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Relativistic treatment of (p,p′γ) reactions

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We establish the formal connection between the spin difference function and the loss of symmetry of the correlation function in (p,p′γ) experiments. The same sensitivity to relativistic effects present in ($\vec{p},\vec{p}\,'$) spin observable differences is also found in (p,p′γ) reactions. We performed a relativistic distorted wave calculation of the asymmetry and found sizable effects.

I. INTRODUCTION

Spin observables have been to date the biggest success of relativistic approaches to proton-nucleus scattering. Excellent description of elastic scattering data was obtained by fitting scalar and vector potentials in a Dirac optical model.[1] Subsequently, the formalism was put on much firmer ground when McNeil, Shepard, and Wallace showed how these potentials could be obtained from a relativistic-impulse approximation treatment.[2]

Although the successes of the relativistic approach are unquestionable, many objections have been raised.[3] Among these, the absence of a first-principles theory that would lead, by a set of well-defined approximations, to the relativistic impulse approximation formalism. While an extensive effort should be devoted to the formal aspects of the theory, it is essential to keep testing the relativistic formalism by studying many different and diverse processes.

Among the processes studied to date are inelastic proton-nucleus reactions. These inelastic transitions can uncover the full richness contained in spin observables and go beyond the highly constrained elastic phenomena. In fact, the spin difference function $\Delta_s \equiv (Q-B) + i(P-A_y)$, defined in terms of spin observables differences, vanishes in the elastic case but has nevertheless proven to be extremely sensitive to differences between equivalent relativistic and nonrelativistic theories.[4] In particular, the spin difference function vanishes in a nonrelativistic impulse approximation approach and nonzero values are obtained only after the inclusion of nonlocal terms.[5] In contrast, a local relativistic treatment contains enough structure to give, in accordance with experiment,[6] a nonzero value to the spin difference function.[4,7] It is then natural to ask if those parts of the amplitude responsible for the nonzero value of the spin difference function are driving other physical observables. If so, these observables would give as much and as valuable information as the spin difference function does.

Recently, Mobed and Wong have shown that a relativistic treatment of (p,p′γ) reactions breaks a particular symmetry of the correlation function that is otherwise preserved in the nonrelativistic case.[8] In this work we will establish the formal connection between the spin difference function and the symmetry breaking of the correlation function. We will show how the same parts of the amplitude driving the spin difference function are also responsible for the loss of symmetry in the correlation function.

This work will be organized as follows: Section II will contain the formal part of the paper. In it, we will derive model independent results for the correlation function and will establish its connection to the spin difference function. In Sec. III we will show results from a relativistic plane wave impulse approximation (RPWIA) calculation, as well as from a distorted wave calculation (RDWIA) that uses eikonal distorted waves. Our conclusions will then follow in Sec. IV.

II. FORMALISM

In this section we will establish the connection between the spin difference function and the p-γ correlation function. Although the whole formalism can be carried out without any reference to the angular momentum and parity of the excited nuclear state, we will concentrate on the unnatural parity 1^+ states. In Ref. 7, it was proven that the most general amplitude that one can write for the $0^+ \to 1^+$ transition consistent with rotational and parity invariance is given by

$$\hat{A}_{1^+}(\mathbf{p},\mathbf{p}') = A_n(\hat{\boldsymbol{\Sigma}}\cdot\hat{\mathbf{n}}) + A_{nn}(\hat{\boldsymbol{\Sigma}}\cdot\hat{\mathbf{n}})(\boldsymbol{\sigma}\cdot\hat{\mathbf{n}}) + A_{KK}(\hat{\boldsymbol{\Sigma}}\cdot\hat{\mathbf{K}})(\boldsymbol{\sigma}\cdot\hat{\mathbf{K}}) + A_{qq}(\hat{\boldsymbol{\Sigma}}\cdot\hat{\mathbf{q}})(\boldsymbol{\sigma}\cdot\hat{\mathbf{q}}) + A_{qK}(\hat{\boldsymbol{\Sigma}}\cdot\hat{\mathbf{q}})(\boldsymbol{\sigma}\cdot\mathbf{K}) + A_{Kq}(\hat{\boldsymbol{\Sigma}}\cdot\hat{\mathbf{K}})(\boldsymbol{\sigma}\cdot\hat{\mathbf{q}}) , \tag{1}$$

