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Application of Gamow resonances to continuum nuclear spectra

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Neutron and proton Gamow resonances are evaluated in ^{208}Pb and they are used together with bound states as single-particle representation to calculate random phase approximation particle-hole excitations. A good account of the escape width corresponding to the particle-hole giant resonances is obtained.

The treatment of continuum spectra in nuclear physics is an old and outstanding problem. The main reason for this is that one has to somehow discretize the continuum to treat it numerically. One is thus left with an infinite set of coupled equations which is difficult to solve.[1,2] Yet, it is necessary to include the continuum in the study of many nuclear processes, e.g., the formation of the α particle in α decay or the building up of resonant states lying high in the nuclear spectra.

The study of resonances and other processes influenced by the continuum is an actual problem not only in nuclear physics but in other branches of physics as well.[3–5] An important step in the study of the continuum in nuclear physics was given in the 1960's[6–8] with the introduction of the so-called Gamow resonances (GAR), i.e., a solution of the time independent Schrödinger equation with purely outgoing waves at large distances. Such a function (which diverges at infinite) is not square integrable but one can still define the norm of a GAR (Refs. 6 and 9) and an inner product between two GAR.[7,8]

Gamow resonances have recently been applied in molecular and atomic physics also.[4,5] Even from a mathematical point of view the GAR are being studied as probable powerful tools in spectral analysis.[10]

An important and rather peculiar property of the GAR is that although they are orthogonal to any real continuum state, they are generally not orthogonal to wave packets formed from a superposition of the continuum states. Moreover, the GAR have a large overlap with wave packets that are peaked at the resonance energy.[11] Therefore, from a physical point of view it makes sense to use GAR to describe states which are immersed in the continuum, especially considering that bound states, GAR, and an integral over a deformed path L in the complex E plane provide a completeness relation.[7] Recently it has been shown that an expansion on this basis can be defined in different ways both in spherical[12–15] and in deformed systems.[16] In this paper we will use the completeness relation of Ref. 7, i.e.,

$$\sum_m |\, m \,\rangle\langle\, m \,| + \sum_n |\, n \,\rangle\langle\, \tilde{n} \,| + \mathrm{Int}(L) = 1 \;, \tag{1}$$

where m (n) labels bound states (GAR) and $\mathrm{Int}(L)$ is the contribution from the integral over the contour L. Due to the flexibility in choosing this contour one can include in the basis the resonances which fit best the physical problem under consideration. The widths of observable resonances are usually smaller than their energy centroids. Therefore, one expects that rather narrow GAR in the energy range of interest play an important role while the others should have little effect on the results. In this paper we will use as basis vectors only bound states and GAR with imaginary parts not larger (in absolute value) than the corresponding real parts, i.e., GAR not broader than twice their energies, and we neglect the continuum integral in Eq. (1). In this way we expect to include in a feasible manner the most important effects induced by the continuum on observable nuclear spectra. We shall see below that the results confirm the soundness of this choice.

In this paper we will study resonances in ^{16}O and in ^{208}Pb, but we will present details of the calculations only for the more complex case of ^{208}Pb. We will first analyze the single-particle spectrum, i.e., the bound states and GAR. For this we solve the one-particle Schrödinger equation with a Woods-Saxon potential, including the spin-orbit term, by using the computer code GAMOW.[17] In ^{16}O we obtained single-particle states which fit the experimental spectrum rather well.[18] In particular, a 96 keV wide $d_{3/2}$ neutron state has been observed in the experiment at 5.085 MeV, while our calculation gives 5.201 MeV for the energy of the $0d_{3/2}$ state and 40 keV for its width. Since this is only the escape width, one can consider the agreement to be rather good. In ^{208}Pb we obtained the single particle states shown in Table I. As one would have expected the absolute value of the imaginary part of the single particle energies (decay widths) are smaller for protons than for neutrons due to the Coulomb barrier. In the same way states with large orbital angular momenta have small decay widths due to the centrifugal barrier. Together with the calculated states we give in Table I the experimental values which are uncertain for high lying states. When comparing the experimental and calculated values in Table I one has to keep in mind that the experimental values do not refer to pure single-particle states.[20] In fact, the calculated bound single-particle energies in Table I are approximately the same as those obtained by similar previous calculations.[21] From the point of view of this paper, the interesting feature is the set of unbound states. Although the proton resonances are narrow, the width of the neutron resonances varies widely,

TABLE I. Single-particle states used in the calculations for ^{208}Pb. The proton (neutron) energies E_p (E_n) are in MeV. The experimental data are from Ref. 19. Note that all imaginary values are negative, as it should be for outgoing (decaying) resonant states (Ref. 11). The parameters in the Woods-Saxon potential are $a=0.75$ (0.70) fm, $r_0=1.19$ (1.27) fm, $V_0=66.0$ (44.4) MeV, and $V_{so}=9.5$ (8.25) MeV for protons (neutrons).

