Total radiative capture rates for three- and four-nucleon pionic atoms

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A corrected closure approximation is used to calculate the reduced rates for radiative capture of negative pions from 1*s* and 2*p* orbitals in ^{3}H, ^{3}He, and ^{4}He. For ^{3}He and ^{4}He the calculated rates are in good agreement with experimental data. In the case of ^{3}H, the calculated value for 1*s* capture is combined with the measured radiative capture branching ratio to obtain a value of $\Gamma_{\rm tot}^{1s}=2.2\pm0.4$ eV for the total width of the 1*s* level in the ^{3}H pionic atom. This value when compared to the measured total width of the 1*s* level in pionic ^{3}He implies a small but definite contribution of singlet spin nucleon pairs to absorption of *s*-wave pions.

I. INTRODUCTION

It is well established that the process of radiative pion capture in complex nuclei is a one body process[1] that can be treated in the impulse approximation analogously to muon capture. Bernabeu showed[2,3] that the strong dependence on an unknown "average" energy transfer in the closure approximation to the total muon capture rate could be largely eliminated by calculating the next higher energy moment of the distribution of transition strengths. Excellent agreement with the measured muon capture rates in ^{4}He, ^{6}Li, and a number of other nuclei was thereby achieved. Application of these techniques for the purpose of calculating total radiative pion capture rates is extended in the present work to the non-self-conjugate nuclei ^{3}H and ^{3}He through the introduction of isospin projection operators[4,5] which exclude physically unrealizable final states. These states are those which would be coupled to the ground state by radiative capture of *positive* pions and implicitly occur in the usual double commutator introduced in the lowest order correction to closure.

Radiative pion capture rates in pionic atoms cannot be measured directly but are obtained as branching ratios for emitting high energy gammas as compared to all processes which a stopped pion can undergo. A knowledge of the total width of a pionic atom level is required to convert a radiative capture rate into a branching ratio. The calculation of a branching ratio is generally complicated by the circumstance that atomic levels of two or more different orbital angular momenta are normally involved in the primary reaction, pion absorption, and in radiative capture. The radiative capture branching ratios in the *s*-shell nuclei were measured more than a decade ago. However, more recent determinations of total 1*s* widths in ^{3}He and ^{4}He (Refs. 6–8) and the extraction of 2*p* widths and capture schedules in the same nuclei from x ray yield data[9] allow the meaningful comparison of theoretical and experimental branching ratios.

A principal motivation of the present work was to obtain a value for the 1*s* absorption width of ^{3}H in order to gain insight into the absorption process of low energy pions on nucleon pairs. If the singlet spin pp pair in ^{3}He plays no role in the 1*s* absorption process one would expect the 1*s* absorption widths in ^{3}H and ^{3}He to be the same,[10] aside from the effects of differing probability densities of the pionic wave functions at the position of the nuclei. While sufficiently high resolution spectrometers exist[11] with which the total 1*s* width could, in principle, be measured [$\Gamma_{\rm tot}^{1s}(^3{\rm H})\simeq 1$ eV], the background from the radioactivity of a tritium target prevents such a measurement.[12] The value of the ^{3}H 1*s* width which we obtain from the calculated total radiative capture rate and the measured branching ratio does, however, indicate a small contribution from the singlet spin nucleon pairs. The size of the contribution is qualitatively consistent with the Silbar and Piasetzsky[13] (π^-,pn) and (π^+,pp) absorption cross sections on ^{3}He and ^{4}He.

II. GENERAL FORMULATION

The radiative pion capture process

$$(A,Z)_a+\pi^-\rightarrow(A,Z-1)_b+\gamma \tag{1}$$

occurs when a pion in an atomic state *nl* is captured by a nucleus in state *a* and a high energy gamma is emitted leaving the nucleus in state *b*. This interaction is supposed to occur between the pion and a single initial proton and direct application of the impulse approximation leads to the following expression for the rate of the transition:[1]

$$\Lambda_\gamma^{nl}(a\rightarrow b)=K(k_{ba})N_{nl}^2(Z)Z\Lambda_r^{nl}(a\rightarrow b)\ ,$$

where (2)

$$K(k)=\frac{(2l+1)(1+k/M)(1+\mu/M)2\tilde{A}^{\,2}}{(1+k/MA)\mu^{2l}}\ ,$$

N_{nl}, the normalization constant of the atomic wave function is defined by $\phi_{nl}(\mathbf{r})=N_{nl}r^l Y_l^m(\hat{\mathbf{r}})R_{nl}(r)$, M and μ are the masses of a nucleon and pion, respectively, and $\tilde{A}$ is the strength in units of μ^{-1} of the $\tau^+\sigma\cdot\hat{\varepsilon}_\lambda$ term in the photoproduction amplitude. Consistent with the formalism of Bernabeu[2] the reduced rate for radiative pion capture is defined by

