Four-body calculation of the 0^+-1^+ binding energy difference in the $A=4$ Λ hypernuclei

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The coupled, two-variable integral equations that determine the ${}^4_\Lambda$He and ${}^4_\Lambda$H ground and first excited states, when the NN and ΛN-ΣN interactions are represented by separable potentials, are solved numerically. We find the spin-isospin suppression of the Λ-Σ conversion due to the composite nature of the ^{3}He, ^{3}H nuclear cores to be a significant factor in understanding the 0^+-1^+ binding energy difference.

I. INTRODUCTION

The binding energy systematics of the $A=4$ Λ hypernuclei offer a unique opportunity for testing our understanding of the s-wave properties of the fundamental hyperon-nucleon (YN) force. We have previously explored[1–4] the consequences of the small charge-symmetry-breaking differences between the Λp and Λn interactions[5–7] for the ground-state ($J^\pi=0^+$) Λ-separation energies of ${}^4_\Lambda$He and ${}^4_\Lambda$H:[8]

$$B_\Lambda({}^4_\Lambda\mathrm{He})=B({}^4_\Lambda\mathrm{He})-B({}^3\mathrm{He})\approx 2.39\pm 0.03 \text{ MeV}$$

and

$$B_\Lambda({}^4_\Lambda\mathrm{H})=B({}^4_\Lambda\mathrm{H})-B({}^3\mathrm{H})\approx 2.04\pm 0.04 \text{ MeV} .$$

We concluded that the Λ-separation energy difference in this hypernuclear isodoublet does appear to be consistent with the charge-symmetry breaking reflected in the low-energy ΛN scattering parameters of meson-theoretic YN potentials (the scattering lengths and effective ranges) when one utilizes a proper four-body theory in contrast to an effective two-body approximation. In addition to charge symmetry breaking, one may also investigate the complications that can arise in calculations of the properties of bound systems in which one baryon (here the Λ) with a given isospin ($T=0$) couples strongly to another (the Σ) with a different isospin ($T=1$).[3,4] (The implications for NN-NΔ coupling in nonstrange nuclei will be apparent to the reader.) In particular, the energies of the ($J^\pi=1^+$) spin-flip states are quite sensitive to the strength of the ΛN-ΣN coupling in the YN interaction.

Exact equation calculations have played an important role in the past in elucidating novel points of physics not readily apparent from effective two-body formulations of few-body problems. The binding energy of a two-body system decreases as the two-body effective range becomes smaller, whereas those of the corresponding three-body and four-body bound states increase.[9,10] This was the essence of the variational argument made by Thomas to show that the nuclear force must be of finite (nonzero) range or the triton would collapse to a point.[11,12] It is this same property of few-body equations that allowed us to reconcile the charge symmetry breaking in the low-energy ΛN scattering parameters and the Λ-separation energy difference observed in the $A=4$ isodoublet ground states.[1,2] Understanding why one sees almost as many deuterons emitted from the ^{3}He(γ,d)p reaction as neutrons from the ^{3}He(γ,n)2p reaction at low energy ($E_\gamma<20$ MeV), when the d + p final state contains only $T=\frac{1}{2}$ components while the n + p + p final state contains both $T=\frac{1}{2}$ and $T=\frac{3}{2}$ components, was not achieved until an analysis of the $A=3$ photodisintegration reaction, in terms of exact equations, was published.[13–15] The intimate connection between the two final states was not appreciated prior to the exact equation studies. The use of exact equation formulations can be a necessity when one is interested in small quantities such as binding energy differences or in the effects of unitarity in the continuum.

Measurement of the $M1$ γ-deexcitation energies in the $A=4$ hypernuclei has yielded[8]

$$E_\gamma({}^4_\Lambda\mathrm{H})=B_\Lambda({}^4_\Lambda\mathrm{H})-B({}^4_\Lambda\mathrm{H}^*)\approx 1.04\pm 0.04 \text{ MeV} ,$$

and

$$E_\gamma({}^4_\Lambda\mathrm{He})=B_\Lambda({}^4_\Lambda\mathrm{He})-B({}^4_\Lambda\mathrm{He}^*)\approx 1.15\pm 0.04 \text{ MeV} .$$

Bound-state transitions of this type provide invaluable data on nuclear structure because our ability to treat bound systems correctly is much more highly developed than it is for the continuum. Furthermore, the experimental precision possible in such measurements is normally much higher than one can achieve in scattering experiments. The approximately 1 MeV excitation energy for each member of the isodoublet implies that the mechanism leading to this 0^+-1^+ splitting must be similar in each case. The question which we address is whether E_γ can be understood, at least qualitatively, in terms of the known properties of the free YN interaction. As we shall see, if one represents the free ΛN interaction in terms of one-channel ΛN central potentials, assuming that the Λ-Σ conversion is unaffected by the composite nature of the nuclear core to which the hyperon binds, then the resulting 0^+ (ground) state and 1^+ (excited) state are inversely ordered with respect to binding energy compared with the experimental data. The 1^+ state is more bound.[16] It is the "Σ-suppression" that results from the reduction in the strength of the ΛN-ΣN off-diagonal coupling, when the YN interaction involves a nucleon bound in a $T=\frac{1}{2}$, $J^\pi=\frac{1}{2}^+$ trinucleon core, that we investigate. We find this spin-isospin suppression of the Λ-Σ conversion due to the composite nature of the nuclear cores of ${}^4_\Lambda$He and ${}^4_\Lambda$H to be a significant factor in understanding the 0^+-1^+ binding energy difference.

In what follows, we discuss the hypernuclear four-body problem in terms of (i) four-body equations which are equivalent to those of Yakubovsky in their separable potential formulation[17–19] and (ii) YN separable potentials[20] whose parameters are determined from the low-energy scattering properties of the Nijmegen meson-theoretic potential of Ref. 7. Our exact four-body equations are derived directly from the Schrödinger equation[1] rather than making a reduction from the t-matrix formulation of Yakubovsky as was done in Refs. 18 and 19. The resulting two-variable, coupled, linear integral equations are solved directly without resort to separable expansion of the three-body kernels. In Sec. II we discuss the two-body separable potential model that we employ along with the low-energy scattering parameters which define the potential model. Expressions for the scattering length and effective range of the ΛN-ΣN potential are derived in an Appendix. In Sec. III we outline the $J^\pi=1^+$ ($A=4$) bound-state equations for spin-dependent, rank-one separable potentials in the case in which one baryon is distinguishable. We also recall the $J^\pi=0^+$ equations for comparison. Our numerical results for the $A=4$ 0^+-1^+ binding energy difference are compiled and discussed in Sec. IV. Our conclusions are briefly summarized in Sec. V.

II. THE POTENTIAL MODEL

Lack of precision YN scattering data has severely limited our ability to characterize that sector of the baryon-baryon interaction. Commendable efforts have been made to parametrize potentials using a combined analysis of all existing YN data plus the extensive NN data base in conjunction with various symmetry assumptions about the meson couplings in a one-boson-exchange (OBE) model description of the YN and NN interaction.[5–7,21–24] A Jülich group has also initiated a program to extend their meson-exchange model of the NN sector to provide a model of the YN interaction.[25] Such models allow one to attempt to circumvent the large uncertainties in the low-energy scattering parameters extracted directly from the sparse YN scattering data. Because we use separable potentials for convenience of calculation, we employ the ΛN-ΣN model of Stepien-Rudza and Wycech[20] in our four-body calculations. Their separable potential is based upon the main features of the OBE model of Ref. 7. We have also investigated the six ΛN-ΣN separable potential models of Toker, Gal, and Eisenberg,[26] which were constructed for the purpose of studying in detail the $K^-d\rightarrow YN\pi$ reaction. However, the ΣN interaction is so strong in these potentials (in some cases supporting by design a ΣN bound state in the absence of ΛN-ΣN coupling) that the 1^+ state, which suffers from strong suppression of the ΛN-ΣN coupling, was unbound in our $A=4$ model calculations in which we retain only the spatially symmetric nuclear core states.