where $\hat{\mathbf{q}}$ and $\hat{\mathbf{K}}$ are unit vectors along the direction of momentum transfer $\mathbf{q}=(\mathbf{p}-\mathbf{p}')$ and average momentum $\mathbf{K}=(\mathbf{p}+\mathbf{p}')/2$, respectively; $\hat{\mathbf{n}} \equiv \hat{\mathbf{q}} \times \hat{\mathbf{K}}$; $\boldsymbol{\sigma}$ is the spin operator of the projectile; and $\hat{\boldsymbol{\Sigma}}$ is the polarization (axial) vector operator of the target defined by

$$\hat{\boldsymbol{\Sigma}}_M \equiv | 1^+, M\rangle\langle 0^+ | \ . \tag{2}$$

The individual amplitudes are scalar functions of the energy and momentum transfer and can be used to write all physical observables in a model independent way. Subsequently, one may calculate these amplitudes using relativistic and nonrelativistic models, and then concentrate on those observables that isolate those amplitudes sensitive to relativistic effects.

In fact, this is precisely the way we proceeded in our study of proton spin observables.[7,9] We calculated the above amplitudes using equivalent relativistic and nonrelativistic formalisms and found large differences in the predicted values for the cross term amplitudes A_{qK} and A_{Kq}. In the nonrelativistic calculation these amplitudes were identically zero, while in the relativistic case nonzero values appeared due to the presence of lower components in the wave function. This fact naturally signaled out the spin difference function

$$\Delta_s \equiv (Q-B)+i(P-A_y) = \left[4\Big/\frac{d\sigma}{d\Omega}\right](A_{KK}A_{Kq}^{*}+A_{qK}A_{qq}^{*}) \tag{3}$$

as the most useful observable in the study of relativistic effects in proton-nucleus scattering.

Unfortunately, measurement of the spin difference function is not an easy task. Extremely good energy resolution is needed to properly identify the excited nuclear state. Adding to that, one is faced with the challenge of polarizing the incident proton beam, as well as detecting their final polarization by means of a second scattering experiment. Alternative physical processes, addressing these same issues, are clearly welcome.

An extremely good alternative is provided by (p,p$'\gamma$) reactions. In these experiments one is spared from having to prepare and detect proton polarizations, in exchange, of course, for a coincidence measurement. We now show how the A_{qK} and A_{Kq} amplitudes can be isolated by performing coincidence measurements in some carefully chosen kinematic region.

The correlation function for $0^+(\mathrm{p,p'})1^+(\gamma)0^+$ reactions is defined by[10]

$$W_\lambda(E,q;\hat{\mathbf{k}})=\frac{4\pi}{\Gamma(k)}\frac{\mathrm{Tr}[\hat{\mathbf{F}}_\lambda(\mathbf{k})\hat{\rho}(E,q)\hat{\mathbf{F}}_\lambda(\mathbf{k})^\dagger]}{\mathrm{Tr}\hat{\rho}(E,q)}\ , \tag{4}$$

where E and q are the energy and momentum transfer in the collision, and $\mathbf{k}$ and λ are the momentum and polarization of the emerging photon, respectively. The density operator $\hat{\rho}(E,q)$ describes the polarization of the 1^+ state after the collision has occurred. For an initially unpolarized beam it is given by

$$\hat{\rho}(E,q)=\tfrac{1}{2}\mathrm{Tr}_\sigma[\hat{A}(1^+)\hat{A}(1^+)^\dagger]\ , \tag{5}$$

