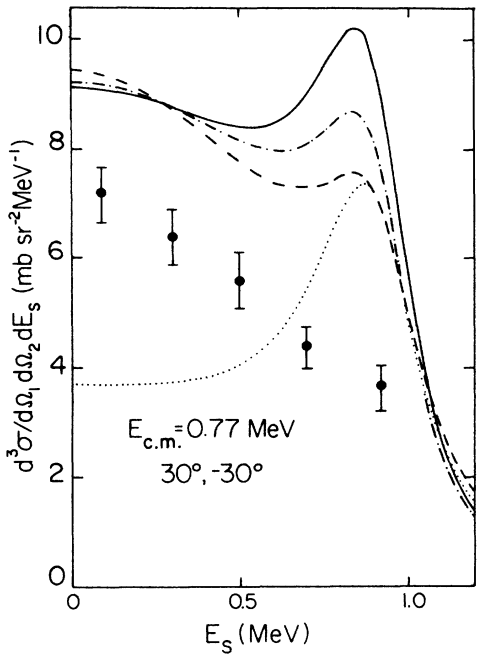

FIG. 12. Comparison of the experimental data (from Ref. 3) with the n-d (dashed line) plus final-state Coulomb corrections calculation (dotted line), S-P-D Coulomb (dashed-dotted line), and S-P-D-F Coulomb (solid line). The data are scaled by a factor of 2.

$E_{\text{c.m.}}=0.77$ MeV. The interactions were the same as in calculation (1) at $E_{\text{c.m.}}=0.243$ MeV. The results are shown in Figs. 11 and 12.

At first it seems that the effect of the higher partial-wave components is declining (Fig. 11). However, the inclusion of the F-wave part of the cut-off Coulomb interaction produced a dramatic effect in the n-p FSI region (around $E_S=0.8-0.9$ MeV). This is an indication that the use of the cut-off Coulomb interaction like any other short-range potential will not work because even at relatively small breakup final state energies we have to include already a large number of partial-wave components. This will increase the size of the Faddeev equations to practically unsolvable values.

However, in spite of the failure to describe the 0.77-MeV breakup (although with some extra effort we would be able to include a few more higher partial-wave components of the cut-off Coulomb interaction), there is a lesson to be learned: at this energy the final-state Coulomb correction gives absolutely wrong results in the $E_S=0-0.5$ MeV region (outside the n-p FSI region). It contradicts not only the cut-off Coulomb calculation, which seems to converge in this region, but also the p-d breakup measurements.[11] Since the final-state Coulomb correction gave reasonably good results at higher energies, we have to conclude that there is a strong Coulomb influence on the formation of the breakup state around this 0.77-MeV-energy region (which is not surprising).

It is now reasonably certain that the deviations shown by the experimental data in Figs. 7, 8, and 9 are not explained by a cut-off Coulomb Faddeev calculation based on two-body forces, which is reasonably reliable as discussed above at length. This is not inconsistent with the hypothesis forwarded elsewhere[3,12] about the identification of such deviations with effects due to the main component of the three-body force in a three-nucleon system, which produces enhancements in the same regions of phase space where anomalies are found. More accurate experimental data at 0.243 MeV in the c.m. and below are certainly necessary. Additional theoretical work is also required at higher kinetic energies, short of "exact" Coulomb Faddeev calculations.

Note added. It is appropriate to discuss the validity of the model used in this paper, based on a cut-off Coulomb potential and its separable expansion. The use of a cut-off is a particular form of the general screening procedure. The latter was shown to be mathematically rigorous by Prugovecki and Zorbas[13] and was applied to the p + d system in the numerical calculations.[14] The question is the quality of the calculations with a separable expansion of the screened potential for a given screening radius (cut-off radius as called here). Our results show a reasonable stability and represent a compromise in an effort to balance the error in the two-body phase shifts and three-body cross sections, against the computational difficulty, which increases rapidly with the number of separable terms.

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*Permanent address: Central Institute for Physics, Hungarian Academy of Sciences, Budapest, Hungary.

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