$$Z\Lambda_r^{nl}(a\rightarrow b)=\frac{k_{ba}}{\mu}\frac{1}{\tilde{A}^2}\frac{1}{4}\,|\langle b\,|\sum_{j=1}^{A}O_{j\lambda}^-(\mathbf{k}_{ba})\,|\,a\,\rangle\,|^2\;, \tag{3}$$

where averages over initial spin states and sums over final spin states are understood. The operator O_λ^- is defined by

$$O_\lambda^-(\mathbf{k}_{ba})=\left[\frac{4\pi}{2l+1}\right]^{1/2}\exp(-i\mathbf{k}_{ba}\cdot\mathbf{r})\times\tau^-(\sigma_\lambda\cdot\mathbf{K}_\lambda+L_\lambda)(\mu r)^l Y_l^m(\hat{\mathbf{r}})R_{nl}(r)\;. \tag{4}$$

The K_λ and L_λ are derived from photoproduction amplitudes which, for the present application, contain no terms of higher order than linear[1] in either the pion or photon momenta. For the most important case of s-wave pion capture $K_\lambda=\tilde{A}\,\hat{\varepsilon}_\lambda^*$, $L_\lambda=0$, λ being the polarization state of the emitted photon. The momentum of the photon is $\mathbf{k}_{ba}$ and the function $R_{nl}(r)$ is defined such that for hydrogenic wave functions $R_{nl}(0)=1$.

The *total* reduced capture rate $Z\Lambda_r^{nl}$ is found by summing over all energetically accessible nuclear final states b. Expanding each term in the sum around a fixed value $\bar{k}$ of the photon momentum and retaining the leading two contributions to the summation over final states one arrives at

$$Z\Lambda_r^{nl}\simeq\frac{1}{4\tilde{A}^2}\sum_b\left[\frac{\bar{k}}{\mu}\left|\left\langle b\left|\sum_j O_{j\lambda}^-(\bar{k})\right|a\right\rangle\right|^2+(k_{ba}-\bar{k})\frac{d}{d\bar{k}}\frac{\bar{k}}{\mu}\times\left|\left\langle b\left|\sum_j O_{j\lambda}^-(\bar{k})\right|a\right\rangle\right|^2\right]\;. \tag{5}$$

The first term within the large square brackets constitutes the closure approximation, $Z\Lambda_r^{nl}(\bar{k})$. If the photon energy is related to the nuclear excitation energy ω_{ba}, $k_{ba}=\mu-\omega_{ba}(1-\mu/MA)$, where MA in the recoil correction is the mass of the final nucleus, the total reduced capture rate can be expressed as

$$Z\Lambda_r^{nl}=\left\{\left[1+(1-\mu/MA)\bar{\omega}\frac{d}{d\bar{k}}\frac{\bar{k}}{\mu}\right]Z\Lambda_r^{nl}(\bar{k})-(1-\mu/MA)\frac{d}{d\bar{k}}\frac{\bar{k}}{\mu}\frac{1}{4\tilde{A}^2}\times\sum_b\omega_{ba}\,|\langle b\,|\,O_\lambda^-(\bar{k})\,|\,a\,\rangle\,|^2\right\}\;. \tag{6}$$

Using the Wigner-Eckart theorem to convert $O_{j\lambda}^-$ to $O_{j\lambda}^z$ one obtains

$$\sum_b\omega_{ba}\left|\left\langle b\left|\sum_j O_{j\lambda}^-(\bar{k})\right|a\right\rangle\right|^2=\sum_{T_b}\frac{(1-1T_aT_{az}\,|\,T_bT_{bz}-1)^2}{(10T_aT_{az}\,|\,T_bT_{az})^2}\times\sum_b\omega_{ba}\left|\left\langle b,T_b\left|\sum_j O_{j\lambda}^z(\bar{k})\,\right|a,T_a\right\rangle\right|^2\;, \tag{7}$$

where the sum over final T_b is necessary for $T_a\neq 0$ nuclei. Because the summation over energy states b on the right-hand side (rhs) of Eq. (7) also implies averages over initial magnetic quantum numbers and sums over final ones, it is equal to

$$\sum_b\omega_{ba}\left|\left\langle b,T_b\left|\sum_j O_{j\lambda}^{z\dagger}(k)\right|a,T_a\right\rangle\right|^2\;. \tag{8}$$

This equality is usually a sufficient condition[3] for writing the rhs of Eq. (7) as a standard double commutator of the nuclear Hamiltonian H with $O^z,O^{z\dagger}$. Unfortunately, O^z and $O^{z\dagger}$ generate transitions to final T states which cannot be connected to the initial isospin state by O^-. An example of such an unphysical T would be $T_b=\frac{1}{2}$ in the case of radiative capture on ^{3}H ($T_a=\frac{1}{2}$, $T_{az}=-\frac{1}{2}$), the final state of three neutrons being restricted to a single isospin state, $T_b=\frac{3}{2}$, $T_{bz}=-\frac{3}{2}$. If projection operators $P(T)$ for the physically allowed final isospin states[4] are employed a new transition operator A_λ^z can be introduced,

$$A_\lambda^z=[P(T_a)O_\lambda^zP(T_b)+P(T_b)O_\lambda^zP(T_a)]\times[1-\delta_{T_bT_a}]+P(T_a)O_\lambda^zP(T_a)\delta_{T_bT_a}\;. \tag{9}$$