where $\hat{A}(1^+)$ is the $0^+\to 1^+$ amplitude (1), and Tr_σ denotes a trace over the proton spin degrees of freedom. The electromagnetic operator $\hat{F}_\lambda(\mathbf{k})$,

$$\hat{F}_\lambda(\mathbf{k})=\sum_M F_{M\lambda}(\mathbf{k})\hat{\Sigma}_M^\dagger\ ,$$
$$F_{M\lambda}(\mathbf{k})=-\left[\frac{k}{2\pi}\right]^{1/2}\lambda\langle 1^+\|\hat{T}_1^{(m)}\|0^+\rangle D_{M\lambda}^{(1)*}(\hat{\mathbf{k}})\ , \tag{6}$$

contains all information related to the electromagnetic decay, and is written in terms of Wigner D functions and the M1 reduced matrix element $\langle 1^+\|\hat{T}_1^{(m)}\|0^+\rangle$. Finally, $\Gamma(k)$ is the total width of the decay,

$$\Gamma(k)=\sum_\lambda \tfrac{1}{3}\int d\hat{\mathbf{k}}\,\mathrm{Tr}[\hat{F}_\lambda(\mathbf{k})\hat{F}_\lambda(\mathbf{k})^\dagger]\ . \tag{7}$$

The formal evaluation of the correlation function is straightforward. The numerator in Eq. (4) is simply given by

$$\mathrm{Tr}[\hat{F}_\lambda(\mathbf{k})\hat{\rho}(E,q)\hat{F}_\lambda(\mathbf{k})^\dagger]=\sum_{MM'}F_{M\lambda}(\mathbf{k})\rho_{MM'}F_{M'\lambda}^{*}(\mathbf{k})$$
$$=\sum_{KQ}B_K(k)\rho_{KQ}Y_{KQ}(\hat{\mathbf{k}})\ , \tag{8a}$$

where

$$B_K(k)=\frac{k}{2\pi}\left[\frac{4\pi}{2K+1}\right]^{1/2}\times\langle 1\lambda;1-\lambda\,|\,K0\rangle\,|\langle 1^+\|\hat{T}_1^{(m)}\|0^+\rangle|^2\ , \tag{8b}$$

and ρ_{KQ} is the statistical tensor defined by

$$\rho_{KQ}\equiv\sum_{MM'}(-1)^{1-M'}\langle 1M,1-M'\,|\,KQ\rangle\rho_{MM'}\ . \tag{8c}$$

The expression for the correlation function is even simpler. Dynamical quantities related to the electromagnetic decay (e.g., the M1 reduced matrix element) factor out from the ratio in Eq. (4), and the correlation function depends exclusively on the collision part of the process,

$$W_\lambda(E,q;\hat{\mathbf{k}}) =\tfrac{1}{2}\left[1+\sqrt{4\pi}\sum_{K>0,Q}\langle 1\lambda;K0\,|\,1\lambda\rangle\frac{\rho_{KQ}}{\rho_{00}}Y_{KQ}(\hat{\mathbf{k}})\right]\ . \tag{9}$$

Our task now becomes clear. Evaluate the density matrix in terms of the scalar amplitudes defined in Eq. (1). Identify those matrix elements dependent on the cross term amplitudes A_{qK} and A_{Kq} and therefore sensitive to relativistic effects. Finally, choose appropriate kinematic conditions (e.g., photon direction) in the evaluation of the correlation function to highlight those matrix elements of interest.