No.	State	E_p	Expt.	E_n	Expt.
1	$0f_{7/2}$	-22.67			
2	$0f_{5/2}$	-20.17			
3	$1p_{3/2}$	-18.32			
4	$1p_{1/2}$	-17.33			
5	$0g_{9/2}$	-16.23		-20.99	
6	$0g_{7/2}$	-12.37	-11.48	-18.06	
7	$1d_{5/2}$	-11.04	-9.68	-17.06	
8	$0h_{11/2}$	-9.26	-9.35	-14.96	
9	$1d_{3/2}$	-9.10	-8.36	-15.51	
10	$2s_{1/2}$	-8.71	-8.01	-15.30	
11	$0h_{9/2}$	-3.78	-3.80	-10.69	-10.78
12	$1f_{7/2}$	-3.54	-2.90	-10.49	-9.71
13	$0i_{13/2}$	-1.84	-2.19	-8.57	-9.00
14	$2p_{3/2}$	-0.69	-0.68	-8.35	-8.27
15	$1f_{5/2}$	-0.52	-0.98	-8.08	-7.94
16	$2p_{1/2}$	0.49	-0.17	-7.41	-7.37
17	$1g_{9/2}$	$4.03-i0.00$		-3.93	-3.94
18	$0i_{11/2}$	$5.43-i0.00$		-2.80	-3.16
19	$0j_{15/2}$	$5.96-i0.00$		-1.88	-2.51
20	$2d_{5/2}$	$6.75-i0.00$		-2.07	-2.37
21	$3s_{1/2}$	$7.84-i0.04$		-1.44	-1.90
22	$1g_{7/2}$	$8.09-i0.00$		-0.77	-1.45
23	$2d_{3/2}$	$8.53-i0.03$		-0.78	-1.40
24	$2f_{7/2}$			$2.10-i0.87$	
25	$1h_{11/2}$			$2.25-i0.03$	
26	$2f_{5/2}$			$2.70-i2.32$	
27	$0k_{17/2}$			$5.03-i0.00$	
28	$1h_{9/2}$			$5.40-i0.73$	
29	$0j_{13/2}$			$5.41-i0.01$	
30	$1i_{13/2}$			$7.66-i1.04$	

from states with practically negligible widths up to the broad state $2f_{5/2}$, for which the width (4.64 MeV) is nearly twice the energy.

With bound states and GAR as single-particle representation we are in a position to analyze more complex excitations. We will present here random phase approximation (RPA) particle-hole calculations done using a separable multipole-multipole interaction.[22,23] The isoscalar strength of the interaction was fixed, as usual, by fitting the experimental energy of the first excited state. Thereby we calculated the isovector strength as in Ref. 22. In order to calculate the RPA matrix and the energy weighted sum rule (EWSR) for electromagnetic operators we had to determine the single-particle matrix elements of $r\partial V/\partial r$ and r^{λ}, respectively, between all combinations of bound and resonant states. Due to the divergent character of the GAR we had to apply a special procedure for the calculation of the radial integrals. The idea is the same as the one used in Ref. 9, which later became known as "exterior complex scaling."[24] With the interaction matrix elements thus calculated we diagonalized the corresponding RPA matrix making use of the complex diagonalization routine F02BDF.[25]

The calculated RPA energies are complex. The real part corresponds to the position of the resonance while the imaginary part is related to its width. The experimental width consists of the escape width, produced by particle emission, and the spreading width, due to mixing with more complicated configurations (including collisional damping[26]). It is usually the spreading width that one calculates.[28] The calculation of the escape width requires a proper inclusion of the continuum.[1,27,29] In our case, however, the imaginary part of the energies corresponds to the escape width as it is implied in the definition of the GAR.

In the rather simple case of ^{16}O we calculated the giant dipole resonance (GDR) adjusting the strength of the interaction to obtain a zero-energy solution (the spurious 1^- state). We then obtained the GDR at an energy of $(23.69-i0.28)$ MeV, exhausting 94% of the EWSR, in fairly good agreement with experiment.[18] We also tested the reliability of our method by comparing with the classical calculation of Ref. 1, where the continuum is treated exactly. For this we chose the potential parameters as in Ref. 1 (case without absorption so that only processes that contribute to the escape width are included). The results thus obtained agree with those in Ref. 1 within 100 keV. Moreover, we also compared with the continuum RPA calculation of Ref. 27 (here we used the particle-hole Migdal force of Ref. 27) and the results of both calculations agree within 150 keV.

