Effects of final-state interaction on the ^{12}C(e,e′p)^{11}B reaction

Toshio Suzuki
Department of Physics, College of Humanities and Sciences, Nihon University, Sakurajousui 3-25-40, Setagaya-ku, Tokyo 156, Japan
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Effects of the spin-orbit part of the proton distorting potential on the ratio of the transverse and longitudinal response functions are investigated in the exclusive ^{12}C(e,e′p)^{11}B reaction as well as in the inclusive reaction. The effect is found to explain the observed data for the parallel exclusive kinematics while it fails to explain the inclusive data. Exclusive experiments at nonparallel kinematics are recommended for further study.

Recently, a longitudinal-transverse separation has been carried out for the coincident quasifree proton knockout reaction ^{12}C (e,e′p)^{11}B$_{\mathrm{g.s.}}$ in parallel kinematics.[1] The ratio of transverse and longitudinal (Coulomb) response functions is deduced and compared[1] with an impulse approximation as well as with certain models[2,3] which assume possible modifications of the nucleon inside the nuclear medium. The observed ratio is larger than the ratio of magnetic and electric form factors of a free proton: G_M/G_E. A similar result is obtained for the inclusive data of ^{12}C.[1] In the impulse approximation, the ratio remains as G_M/G_E if we assume that the distorting potential for the outgoing proton has no spin-orbit term and that the orbital part of the operator can be neglected in the transverse response function. The latter assumption is well justified for a high momentum transfer region, for example, as high as 1 fm^{-1} for parallel kinematics.

If the distorting potential has a spin-orbit term, the ratio can deviate from G_M/G_E even in the impulse approximation. This effect should be investigated carefully before we try to find other reasons, such as an enlargement of the nucleon radius in the nuclear medium[2] or relativistic effects given by a simple version of the σ-ω model,[3] which are found to give improved description of the data.[1] The purpose of this work is to estimate possible effects of the spin-orbit potential of the outgoing proton on the ratio of the transverse and longitudinal response functions. The effect is found to be large enough to reproduce the observed ratio for the exclusive reaction on ^{12}C while it fails to explain the inclusive data.

Longitudinal and transverse response functions at momentum transfer $\mathbf{q}$ are expressed as

$$R_L=\sum_{m_s m_h} f^{\dagger}_{m_s m_h} f_{m_s m_h} ,$$
$$R_T=\tfrac{1}{2}\sum_{\lambda=\pm1}\sum_{m_s m_h} f^{\lambda\dagger}_{m_s m_h} f^{\lambda}_{m_s m_h} , \tag{1}$$

using the transition matrix elements

$$f_{m_s m_h}=G_E(q^2)\sum_{m_s'}\int a^{m_s-m_s'}_{m_s}(\mathbf{r})\chi^{\dagger}_{m_s'}e^{i\mathbf{q}\cdot\mathbf{r}}\phi^{m_h}_{l_h j_h}(\mathbf{r})d\mathbf{r} ,$$
$$f_{m_s m_h}=\frac{1}{2m}G_M(q^2)\sum_{m_s'}\int a^{m_s-m_s'}_{m_s}(\mathbf{r})\chi^{\dagger}_{m_s'}(\sigma\times\mathbf{q})_\lambda \times e^{i\mathbf{q}\cdot\mathbf{r}}\phi^{m_h}_{l_h j_h}(\mathbf{r})d\mathbf{r} , \tag{2}$$

with

$$a^{m_s-m_s'}_{m_s}(\mathbf{r})=4\pi\sum_{l_p j_p m_p} i^{-l_p}\chi_{l_p j_p}(pr)C^{m_s m_s'}_{l_p j_p m_p} \times Y^{m_p+m_s-m_s'*}_{l_p}(\hat{\mathbf{r}})Y^{m_p}_{l_p}(\hat{\mathbf{p}}) , \tag{3}$$

$$C^{m_s m_s'}_{l_p j_p m_p}=(l_p m_p \tfrac{1}{2} m_s \mid j_p m_j)(l_p m_p+m_s-m_s' \tfrac{1}{2} m_s' \mid j_p m_j) .$$