In the evaluation of the density matrix we will choose our coordinate system with the z axis defined in the direction of the average momentum $\hat{\mathbf{K}}$. This represents the natural choice of coordinate system whenever distorted waves are calculated in the eikonal approximation. If we also neglect the Q value of the reaction, then $\hat{\mathbf{n}}$, $\hat{\mathbf{q}}$, and $\hat{\mathbf{K}}$ form a right-handed orthonormal system. We expect this to be an excellent approximation for the physical observables of interest. Keeping the Q value of the reaction introduces a correction term, proportional to $\hat{\mathbf{q}}\cdot\hat{\mathbf{K}}$, capable of breaking the symmetry of the correlation function, as well as generating a nonzero value for the spin difference function, even in a nonrelativistic treatment. For small q, however, both of these observables are proportional to q and therefore vanish at forward angles. Furthermore, in the medium energy region $\hat{\mathbf{q}}\cdot\hat{\mathbf{K}}$ becomes negligible beyond $q \simeq 0.5$ fm^{-1}. Corrections due to a finite Q value are therefore expected to be small and will be neglected throughout this work.

The hermiticity of the density matrix, $\rho_{MM'}=\rho^*_{M'M}$, as well as the frame-dependent relation $\rho_{MM'}=\rho_{-M,-M'}$, limit to four the number of independent matrix elements. These can be written in terms of the scalar amplitudes as

$$\begin{aligned}
\rho_{11}&=\tfrac{1}{2}[\,|A_n|^2+|A_{nn}|^2+|A_{qq}|^2+|A_{qK}|^2]\ ,\\
\rho_{10}&=\frac{i}{\sqrt{2}}[A_{qq}A^*_{Kq}+A_{qK}A^*_{KK}]\ ,\\
\rho_{1-1}&=-\tfrac{1}{2}[\,|A_n|^2+|A_{nn}|^2-|A_{qq}|^2-|A_{qK}|^2]\ ,\\
\rho_{00}&=[\,|A_{KK}|^2+|A_{Kq}|^2]\ .
\end{aligned}\tag{10}$$

Clearly, ρ_{10} is by far the most interesting matrix element. It is linear in the cross term amplitudes A_{qK} and A_{Kq}, and therefore vanishes in a local nonrelativistic treatment. All physical observables written exclusively in terms of ρ_{10} will therefore enjoy the same special status as the spin difference function does. Isolating this term in the correlation function will be our next step.

In a polarization insensitive measurement the experimentally determined quantity is the unpolarized correlation function $W(\hat{\mathbf{k}})$ defined by

$$W(\hat{\mathbf{k}})\equiv W_{\lambda=1}(E,q;\hat{\mathbf{k}})+W_{\lambda=-1}(E,q;\hat{\mathbf{k}})\ ,\tag{11}$$

and given, according to Eq. (9), by

$$\begin{aligned}
W(\hat{\mathbf{k}})=1+\frac{3}{2\,\mathrm{Tr}\rho}[\tfrac{1}{3}(\rho_{11}-\rho_{00})(3\cos^2\theta-1)\\
-\sqrt{2}\,\mathrm{Im}(\rho_{10})\sin 2\theta\sin\phi\\
+\rho_{1-1}\sin^2\theta\cos 2\phi]\ ,
\end{aligned}\tag{12}$$

where θ and ϕ are the polar and azimuthal photon angles, respectively, and we have suppressed the dependence of the correlation function on the proton variables. By setting the photon detector at a polar angle defined by $\cos^2\theta_0=\frac{1}{3}$, and azimuthal angle $\phi_0=\pi/4$, one is able to isolate the ρ_{10} term. The correlation function now becomes a linear function of the cross term amplitudes, as desired,

$$W(\theta_0,\phi_0)=1-\frac{\mathrm{Re}[A_{qq}A^*_{Kq}+A_{qK}A^*_{KK}]}{\mathrm{Tr}\rho}\ .\tag{13}$$

This fact places the correlation function on the same footing as the spin difference function Eq. (3), another physical observable extremely sensitive to relativistic effects, and perhaps even more accessible to experiment.