In order to make clear the role of Λ-Σ conversion in understanding the qualitative features of the $A=4$ Λ hypernuclei and in our model calculations, let us first consider the model that results when one *assumes* that the YN force is independent of explicit ΛN-ΣN coupling effects. Restated, the assumption is that the YN force is unmodified from its free form even when the nucleon involved in the interaction is contained in a composite nuclear system. This model approach has been used extensively for s-shell hypernuclear studies. Such an ansatz leads to the following average spin-isospin combinations of the effective ΛN spin-singlet and spin-triplet potentials $\bar{V}^s_{\Lambda N}$ and $\bar{V}^t_{\Lambda N}$:[27,28]

$$ {}^4_\Lambda\mathrm{H}:\quad V_{YN}=\tfrac{1}{2}\bar{V}^s_{\Lambda N}+\tfrac{1}{2}\bar{V}^t_{\Lambda N}\,, \tag{1a}$$

$$ {}^4_\Lambda\mathrm{H}^*:\quad V_{YN}=\tfrac{1}{6}\bar{V}^s_{\Lambda N}+\tfrac{5}{6}\bar{V}^t_{\Lambda N}\,. \tag{1b}$$

In arriving at these average potentials in the ground and excited states, it is assumed that the spin-singlet force is stronger than the spin-triplet force so that the ground state is 0^+. (Any charge-symmetry-breaking differences between the Λp and Λn interactions are neglected so that ${}^4_\Lambda\mathrm{H}={}^4_\Lambda\mathrm{He}$.) The YN subscript denotes the fact that the potential describes the full *effective* ΛN-ΣN interaction. The implicit assumption is that the ΛN-ΣN coupling is identical in each spin state. That is, one has assumed that the 2×2 matrix potentials

$$ V^s_{YN}=\begin{bmatrix} V^s_{\Lambda N} & V^s_{XN} \\ V^s_{XN} & V^s_{\Sigma N}\end{bmatrix} \tag{2a}$$

and

$$ V^t_{YN}=\begin{bmatrix} V^t_{\Lambda N} & V^t_{XN} \\ V^t_{XN} & V^t_{\Sigma N}\end{bmatrix} \tag{2b}$$

can be represented by unique, average potentials $\bar{V}^s_{\Lambda N}$ and $\bar{V}^t_{\Lambda N}$ independently of the spin and isospin of the hypernuclear state in which the Λ is embedded. Such is not necessarily the case. The free space ΛN-ΣN potentials do have the form given in Eq. (2). However, for the $A=4$, $J^\pi=0^+$ state built on a $T=\frac{1}{2}$, $J^\pi=\frac{1}{2}^+$ trinucleon core with spatial symmetry, the YN interactions are of the form

$$ V^s_{YN}(0^+)=\begin{bmatrix} V^s_{\Lambda N} & -\tfrac{1}{3}V^s_{XN} \\ -\tfrac{1}{3}V^s_{XN} & V^s_{\Sigma N}\end{bmatrix} \tag{3a}$$

and

$$ V^t_{YN}(0^+)=\begin{bmatrix} V^t_{\Lambda N} & V^t_{XN} \\ V^t_{XN} & V^t_{\Sigma N}\end{bmatrix}, \tag{3b}$$

whereas for the $J^\pi=1^+$ state the YN interactions are of the form

$$V^s_{\rm YN}(1^+)=\begin{bmatrix} V^s_{\Lambda \rm N} & V^s_{\rm XN} \\ V^s_{\rm XN} & V^s_{\Sigma \rm N}\end{bmatrix} \tag{4a}$$

and

$$V^t_{\rm YN}(1^+)=\begin{bmatrix} V^t_{\Lambda \rm N} & \frac{1}{5}V^t_{\rm XN} \\ \frac{1}{5}V^t_{\rm XN} & V^t_{\Sigma \rm N}\end{bmatrix} . \tag{4b}$$

(For the discussion of the spin-isospin coupling see, for example, Refs. 29 and 30). The essence of the derivation is that in coupling a ($T=0$) Λ to a composite $T=\frac{1}{2}$ core (i.e., three $T=\frac{1}{2}$ nucleons coupled to $T=\frac{1}{2}$) one can couple the Λ and any one nucleon only to $T=\frac{1}{2}$, whereas in coupling a ($T=1$) Σ to a composite $T=\frac{1}{2}$ core one can couple the Σ and a nucleon to $T=\frac{1}{2}$ or $T=\frac{3}{2}$ because the other pair of nucleons can be coupled to $T=0$ or $T=1$. However, only the $T=\frac{1}{2}$ $\Sigma\times$N component has a nonzero overlap with the $T=\frac{1}{2}$ $\Lambda\times$N component, which leads to a reduction in the strength of the ΛN$\rightarrow\Sigma$N transition from the free space situation.

In neither case is the coupling of the Λ-Σ hyperon system to the nucleons in the composite isospin-$\frac{1}{2}$ trinucleon core the same as the coupling to a free isospin-$\frac{1}{2}$ nucleon. The *singlet* potential differs from the free interaction in the 0^+ *ground* state. The *triplet* potential differs from the free interaction in the 1^+ *excited* state. In each case the magnitude of the ΛN-ΣN coupling is reduced, weakening the YN interaction relative to its free strength.[4] Both 0^+ and 1^+ state binding energies are smaller than those which would result from calculations based entirely upon free ΛN interaction parameters. Furthermore, it is not possible to use the same effective, average interactions ($\bar{V}^s_{\Lambda \rm N}$ and $\bar{V}^t_{\Lambda \rm N}$) in any calculation of the 0^+ and 1^+ binding energies. The result of such an erroneous procedure would be to bind the 1^+ state more than the 0^+ ground state in an exact equation $A=4$ calculation, as we demonstrate in Sec. IV.

In order to expedite our calculations within the context of an exact four-body formalism, we represent both the NN and YN interactions in terms of simple separable potentials. We utilize the rank-one Yamaguchi form[31,32]

$$V_n(k,k')=-\frac{\lambda_n}{2\mu}g_n(k)g_n(k'),\quad n=s,t\ . \tag{5}$$

The form factors are

$$g_n(k)=(k^2+\beta_n^2)^{-1}\ , \tag{6}$$

if there is no tensor component, and

$$g_t(k)=g_c(k)+\frac{S_{ij}}{\sqrt{8}}g_T(k)\ , \tag{7a}$$

$$g_c(k)=(k^2+\beta_c^2)^{-1}\ , \tag{7b}$$

$$g_T(k)=\xi_T k^2(k^2+\beta_T^2)^{-2}\ , \tag{7c}$$

$$S_{ij}=3\sigma_i\cdot\hat{\mathbf{k}}\sigma_j\cdot\hat{\mathbf{k}}-\sigma_i\cdot\sigma_j\ , \tag{7d}$$

when the triplet potential has a tensor force. The quantity μ is the appropriate two-body reduced mass: $m_i m_j/(m_i+m_j)$.

In our previous ground-state investigations,[1-4] we restricted our consideration to rank-one effective YN potentials for which the potential parameters were chosen to describe the low-energy free ΛN scattering data. For the singlet potential this was justified on the basis that the ΛN-ΣN coupling in that channel is very weak,[33,34] so that $V^s_{\rm XN}=0$ was a good approximation; see also Refs. 3–7. Although the singlet transition Λp$\rightarrow\Sigma$n is suppressed, $V^s_{\rm XN}$ is not necessarily small. Therefore, in this investigation we (1) utilize for the free space YN interactions the rank-two potentials of Ref. 20; (2) modify the off-diagonal coupling terms as indicated in Eq. (3) for the ground state ($V^s_{\rm XN}$ is replaced by $-\frac{1}{3}$ $V^s_{\rm XN}$) and in Eq. (4) for the excited state ($V^t_{\rm XN}$ is replaced by $\frac{1}{5}V^t_{\rm XN}$); and (3) generate effective rank-one potentials which yield the same scattering length (a) and effective range (r) as the corresponding rank-two potentials (see the Appendix for details). The result is a tractable model which provides a reasonable qualitative description of the spin-isospin suppression (compared with the free space interaction) that occurs in our model of the ground state and the excited state of the $A=4$ Λ-hypernuclear isodoublet.

Our YN potential parameters, which come from the model of Ref. 20, are listed in Table I. The first five lines define the coupled-channel potentials which yield the ΛN scattering lengths and effective ranges given. The last two lines contain the strengths and ranges of the equivalent rank-one potentials which yield the same

TABLE I. The YN potential parameters with corresponding scattering lengths and effective ranges.

	$V^s_{\rm YN}$	$V^t_{\rm YN}$	$V^s_{\rm YN}(0^+)$	$V^t_{\rm YN}(1^+)$
λ_Λ (fm^{-3})	-0.7251	-0.5298	-0.7251	-0.5298
λ_X (fm^{-3})	-1.0970	-0.6777	0.3657	-0.1355
λ_Σ (fm^{-3})	0.8916	-0.9871	0.8916	-0.9871
β_Λ (fm^{-1})	1.18	1.6	1.18	1.6
β_Σ (fm^{-1})	1.44	2.0	1.44	2.0
a (fm)	-1.97	-1.95	-1.33	-0.95
r (fm)	3.90	2.43	4.68	3.50
λ (fm^{-3})	0.0952	0.3262	0.0739	0.1814
β (fm^{-1})	1.2011	1.7251	1.1828	1.6061

values for a and r. The first two columns describe the free space YN singlet and triplet interactions. The potentials, as modified for the 0^+ and 1^+ state calculations due to the coupling of the ΛN-ΣN system to the composite trinucleon cores, are described in the last two columns. Note that the strength definitions which we use differ from those of Ref. 20 (see the Appendix for details). Also, our calculations of a and r for the free interactions differ slightly from those reported in Ref. 20.