In Eq. (2), $\chi_{m_s'}$ is the two-component spin functions of the outgoing proton with the spin direction m_s'. $\chi_{l_p j_p}(pr)$'s in Eq. (3) are proton distorted partial waves with angular momenta l_p and j_p and the asymptotic momentum $\mathbf{p}$. The spin-orbit potential can flip proton spin after the photon coupling and the asymptotic spin direction m_s is not necessarily the same as that m_s' at the virtual photon exchange. If there is no spin-orbit potential, distorted waves are independent of total angular momentum j_p and only contributions from the non-spin-flip component $a^0_{1/2}=a^0_{-1/2}$ remain. In this case, the ratio

$$\left[\frac{4m^2}{q^2}\frac{R_T}{R_L}\right]^{1/2}=\frac{G_M}{G_E}\left[\frac{\frac{1}{2}(\sigma\times\hat{\mathbf{q}})\sum_{m_s}\chi_{m_s}\chi^{\dagger}_{m_s}(\sigma\times\hat{\mathbf{q}})}{\sum_{m_s}\chi_{m_s}\chi^{\dagger}_{m_s}}\right]^{1/2} \tag{4}$$

is equal to G_M/G_E owing to the completeness relation $\sum_{m_s}\chi_{m_s}\chi^{\dagger}_{m_s}=1$.

In the general case, the response functions can be written as

$$
\begin{aligned}
R_L &= G_E^2 \sum_{m_h} \int d\mathbf{r} \int d\mathbf{r}' \phi^{m_h *}_{l_h j_h}(\mathbf{r}') e^{-i\mathbf{q}\cdot\mathbf{r}'} [\cdots] e^{i\mathbf{q}\cdot\mathbf{r}} \phi^{m_h}_{l_h j_h}(\mathbf{r}) , \\
R_T &= \left[\frac{q}{2m} G_m \right]^2 \sum_{m_h} \int d\mathbf{r} \int d\mathbf{r}' \phi^{m_h *}_{l_h j_h}(\mathbf{r}') e^{-i\mathbf{q}\cdot\mathbf{r}'} \tfrac{1}{2} \\
&\quad \times \sum_{\lambda} (\sigma \times \hat{\mathbf{q}})^{\dagger}_{\lambda} [\cdots] (\sigma \times \hat{\mathbf{q}})_{\lambda} \\
&\quad \times e^{i\mathbf{q}\cdot\mathbf{r}} \phi^{m_h}_{l_h j_h}(\mathbf{r}) ,
\end{aligned} \tag{5}
$$

with

$$
[\cdots] = \sum_{m_s \tilde{m}_s' m_s'} a^{m_s - \tilde{m}_s' *}(\mathbf{r}') \chi_{\tilde{m}_s'} \chi^{\dagger}_{m_s'} a^{m_s - m_s'}(\mathbf{r}) . \tag{6}
$$

Note that the distorted waves depend on spin flips $m_s - m_s'$ or $m_s - \tilde{m}_s'$. Summation over m_s, $\tilde{m}_s'$, and m_s' cannot be carried out independent of distorted waves. Equation (6) is a matrix in spin space,

$$
[\cdots] = \begin{bmatrix} D & N \\ -N & D \end{bmatrix} , \tag{7}
$$

where

$$
\begin{aligned}
D(\mathbf{r}',\mathbf{r}) &= a^{0*}_{1/2}(\mathbf{r}') a^{0}_{1/2}(\mathbf{r}) + a^{1*}_{1/2}(\mathbf{r}') a^{1}_{1/2}(\mathbf{r}) , \\
N(\mathbf{r}',\mathbf{r}) &= a^{0*}_{1/2}(\mathbf{r}') a^{1}_{1/2}(\mathbf{r}) - a^{1*}_{1/2}(\mathbf{r}') a^{0}_{1/2}(\mathbf{r}) .
\end{aligned} \tag{8}
$$

Here, use has been made of relations $a^{0}_{-1/2} = a^{0}_{1/2}$ and $a^{-1}_{-1/2} = -a^{1}_{1/2}$. N vanishes unless there are spin flips, while D has both spin-flip and non-spin-flip contributions. The spin operator $(\sigma \times \mathbf{q})_{\lambda}$ mixes up the components of the matrix, which results in a simple form for the operator

$$
\tfrac{1}{2} \sum_{\lambda} (\sigma \times \hat{\mathbf{q}})^{\dagger}_{\lambda} \begin{bmatrix} D & N \\ -N & D \end{bmatrix} (\sigma \times \hat{\mathbf{q}})_{\lambda} = \begin{bmatrix} D & 0 \\ 0 & D \end{bmatrix} \tag{9}
$$

in the coordinate system $q \parallel z$. Summation over m_h for $p_{3/2}$ hole leads to the following result for the ratio:

$$
\left[\frac{4m^2}{q^2} \frac{R_T}{R_L} \right]^{1/2} = \frac{G_M}{G_E} \left[\frac{\langle D \rangle}{\langle D \rangle - \langle N \rangle} \right]^{1/2} \tag{10}
$$