Since A_λ^z does not connect the initial nuclear state to unphysical final states, the standard replacement of the sum over excited states by a double commutator can be carried through to give

$$\sum_b\omega_{ba}\,|\langle b,T_b\,|\,O_\lambda^z(\bar{k})\,|\,a,T_a\,\rangle\,|^2=\langle a,T_a\,|\,\tfrac{1}{4}\{[A_\lambda^{z\dagger},(H,A_\lambda^z)]+[(A_\lambda^{z\dagger},H),A_\lambda^z]\}\,|\,a,T_a\,\rangle\;. \tag{10}$$

III. APPLICATION TO S-SHELL NUCLEI

A. Closure approximation

Ignoring momentum dependent terms in the photoproduction amplitude, the closure approximation to the reduced total radiative capture rate for negative pions in the 1s orbital is the same as for negative muon capture[14]

$$Z\Lambda_r^{1s}(k)=\frac{Z\bar{k}}{\mu}\langle R_{1s}^2(r)\rangle[1-\beta(A,Z)F_{\rm rel}(\bar{k})]\;. \tag{11}$$

An average over $R_{nl}(r_1)R_{nl}(r_2)\exp(i\mathbf{k}\cdot\mathbf{r}_{12})$ has been approximated by $\langle R_{1s}^2(r)\rangle F_{\rm rel}(\bar{k})$. The relative form factor is taken over the relative neutron-proton wave function in the ground state of the nucleus. The parameter $\beta(A,Z)=\frac{1}{3}$ (^{2}H), $=\frac{1}{2}$ (^{3}He), and $=1$ (^{3}H and ^{4}He).

The closure approximation to the reduced total radiative pion capture rate from the $2p$ state is given by

$$Z\Lambda_r^{2p}(\bar{k})=\frac{1}{3}\frac{Z\bar{k}}{\mu}\langle R_{2p}^2\rangle\left\{\left[\mu^2\overline{r_m^2}+\frac{\bar{k}^2}{\mu^2}(B^2+C^2+D^2)/\tilde{A}^2\right][1-\beta(A,Z)F_{\rm rel}(\bar{k})]+\beta(A,Z)\left[-\frac{\nabla_k^2}{4}+\gamma(A,Z)\overline{r_m^2}+\frac{B}{\tilde{A}}\bar{k}\frac{\partial}{\partial\bar{k}}\right]F_{\rm rel}(\bar{k})\right\}, \tag{12}$$

where $\overline{r_m^2}$ is the rms matter radius of a point nucleon in the ground state and B, C, and D are the amplitudes of the terms linear in pion momentum in the photoproduction amplitude.[1] The parameter $\gamma(A,Z)$ takes on the values $\gamma(A,Z)=1$ (^{2}H), $=\frac{3}{4}$ (^{3}H and ^{3}He), and $=\frac{2}{3}$ (^{4}He). Its significance is that $[1-\gamma(N,Z)]\overline{r_m^2}$ is the mean square radius of an np pair in the ground state.

B. Double commutator

The specific form of the closure approximation obtained above depends on retaining only the dominant completely symmetric principal S state in the ground states. Consistent with this simplification the nuclear Hamiltonian need contain only central potential terms which can be expressed by the the usual four exchange terms, $V=V_W+V_MP_x-V_HP_\tau+V_BP_\sigma$. Then the double commutators for capture of $1s$ pions in the three nuclei of interest are

$$^3\text{H:}\quad \frac{1}{4\tilde{A}^2}\sum_b\omega_{ba}\,|\langle b\,|\,O_\lambda^-(\bar{k})\,|\,a\rangle|^2=\langle R_{1s}^2\rangle\left[\frac{\bar{k}^2}{2M}+\langle\Phi\,|\,-3(V_H+V_M)+(-2V_B+V_H+3V_M)\exp i\bar{\mathbf{k}}\cdot\mathbf{r}_{12}\,|\,\Phi\rangle\right], \tag{13a}$$

$$^3\text{He:}\quad \frac{1}{4\tilde{A}^2}\sum_b\omega_{ba}\,|\langle b\,|\,O_\lambda^-(\bar{k})\,|\,a\rangle|^2=\langle R_{1s}^2\rangle\left[\frac{\bar{k}^2}{M}+\langle\Phi\,|\,4V_B-V_H-5V_M+(-2V_B+3V_H+5V_M)\exp i\bar{\mathbf{k}}\cdot\mathbf{r}_{12}\,|\,\Phi\rangle\right], \tag{13b}$$

$$^4\text{He:}\quad \frac{1}{4\tilde{A}^2}\sum_b\omega_{ba}\,|\langle b\,|\,O_\lambda^-(\bar{k})\,|\,a\rangle|^2=\langle R_{1s}^2\rangle\left\{\frac{\bar{k}^2}{M}+4\langle\Phi\,|\,[V_B-V_H-2V_M][1-\exp i\bar{\mathbf{k}}\cdot\mathbf{r}_{12}]\,|\,\Phi\rangle\right\}. \tag{13c}$$

Note that the sum of the potential energy contributions to the double commutators for ^{3}H and ^{3}He are equal to the double commutator for ^{4}He. This equality illustrates the point made in Ref. 10 that the sum over the rates for both $A=3$ nuclei is equivalent to evaluating the double commutator of H and O_λ^z for the self-conjugate nucleus with six np pairs.