It is the rank-one effective potentials defined by the last two rows of Table I which were utilized in generating our numerical estimates of the $A=4$ binding energies. From our previous investigations involving solutions of exact four-body equations,[1,2,10] the following can be anticipated: the singlet potential (column 1) is stronger than the triplet potential (column 2) in the two-body sense: $|a_s|>|a_t|$ and $r_s>r_t$. But since $a_s \sim a_t$, $r_s>r_t$ implies that the reverse is true (the triplet potential is stronger than the singlet potential) for a true four-body calculation. In fact, the significant difference in size between r_s and r_t ensures that the triplet-potential dominated 1^+ state will be more bound than the 0^+ state in a true four-body calculation in which one uses average potentials fitted to the free ΛN low-energy scattering parameters and spin-independent equations such as those derived in Ref. 1. We will return to this point in Sec. IV.

III. FOUR-BODY EQUATIONS

The Schrödinger equation formalism that we use has been described in detail for the four-nucleon problem in Ref. 10 and for the $A=4$ hypernuclear ground state in Ref. 1. We employ the equations of Ref. 1 in this investigation for our 0^+ calculation. We can do so because we approximate the rank-two coupled channel ΛN-ΣN potentials by rank-one potentials as described in the previous section. We summarize those equations here to establish our notation and to permit a detailed comparison with the 1^+ equations below.

The Schrödinger wave function Ψ for a bound state of the Λ plus three nucleons can be expressed in the spin-independent limit as a sum of 18 amplitudes; 12 of these are of the symmetry type $\Psi(ij,k;l)$ which are symmetric under the interchange of i and j but have no special symmetry under any other index permutation. The remaining six amplitudes are of the type $\Psi(ij,kl)$, which are symmetric under the interchange of i and j or of k and l but which have no other special permutation symmetry. The former amplitudes are of [3,1] character and describe asymptotically the separation of one baryon from the subcluster defined by the other three; the latter amplitudes are of [2,2] character and describe asymptotically the separation of two subclusters, each containing two baryons. Because the three nucleons are indistinguishable, there are only three distinct $(ij,k;l)$ amplitudes and two distinct (ij,kl) amplitudes. The partial-wave projected, spin-independent amplitudes are conveniently expressed in terms of the two-body potential form factors, energy denominators having the form $\Sigma k_i^2/2m_i+B$, where B is the total binding energy of the four-body system, and five functions of two variables which describe the positions of the two spectator baryons relative to the interacting pair:

$$\Psi(12,3,\hat{4})=g(k)A(\mathbf{p},\mathbf{q})/\Delta_A \;, \tag{8a}$$

$$\Psi(12,\hat{3},4)=g(k)B(\mathbf{p},\mathbf{q})/\Delta_B \;, \tag{8b}$$

$$\Psi(1\hat{2},3,4)=\hat{g}(k)C(\mathbf{p},\mathbf{q})/\Delta_C \;, \tag{8c}$$

$$\Psi(12,3\hat{4})=g(k)D(\boldsymbol{\kappa},\mathbf{s})/\Delta_D \;, \tag{8d}$$

$$\Psi(1\hat{2},34)=\hat{g}(k)F(\boldsymbol{\kappa},\mathbf{s})/\Delta_F \;. \tag{8e}$$

The caret (^) over a particle index denotes the Λ; over a form factor it indicates that for the ΛN interaction. The momentum variables are the normal Jacobi relative coordinates defined in terms of the mass difference $\delta=(m_\Lambda/m-1)$:

$$\mathbf{k}=\tfrac{1}{2}(\mathbf{k}_1-\mathbf{k}_2)\;,\quad \mathbf{p}=\tfrac{1}{3}(\mathbf{k}_1+\mathbf{k}_2-2\mathbf{k}_3)\;,\quad \mathbf{q}=\frac{1+\delta}{4+\delta}(\mathbf{k}_1+\mathbf{k}_2+\mathbf{k}_3)-\frac{3}{4+\delta}\mathbf{k}_4\;, \tag{9a}$$

with

$$0=\mathbf{k}_1+\mathbf{k}_2+\mathbf{k}_3+\mathbf{k}_4\;,$$

or

$$\mathbf{k}=\tfrac{1}{2}(\mathbf{k}_1-\mathbf{k}_2)\;,\quad \boldsymbol{\kappa}=\frac{1+\delta}{2+\delta}\mathbf{k}_3-\frac{1}{2+\delta}\mathbf{k}_4\;,\quad \mathbf{s}=\frac{2+\delta}{4+\delta}(\mathbf{k}_1+\mathbf{k}_2)-\frac{2}{4+\delta}(\mathbf{k}_3+\mathbf{k}_4)\;. \tag{9b}$$

These momenta are depicted in Fig. 1. The energy denominators, expressed in terms of these momenta, are

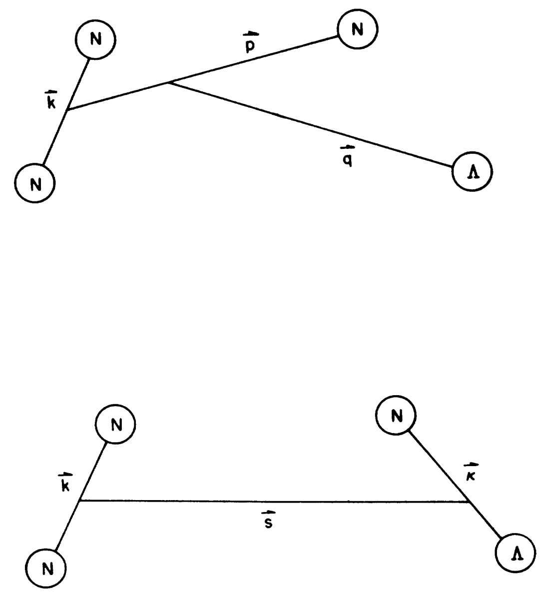

FIG. 1. Momentum coordinates as defined in Eq. (9) of the text.

$$\Delta_A = k^2 + 3p^2/4 + \frac{4+\delta}{6(1+\delta)} q^2 + mB \ , \tag{10a}$$

$$\Delta_B = k^2 + \frac{3+\delta}{4(1+\delta)} p^2 + \frac{4+\delta}{2(3+\delta)} q^2 + mB \ , \tag{10b}$$

$$\Delta_C = \frac{2+\delta}{2(1+\delta)} k^2 + \frac{3+\delta}{2(2+\delta)} p^2 + \frac{4+\delta}{2(3+\delta)} q^2 + mB \ , \tag{10c}$$

$$\Delta_D = k^2 + \frac{2+\delta}{2(1+\delta)} \kappa^2 + \frac{4+\delta}{4(2+\delta)} s^2 + mB \ , \tag{10d}$$

$$\Delta_F = \frac{2+\delta}{2(1+\delta)} k^2 + \kappa^2 + \frac{4+\delta}{4(2+\delta)} s^2 + mB \ . \tag{10e}$$

The unknown spectator functions $A \ldots F$ are determined by a system of five coupled, two-variable homogeneous integral equations: The equation for $A(\mathbf{p},\mathbf{q})$ is

$$A(\mathbf{p},\mathbf{q}) = \tau_A \left[z - 3p^2/4 - \frac{4+\delta}{6(1+\delta)} q^2 \right] \int d^3k \left[X \left[\mathbf{p},\mathbf{k}+\mathbf{q}/3; z - \frac{4+\delta}{6(1+\delta)} q^2 \right] B \left[\mathbf{q} + \frac{1+\delta}{3+\delta}\mathbf{k},\mathbf{k} \right] + X \left[\mathbf{p},\mathbf{k}-2\mathbf{q}/3; z - \frac{4+\delta}{6(1+\delta)} q^2 \right] D \left[\mathbf{q} - \frac{1+\delta}{2+\delta}\mathbf{k},\mathbf{k} \right] \right] , \tag{11}$$

where the propagator is

$$[m\tau_A(z)]^{-1} = \lambda^{-1} + \int d^3k \frac{g^2(k)}{z-k^2} \ , \tag{12}$$

and $z = -mB$. The kernel of the integral equation is, itself, the solution of an inhomogeneous integral equation, one which describes the underlying off-shell three-body problem

$$X(\mathbf{p},\mathbf{p}';z) = U(\mathbf{p},\mathbf{p}';z) + \int d^3p'' U(\mathbf{p},\mathbf{p}'';z) \times \tau_A(z-3p''^2/4) X(\mathbf{p}'',\mathbf{p}';z) \ ,$$

with a driving term defined in terms of the NN separable potential form factor as

$$U(\mathbf{p},\mathbf{p}';z) = -2m \frac{g(\mathbf{p}'+\mathbf{p}/2) g(\mathbf{p}+\mathbf{p}'/2)}{z - (p^2 + p'^2 + \mathbf{p}\cdot\mathbf{p}')} \ . \tag{13}$$

