$^{20}Ne(\alpha,2\alpha)^{16}O$ reaction

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The $^{20}Ne(\alpha,2\alpha)^{16}O$ reaction at 140 MeV incident energy is analyzed in the framework of the distorted-wave impulse approximation. The bound state α wave functions in ^{20}Ne are generated using the orthogonal condition model. The predicted results agree with the experimental data. They are also in rough accord with the results obtained with the Woods-Saxon α wave function.

There is ample evidence that, in addition to the complimentary independent particle model and the collective model, nuclei possess cluster structure. These clusters, due to the reduced importance of the Pauli blocking, are confined to the low density surface region of nuclei. The size and composition of these clusters are determined by their intrinsic binding energies. Due to this, α particles, which have maximum binding energy, stand the greatest change of being found as recognizable clusters. To explore their distribution quantitatively, the knock-out reactions, like $(\alpha,2\alpha)$, have been performed experimentally on various nuclei. It is expected that the analysis of these data will yield reliable information about the α cluster probability in nuclei. However, initially there were some setbacks in this respect, when it was found that calculated distorted-wave impulse approximation (DWIA) cross sections, for the known α structure nuclei, were two orders of magnitude smaller than the corresponding measured values.[1,2] Soon the causes for this large discrepancy were identified by some of us (N.R.S. and B.K.J.) and a modified procedure was proposed.[3] This procedure exploited the peripheral nature of the $(\alpha,2\alpha)$ reaction and described the α distorted waves directly in terms of the measured α nucleus phase shifts, in contrast to the earlier calculations where the α distorted waves were generated through an optical potential. Describing the α cluster in the nucleus as a single particle moving in a certain orbit (given by the conservation of the oscillator quanta) of the α core potential this formalism successfully reproduced the experimental data on various nuclei.

These results, of course, had some uncertainty due to a somewhat arbitrary choice in the α core binding potential. This potential was assumed to have the Woods-Saxon form with the radius parameter being guided by a "folding" model and an arbitrarily chosen diffuseness parameter. In the present paper we remove this uncertainty by generating the bound state α wave function microscopically. As a test case we present the results for the ^{20}Ne target nucleus as there is ample evidence to suggest that this nucleus is a dominant structure of α and ^{16}O. The binding potential is taken to be an orthogonality condition model (OCM) potential whose parameters are fixed by comparing the energy surface calculated within the OCM to that calculated using the generator coordinate method (RGM).[4-6] The $(\alpha,2\alpha)$ cross section is calculated at 140 MeV incident energy as the uncertainties due to off-shell effects become negligible around and above this energy.[7]

In the factorized on shell version of the DWIA the differential cross section for the $(\alpha,2\alpha)$ reaction can be written as

$$\frac{d^3\sigma}{d\Omega_1 d\Omega_2 dE_1} = F_K \sigma_{\alpha\alpha}(\bar{E},\bar{\theta}) \times \sum_M S_\alpha(L)(2L+1)^{-1} |P_{LM}(\mathbf{Q})|^2 , \quad (1)$$

where S_α is the spectroscopic factor, F_k is the kinematic factor, and $\sigma_{\alpha\alpha}$ is the free α-α cross section in its center of mass at the appropriate laboratory energy $\bar{E}$ and scattering angle $\bar{\theta}$. In the eikonal approximation and by use of the surface localization of the $(\alpha,2\alpha)$ reaction, the "distorted" momentum distribution, $P_{LM}(\mathbf{Q})$ of the initially bound particle is given by (for details see Ref. 3)

$$P_{LM}(\mathbf{Q}) = (2\pi)^{-1} \int e^{i\lambda z} J_M(k^\perp b) F_{NL}(\gamma) \Theta_{LM}(\theta) \times D_{\mathbf{k}_0}(yb,yz) D_{\mathbf{k}_1}(b,z) \times D_{\mathbf{k}_2}(b,z) b\, db\, dz , \quad (2)$$

with the nuclear part of the distortion factor D_k expressed as

$$D_{\mathbf{k}}(b,z) = \exp\{\tfrac{1}{2} i\chi(b)[1+\mathrm{erf}(z/\sqrt{2ab})]\} , \quad (3)$$

and

$$y = (A-4)/A ,$$

where $\chi(b)$ $[=2\delta(b)]$ are the α nucleus phase shifts for the angular momentum l $(\simeq kb-\frac{1}{2})$ and a is the measure of the diffuseness of the nuclear surface. λ and $k^\perp$

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TABLE I. Best fit parameters for α-^{16}O phase shifts at various energies.