The importance of radiative pion capture from the $2p$ orbital is presumably nil in ^{3}H, small in ^{3}He, but significant in ^{4}He. We therefore give the double commutator result for $2p$ pion capture solely for ^{4}He:

$$^4\text{He:}\quad \frac{1}{4\tilde{A}^2}\sum_b\omega_{ba}\,|\langle b\,|\,O_\lambda^-(\bar{k})\,|\,a\rangle|^2=\frac{1}{3}\langle R_{2p}^2\rangle\left\{\frac{\mu^2}{M}(3+\bar{k}^2\overline{r_m^2}-2\frac{B}{\tilde{A}}\frac{\bar{k}^2}{\mu}+\frac{\bar{k}^4(B^2+C^2+D^2)}{\mu^3\tilde{A}^2}+2\langle\Phi\,|\left\{\left[\mu^2(r_1^2+r_2^2)+\mu^2\nabla_{\bar{k}}^{\,2}-2\frac{B}{\tilde{A}}\bar{\mathbf{k}}\cdot\nabla_k+\frac{2\bar{k}^2(B^2+C^2+D^2)}{\mu^2\tilde{A}^2}\right]\times(V_B-V_H-2V_M)-\frac{2\bar{k}^2D^2}{\mu^2\tilde{A}^2}V_B\right\}[1-\exp(i\bar{\mathbf{k}}\cdot\mathbf{r}_{12})]\,|\,\Phi\rangle\right\}. \tag{14}$$

The kinetic energy part of the double commutator for $2p$ capture agrees with the result of Lipparini *et al.*[16] up to a factor of $\frac{1}{2}$ which seems to have been inadvertently introduced by these authors in converting matrix elements of the operator O_λ^- into matrix elements of O_λ^z. The potential energy parts cannot be compared because a momentum dependent Skyrme interaction containing delta functions of position $\delta(\mathbf{r}_1-\mathbf{r}_2)$ was used in Ref. 15 to simplify the evaluation of potential energy parts of the double commutator. Note that the L_λ part of the photoproduction amplitude, proportional to D, commutes with $V_B P_\sigma$ since it is spin independent.

C. Evaluation of reduced capture rates

A critical element in the evaluation of the reduced total capture rates is a suitable representation for the relative form factors $F_{\rm rel}(k)$ which are the normalized expectation values $\langle \boldsymbol{\sigma}_1\cdot\boldsymbol{\sigma}_2 \exp[i\mathbf{k}\cdot(\mathbf{r}_1-\mathbf{r}_2)]\rangle/\langle \boldsymbol{\sigma}_1\cdot\boldsymbol{\sigma}_2\rangle$ taken over np pairs in the ground state. While detailed charge and magnetic form factors for the $A=3$ nuclei have been published there does not exist a similar tabulation for the desired relative form factors. If both the neutrons and protons in the ground state are assumed to have identical distributions and the constraint that the center of mass of the system be fixed is imposed it can be shown that

$$\langle r_{12}^2\rangle = \frac{2A}{A-1}\langle(\mathbf{r}_1-\mathbf{R})^2\rangle , \tag{15}$$

where $\mathbf{R}$ is the position coordinate of the center of mass. This suggests that for low momentum transfers the relative form factor can be related to the charge form factor through

$$F_{\rm rel}(k^2)=F_c\left[\frac{2A}{A-1}k^2\right]\Big/F_{\rm p}\left[\frac{2A}{A-1}k^2\right] , \tag{16}$$

where A is the atomic weight of the nucleus and $F_{\rm p}$ is the proton form factor.

A two-body correlation function for nucleons derived from the completely space symmetric component of an $A=3$ bound state wave function utilizing the Reid soft core potential[17] has been tabulated in Ref. 18. From the table we obtain $[\langle r_{12}^2\rangle/3]^{1/2}=1.9$ fm, in qualitative agreement with Eq. (15), but quantitative agreement cannot be demonstrated because the single particle rms radii of the symmetric component is not given in either of the publications referenced above.