In the case of ^{208}Pb we calculated the quadrupole states within the basis of Table I. In Table II we present the calculated correlated states up to an excitation energy of 13 MeV. There are some striking features in Table II worthwhile to be commented. The escape width of

TABLE II. Correlated particle-hole energy E_c up to 13 MeV and the corresponding contribution to the isoscalar energy weighted sum rule S_c for the operator $r\partial V/\partial r$ in ^{208}Pb. The energies are in MeV and the sum rule is given in arbitrary units. E_p (E_n) are in MeV. The experimental data are from Ref. 19.

E_c	S_c
$4.09-i0.00$	$67-i5$
$5.44-i0.00$	$10-i3$
$5.51-i0.00$	$5-i2$
$5.81-i0.00$	$0+i0$
$10.13-i2.33$	$2-i2$
$10.18-i0.87$	$0+i0$
$10.44-i0.06$	$349-i23$
$10.49-i0.86$	$10-i10$
$10.78-i2.33$	$0+i0$
$11.06-i2.33$	$0+i0$
$12.59-i0.88$	$0+i0$
$12.78-i0.03$	$0+i0$
$12.79-i0.87$	$0+i0$
$12.94-i0.03$	$0+i0$

the GDR in heavy nuclei is small, only about 15% of the total width.[30] The same value is usually assumed for the isoscalar quadrupole giant resonance (SQGR) in those nuclei.[28] The experimental SQGR in ^{208}Pb exhausts about 70% of the EWSR and is located at 10.6 MeV with a total width of about 2.0 MeV.[31] Thus, according to the previous argument the experimental escape width would be 0.30 MeV.

In our case, between 10 and 11 MeV of excitation energy in the uncorrelated spectrum all states are very broad, either 1.74 or 4.64 MeV wide. Thus, it would seem that our calculation would predict a too wide giant resonance. Even more, up to 13 MeV in that spectrum the values of the corresponding sum rule are so small that it is not clear whether there would be any collective state at all between 10 and 11 MeV. However, in the correlated spectrum of Table II a very collective state appears (it exhausts 76% of the EWSR) at 10.44 MeV with a width of only 0.12 MeV just engulfed by noncollective, but very wide, states. This is the SQGR. Although already this in itself is a surprising result, it is also remarkable that the giant resonance is not the lowest state among the $\Delta N = 2$ excited states as it would be within a real (bound) basis. To analyze this in more detail we also calculated the quadrupole states using the computer code RPAPH (Ref. 32) within a harmonic oscillator representation with standard parameters.[33] We found that bound states are about the same in both calculations, but the SQGR is indeed the first $\Delta N = 2$ excited state in the harmonic oscillator case. It lies at 8.66 MeV of excitation energy. This value is very low but agrees with the one given in Ref. 34. In our case, the first $\Delta N = 2$ excited state is the very broad one at 10.13 MeV, which is only weakly excited by the external field. This indicates that our method differs considerably from the usual treatment of resonant states.

In general, the main features of the SQGR mentioned above appear for the isovector case as well.

Just opposite to what one would have predicted from the uncorrelated spectrum, our calculated giant resonances turn out to be narrow. The main reason for this is that the most important components of these states are built upon high spin single-particle states, all of which are either bound or narrow.

Although the calculated position of the SQGR agrees well with the corresponding experimental value, the calculated width seems to be too small. However, one has to consider that the escape width extracted from the experimental spectrum is rather uncertain. Moreover, only a small mixing of the giant resonance with the near lying wide states would enlarge it considerably.

Our strength function is also complex. While the imaginary part of the energy gives the width of the state concerned, it is not clear what meaning should be assigned to the imaginary part of the strength function and, in general, to the imaginary part of any transition probability. A reasonable interpretation of this quantity is that it is related to the interference between the resonance and the background of the process being studied.[35] In our case, however, the EWSR has small imaginary components.

We also calculated the sum rule corresponding to an electromagnetic field to see if the long range components of this field would be unrealistically enhanced by the divergent GAR but we found that the structure of the corresponding strength function is very similar to the $r\partial V/\partial r$ case analyzed above.

To see the effect of the size of the basis upon the results we increased the number of states in Table I by 20% but the complex energies of the giant resonances remained practically the same while the total EWSR increased by only 3%. We also verified that a known property of the RPA energy weighted sum rule is satisfied, namely the total values of the uncorrelated and correlated EWSR exactly coincide.[36]

Finally, it is worthwhile to point out that graphical calculations of the spreading width may be conveniently done within the formalism presented in this paper. Since the energies can now be complex, one may avoid the divergences associated with zero energy denominators.[34]

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