Energy (MeV)	ε_1	ε_2	L_1	L_2	Δ_1	Δ_2	A_μ
146	0.3315	0.144	21.47	15.207	0.7458	1.702	2.09
69.5	0.0	0.0	14.94	14.94	0.599	0.599	2.10
49.5[a]	0.0	0.0	12.82	12.82	0.494	0.494	1.906
39.3	0.0	0.0	11.34	11.34	0.584	0.584	1.598
19.8[a]	0.0	0.0	9.63	9.63	0.358	0.358	2.297

[a]Results not shown in Fig. 1 but used in calculating cross sections in Fig. 2.

are the longitudinal and transverse components of the recoil momentum Q, respectively. F_{NL} is the radial part of the α-particle bound state wave function which is normalized to unity. The Coulomb part of the distortion is evaluated using the appropriate Coulomb potential.

For the elementary free α-α cross section, we have used the final energy prescription to fix $\bar{E}$. $\sigma_{\alpha\alpha}$ is taken from the measured values at these energies.

The nuclear phase shifts needed to describe the distortion in the initial and final states are parametrized as

$$S_l \equiv \exp(2i\delta_l) = \mathrm{Re}S_l + i\,\mathrm{Im}S_l \ , \tag{4}$$

where

$$\mathrm{Re}S_l = \varepsilon_1 + (1-\varepsilon_1)\{1+\exp[(l-L_1)/\Delta_1]\}^{-1} \ , \tag{5}$$

$$\mathrm{Im}S_l = A_\mu\{\exp[(l-L_2)/2\Delta_2] + \exp[(L_2-l)/2\Delta_2]\}^{-2} + \varepsilon_2\{1+\exp[(L_1-l)/\Delta_1]\}^{-1} \ . \tag{6}$$

The parameters in these expressions are numerically searched by fitting to the $\alpha-{}^{16}$O elastic scattering data at 146, 69.3, and 39.3 MeV.[8] For 140 MeV incident energy the energy of the outgoing α particles in an energy sharing experiment at the quasielastic angle centers around 65 MeV. The best fit parameters are listed in Table I and the computed α-^{16}O cross section along with the experimental data are shown in Fig. 1.

For the radial wave function, $F_{NL}(\gamma)$, the bound state α cluster wave function in ^{20}Ne, is generated in the orthogonality condition model. In this model the cluster wave function, $\psi_\alpha(\mathbf{r})$, is related to the solution $\phi_\alpha(\mathbf{r})$ of the following single particle equation:

$$\Lambda(T+V)\phi_\alpha = E\phi_\alpha \ , \tag{7}$$

where V is a local potential and the operator Λ projects on to the Pauli-allowed state. $\psi_\alpha(\gamma)$ is related to $\phi_\alpha(\gamma)$ through[5]

$$\psi_\alpha(\gamma) = \hat{A}^{1/2}\phi_\alpha(\gamma) \ , \tag{8}$$

where the integral operator $A^{1/2}$ is defined as

$$\hat{A}^{1/2} = \int d\gamma' A(\gamma,\gamma') \ , \tag{9}$$

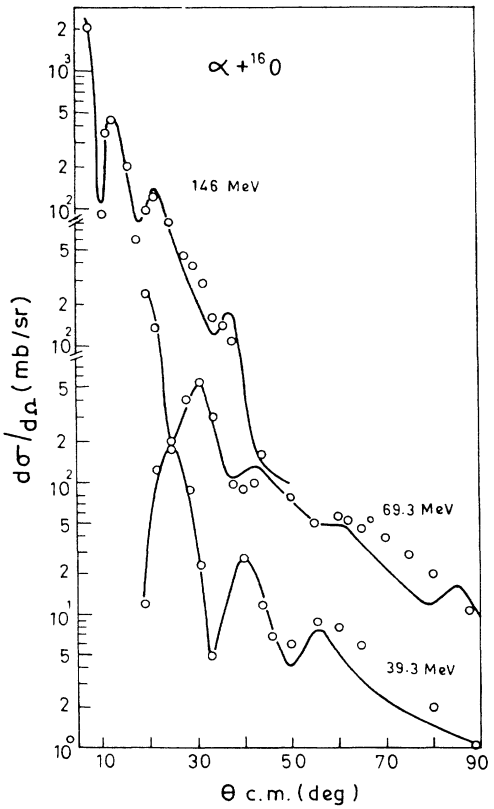

FIG. 1. Parametrized phase shift fits to the α-^{16}O elastic scattering data at various energies.

