Nuclear charge symmetry breaking and the $^3H-^3He$ binding energy difference

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(Received 29 September 1987)

We study the $^3H-^3He$ binding energy difference, taking into account the Coulomb interaction and charge symmetry breaking of the nuclear force consistent with recent NN experimental data. Realistic interactions are generated which describe the charge symmetry violations reflected in the different nucleon-nucleon scattering lengths. The influence of nuclear charge symmetry breaking on the perturbative Coulomb contribution to the ^3He binding energy is discussed. It is shown that the experimental mass difference can be explained by these and theoretical estimates of other known effects.

I. INTRODUCTION

In the early history of nuclear physics, the binding energy difference between ^3H and ^3He was used to show that the nuclear force was approximately charge symmetric.[1] In fact, in those days this difference was considered one of the most convincing pieces of evidence for charge symmetry.[2] Today this same difference is used to gauge the charge asymmetry of the nuclear force.[3] Indeed, the difference in the binding energies of the $A=3$ nuclei may be considered the best present evidence for charge symmetry breaking (CSB).

Although the two-nucleon system might seem *a priori* more appropriate for studying CSB, this has not been the case.[4,5] The simplest instance of nuclear CSB would be a difference in the interaction between two neutrons and that of two protons after the latter has been corrected for the obvious CSB due to the Coulomb force. The singlet state nucleon-nucleon (NN) scattering length is the classic focus for studies of this kind. Because this state is almost bound, the absolute value of the (negative) scattering length is large and is extremely sensitive to small changes in the strength of the force, so that, in principle, this would be a favorable quantity to investigate. However, there are uncertainties in the proton-proton scattering length a_{pp} due to some model dependence in the electromagnetic corrections, and in the experimental determination of the neutron-neutron scattering length a_{nn}. For a long time different types of experiments gave substantially different results[4–6] for this later quantity. The differences were so large that even the sign of the nuclear CSB effect was uncertain.

Recently the situation seems to have changed. It appears that the results for a_{nn}, as obtained from different types of experiments, are finally beginning to converge (see Table I). In view of this, it is worthwhile to once again question whether the charge asymmetry apparently evidenced in the NN system is consistent with that seen in the trinucleon bound states.

In Sec. II, the experimental and theoretical status of charge asymmetry in the NN system is briefly reviewed. Section III contains our results for the binding energy differences between the trinucleon states due to the Coulomb interaction and to a charge asymmetry in the NN force. A concluding discussion of the results may be found in Sec. IV.

II. CHARGE ASYMMETRY IN THE NN SYSTEM

A. Empirical evidence

Whereas the neutron-proton and the Coulomb uncorrected proton-proton scattering lengths, a_{np} and a_{pp}, respectively, can be obtained directly from the corresponding low-energy, two-body data, the neutron-neutron scattering length a_{nn} is more elusive since direct nn scattering experiments have not been feasible. The experiments from which this scattering length has been extracted can be subdivided into those with either two nucleons or three nucleons in the final state. An example of the former type of experiment is the reaction $\pi^- d \rightarrow nn\gamma$, while an instance of the latter is the $nd \rightarrow nnp$ reaction. A decade ago a value of $a_{nn}=-16.4\pm1.2$ fm was obtained from the analysis of a large number of results from both types of experiments.[5] Recently, however, an experimental value of $a_{nn}=-18.51\pm0.42$ fm has been recommended—again based on data from both types of experiments.[7,8,10] This variation in the quoted value of such an important physical observable reflects the difficulties in the performance and analysis of such experiments. A summary of the experimental values for the $^1S_0$ scattering lengths is given in Table I.

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TABLE I. Singlet S-state low energy NN scattering parameters.

	Experiment			Theory[a]	
	a (fm)[b]	r (fm)[c]	Reference	a (fm)[b]	r (fm)[c]
np	-23.748 ± 0.010	2.75 ± 0.05	7	-23.744	2.704
nn	-18.45 ± 0.46	2.83 ± 0.16	7[d]	-18.42	2.78
	-18.8 ± 1.0		8[e]		
pp[f]	-17.1 ± 0.3	2.83 ± 0.03	4,6,9	-17.11	2.80

[a]Predictions from the charge-dependent potentials applied in this work.
[b]Scattering length.
[c]Effective range.
[d]From the reaction $\pi^- d \rightarrow \gamma nn$.
[e]From the reaction $nd \rightarrow pnn$.
[f]Corrected for electromagnetic effects.