with

$$
\begin{aligned}
\langle D \rangle &= 2 \int d\mathbf{r} \int d\mathbf{r}' R_p(r') R_p(r) e^{-i\mathbf{q}\cdot\mathbf{r}'} e^{i\mathbf{q}\cdot\mathbf{r}} D(\mathbf{r}',\mathbf{r}) \\
&\quad \times [\tfrac{1}{3} Y^{0*}_{1}(\hat{\mathbf{r}}') Y^{0}_{1}(\hat{\mathbf{r}}) + \tfrac{5}{3} Y^{1*}_{1}(\hat{\mathbf{r}}') Y^{1}_{1}(\hat{\mathbf{r}})] , \\
\langle N \rangle &= 2 \int d\mathbf{r} \int d\mathbf{r}' R_p(r') R_p(r) e^{-i\mathbf{q}\cdot\mathbf{r}'} e^{i\mathbf{q}\cdot\mathbf{r}} N(\mathbf{r}',\mathbf{r}) \\
&\quad \times \tfrac{2}{3} [Y^{0*}_{1}(\hat{\mathbf{r}}') Y^{1}_{1}(\hat{\mathbf{r}}) - Y^{1*}_{1}(\hat{\mathbf{r}}') Y^{0}_{1}(\hat{\mathbf{r}})] .
\end{aligned} \tag{11}
$$

Considering the fact that $\langle N \rangle$ comes entirely from spin flip, we find that the Coulomb response is more sensitive to spin flip; that is, more influenced by the spin-orbit potential than the transverse response. This will soon be confirmed by the actual calculation. We also notice that, for the knockout from an LS-closed core, the corresponding spin-flip contribution $\langle N \rangle$ vanishes if we have contributions from both the $j = l + \frac{1}{2}$ and $j = l - \frac{1}{2}$ shells and assume that they have the same binding energies and radial shapes for the wave functions. Inclusive knockout from ^{16}O and ^{40}Ca cores, as well as knockout from the ^{4}He core, are examples of such processes if the difference between spin-orbit partners are neglected.

Calculations including the spin-orbit term of the distorting potential are carried out and the results are shown in Fig. 1 for the parallel kinematics as well as for the inclusive case. The orbital part is also taken into account for the transverse operator;

$$
O_T = \frac{1}{2m} [G_M i \sigma \times \mathbf{q} + F_1(\mathbf{p}_i + \mathbf{p}_f)] e^{i\mathbf{q}\cdot\mathbf{r}} . \tag{12}
$$

Observed data are taken from Ref. 1. Here we plot the values of

$$
R_G = \left[\frac{4m^2}{q^2} \frac{R_T}{R_L} \right]^{1/2}
$$

instead of

$$
\left[\frac{4m^2}{Q^2} \frac{W_T}{W_L} \right]^{1/2} = \frac{q^2}{Q^2} R_G
$$

in Ref. 1, where $Q^2 = q^2 - \omega^2$. The latter choice is convenient for the off-shell single nucleon current due to de Forest.[4] It is more appropriate to adopt R_G for the present operators whose off-shell behavior is different from de Forest's. Here, ^{12}C is considered to be a closed core of $s_{1/2}$ and $p_{3/2}$ shells and proton knockout from the $p_{3/2}$ shell is taken into account for both cases. Knockout from the s shell is also included for the inclusive case. Energy transfer is fixed to be $\omega = 86.4$ MeV in the calculations. The p shell part of the inclusive contribution is obtained just by integrating the exclusive result over the outgoing proton angle assuming that all the $p_{3/2}$ strength is exhausted by the transition to the

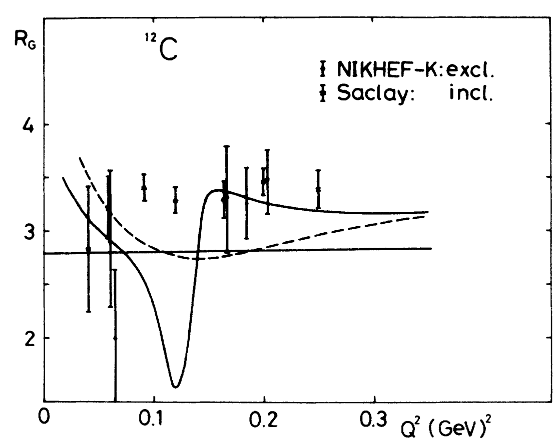

FIG. 1. Ratios $R_G = [(4m^2/q^2)(R_T/R_L)]^{1/2}$ for the exclusive reaction $^{12}C(e,ep')^{11}B_{g.s.}$ at parallel kinematics (solid curve) as well as for the inclusive case (dashed curve). The straight solid line denotes the values of G_M/G_E. Experimental data are taken from Ref. 1.