A satisfactory common functional representation of the charge form factors of the S-shell nuclei for momentum transfers below 2 $\rm fm^{-1}$ is[19]

$$F_c(k^2)=(1+ak^4)\exp(-k^2r_c^2/6) , \tag{17}$$

where r_c is the rms charge radius of the nucleus. The proton form factor is taken as[20,21]

$$F_{\rm p}(k^2)=(1+d^2k^2)^{-2} ,\quad d^2=1/18.23\ {\rm fm}^2 . \tag{18}$$

The rms charge radius of ^{3}He has been measured[19] to be $r_c(^3{\rm He})=1.93\pm0.03$ fm, somewhat larger than the charge radius[20] of ^{3}H, $r_c(^3{\rm H})=1.81\pm0.05$ fm. What is really needed in the present calculation is $\langle r_{12}^2\rangle$ for the np pairs in each nucleus and it has been obtained for both $A=3$ nuclei using the relationship[4]

$$r^2=\langle r_{12}^2\rangle/3=2\langle r_u^2\rangle/3+\langle r_l^2\rangle/3 , \tag{19}$$

where u refers to the rms matter radius of the unlike nucleon (the proton in ^{3}H, the neutron in ^{3}He) and l refers to the like nucleon matter radius. The matter radii of the neutrons in both ^{3}H and ^{3}He are unknown but can be determined from the proton charge radii in ^{3}He and ^{3}H, respectively. Neglecting Coulomb effects $r_{\rm n}(^3{\rm H})=r_{\rm p}(^3{\rm He})$, $r_{\rm n}(^3{\rm He})=r_{\rm p}(^3{\rm H})$. However, Gibson, Payne, and Friar[22] have calculated that Coulomb repulsion between the two protons dilates all radii in ^{3}He by about 2% so that we have used $r_{\rm n}(^3{\rm H})=1.89$ fm, $r_{\rm n}(^3{\rm He})=1.85$ fm. In applying Eqs. (17) and (18) for the purpose of evaluating $F_{\rm rel}(\bar{k})$ the following parameters were used in the one body form factor:

$$\begin{aligned} &^3{\rm H}:\ a=0.038\ {\rm fm},\ r_c=1.84\ {\rm fm} ,\\ &^3{\rm He}:\ a=0.038\ {\rm fm},\ r_c=1.88\ {\rm fm} ,\\ &^4{\rm He}:\ a=0.0057\ {\rm fm},\ r_c=1.68\ {\rm fm} . \end{aligned} \tag{20}$$

The ^{4}He parameters were obtained from fitting low-energy electron scattering data from a number of experiments.[23–26] The inclusion of a k (Ref. 4) term in the ^{4}He form is necessary to fit the form factor in the $1\le k\le 2$ $\rm fm^{-1}$ range.

The potential energy terms in the double commutator expressions of Eqs. (13) and (14) were evaluated using effective potentials and corresponding two component Gaussian wave functions which are consistent with the binding energies and rms radii of the $A=3$ and $A=4$ nuclei.[27,28]

The reduced radiative capture rates $Z\Lambda_r^{nl}(A,Z)$ as defined in Eq. (5) were calculated as a function of $\bar{k}$ and are plotted in Fig. 1 ($A=3$) and Fig. 2 ($A=4$). The curves for 1s capture behave similarly to their counterparts for muon capture,[2,3] but the 2p "closure + correction" behaves quite differently. The procedure adopted in the muon capture rate was to take $Z\Lambda_r^{1s}(A,Z)$ equal to its value on the "closure plus correction" curve where its derivative of with respect to $\bar{k}$ equals zero. In the case of 1s capture in ^{3}H and ^{4}He one sees from Fig. 1 and 2 that this value is essentially the same as the value of $Z\Lambda_r^{1s}$ where the "closure" and "closure plus correction" curves intersect, the point at which the first order correction to closure vanishes. On the other hand, with ^{3}He the $Z\Lambda_r^{1s}$ value at the zero derivative point and the intersection point differ by about 8%. Fortunately, in the ^{3}He case there exists the detailed calculation of Phillips and Roig[29] on the 1s radiative capture rate with which to capture our calculation. Their total radiative rates range from 4.49 to 4.65 eV, in excellent agreement with our value of $\Lambda_\gamma^{1s}(^3{\rm He})=4.61$ eV listed in Table I of this paper. They give $\bar{k}=116$ MeV as the gamma energy at which their own closure calculation is equal to the total radiative rates which they calculate, while from Fig. 1 of this paper one sees that $\bar{k}=114$ MeV is the gamma energy at which the maximum value of $Z\Lambda_r^{1s}$ along the "closure plus correction" curve equals the "closure" value. One can conclude that for the s-shell nuclei under study the stationary value of $Z\Lambda_r^{1s}$ along the "closure plus correction" curve is the proper point at which to select $Z\Lambda_r^{1s}$.