B. Status of theory

The sources of CSB in the NN system are several. The first that come to mind are the purely electromagnetic effects, the most important of which is the Coulomb interaction, present in the pp system and missing in the nn system. The separation of the Coulomb from the nuclear effects in pp scattering is not totally unambiguous and leads to the model dependence in the Coulomb-corrected single-S scattering length a_{pp} alluded to above. Another source is the difference in the kinetic energy of the nn and pp systems due to the n−p mass difference. All CSB effects other than these are usually referred to as CSB of the NN interaction.

Theoretical investigations of CSB in the NN potential often give conflicting results, so that the theoretical situation over the last several decades has been more uncertain than the experimental one. Calculated values of the contribution to the difference in scattering lengths $\Delta a = a_{pp} - a_{nn}$ from the intermediate-range 2π-exchange part of the nuclear force vary from $+0.3$ fm (Ref. 11) to ≈ -2.0 fm (Ref. 12). The recent result of Ref. 11 takes Δ isobars into account and appears more comprehensive. More systematic work is clearly needed. The longest range contribution to CSB comes from irreducible meson-photon exchange. This effect has been studied in the literature, but again, there is little agreement between various authors. Estimates of this effect on Δa range from a small fraction of a femtometer[12,13] to -1.31 fm.[14] Finally, electromagnetic mixing of neutral mesons of the same spin and parity, but of different isospin, can contribute to CSB. The best studied examples are π-η and ρ-ω mixing.[15,16] A contribution of the order of 1 fm to Δa from the mixing of the ρ and the ω mesons was found, whereas the π-η mixing contributes negligibly. A more complete discussion of the topics of this section can be found in review articles.[4,6,17]

III. THE ^{3}H − ^{3}He BINDING ENERGY DIFFERENCE

A. NN charge asymmetry

The effect of CSB of the NN potential on the ^{3}H − ^{3}He binding energy difference can be expected to come almost entirely from the $^1S_0$ NN interaction, since the bulk of the binding energy in the three-nucleon system comes from the $^1S_0$ and $^3S_1$-$^3D_1$ NN potential amplitudes.[18] Higher partial waves, where CSB effects are possible but problematic, contribute very little to the three-nucleon binding energy, so the neglect of those partial waves for the binding-energy difference appears to be justified. Therefore, only the $^1S_0$ and $^3S_1$-$^3D_1$ partial waves are included in the three-nucleon calculations of this section. We base our CSB study solely on the empirical NN data by using three potentials fitted to sets of NN observables which differ only in the $^1S_0$ scattering lengths and effective ranges. For the np potential, we use the momentum space one-boson-exchange potential (OBEPQ) of Ref. 19. The nn and pp potentials are modified versions of this static Bonn potential adjusted to fit the currently accepted values of a_{nn}, a_{pp}, and the corresponding effective ranges. These potentials differ from OBEPQ only in the coupling constant for the effective σ meson, and then only for the isospin-one potentials (see Table 5 of Ref. 19). The coupling constants used for the nn and pp potential are 8.205 and 8.1797, respectively. These charge-dependent potentials describe well the currently accepted low-energy scattering observables (see Table I).

For our three-nucleon Faddeev calculations we assume identical, equal mass particles. The NN interaction, although dependent on the two-nucleon isospin projection quantum number M_T, is assumed to conserve spin and isospin. Thus, the NN interaction has the general property

$$\langle (LS)JTM_T \,|\, V \,|\, (L'S')J'T'M_{T'} \rangle = \delta_{SS'}\delta_{TT'}\delta_{M_T M_{T'}}\delta_{JJ'} V_{LL'}^{JSTM_T} \,. \quad (1)$$