The equations for $B(\mathbf{p},\mathbf{q})$ and $C(\mathbf{p},\mathbf{q})$ are directly coupled:

$$\begin{aligned} B(\mathbf{p},\mathbf{q}) = \tau_B & \left[z - \frac{3+\delta}{4(1+\delta)} p^2 - \frac{4+\delta}{2(3+\delta)} q^2 \right] \\ & \times \int d^3k \left[X_{\mathrm{NN}} \left[\mathbf{p},\mathbf{k} + \frac{1+\delta}{3+\delta}\mathbf{q}; z - \frac{4+\delta}{2(3+\delta)} q^2 \right] A(\mathbf{q}+\mathbf{k}/3,\mathbf{k}) \right. \\ & + X_{\mathrm{NN}} \left[\mathbf{p},\mathbf{k} - \frac{2}{3+\delta}\mathbf{q}; z - \frac{4+\delta}{2(3+\delta)} q^2 \right] D \left[\mathbf{q} - \frac{1}{2+\delta}\mathbf{k},\mathbf{k} \right] \\ & + X_{\mathrm{N}\Lambda} \left[\mathbf{p},\mathbf{k} + \frac{1}{3+\delta}\mathbf{q}; z - \frac{4+\delta}{2(3+\delta)} q^2 \right] C \left[\mathbf{q} - \frac{1}{3+\delta}\mathbf{k},\mathbf{k} \right] \\ & \left. + X_{\mathrm{N}\Lambda} \left[\mathbf{p},\mathbf{k} - \frac{2+\delta}{3+\delta}\mathbf{q}; z - \frac{4+\delta}{2(3+\delta)} q^2 \right] F(\mathbf{q}-\mathbf{k}/2,\mathbf{k}) \right] , \end{aligned} \tag{14}$$

$$\begin{aligned} C(\mathbf{p},\mathbf{q}) = \tau_C & \left[z - \frac{3+\delta}{2(2+\delta)} p^2 - \frac{4+\delta}{2(3+\delta)} q^2 \right] \\ & \times \int d^3k \left[X_{\Lambda\mathrm{N}} \left[\mathbf{p},\mathbf{k} + \frac{1+\delta}{3+\delta}\mathbf{q}; z - \frac{4+\delta}{2(3+\delta)} q^2 \right] A(\mathbf{q}+\mathbf{k}/3,\mathbf{k}) \right. \\ & + X_{\Lambda\mathrm{N}} \left[\mathbf{p},\mathbf{k} - \frac{2}{3+\delta}\mathbf{q}; z - \frac{4+\delta}{2(3+\delta)} q^2 \right] D \left[\mathbf{q} - \frac{1}{2+\delta}\mathbf{k},\mathbf{k} \right] \\ & + X_{\Lambda\Lambda} \left[\mathbf{p},\mathbf{k} + \frac{1}{3+\delta}\mathbf{q}; z - \frac{4+\delta}{2(3+\delta)} q^2 \right] C \left[\mathbf{q} + \frac{1}{3+\delta}\mathbf{k},\mathbf{k} \right] \\ & \left. + X_{\Lambda\Lambda} \left[\mathbf{p},\mathbf{k} - \frac{2+\delta}{3+\delta}\mathbf{q}; z - \frac{4+\delta}{2(3+\delta)} q^2 \right] F(\mathbf{q}-\mathbf{k}/2,\mathbf{k}) \right] , \end{aligned} \tag{15}$$

where the propagators are

$$[m\tau_B(z)]^{-1}=\lambda^{-1}+\int d^3k\frac{g^2(k)}{z-k^2}\,, \tag{16}$$

and

$$[m\tau_C(z)]^{-1}=(\zeta\hat{\lambda})^{-1}+\int d^3k\frac{\hat{g}^2(k)}{z-\zeta k^2}\,, \tag{17}$$

with $\zeta=(2+\delta)/2(1+\delta)$. The kernels of this set of coupled equations are solutions of the coupled set of inhomogeneous equations

$$X_{\alpha\beta}(\mathbf{p},\mathbf{p}';z)=U_{\alpha\beta}(\mathbf{p},\mathbf{p}';z)+\sum_{\gamma\epsilon}\int d^3p''U_{\alpha\gamma}(\mathbf{p},\mathbf{p}'';z)\tau_{\gamma\epsilon}(z-\alpha_{\gamma\epsilon}p''^2)X_{\epsilon\beta}(\mathbf{p}'',\mathbf{p}';z)\,, \tag{18}$$

where $\alpha_{NN}=(3+\delta)/4(1+\delta)$ and $\alpha_{\Lambda\Lambda}=(3+\delta)/2(2+\delta)$. The Born matrix has the components

$$U_{NN}(\mathbf{p},\mathbf{p}';z)=0\,, \tag{19a}$$

$$U_{N\Lambda}(\mathbf{p},\mathbf{p}';z)=-2m\frac{g(\mathbf{p}'+\mathbf{p}/2)\hat{g}\left[\mathbf{p}+\frac{1+\delta}{2+\delta}\mathbf{p}'\right]}{z-\left[p^2+\frac{2+\delta}{2(1+\delta)}p'^2+pp'x\right]}\,, \tag{19b}$$

$$U_{\Lambda N}(\mathbf{p},\mathbf{p}';z)=-m\frac{\hat{g}\left[\mathbf{p}'+\frac{1+\delta}{2+\delta}\mathbf{p}\right]g(\mathbf{p}+\mathbf{p}'/2)}{z-\left[\frac{2+\delta}{2(1+\delta)}p^2+p'^2+pp'x\right]}\,, \tag{19c}$$

$$U_{\Lambda\Lambda}(\mathbf{p},\mathbf{p}';z)=-m\frac{\hat{g}\left[\mathbf{p}'+\frac{1}{2+\delta}\mathbf{p}\right]\hat{g}\left[\mathbf{p}+\frac{1}{2+\delta}\mathbf{p}'\right]}{z-\left[\frac{2+\delta}{2(1+\delta)}(p^2+p'^2)+\frac{1}{1+\delta}pp'x\right]}\,, \tag{19d}$$

where $g(p)$ is the form factor for the NN potential and $\hat{g}(p)$ is the form factor for the ΛN potential. Thus the channel coupling coefficient matrix is of the form

$$C_{\alpha\beta}=\begin{bmatrix}0 & 2\\ 1 & 1\end{bmatrix} \tag{20}$$

for the NNΛ cluster three-body problem in the spin-independent limit. The equations for the $D(\boldsymbol{\kappa},\mathbf{s})$ and $F(\boldsymbol{\kappa},\mathbf{s})$ functions are also coupled:

$$\begin{aligned}D(\boldsymbol{\kappa},\mathbf{s})=\tau_D&\left[z-\frac{2+\delta}{2(1+\delta)}\kappa^2-\frac{4+\delta}{4(2+\delta)}s^2\right]\\ &\times\int d^3k\left[Y_{NN}\left[\boldsymbol{\kappa},-\mathbf{k}+\frac{1+\delta}{2+\delta}\mathbf{s};z-\frac{4+\delta}{4(2+\delta)}s^2\right]A(-2\mathbf{k}/3+\mathbf{s},\mathbf{k})\right.\\ &\quad+Y_{NN}\left[\boldsymbol{\kappa},\mathbf{k}+\frac{1}{2+\delta}\mathbf{s};z-\frac{4+\delta}{4(2+\delta)}s^2\right]B\left[-\frac{2}{3+\delta}\mathbf{k}+\mathbf{s},\mathbf{k}\right]\\ &\quad\left.+2Y_{N\Lambda}\left[\boldsymbol{\kappa},-\mathbf{k}-\mathbf{s}/2;z-\frac{4+\delta}{4(2+\delta)}s^2\right]C\left[-\frac{2+\delta}{3+\delta}\mathbf{k}-\mathbf{s},\mathbf{k}\right]\right]\,,\end{aligned} \tag{21}$$

$$\begin{aligned}F(\boldsymbol{\kappa},\mathbf{s})=\tau_F\left[z-\kappa^2-\frac{4+\delta}{4(2+\delta)}s^2\right]\int d^3k&\left[Y_{\Lambda N}\left[\boldsymbol{\kappa},-\mathbf{k}-\frac{1+\delta}{2+\delta}\mathbf{s};z-\frac{4+\delta}{4(2+\delta)}s^2\right]A(-2\mathbf{k}/3-\mathbf{s},\mathbf{k})\right.\\ &+Y_{\Lambda N}\left[\boldsymbol{\kappa},\mathbf{k}+\frac{1}{2+\delta}\mathbf{s};z-\frac{4+\delta}{4(2+\delta)}s^2\right]B\left[-\frac{2}{3+\delta}\mathbf{k}-\mathbf{s},\mathbf{k}\right]\\ &\left.+2Y_{\Lambda\Lambda}\left[\boldsymbol{\kappa},-\mathbf{k}+\tfrac{1}{2}\mathbf{s};z-\frac{4+\delta}{4(2+\delta)}s^2\right]C\left[-\frac{2+\delta}{3+\delta}\mathbf{k}+\mathbf{s},\mathbf{k}\right]\right]\,,\end{aligned} \tag{22}$$

where the propagators are

$$[m\tau_D(z)]^{-1}=\lambda^{-1}+\int d^3k\frac{g^2(k)}{z-k^2}\,, \tag{23}$$

and

$$[m\tau_F(z)]^{-1}=(\zeta\hat{\lambda})^{-1}+\int d^3k\frac{\hat{g}^2(k)}{z-\zeta k^2}\,. \tag{24}$$