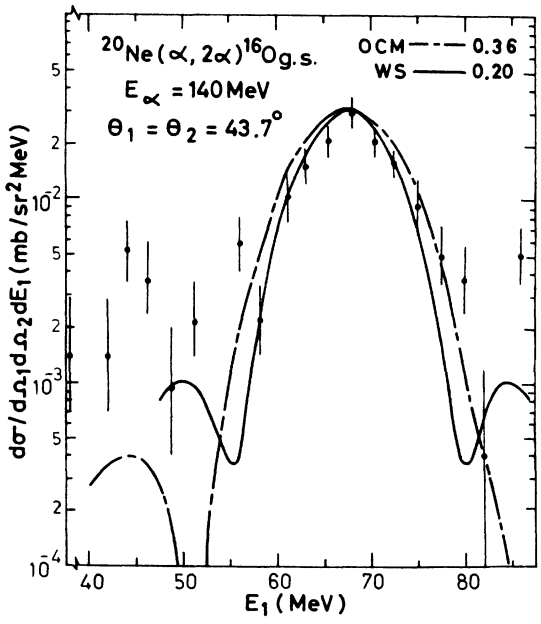

FIG. 2. Energy-sharing distribution for the ^{20}Ne$(\alpha,2\alpha)^{16}$O reaction at 140 MeV (quasifree angle pair 43.7°–43.7°).

with the kernel, $A(\gamma,\gamma')$, given by

$$A(\gamma,\gamma')=\langle \mathcal{A}[\Phi_c\Phi_\alpha\delta(\gamma-\gamma_{c\alpha})]\,|\,\mathcal{A}[\Phi_c\Phi_\alpha\delta(\gamma'-\gamma_{c\alpha})]\rangle \ . \tag{10}$$

Here Φ's are the internal cluster wave functions and $\mathcal{A}$ is the intercluster antisymmetrizer. The radial function $F_{NL}(\gamma)$ is related to $\psi_\alpha(\gamma)$ through

$$\psi_\alpha(\gamma)=S_\alpha^{1/2}(L)F_{NL}(\gamma)\Theta_{LM}(\theta) \ , \tag{11}$$

where S_α, as used in Eqs. (1) and (2), is the spectroscopic factor and $F_{NL}(\gamma)$ is normalized to unity.

Eq. (7) is solved with the potential $V(\gamma)$ parametrized as

$$V(\gamma)=-V_0\exp[-(\gamma/\gamma_0)^2] \ . \tag{12}$$

The parameters V_0 and γ_0 are fixed by comparing the energy surfaces calculated with this potential within the OCM to that calculated microscopically (i.e., the generator coordinate method). (The energy surface is the expectation value of the Hamiltonian of the two cluster systems, with their respective harmonic oscillator potentials fixed in space, as a function of their relative distance.)[6] The microscopic energy surfaces have been calculated with the Volkov 2 two body interaction. The Majorana admixture in this force was taken to be 0.638. This force yields almost the correct separation energy for the $\alpha+{}^{16}$O system. The resulting values of the parameters V_0 and γ_0 are 169.76 MeV and 2.703 fm, respectively.

With the above wave function the calculated cross sections for ^{20}Ne $(\alpha,2\alpha)$ $^{16}O_{g.s.}$ at 140 MeV incident energy along with the experimental data[1] are shown in Fig. 2 (dashed curve). The shape of the energy spectrum around the central maximum is reproduced. The lack of agreement away from this region is probably due to significant contributions coming from the reaction mechanisms different from the single step knock-out mechanism. The value of the spectroscopic factor determined by normalizing the calculated peak cross section to the corresponding measured cross section turns out to be 0.36. The theoretical value of the spectroscopic factor from the above OCM wave function is 0.54.

Considering the present uncertain status of the field and the fact that the above results are practically parameter free the extent of agreement arrived above with the experimental data is quite satisfactory. It definitely generates confidence that the procedure advocated in Ref. 3 for the analysis of the $(\alpha,2\alpha)$ data at medium and high energies is useful.

In Fig. 2 we have also shown results corresponding to one more wave function. The solid curve corresponds to the $4S$ α wave function generated in a Woods-Saxon potential. The half-value radius parameter of this potential is fixed from

$$\langle R^2\rangle=\tfrac{3}{5}R_{1/2}^2+\tfrac{7}{5}\pi^2(a')^2 \ , \tag{13}$$

where a' is taken equal to 0.65. $\langle R^2\rangle$ is obtained from the folding model potential.[9] However, to take account of the antisymmetrization effect, the folding model value is arbitrarily increased by 1.0 fm.[10] As shown in Fig. 2 the spectroscopic factor obtained with this wave function is 0.20 fm. The quality of the shape agreement with the data is similar to that obtained with the OCM wave function. This means that in ^{20}Ne, so far as the $(\alpha,2\alpha)$ reaction on it is concerned, the wave function generated in the Woods-Saxon potential (with parameters fixed as above) and that obtained in the OCM are similar to a great extent. If we extrapolate this observation, it may be reasonable to suggest that in those situations where the microscopically generated α wave functions are not available it should be possible to extract the spectroscopic factors from the experimental data using the α wave functions generated in a phenomenological Woods-Saxon potential with appropriate parameters.

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