In order to make explicit the approximations which we use here, we sketch the development of the Faddeev equations, insofar as it differs from the usual case of charge-independent interactions. The formal equation for the Faddeev amplitude for identical particles has the form

$$\Psi = tG_0P\Psi \,, \quad (2)$$

where P is the sum of the cyclic and anticyclic permutation operators, t is the two-nucleon t matrix (in the three-nucleon Hilbert space), and G_0 is the three-nucleon free Green's function. We define the Faddeev amplitude

$$\Psi(p,q,\alpha)\equiv\Psi(p,q,\beta;\mathcal{J},\mathcal{J}_z,\mathcal{T},\mathcal{T}_z)=\langle p,q,\alpha \mid \Psi\rangle , \tag{3}$$

where

$$\alpha=\{[(LS)J(l\tfrac{1}{2})j]\mathcal{J},\mathcal{J}_z;(T\tfrac{1}{2})\mathcal{T}\mathcal{T}_z\} \tag{4}$$

is the set of quantum numbers which, together with the Jacobi momenta p and q, specify the basis states $|p,q,\alpha\rangle$, and where β represents the quantum numbers $\{LSJTlj\}$. The spectator momentum is q and the spectator quantum numbers are shown in lower case. Using the completeness of the state $|pq\alpha\rangle$, we rewrite Eq. (2) as

$$\Psi(p,q,\beta;\mathcal{J},\mathcal{J}_z,\mathcal{T},\mathcal{T}_z)=\sum_{\alpha'\alpha''}\int p'^2dp'q'^2dq'p''^2dp''q''^2dq''\langle p,q,\alpha \mid t \mid p',q',\alpha'\rangle \times\frac{\langle p',q',\alpha' \mid P \mid p'',q''\alpha''\rangle}{(E-\frac{3}{4}q'^2-p'^2)}\Psi(p'',q'',\alpha'') . \tag{5}$$

The evaluation of the matrix element of the permutation operators is exactly the same as for the charge-independent case (we use the particular notation of Ref. 20). This matrix element is given by

$$\langle p',q',\alpha' \mid P \mid p,q,\alpha\rangle=\delta_{\mathcal{J}'\mathcal{J}}\delta_{\mathcal{J}'_z\mathcal{J}_z}\delta_{\mathcal{T}'\mathcal{T}}\delta_{\mathcal{T}'_z\mathcal{T}_z}\int_{-1}^{1}dx\, G_{\beta'\beta}^{\mathcal{J}\mathcal{T}}(q',q,x)\frac{\delta(p'-p_1(q',q,x))}{p_1^{L'+2}(q',q,x)}\frac{\delta(p-p_1(q,q',x))}{p_1^{L+2}(q,q',x)} , \tag{6}$$

where the function p_1 is defined as

$$p_1(a,b,c)=(\tfrac{1}{4}a^2+b^2+abc)^{1/2} \tag{7}$$

and $x\equiv(\mathbf{q}\cdot\mathbf{q}')/(qq')$. The geometrical coefficient $G_{\beta'\beta}^{\mathcal{J}\mathcal{T}}$ is given explicitly in Ref. 20. Using the properties of the NN potential as given in Eq. (1), we may express the matrix element of t in the form

$$\langle p,q,\alpha \mid t \mid p',q',\alpha'\rangle=\frac{\delta(q-q')}{q^2}\delta_{\mathcal{J}\mathcal{J}'}\delta_{\mathcal{J}_z\mathcal{J}'_z}\delta_{\mathcal{T}_z\mathcal{T}'_z}\sum_{M_T,\theta}t_{LL'}^{JSTM_T}(p,p';E-\tfrac{3}{4}q^2)\langle T\tfrac{1}{2}M_T\theta \mid \mathcal{T}\mathcal{T}_z\rangle\langle T\tfrac{1}{2}M_T\theta \mid \mathcal{T}'\mathcal{T}_z\rangle , \tag{8}$$

where $\langle T\frac{1}{2}M_T\theta \mid \mathcal{T}\mathcal{T}_z\rangle$ is a Clebsch-Gordan coefficient. The final form of the integral equation for the Faddeev amplitude is