No such stationary point exists for the corresponding 2p curve in Fig. 2, so the intersection point was chosen as the point at which to select $Z\Lambda_r^{2p}$ for ^{3}He the closure

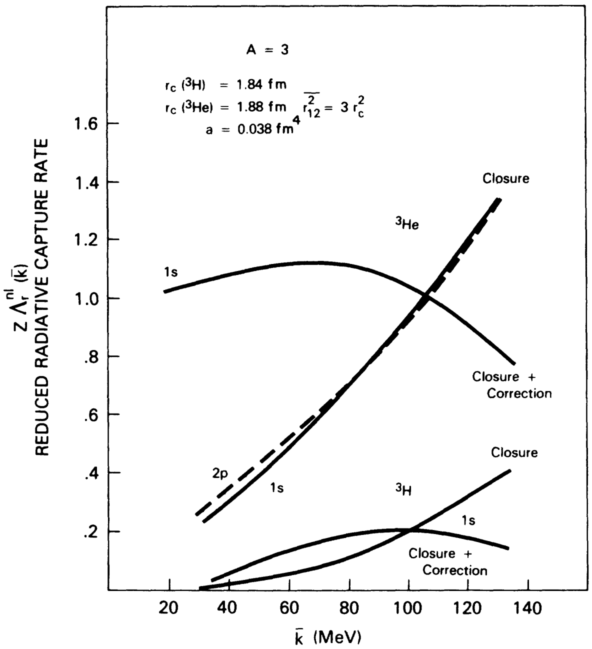

FIG. 1. Dependence of the labeled reduced radiative capture rates on the gamma energy $\bar{k}$. The closure approximation equates $Z\Lambda_r^{nl}$ to the first term on the rhs of Eq. (5) while the closure plus correction curve includes both terms on the rhs of the same equation. The $1s$ and $2p$ closure rates were calculated from Eqs. (11) and (12), respectively, while the double commutator corrections to the $1s$ capture rates were calculated per Eqs. (13a) and (13b) for ^{3}H and ^{3}He, respectively. The relative form factors on which the closure approximation depends were calculated from $F_{\rm rel}(k)=F_c(\overline{r_{12}^2}k^2/r_c^2)/F_{\rm p}(\overline{r_{12}^2}k^2/r_c^2)$ with $F_c(k^2)=(1+a^4k^4)\exp(-k^2r_c^2/6)$. The constant factor $R_{nl}^2(0)$ is not included in the functions graphed above.

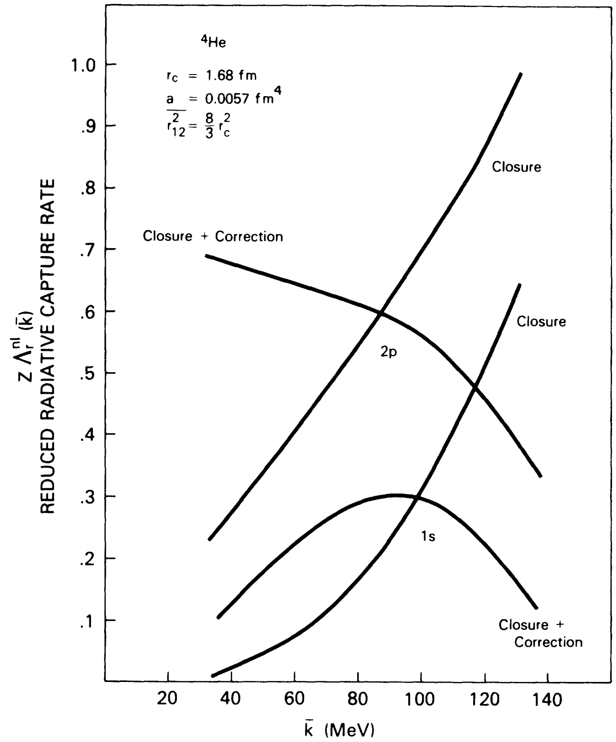

FIG. 2. Same as in Fig. 1 except that the double commutator corrections to the $1s$ and $2p$ rates were calculated according to Eqs. (13c) and (14), respectively.

value for $Z\Lambda_r^{2p}$ at $\bar{k}=114$ MeV appears in Table I.

The reduced capture rates and the corresponding radiative capture widths Λ_γ^{nl} are displayed in Table I. Excluding ^{3}H, the experimental total widths $\Gamma_{\rm tot}^{nl}$ (contained within brackets) and capture probabilities w_{nl} were used to calculate a "theoretical" branching ratio from the relation

$$R_B=\sum_{n,l}\Lambda_\gamma^{nl}/\Gamma_{\rm tot}^{nl}\,w_{nl} \tag{21}$$

using[9] $w_{\rm p}$ (^{3}He)$=0.1\pm0.06$ and $w_{\rm p}$ (^{4}He)$=0.21\pm0.10$. The agreement between the calculated branching ratios and the experimental ones, enclosed within brackets in

TABLE I. Total radiative capture rates and branching ratios. The theoretical radiative capture rates were calculated from $\Lambda_\gamma^{nl}=K(\bar{k})N_{nl}^2(Z)Z\Lambda_r^{nl}$ [see Eq. (2) and following]. The branching ratios were calculated from the expression $BR=w_{1s}\Lambda_\gamma^{1s}/\Gamma_{\rm tot}^{1s}+w_{wp}\Lambda_\gamma^{2p}/\Gamma_{\rm tot}^{2p}$ using theoretical radiative capture rates but experimental capture probabilities and total widths. The exception is the nucleus ^{3}H for which no measurements of $\Gamma_{\rm tot}^{1s}$ exist. Experimental quantities are enclosed in square brackets.