A nonrelativistic treatment of (p,p′γ) reactions predicts an invariance of the correlation function under rotations of the photon wave vector by an angle π around the $\hat{\mathbf{K}}$ direction.[11] Recently, Mobed and Wong have shown that a relativistic treatment breaks this symmetry, even in plane wave.[8] Not surprisingly, the symmetry breaking of the correlation function has the same origin as the nonzero value for the spin difference function, namely, the nonzero values of the cross term amplitudes A_{qK} and A_{Kq} predicted in a relativistic treatment. Mobed and Wong have introduced a physical observable to quantify this loss of symmetry,

$$\Delta W(\theta,\phi)\equiv\frac{W\left[\theta,\phi-\frac{\pi}{2}\right]-W\left[\theta,\phi+\frac{\pi}{2}\right]}{W\left[\theta,\phi-\frac{\pi}{2}\right]+W\left[\theta,\phi+\frac{\pi}{2}\right]}\ .\tag{14}$$

Evaluation of this expression at its maximum value, namely $\theta=\pi/4$ and $\phi=0$, yields

$$\Delta W(\theta,\phi)=\frac{2\sqrt{2}\,\mathrm{Im}(\rho_{10})}{3\rho_{11}+\rho_{00}-\rho_{1-1}}\ .\tag{15}$$

As expected, the photon asymmetry $\Delta W(\theta,\phi)$ is linear in ρ_{10} and consequently linear in the cross term amplitudes A_{qK} and A_{Kq}.

We note, however, that a polarization insensitive experiment can only determine the imaginary part of ρ_{10}. For a determination of the real part of ρ_{10}, one needs a polarization sensitive (p,p′$\vec{\gamma}$) measurement. In this case the correlation function is given by

$$\vec{W}(\hat{\mathbf{k}})=W(\hat{\mathbf{k}})+3\sqrt{2}(\epsilon_+-\epsilon_-)\frac{\mathrm{Re}(\rho_{10})}{\mathrm{Tr}\rho}\sin\theta\cos\phi\ ,\tag{16}$$

where ϵ_+ and ϵ_- are the detection efficiencies for the right- and left-handed polarized photons, respectively.

We want to conclude this section by stressing that, aside from neglecting the Q value of the reaction, all results in this section are model independent and free from approximations.

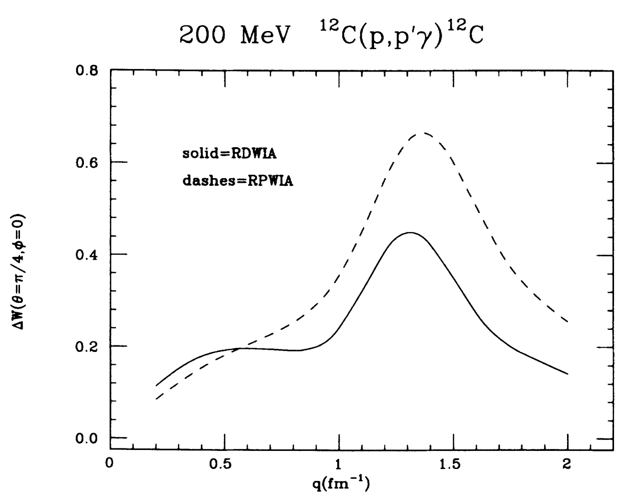

FIG. 1. Photon asymmetry $\Delta W(\theta=\pi/4,\phi=0)$ as a function of momentum transfer q at $T_{\mathrm{lab}}=200$ MeV. Photons are product of the electromagnetic decay of the 12.71 MeV, $(J^\pi,T)=(1^+,0)$ state in ^{12}C.