$$\Psi(p,q,\beta;\mathcal{J},\mathcal{J}_z\mathcal{T},\mathcal{T}_z)=\sum_{L',\mathcal{T}'}\int_0^\infty q''^2dq''\int_{-1}^{1}dx\left[\sum_{M_T,\theta}\langle T\tfrac{1}{2}M_T\theta \mid \mathcal{T}\mathcal{T}_z\rangle\langle T\tfrac{1}{2}M_T\theta \mid \mathcal{T}'\mathcal{T}_z\rangle \times\frac{t_{LL'}^{JSTM_T}(p,p_1(q,q'',x);E-\frac{3}{4}q^2)}{p_1^{L'}(q,q'',x)}\right] \times\sum_{\beta''}\frac{G_{\bar{\beta}\beta''}^{\mathcal{J}\mathcal{T}}(q,q'',x)}{[E-\frac{3}{4}q^2-p_1^2(q,q'',x)]}\frac{\Psi(p_1(q'',q,x),q'',\beta'';\mathcal{J},\mathcal{J}_z,\mathcal{T}',\mathcal{T}_z)}{p_1^{L''}(q,'',q,x)} , \tag{9}$$

where $\bar{\beta}\equiv\{L'SJTlj\}$.

If one were to assume charge independence of the potential, so that the two nucleon amplitude t is not dependent on the isospin projection M_T,the sums over the isospin magnetic quantum numbers collapse and we regain the usual form of the Faddeev equations.[20] In the more realistic case which we consider here, isospin is no longer a good quantum number. One sees from Eq. (9) that isospin-$\frac{3}{2}$ admixtures are present in the trinucleon wave functions. We shall assume here that these isospin-$\frac{3}{2}$ admixtures are small and have a negligible effect on the binding energy, and arbitrarily restrict our wave functions to be pure isospin-$\frac{1}{2}$ states. This assumption has been shown to be valid when only the Coulomb potential is responsible for isospin mixing.[21] With this simplifying approximation, we can cast Eq. (9) in the same form as the Faddeev equations with a charge-independent potential be defining the isospin-averaged t matrix to be

$$\bar{t}^{\,T=1}=\sum_{M_T\theta}|\langle 1\tfrac{1}{2}M_T\theta \mid \tfrac{1}{2}\mathcal{T}_z\rangle|^2 t^{T=1,M_T} \tag{10}$$

$$=\tfrac{1}{3}t^{T=1}(\mathrm{n,p})+\tfrac{2}{3}t^{T=1}(\mathrm{p,p}) \quad (\text{for } ^3\mathrm{He}) \tag{11}$$

$$=\tfrac{1}{3}t^{T=1}(\mathrm{n,p})+\tfrac{2}{3}t^{T=1}(\mathrm{n,n}) \quad (\text{for } ^3\mathrm{H}) . \tag{12}$$

Using the potentials of Table I, we construct the appropriate $\bar{t}$ for ^{3}He and ^{3}H, and solve the corresponding five channel Faddeev equations. Our calculated values for the binding energies are, respectively, 8.111 and 8.170 MeV, which gives a contribution of 59 keV to ΔE from the NN charge symmetry breaking.

B. The Coulomb energy

The largest contribution to the ^{3}H$-$ ^{3}He binding energy difference comes from the Coulomb potential. We evaluate the Coulomb contribution to the ^{3}He binding energy in first order perturbation theory. This is expected to overestimate the result by about 1%.[21] The Coulomb potential between nucleons in the coordinate representation is given by

$$\langle \mathbf{r}_1',\mathbf{r}_2',\mathbf{r}_3' \,|\, V^c(23,1) \,|\, \mathbf{r}_1,\mathbf{r}_2,\mathbf{r}_3 \rangle = \left[\frac{1+\tau_z(2)}{2}\right]\left[\frac{1+\tau_z(3)}{2}\right]\delta(\mathbf{r}_1'-\mathbf{r}_1)\delta(\mathbf{r}_2'-\mathbf{r}_2)\delta(\mathbf{r}_3'-\mathbf{r}_3)\int d^3x\, d^3y \frac{\rho(\mathbf{x}-\mathbf{r}_2)\rho(\mathbf{y}-\mathbf{r}_3)}{|\mathbf{x}-\mathbf{y}|}, \tag{13}$$

where ρ describes the charge distribution of the proton. The expectation value of the Coulomb potential between two pointlike protons in ^{3}He is evaluated in momentum space as