The kernels of this set of coupled equations are the solutions of

$$Y_{\alpha\beta}(\boldsymbol{\kappa},\mathbf{k};z)=W_{\alpha\beta}(\boldsymbol{\kappa},\mathbf{k};z)+\sum_{\gamma\epsilon}\int d^3k'W_{\alpha\gamma}(\boldsymbol{\kappa},\mathbf{k}';z)\times\tau_{\gamma\epsilon}(z-\alpha_{\gamma\epsilon}k'^2)\times Y_{\epsilon\beta}(\mathbf{k}',\mathbf{k};z)\,, \tag{25}$$

where we have defined $\alpha_{NN}=\zeta$ and $\alpha_{\Lambda\Lambda}=1$. The matrix of driving terms contains the elements

$$W_{NN}(\kappa,k;z)=0\ , \tag{26a}$$

$$W_{N\Lambda}(\kappa,k;z)=\frac{-mg(k)\hat{g}(\kappa)}{z-\left[k^2+\frac{2+\delta}{2(1+\delta)}\kappa^2\right]}\ , \tag{26b}$$

$$W_{\Lambda N}(\kappa,k;z)=\frac{-m\hat{g}(k)g(\kappa)}{z-\left[\frac{2+\delta}{2(1+\delta)}k^2+\kappa^2\right]}\ , \tag{26c}$$

$$W_{\Lambda\Lambda}(\kappa,k;z)=0\ . \tag{26d}$$

The channel coupling coefficient matrix here is of the form

$$\bar{C}_{\alpha\beta}=\begin{bmatrix}0 & 1\\ 1 & 0\end{bmatrix}\ ; \tag{27}$$

that is, the driving mechanism is the off-diagonal exchange of a Λ between the two-body subclusters.

Schematically, we summarize these results in a form that makes the extension to the spin-dependent 0^+ and 1^+ equations more transparent:

$$A=X(B+D) \tag{28}$$

with

$$X=U+U\tau X,\quad U=-Cg^2/D\ ; \tag{29}$$

and

$$\begin{bmatrix}B\\ C\end{bmatrix}=\begin{bmatrix}X_{NN} & X_{N\Lambda}\\ X_{\Lambda N} & X_{\Lambda\Lambda}\end{bmatrix}\begin{bmatrix}A+D\\ C+F\end{bmatrix} \tag{30}$$

with

$$X_{\alpha\beta}=U_{\alpha\beta}+\sum_{\gamma}U_{\alpha\gamma}\tau_{\gamma}X_{\gamma\beta},\quad U_{\alpha\beta}=C_{\alpha\beta}\frac{g_\alpha g_\beta}{D_{\alpha\beta}} \tag{31}$$

and

$$\begin{bmatrix}D\\ F\end{bmatrix}=\begin{bmatrix}Y_{NN} & Y_{N\Lambda}\\ Y_{\Lambda N} & Y_{\Lambda\Lambda}\end{bmatrix}\begin{bmatrix}A+B\\ 2C\end{bmatrix} \tag{32}$$

with

$$Y_{\alpha\beta}=W_{\alpha\beta}+\sum_{\gamma}W_{\alpha\gamma}\tau_{\gamma}Y_{\gamma\beta},\quad W_{\alpha\beta}=\bar{C}_{\alpha\beta}\frac{g_\alpha g_\beta}{D_{\alpha\beta}}\ . \tag{33}$$

Before leaving the spin-independent equations, we should point out that it is the A amplitude that corresponds to constructing ${}^4_\Lambda$H and ${}^4_\Lambda$He as a Λ bound to a trinucleon core. The other four amplitudes correspond to other means of building the $A=4$ Λ-hypernucleus. For the naive $\Lambda\times N^3$ picture to be valid, the $B\ldots F$ amplitudes must be negligible compared to the A amplitude.

A. 0^+ state

The 0^+ Λ-hypernuclear state has exactly the same spin structure as the 0^+ alpha particle. It corresponds to the substitution of a Λ with $s=\frac{1}{2}$ for a neutron or a proton in ^{4}He. Thus, it is not surprising that the number of spectator functions in our model doubles from five to ten when the model is extended to include spin-dependent interactions. Indeed, the structure of the equations is identical to that of the spin-independent equations above. All quantities develop spin indices $(i,j=s,t)$ which indicate whether the interacting pair is in a spin-singlet or a spin-triplet state.

Schematically, the resulting set of equations can be written as

$$\begin{bmatrix}A_t\\ A_s\end{bmatrix}=\begin{bmatrix}X^{tt} & X^{ts}\\ X^{st} & X^{ss}\end{bmatrix}\begin{bmatrix}B_t+D_t\\ B_s+D_s\end{bmatrix} \tag{34}$$

with

$$X^{ij}=U^{ij}+\sum_k U^{ik}\tau^k X^{kj},\quad U^{ij}=-C^{ij}\frac{g_i g_j}{D} \tag{35}$$

$$C^{ij}=\begin{bmatrix}\frac{1}{2} & -\frac{3}{2}\\ -\frac{3}{2} & \frac{1}{2}\end{bmatrix}\ , \tag{36}$$

and

$$\begin{bmatrix}B_t\\ B_s\\ C_t\\ C_s\end{bmatrix}=\begin{bmatrix}X^{tt}_{NN} & X^{ts}_{NN} & X^{tt}_{N\Lambda} & X^{ts}_{N\Lambda}\\ X^{st}_{NN} & X^{ss}_{NN} & X^{st}_{N\Lambda} & X^{ss}_{N\Lambda}\\ X^{tt}_{\Lambda N} & X^{ts}_{\Lambda N} & X^{tt}_{\Lambda\Lambda} & X^{ts}_{\Lambda\Lambda}\\ X^{st}_{\Lambda N} & X^{ss}_{\Lambda N} & X^{st}_{\Lambda\Lambda} & X^{ss}_{\Lambda\Lambda}\end{bmatrix}\begin{bmatrix}A_t+D_t\\ A_s+D_s\\ C_t+F_t\\ C_s+F_s\end{bmatrix} \tag{37}$$

with

$$X^{ij}_{\alpha\beta}=U^{ij}_{\alpha\beta}+\sum_{k,\gamma}U^{ik}_{\alpha\gamma}\tau^k_\gamma X^{kj}_{\gamma\beta},\quad U^{ij}_{\alpha\beta}=-C^{ij}_{\alpha\beta}\frac{g_{i\alpha}g_{j\beta}}{D_{\alpha\beta}} \tag{38}$$

$$C^{ij}_{\alpha\beta}=\begin{bmatrix}0 & 0 & \frac{1}{2} & -\frac{3}{2}\\ 0 & 0 & -\frac{3}{2} & \frac{1}{2}\\ \frac{1}{4} & -\frac{3}{4} & \frac{1}{4} & -\frac{3}{4}\\ -\frac{3}{4} & \frac{1}{4} & -\frac{3}{4} & \frac{1}{4}\end{bmatrix}\ , \tag{39}$$

and

$$\begin{bmatrix}D_t\\ D_s\\ F_t\\ F_s\end{bmatrix}=\begin{bmatrix}Y^{tt}_{NN} & 0 & Y^{tt}_{N\Lambda} & 0\\ 0 & Y^{ss}_{NN} & 0 & Y^{ss}_{N\Lambda}\\ Y^{tt}_{\Lambda N} & 0 & Y^{tt}_{\Lambda\Lambda} & 0\\ 0 & Y^{ss}_{\Lambda N} & 0 & Y^{ss}_{\Lambda\Lambda}\end{bmatrix}\begin{bmatrix}A_t+B_t\\ A_s+B_s\\ 2C_t\\ 2C_s\end{bmatrix} \tag{40}$$

with

$$Y^{ij}_{\alpha\beta}=W^{ij}_{\alpha\beta}+\sum_{k,\gamma}W^{ik}_{\alpha\gamma}\tau^k_\gamma Y^{kj}_{\gamma\beta},\quad W^{ij}_{\alpha\beta}=-\bar{C}^{ij}_{\alpha\beta}\frac{g_{i\alpha}g_{j\beta}}{D_{\alpha\beta}}\delta_{ij} \tag{41}$$

$$\bar{C}^{ij}_{\alpha\beta}=\begin{bmatrix}0 & 0 & 1 & 0\\ 0 & 0 & 0 & 1\\ 1 & 0 & 0 & 0\\ 0 & 1 & 0 & 0\end{bmatrix}\ . \tag{42}$$

The i,j are spin indices (s,t), so that there are ten spectator functions and corresponding amplitudes instead of five in the spin-independent limit. The coupling coefficient matrices contain the essential differences between these spin-dependent equations and the spin-independent equations above. Both C^{ij} and $C^{ij}_{\alpha\beta}$ are direct products of $C=2$ and $C_{\alpha\beta}$ of Eq. (20) with the spin-isospin coupling matrix

$$\begin{bmatrix} \frac{1}{4} & -\frac{3}{4} \\ -\frac{3}{4} & \frac{1}{4} \end{bmatrix} .$$

The sign differences in the off-diagonal elements between these matrices and those of Ref. 1 are due to the use here of the more standard convention in defining the spin-isospin coupling coefficients. However, there was an error in the derivation of the $C^{ij}_{\alpha\beta}$ appearing in Ref. 1; the form of $C^{ij}_{\alpha\beta}$ given here in Eq. (39) is the correct one. Because the spin dependence of the ΛN interaction is weak, the numerical significance of the error proved small; both sets of equations reduce to the proper five in the spin-independent limit. The $\bar{C}^{ij}_{\alpha\beta}$ is a direct product of the channel coupling matrix $\bar{C}_{\alpha\beta}$ and the spin-isospin coupling unit matrix. The spin-isospin coupling is diagonal because the spins of the two-body subclusters (s and t) must combine to an overall spin 0 for the 0^+ state. One can anticipate that this coupling cannot be diagonal for the 1^+ state.