Nucleus	R_{1s}^2	$Z\Lambda_r^{1s}$	Λ_γ^{1s} eV	$Z\Lambda_r^{2p}$	Λ_γ^{2p} meV	$\Gamma_{\rm tot}^{1s}$ eV	$\Gamma_{\rm tot}^{2p}$ meV	R_B ratio (%)
^{3}H	1	0.205	0.100			2.2 ± 0.4		[4.5 ± 0.8][a]
						(1.02)[b]		
^{3}He	1.05[c]	1.17	4.61	1.09	0.026	[28 ± 7][d]	[0.7 ± 0.2][e]	15.2 ± 4.5
						[36 ± 7][f]		11.9 ± 2.8
								[14.0 ± 1][g]
^{4}He	0.77[h]	0.230	0.927	0.593	0.015	[45 ± 3][i]	[2.1 ± 0.3][e]	1.78 ± 0.18
						[51 ± 9][f]		1.59 ± 0.24
								[1.5 ± 0.3][j]

[a]Reference 31.
[b]References 29 and 30, theoretical value.
[c]Fit to energy shift -32 ± 3 eV using method of Reference 32.
[d]Reference 6.
[e]Reference 9.
[f]Reference 7.
[g]Reference 33.
[h]Reference 34.
[i]Reference 8
[j]Reference 35.

the last column, is satisfactory. The inclusion of the weighted p-state rate in calculating the ^{4}He branching ratios is essential to obtaining agreement within errors.

There is no experimental Γ_{tot}^{1s} (^{3}H) so the calculated total radiative capture rate and the branching ratio were combined to estimate this quantity. It is compared in Table I with an earlier estimate (enclosed within parenthesis) by Phillips and Roig.[30] They used their calculated partial capture rate to three nucleon final states in ^{3}He, the $T=\frac{3}{2}$ component of which is directly related to the ^{3}H capture rate, to calculate Γ_{tot}^{1s} (^{3}H). As the authors themselves point out[29] their theoretical values for the sum of the n + ^{2}H and 2n + p final states are much below the measured rates for ^{3}He and this circumstance is the cause of the discrepancy between our total width and theirs.

IV. ABSORPTION OF LOW ENERGY PIONS

From the total 1s width for ^{3}H in Table I and the 95.5±0.8 % branching ratio for absorption one obtains an absorption width of

$$\Gamma_{ab}^{1s}(^3\text{H})=2.12\pm0.38 \text{ eV} . \quad (22)$$

In an earlier work[10] it was pointed out that the 1s absorption widths for ^{3}H and ^{3}He, aside from renormalization effects due to the differing scale lengths of the atomic wave functions, would be the same if low energy pion absorption occurred only on spin triplet (isospin singlet) nucleon pairs. Since the total isospin of the $\pi+(A=3)$ system is conserved the ratio of the ^{3}H and ^{3}He widths (with renormalization effects removed) can be rewritten as $\Gamma_{ab}^{1s}(T=\frac{3}{2})/[\frac{2}{3}\Gamma_{ab}^{1s}(T=\frac{1}{2})+\frac{1}{3}\Gamma_{ab}^{1s}(T=\frac{3}{2})]$ where both the $T=\frac{1}{2}$, $\frac{3}{2}$ widths may be considered to refer to the ^{3}He pionic atom. Using the formula

$$\frac{\Gamma_{ab}^{1s}(T=3/2)}{2/3\Gamma_{ab}^{1s}(T=1/2)+1/3\Gamma_{ab}^{1s}(T=3/2)} = \frac{(2)^3R_{1s}^2(^3\text{He})}{R_{1s}^2(^3\text{H})}\frac{\Gamma_{ab}^{1s}(^3\text{H})}{\Gamma_{ab}^{1s}(^3\text{He})} , \quad (23)$$

we find that

$$\frac{\Gamma_{ab}^{1s}(T=3/2)}{2/3\Gamma_{ab}^{1s}(T=1/2)+1/3\Gamma_{ab}^{1s}(T=3/2)} = \begin{matrix} 0.94\pm0.44^6 \\ 0.73\pm0.29^7 \end{matrix} \quad (24)$$

after deducing absorption widths from the two different values of the total 1s widths listed in Table I.