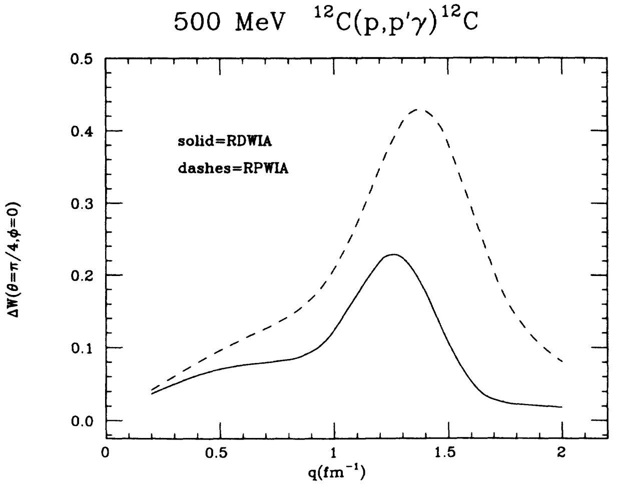

FIG. 2. Same as Fig. 1, except for $T_{\rm lab}=500$ MeV incident protons.

III. CALCULATIONS AND RESULTS

In the preceding section we have recognized the importance of (p,p$'\gamma$) reactions and have established its relation to ($\vec{p},\vec{p}'$) processes. In particular we have shown that those amplitudes responsible for a nonzero value of the spin difference function are also responsible for the loss of symmetry in the correlation function. Having previously calculated the $0^+\rightarrow 1^+$ transition amplitude to the isoscalar 12.71 MeV state in ^{12}C, it has resulted straightforward to extend our calculation to (p,p$'\gamma$) reactions.

We have calculated the correlation function by performing both relativistic plane wave and relativistic distorted wave impulse approximation calculations. Eikonal distorted waves[12] were calculated using optical potentials with strengths and ranges chosen to reproduce elastic scattering data.[13] Transition densities were calculated by assuming a simple $p^{3/2}\rightarrow p^{1/2}$ single particle excitation. The upper component of the Dirac bound states was given by a nonrelativistic harmonic oscillator wave function. Lower components were subsequently obtained with the use of the Dirac equation. Finally, we used a relativistic parametrization of the NN interaction[14] with the Lorentz invariant amplitudes evaluated at their optimal value.[14,15] For a complete description of the procedure, along with the numerical values used in the calculation, we refer the reader to previous publications.[7,9]

Figures 1 and 2 contain our predictions for the photon asymmetry $\Delta W(\theta,\phi)$, at $T_{\rm lab}=200$ MeV and $T_{\rm lab}=500$ MeV proton energies, respectively. The plane wave results predict large asymmetries in the correlation function. Although distortions tend to reduce the effect, the qualitative behavior is not changed. It is also interesting to note that while the 500 MeV result predicts a smaller photon asymmetry, this result might be more significant (photon) since it will survive corrections coming from exchange and Pauli blocking. Some of these corrections essential for the prediction of nontrivial effects in nonrelativistic theories.

IV. CONCLUSIONS

The central point of the present work was to show that (p,p$'\gamma$) reactions offer a very attractive alternative to ($\vec{p},\vec{p}'$) experiments in the study of relativistic effects. We showed that the same sensitivity to relativistic effects present in ($\vec{p},\vec{p}'$) reactions can be found in (p,p$'\gamma$) experiments, with the added bonus of never having to prepare, nor detect, proton polarizations. We explicitly showed how those amplitudes responsible for a nonzero value of the spin difference function are also responsible for the asymmetry in the correlation function.

The inability of nonrelativistic treatments to generate nonzero values for these amplitudes will result in no loss of symmetry in the correlation function in the same way as it resulted in a null prediction for the spin difference function. In contrast, relativistic calculations predict a large asymmetry. We calculated photon asymmetries using local, relativistic plane wave, and distorted wave formalisms. Inclusion of distortions does not change the shape of the photon asymmetry, although in some regions it reduces the asymmetry by up to a factor of 2.

As we mentioned in the Introduction, it is essential to subject the relativistic formalism to the most stringest tests. In fact, it is our belief that only by testing the relativistic predictions for many different and diverse processes, while at the same time working towards a formal justification of the theory, that a clear and satisfactory picture will ever emerge.

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