B. 1^+ state

The spin structure of the 1^+ Λ-hypernuclear state differs from the familiar alphalike structure of the 0^+ state in that $S=\frac{3}{2}$ three-baryon subclusters appear. Although one might consider it tempting to neglect the $S=\frac{3}{2}$ trinucleon excited states that are found in the A type of spectator function, the $S=\frac{3}{2}$ hypertriton excited state lies very close in energy to the barely bound $S=\frac{1}{2}$ hypertriton and cannot be neglected in the B and C type amplitudes. Thus, we retain these additional functions throughout. The appearance of $S=\frac{3}{2}$ subclusters means that we must distinguish these quartet functions with spin-triplet interacting pairs from the doublet functions with spin-triplet interacting pairs as exist in the 0^+ state equations. We denote the doublet functions with a subscript "d" and the quartet functions with a subscript "q." Thus, there are 15 spectator functions $A_i \ldots F_i$ with index i running over q,d,s compared with 10 ($i=s,t$) in the 0^+ case.

The full derivation of the 1^+ equations is tedious and will be presented elsewhere.[35] However, the equations readily follow using the procedures of Refs. 1 and 36. In analogy with Eq. (8), the following 15 terms are needed to construct the wave function Ψ by means of standard permutation operations:[37]

$$\begin{aligned}
&g_t(12)A_q(3,4)/\Delta_A\ [\chi^s(\overline{123})\times\tfrac{1}{2}]^{[1]}\eta'(\hat{12},3)\ ,\\
&g_t(12)A_d(3,4)/\Delta_A\ [\chi''(\overline{12},3)\times\tfrac{1}{2}]^{[1]}\eta'(\hat{12},3)\ ,\\
&g_s(12)A_s(3,4)/\Delta_A\ [\chi'(\hat{12},3)\times\tfrac{1}{2}]^{[1]}\eta''(\overline{12},3)\ ,\\
&g_t(12)B_q(4,3)/\Delta_B\ [\chi^s(\overline{124})\times\tfrac{1}{2}]^{[1]}\eta'(\hat{12},3)\ ,\\
&g_t(12)B_d(4,3)/\Delta_B\ [\chi''(\overline{12},4)\times\tfrac{1}{2}]^{[1]}\eta'(\hat{12},3)\ ,\\
&g_s(12)B_s(4,3)/\Delta_B\ [\chi'(\hat{12},4)\times\tfrac{1}{2}]^{[1]}\eta''(\overline{12},3)\ ,\\
&\hat{g}_t(34)[C_q(1,2)+C_q(2,1)]/\Delta_C\ \{\sqrt{2/3}[\chi^1(34)\times\chi^0(12)]^{[1]}\eta''(\overline{12},3)+\sqrt{1/3}[\chi^1(34)\times\chi^1(12)]^{[1]}\eta'(\hat{12},3)\}\ ,\\
&\hat{g}_t(34)[C_d(1,2)+C_d(2,1)]/\Delta_C\ \{-\sqrt{1/3}[\chi^1(34)\times\chi^0(12)]^{[1]}\eta''(\overline{12},3)+\sqrt{2/3}[\chi^1(34)\times\chi^1(12)]^{[1]}\eta'(\hat{12},3)\}\ ,\\
&\hat{g}_s(34)\{C_s(1,2)[\chi'(\hat{34},1)\times\tfrac{1}{2}]^{[1]}+C_s(2,1)[\chi'(\hat{34},2)\times\tfrac{1}{2}]^{[1]}\}/\Delta_C\ \eta'(\hat{12},3)\ ,\\
&g_t(12)D_q(34,s)/\Delta_D\ [\chi^1(12)\times\chi^1(34)]^{[1]}\eta'(\hat{12},3)\ ,\\
&g_t(12)D_d(34,s)/\Delta_D\ [\chi^1(12)\times\chi^0(34)]^{[1]}\eta'(\hat{12},3)\ ,\\
&g_s(12)D_s(34,s)/\Delta_D\ [\chi^0(12)\times\chi^1(34)]^{[1]}\eta''(\overline{12},3)\ ,\\
&\hat{g}_t(34)F_q(12,-s)/\Delta_F\ [\chi^1(34)\times\chi^1(12)]^{[1]}\eta'(\hat{12},3)\ ,\\
&\hat{g}_t(34)F_d(12,-s)/\Delta_F\ [\chi^1(34)\times\chi^0(12)]^{[1]}\eta''(\overline{12},3)\ ,\\
&\hat{g}_s(34)F_s(12,-s)/\Delta_F\ [\chi^0(34)\times\chi^1(12)]^{[1]}\eta'(\hat{12},3)\ .
\end{aligned} \tag{43}$$

In Eq. (43), χ' and χ'' (η' and η'') are the doublet spin (isospin) functions of three spin-$\frac{1}{2}$ fermions having mixed symmetry; in χ', particles 1 and 2 are coupled to spin 0 [the caret (^) denotes that they are antisymmetric under the permutation operation] while in χ'', particles 1 and 2 are coupled to spin-1 (the overbar denotes that they are symmetric under the permutation operation). Note the isospin asymmetry between the D_i and F_i amplitudes for

$i=q,d$. The combinations of C_i are those needed to provide correct permutation symmetries assuming even partial waves, as we do. In our numerical solution of the resulting equations, we truncate the partial wave expansion at $l=0$, which is the reason for this choice.

We find it convenient to construct linear combinations of the [3,1] spectator functions in writing the coupled integral equations. In particular, we define

$$A^{(+)}=(2A_d+\sqrt{2}A_q)/3\ ,$$
$$B^{(+)}=(2B_d+\sqrt{2}B_q)/3\ , \qquad (44)$$
$$C^{(+)}=(2C_d+\sqrt{2}C_q)/3\ ,$$

and

$$A^{(-)}=-\sqrt{1/3}(A_d-\sqrt{2}A_q)\ ,$$
$$B^{(-)}=+\sqrt{1/3}(B_d-\sqrt{2}B_q)\ , \qquad (45)$$
$$C^{(-)}=-\sqrt{1/3}(C_d-\sqrt{2}C_q)\ .$$

The B and C amplitudes will appear only in these combinations in the equations below. The orthogonality of the $S=\frac{3}{2}$ and $S=\frac{1}{2}$ trinucleon states and the fact that they are the core states of only the A amplitudes leads to a separation of the (A_d,A_s) coupled equations from the A_q equation. A similar decoupling of the D_q and F_q equations from the (D_d,D_s) and (F_d,F_s) coupled equations also occurs. Thus, we are led to five sets of integral equations to be solved as compared with the three sets in the 0^+ case.

The coupled integral equations which determine the four-body spectator functions $A_i \ldots F_i$ are, schematically,

$$\sqrt{2}A_q=\bar{X}^{tt}[(B^{(+)}+\sqrt{2/3}D_q)+2\sqrt{1/3}(B^{(-)}+D_d)]\ . \qquad (46)$$

$$\begin{bmatrix} A_d \\ A_s \end{bmatrix}=\begin{bmatrix} X^{tt} & X^{ts} \\ X^{st} & X^{ss} \end{bmatrix}\begin{bmatrix} (B^{(+)}+\sqrt{2/3}D_q)-\sqrt{1/3}(B^{(-)}+D_d) \\ B_s+D_s \end{bmatrix}, \qquad (47)$$