$$\Delta E_c = \sum_{i<j}^{3} \langle \Psi_B^{^3\mathrm{He}} \,|\, V^c(ij,k) \,|\, \Psi_B^{^3\mathrm{He}} \rangle$$
$$= \frac{2e^2}{\pi}\sum_{\beta}\delta_{T1}\int_0^\infty\int_0^\infty\int_0^\infty p'^2dp'p^2dp\, q^2dq\, \Psi_B(p',q,\beta;\tfrac{1}{2},\mathcal{J}_z,\tfrac{1}{2},\tfrac{1}{2})Q_L\left[\frac{p^2+p'^2}{2pp'}\right]\Psi_B(p,q,\beta;\tfrac{1}{2},\mathcal{J}_z,\tfrac{1}{2},\tfrac{1}{2}) . \tag{14}$$

Numerically, the logarithmic singularity present in the integrand of Eq. (14) is treated by the Landé subtraction method.[22]

We calculate the Coulomb energy with Eq. (14) using the same five channel ^{3}He wave function discussed in Sec. III A. For the point Coulomb interaction we obtain a value of 687 keV. Repeating the calculation to take into account the finite size of the protons, we find a Coulomb energy of 646 and 649 keV for the proton form factors of Refs. 23 and 24, respectively. We also calculate these quantities using a charge-independent ^{3}He wave function obtained from a 34 channel solution of the Faddeev equations, using as input the Bonn OBEPQ potential.[19] The 34 channel calculation adds 1 keV to the corresponding five channel result. Here the ^{3}He and ^{3}H wave functions—aside from their trivial $\mathcal{T}_z$ dependence—are degenerate. Such a calculation is obviously inconsistent with our treatment of the nuclear CSB effect. However, previous estimates of the Coulomb shift have been done in this manner. The results are 697 keV for point protons and 654 and 657 keV for the form factors of Refs. 23 and 24, respectively. Thus, the assumption of charge-independence (for a potential fit to the np scattering length) leads to an overestimate of the Coulomb energy, comparable in size to the overestimate due to the use of (first-order) perturbation theory.[21] This overestimate of about 1% can be understood as the effect of the stronger attraction between the protons in the charge-independent case. The protons are then closer together, increasing the Coulomb energy.

SUMMARY AND CONCLUSIONS

We summarize in Table II the various contributions to ΔE, as calculated here or taken from the literature.[21,25] The total "theoretical" contributions add up to 742 keV, which is to be compared with the experimental value of 764 keV. We emphasize strongly that the many small contributions in Table II should be considered only as estimates. In particular, the CSB effects, as we have treated them here, are obviously strongly dependent on the current empirical values for the $^1S_0$ nn and Coulomb-corrected pp scattering lengths. Further refinements in the experimental value of the first and a better extraction of the latter will have implications for the precise ^{3}H$-$^{3}He binding-energy difference. In addition, the accuracy of the approximation made in the neglect of isospin-$\frac{3}{2}$ admixtures due to the NN CSB is yet to be established. We are currently considering this latter question.

The conclusion which we draw from this work is that at the *current level of our understanding*, the charge asymmetry observed in the two-nucleon system is consistent with that evidenced in the trinucleon bound-state systems. It is interesting to note that our result is consistent with considerations of heavier nuclei,[26,27] where it was shown that a difference in the scattering lengths of about 1 fm (a value very close to our assumption and present empirical evidence) explains the so-called Coulomb or Nolen-Schiffer anomaly.[28]

TABLE II. Charge asymmetric contributions to the ^{3}H$-$^{3}He binding energy difference.

	ΔE (keV)	Reference
Static Coulomb	687	This work
Charge symmetry breaking forces	59	This work
Finite size effects	-39	This work
p$-$n mass difference in kinetic energy	12	25
Second order perturbation corrections	-6	21
Other electromagnetic effects	29	25
Total (theory)	742	
Experiment	764	

ACKNOWLEDGMENTS

This work was supported by the U.S. Department of Energy and by the U.S. National Science Foundation. The Los Alamos National Laboratory is operated by the University of California for the U.S. Department of Energy under Contract No. W-7405-Eng-36.

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