The same ratio of absorption widths with reduced statistical error can be evaluated directly from the *ratios* of branching ratios for radiative pion capture and absorption in the two nuclei and from our calculated reduced total radiative capture rates:

$$\frac{\Gamma_{ab}^{1s}(T=3/2)}{2/3\Gamma_{ab}^{1s}(T=1/2)+1/3\Gamma_{ab}^{1s}(T=3/2)} = \frac{R_2 Z\Lambda_r^{1s}(^3\text{H})}{R_1 Z\Lambda_r^{1s}(^3\text{He})} = 0.81\pm0.20 , \quad (25)$$

where $R_{1,2}=R_B(\pi,\gamma)/R_B(\pi\text{abs})$ in ^{3}H and ^{3}He, respectively (see Refs. 28 and 31). From Eq. (25) we obtain

$$\Gamma_{ab}^{1s}(T=\tfrac{3}{2})/\Gamma_{ab}^{1s}(T=\tfrac{1}{2})=0.74\pm0.27 . \quad (26)$$

While the value of 1 for the ratios of absorption widths in Eqs. (24), (25), and (26) is included within the limits of error of all three estimates, the central values imply that there is some degree of low energy absorption on singlet spin pairs. Phillips and Roig[36] have calculated the partial absorption rates of stopped π^- in ^{3}He into n + ^{3}H and n + n + p final states using a phenomenological two-nucleon model for s-wave pion absorption. Two constants g_0 and g_1 appear in the effective absorption Hamiltonian which parametrize absorption on isospin singlet (^{2}H) and isospin triplet (nn and np) nucleon pairs, respectively. From fitting low energy two-nucleon pion production data they determined that $g_1^2/g_0^2=0.3\pm0.15$. The $T=\frac{1}{2}$ and $T=\frac{3}{2}$ widths were shown in Ref. 10 to differ by terms of the order g_1^2/g_0^2 in the limit of SU4 invariance of the nucleon-nucleon potential. In this limit the three-body ground state would be completely space symmetric and isospin singlet and triplet final state nucleon pairs would have identical wavefunctions. After adding (np) + n and (nn) + p final states on the same footing as the n + ^{2}H a decomposition of the Phillips and Roig two-body and three-body rates into isospin components was carried out.[37] The ratio 0.74 of the $T=\frac{3}{2}$ to $T=\frac{1}{2}$ width is found to be consistent with the value 0.30 for the ratio of g_1^2/g_0^2 fit to low energy two-nucleon production data.

However, precisely because the difference in the two isospin widths differ by terms of the order of g_1^2/g_0^2 rather than g_1/g_0 it is worthwhile to look for other quantities which are sensitive to the latter ratio. The partial rate for absorption into the n + ^{2}H final state is such a quantity. Recent measurements[6,7] of the total ^{3}He 1s pionic atom width and the observation[38] of a 11.5% branching ratio for the post-absorption n + ^{2}H channel were not available to the authors of Ref. 36. Knowledge of the newer experimental data allows a more accurate determination of the absolute value of the partial width for the n + ^{2}H channel. Using the two experimental total absorption widths of 19.1 and 24.5 eV, respectively, it can be deduced that (expressing the widths as rates per Ref. 36)

$$\Gamma_{ab}^{1s}(\text{n}+{}^2\text{H})=\begin{cases}3.3\pm1.4\times10^{15}\ \text{s}^{-1}{}^{6} \\ 4.3\pm1.8\times10^{15}\ \text{s}^{-1}{}^{7} .\end{cases} \quad (27)$$

These values when compared to the numerical predictions in Table III of Ref. 36 and using the general formula for the partial absorption rate for the n + ^{2}H final state yields $0\le g_1/g_0\le 0.1$, much lower than the ratio from pion production on nucleon pairs.

V. DISCUSSION AND SUMMARY

Total rates for radiative pion capture from the 1s and 2p states in the $A=3$ and $A=4$ nuclei have been calculated using the method of Bernabeu[2] which was used originally to estimate total muon capture rates in light self-conjugate nuclei. In the present work isospin projection operators were introduced to limit sums over intermediate states to those physically coupled to the ground state by the radiative capture operator $O_{\lambda}^{-}(\bar{k})$ and the rates for both $A=3$ nuclei were obtained. The calculated rates are consistent with the measured branching ratios for radiative pion capture and the total pionic atom widths in ^{3}He and ^{4}He. In the case of ^{3}H the calculated radiative capture rate and the measured branching ratio were combined to obtain a reliable estimate of the total 1*s* width of the pionic atom. The width obtained implies that the $T=\frac{1}{2}$ and $T=\frac{3}{2}$ 1*s* pion *absorption* widths of the $A=3$ ground state are related by $\Gamma_{ab}^{1s}(T=\frac{3}{2})/\Gamma_{ab}^{1s}(T=\frac{1}{2})=0.74\pm0.27$.

The sum rule method applied to the 2*p* radiative capture was less satisfactory because the "closure plus correction" approximation for the reduced total rate is not stationary around some average gamma energy $\bar{k}$; the only unique value of $\bar{k}$ is the value where the first order correction vanishes. Branching ratios for radiative capture in atoms of higher *Z* in which 2*p* capture dominates should be investigated in order to clarify the procedure for obtaining the optimal $\bar{k}$.

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