$$\begin{bmatrix} B^{(+)} \\ B^{(-)} \\ B_s \\ C^{(+)} \\ C^{(-)} \\ C_s \end{bmatrix}=\begin{bmatrix} X_{NN}^{++} & X_{NN}^{+-} & X_{NN}^{+s} & X_{N\Lambda}^{++} & X_{N\Lambda}^{+-} & X_{N\Lambda}^{+s} \\ X_{NN}^{-+} & X_{NN}^{--} & X_{NN}^{-s} & X_{N\Lambda}^{-+} & X_{N\Lambda}^{--} & X_{N\Lambda}^{-s} \\ X_{NN}^{s+} & X_{NN}^{s-} & X_{NN}^{ss} & X_{N\Lambda}^{s+} & X_{N\Lambda}^{s-} & X_{N\Lambda}^{ss} \\ X_{\Lambda N}^{++} & X_{\Lambda N}^{+-} & X_{\Lambda N}^{+s} & X_{\Lambda\Lambda}^{++} & X_{\Lambda\Lambda}^{+-} & X_{\Lambda\Lambda}^{+s} \\ X_{\Lambda N}^{-+} & X_{\Lambda N}^{--} & X_{\Lambda N}^{-s} & X_{\Lambda\Lambda}^{-+} & X_{\Lambda\Lambda}^{--} & X_{\Lambda\Lambda}^{-s} \\ X_{\Lambda N}^{s+} & X_{\Lambda N}^{s-} & X_{\Lambda N}^{ss} & X_{\Lambda\Lambda}^{s+} & X_{\Lambda\Lambda}^{s-} & X_{\Lambda\Lambda}^{ss} \end{bmatrix}\begin{bmatrix} A^{(+)}+\sqrt{2/3}D_q \\ A^{(-)}+D_q \\ A_s+D_s \\ C^{(+)}+\sqrt{2/3}F_q \\ C^{(-)}+D_d \\ C_s+D_s \end{bmatrix}, \qquad (48)$$

$$\begin{bmatrix} \sqrt{2/3}D_q \\ \sqrt{2/3}F_q \end{bmatrix}=\begin{bmatrix} \bar{Y}_{NN}^{tt} & \bar{Y}_{N\Lambda}^{tt} \\ \bar{Y}_{\Lambda N}^{tt} & \bar{Y}_{\Lambda\Lambda}^{tt} \end{bmatrix}\begin{bmatrix} A^{(+)}+B^{(+)} \\ 2C^{(+)} \end{bmatrix}, \qquad (49)$$

and

$$\begin{bmatrix} D_d \\ D_s \\ F_d \\ F_s \end{bmatrix}=\begin{bmatrix} Y_{NN}^{tt} & 0 & 0 & Y_{N\Lambda}^{ts} \\ 0 & Y_{NN}^{ss} & Y_{N\Lambda}^{st} & 0 \\ 0 & Y_{\Lambda N}^{ts} & Y_{\Lambda\Lambda}^{tt} & 0 \\ Y_{\Lambda N}^{st} & 0 & 0 & Y_{\Lambda\Lambda}^{ss} \end{bmatrix}\begin{bmatrix} A^{(-)}+B^{(-)} \\ A_s+B_s \\ 2C^{(-)} \\ 2C_s \end{bmatrix}. \qquad (50)$$

These equations have the same generic structure as do the analogous equations for the 0^+ state. The integral equations defining the kernels of Eqs. (46)–(50) are also of the same form as those obtained for the 0^+ state. The kernel equations are

$$\bar{X}^{tt}=U^{tt}+U^{tt}\tau^tX^{tt},\quad U^{tt}=-C^{tt}\frac{g_tg_t}{D} \qquad (51)$$

$$X^{ij}=U^{ij}+\sum_k U^{ik}\tau^kX^{kj},\quad U^{ij}=-C^{ij}\frac{g_ig_j}{D} \qquad (52)$$

$$X_{\alpha\beta}^{ij}=U_{\alpha\beta}^{ij}+\sum_{k,\gamma} U_{\alpha\gamma}^{ik}\tau_\gamma^kX_{\gamma\beta}^{kj},\quad U_{\alpha\beta}^{ij}=-C_{\alpha\beta}^{ij}\frac{g_{i\alpha}g_{j\beta}}{D_{\alpha\beta}} \qquad (53)$$

$$Y_{\alpha\beta}^{tt}=W_{\alpha\beta}^{tt}+\sum_\gamma W_{\alpha\gamma}^{tt}\tau_\gamma^tY_{\gamma\beta}^{tt},\quad W_{\alpha\beta}^{tt}=-\bar{C}_{\alpha\beta}^{tt}\frac{g_{t\alpha}g_{t\beta}}{D_{\alpha\beta}} \qquad (54)$$

$$Y_{\alpha\beta}^{ij}=W_{\alpha\beta}^{ij}+\sum_{k,\gamma} W_{\alpha\gamma}^{ik}\tau_\gamma^kY_{\gamma\beta}^{kj},\quad W_{\alpha\beta}^{ij}=-\bar{C}_{\alpha\beta}^{ij}\frac{g_{i\alpha}g_{j\beta}}{D_{\alpha\beta}}\ . \qquad (55)$$

The spin indices (i,j) in Eq. (53) run over $(+,-,s)$ where $(+,-)$ correspond to a spin-triplet form factor and τ propagator. The spin-isospin coupling matrices for the 1^+ state are

$$C^{tt}=-1\ , \tag{56}$$

$$C^{ij}=\begin{bmatrix} \frac{1}{2} & -\frac{3}{2} \\ -\frac{3}{2} & \frac{1}{2} \end{bmatrix}, \tag{57}$$

$$C^{ij}_{\alpha\beta}=\begin{bmatrix} 0 & 0 & 0 & 0 & -1 & -\sqrt{1/3} \\ 0 & 0 & 0 & \sqrt{3}/2 & \sqrt{3}/2 & -\frac{1}{2} \\ 0 & 0 & 0 & \frac{3}{2} & \frac{1}{2} & \sqrt{3}/2 \\ 0 & 1/2\sqrt{3} & \frac{1}{2} & 0 & \frac{1}{2} & 1/2\sqrt{3} \\ -\frac{3}{4} & \sqrt{3}/4 & \frac{1}{4} & \frac{3}{4} & \frac{1}{4} & \sqrt{3}/4 \\ -\sqrt{3}/4 & -\frac{1}{4} & \sqrt{3}/4 & \sqrt{3}/4 & \sqrt{3}/4 & -\frac{1}{4} \end{bmatrix}, \tag{58}$$

$$\bar{C}^{tt}_{\alpha\beta}=\begin{bmatrix} 0 & -1 \\ -1 & 0 \end{bmatrix}, \tag{59}$$

and

$$\bar{C}^{ij}_{\alpha\beta}=\begin{bmatrix} 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \end{bmatrix}. \tag{60}$$

There is no real analog of this Λ-hypernuclear 1^+ state to be found in the four-nucleon system. Four nucleons in an s state having spin of 1 is not allowed by the Pauli principle. However, the 1^+ equations do reduce to the original set (that is, there exist in the spin-independent limit, three decoupled sets of five equations of which only one set has a nontrivial solution) as do the 0^+ equations in the spin-independent limit.[35] Thus, it is the spin dependence of the ΛN interaction that leads to the splitting of the states observed experimentally. However, we argue in the next section that this spin dependence is much more subtle than heretofore has been understood.

IV. NUMERICAL RESULTS

We solve the exact integral equations outlined in the previous section without resort to expansion of the kernels. The resulting solutions possess the characteristics of true few-body calculations:[9,10] for an attractive potential with a negative scattering length (no bound state), $|a|>|a'|$ implies that V is more attractive than V' in two-body, three-body, and four-body calculations; in contrast $r>r'$ implies that V is more attractive than V' in a two-body calculation, but *less* attractive in both three-body and four-body calculations. Even though this is an oversimplified picture, it does help one to understand qualitatively the results described here for E_γ, the 0^+-1^+ binding energy difference, in terms of the scattering lengths and effective ranges of the various potential models defined in Table I.

In order to make clear the magnitude of the effect of suppression of the Λ-Σ conversion in our true four-body calculations, we consider first the model based upon the free YN interactions defined in columns one and two. The singlet potential is more attractive than the triplet potential in the two-body sense outlined above: $|a_s|>|a_t|$ and $r_s>r_t$. However, the significant difference in the size of r_s compared to r_t means that the calculated values of $E_\gamma=B({}^4_\Lambda\text{H})-B({}^4_\Lambda\text{H}^*)$ is determined principally by the effective range difference rather than the difference in the scattering lengths. Because we have $r_t<r_s$ (which implies that the triplet interaction is stronger than the singlet interaction in a true four-body calculation) and because the triplet interaction dominates the YN interaction for the 1^+ state, the 1^+ state will be more bound than the 0^+ state in this model. These potentials which describe free space ΛN scattering will bind the 1^+ state more than the 0^+ state in an exact equation four-body calculation. Indeed, we find that $E_\gamma\approx-1.0$ MeV for this case, which has the wrong sign.

It is also of interest to note that had we attempted to calculate E_γ in a zero-range type, mean field model (e.g., a shell model calculation based only upon the a_s and a_t scattering length information), we would have found a value of less than 0.1 MeV for E_γ because $a_s\approx a_t$. (However, the sign would be correct.) Alternatively, if we utilized the full potentials in a folding model approach of the Dalitz-Downs type,[38] then a larger value of E_γ would be obtained with the correct sign. (Recall that $|a_s|>|a_t|$ and $r_s>r_t$ imply that V^s is more attractive than V^t in any effective two-body calculation.) However, even *if* such an approximate calculation were to yield a correct value of E_γ, the physics would be *wrong*. [This situation is similar in spirit to that which resulted when incorrect two-body model calculations managed to reproduce the ^{3}He(γ,d)p cross section by distorting the deuteron in the final state, when in fact the cross section enhancement is due to the transfer of strength from the $T=\frac{1}{2}$ three-body channel through the off-shell $n+d^*\to n+d$ rescattering, a process which is entirely omitted from the approximate two-body formalism.[13-15]] Furthermore, if such a model were extended to the $A=5$ system and used to estimate the Λseparation energy of ${}^5_\Lambda$He, it would severely overbind that hypernucleus.[27–29]

To obtain a correct picture, one should take into account the spin-isospin suppression of the off-diagonal potentials outlined in Sec. II. (Σ suppression in ${}^5_\Lambda$He has been considered as a possible explanation of the anomalously small $A=5$ Λ-separation energy for some time.[39,40]) The modified singlet potential (corresponding to $-\frac{1}{3}V^s_{XN}$) of column 3 in Table I is combined with the free triplet potential of column 2 for the 0^+ calculation. Similarly, the modified triplet potential (corresponding to $\frac{1}{5}V^t_{XN}$) of the last column is combined with the free singlet potential of the first column for the 1^+ calculation. These are the potential combinations given in Eqs. (3) and (4) in Sec. II. It should be clear that the modified potentials of columns 3 and 4 are weaker than the free space potentials from which they were generated. Thus, both the 0^+ and 1^+ $A=4$ bound-state energies are reduced compared to the values calculated using the free ΛN scattering potentials. Because the off-diagonal coupling is multiplied by a smaller factor in the triplet case ($\frac{1}{5}$) than in the singlet case ($-\frac{1}{3}$), the 1^+ eigenvalue is reduced more than the 0^+ eigenvalue. In the model calculations that we report here (see Table II), we find

$$\begin{aligned} E_\gamma &= B(0^+) - B(1^+) \\ &= 9.6 - 8.2 \\ &= 1.4 \text{ MeV} . \end{aligned}$$

Hence, the 0^+-1^+ binding energy difference in this true four-body model calculation is qualitatively correct. The value of E_γ has the correct sign and order of magnitude. In addition, the 0^+ ground-state binding energy yields the approximate Λ-separation energy. The ^{3}H binding energy in our model[41] is about 7.1 MeV, such that we obtain

$$\begin{aligned} B_\Lambda(A=4) &= B(0^+) - B({}^3\text{H}) \\ &\approx 2.5 \text{ MeV} , \end{aligned}$$

in reasonable agreement with the experimental values of $B_\Lambda({}^4\text{He})$ and $B_\Lambda({}^4_\Lambda\text{He})$ in view of our neglect of charge symmetry breaking in the YN interactions and the tensor nature of the spin-triplet force.[1]

It is well known that a tensor force is less effective in binding few-body nuclei and hypernuclei than is a central force (see, for example, Refs. 1, 9, 10, and 32). Therefore, it is quite possible that tensor force effects can also help lower the triplet potential dominated 1^+ binding energy below that of the 0^+ binding energy. [It was shown in Ref. 1 that use of a YN tensor force will lower the 0^+ binding by several tenths of an MeV, which would bring the value of $B_\Lambda(A=4)$ obtained above into better agreement with the data.] In such a model, the assumption of complete suppression of ΛN-ΣN coupling through the $T=\frac{3}{2}$ trinuclear core states made here could be relaxed, and an approximation of the suppression could be made by adding the large excitation energy of the $T=\frac{3}{2}$ trinucleon states (estimated[42] to be more that 70 MeV) to the Λ-Σ mass difference. That and solving the full set of equations including $T=\frac{3}{2}$ nuclear core states must be left to the future.

TABLE II. $A=4$ Λ-hypernuclear binding energies calculated with the YN potentials of Table I as indicated by the column numbers. The NN potential is the 7% deuteron D-state model of Ref. 32 in the truncated t-matrix approximation as discussed in Ref. 41. Binding energies are in MeV.

Potential combination	$B(0^+)$	$B(1^+)$
(1,2)	10.7	11.7
(2,3)	9.6	
(1,4)		8.2

V. CONCLUSION

We have examined the $A=4$ 0^+-1^+ binding energy difference in terms of a YN potential model which admits explicit ΛN-ΣN coupling and exact four-body integral equations. Assuming that the free YN interaction acts unmodified in the $A=4$ hypernuclear system leads to an incorrect (inverse) ordering of the 0^+ and 1^+ states. Assuming that the ΛN-ΣN coupling to the $T=\frac{3}{2}$ states of the trinucleon core states is completely suppressed leads to a 0^+-1^+ binding energy difference which is qualitatively correct for the potential model investigated. E_γ has the correct sign and is of the correct magnitude. The physics of the calculation is correct in that the binding energies reflect properly the properties of the YN force model resulting from true four-body calculations.

An exact equation, four-body calculation utilizing the full, coupled ΛN-ΣN potential including tensor components is required to test our hypothesis that coupling through the $T=\frac{3}{2}$ trinucleon core states is so strongly suppressed. Nonetheless, the numerical results presented here are indicative that Σ suppression through this mechanism is a significant factor in understanding s-shell Λ-hypernuclear binding energies.

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APPENDIX: THE ΛN-ΣN POTENTIAL

We assume that the coupled ΛN-ΣN potential is of the form

$$V_{YN} = \begin{bmatrix} V_{\Lambda\Lambda} & V_{\Lambda\Sigma} \\ V_{\Sigma N} & V_{\Sigma\Sigma} \end{bmatrix} \tag{A1}$$

where

$$V_{ij} = c_{ij} g_i(p) g_j(p) . \tag{A2}$$

The c_{ij} matrix is

$$c=\begin{bmatrix} \frac{-\lambda_\Lambda}{2\mu_\Lambda} & \frac{-\lambda_X}{2\mu_\Lambda} \\ \frac{-\lambda_X}{2\mu_\Sigma} & \frac{-\lambda_\Sigma}{2\mu_\Sigma} \end{bmatrix} \quad \text{(A3)}$$

so that the interaction has identical off-diagonal strengths λ_X.

The ΛN scattering amplitude can be expressed as

$$f(k)=-2\mu_\Lambda(2\pi^2)t_{\Lambda N}$$
$$=\frac{g_\Lambda^2(k)}{\det(k)}[c_{\Lambda N}-(c_{\Lambda N}c_{\Sigma N}-c_{XN}^2)I_\Sigma(k)] , \quad \text{(A4)}$$

where the denominator is the determinant

$$\det(k)=(1-c_{\Lambda N}I_\Lambda)(1-c_{\Sigma N}I_\Sigma)+c_{XN}^2 I_\Sigma I_\Lambda . \quad \text{(A5)}$$

The Green's function integrals I_Λ and I_Σ are the usual

$$I_\Lambda=2\mu_\Lambda \int \frac{d^3p\, g_\Lambda^2(p)}{k^2-p^2+i\epsilon} \quad \text{(A6)}$$

and

$$I_\Sigma=2\mu_\Sigma \int \frac{d^3p\, g_\Sigma^2(p)}{\bar{k}^2-p^2}, \quad \bar{k}^2=k^2\mu_\Sigma/\mu_\Lambda-2\mu_\Sigma(m_\Sigma-m_\Lambda) . \quad \text{(A7)}$$

(We assume that one is only interested in the ΛN scattering below the Σ threshold, so that there is no pole in I_Σ in the region of interest.)

In the effective range limit ($k \to 0$), we obtain

$$a_{\Lambda N}^{-1}=\frac{\beta_\Lambda}{2}-\frac{\beta_\Lambda^4}{2\pi^2\lambda_\Lambda}+\frac{\lambda_X^2}{\lambda_\Lambda}\frac{\beta_\Lambda^4}{2\beta_\Sigma(\beta_\Sigma+\Delta)^2}\left[1-(1-\lambda_X^2/\lambda_\Lambda\lambda_\Sigma)\frac{\pi^2\lambda_\Sigma}{\beta_\Sigma(\beta_\Sigma+\Delta)^2}\right]^{-1} ,$$

$$r_{\Lambda N}=\frac{1}{\beta_\Lambda}+\frac{2\beta_\Lambda^2}{\pi^2\lambda_\Lambda}-\left\{\frac{2}{\beta_\Lambda^2}+\frac{\mu_\Sigma/\mu_\Lambda}{\Delta(\beta_\Sigma+\Delta)}\left[1-\left[1-\frac{\lambda_X^2}{\lambda_\Lambda\lambda_\Sigma}\right]\frac{\pi^2\lambda_\Sigma}{\beta_\Sigma(\beta_\Sigma+\Delta)^2}\right]^{-1}\right\}$$
$$\times\left[\frac{\lambda_X}{\lambda_\Lambda}\right]^2\frac{\beta_\Lambda^4}{\beta_\Sigma(\beta_\Sigma+\Delta)^2}\left[1-\left[1-\frac{\lambda_X^2}{\lambda_\Lambda\lambda_\Sigma}\right]\frac{\pi^2\lambda_\Sigma}{\beta_\Sigma(\beta_\Sigma+\Delta)^2}\right]^{-1} , \quad \text{(A8)}$$

where $\Delta^2=2\mu_\Sigma(m_\Sigma-m_\Lambda)$.

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