ground state of ^{11}B. For the s-shell knockout, all the $s_{1/2}$ strength is assumed to be represented by a transition to a single state of ^{11}B at $E_{ex}=20.5$ MeV. The ratio for the s-shell contribution is almost unaffected by the spin-orbit potential. It remains as G_M/G_E if the orbital part of the transverse operator is neglected. The effect for the ratio, therefore, becomes less remarkable for the inclusive case compared with the exclusive case. The optical potential is taken from Ref. 5. The imaginary part is set as zero to keep a sum rule[6] when the inclusive calculation is done. Rapid change of the calculated ratio for the exclusive case around $Q^2=0.14$ GeV^2 is caused by the occurrence of the minima of transverse and longitudinal response functions around $q=1.9$ fm^{-1}. Therefore, the values around this region should not be taken seriously. On the contrary, inclusive response functions have maxima around $q=1.5$ fm^{-1} and oscillatory behavior seen in the exclusive case does not occur.

We see from Fig. 1 that the enhancement of the ratio is already reproduced by the effect of the spin-orbit potential for the exclusive parallel case. This enhancement is obtained by the reduction of the Coulomb response function. Transverse response is almost unaffected by the inclusion of the spin-orbit potential. This result is consistent with the expectation from Eq. (10): the Coulomb response is more sensitive to the spin-orbit potential than the transverse response. For the inclusive case, the peak of the Coulomb response function is shifted toward a higher q region by about 0.07 fm^{-1} with the inclusion of the spin-orbit term. This is consistent with the attractive nature of the interaction; partial waves with $j=l+\frac{1}{2}$ are pulled inward gaining more overlap with the hole wave function. Attractive interaction shifts the response function to the lower ω region for fixed q, which is equivalent to the enhancement of the response function at the higher q region for fixed ω as observed here. The behavior is consistent with the sum rule; the spin-orbit interaction does not change the integral of the Coulomb response function over ω for fixed q. The transverse part is, again, affected very little by the spin-orbit potential. The crossing of the calculated ratio of the inclusive response function over G_M/G_E at $Q^2=0.1$ GeV^2 in Fig. 1 corresponds to the shift of the peak of the Coulomb response function. The observed inclusive data are not reproduced by the inclusion of the spin-orbit interaction.

When the Coulomb response is reduced in one kinematical region such as in parallel kinematics, it must be enhanced in another kinematics if the sum rule is kept. We carried out calculations for exclusive reactions at nonparallel kinematics to confirm this. Results are shown in Fig. 2 for angles between **p** and **q** at $\vartheta=10°$, 20°, and 30°. At 20° and 30°, response functions and their ratios behave similar to the p-shell contribution of the inclusive case. This comes from the fact that the response functions have peaks around 20° for fixed q. The ratio is enhanced below the peak of the Coulomb response function while it is reduced above it.

We find that the effect of the spin-orbit potential of the outgoing proton is large enough to reproduce the experimental data at exclusive parallel kinematics while it

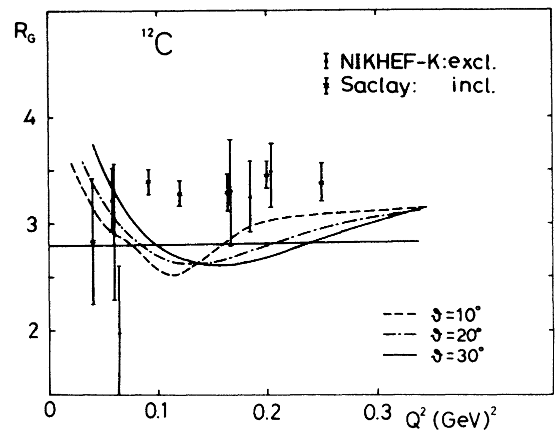

FIG. 2. The same as in Fig. 1 for the ratios for nonparallel exclusive kinematics at $\vartheta=10°$, 20°, and 30°.

is not successful for the inclusive case. The situation does not change when another optical potential[7] is used. It is highly recommended that exclusive experiments be done at nonparallel kinematics and that these be compared with calculations to see how important and essential the spin-orbit effect is for the resolution of the problem. The failure in the inclusive case is directly connected with the long-standing problem of the reduction of the Coulomb response function. To really solve the problem, it is important to go beyond the impulse approximation and take into account the particle-hole rescattering effect[8] and (or) 2p-2h excitation mechanisms.[9] The same thing is expected to be true for the exclusive case, which is now under investigation. On the experimental side, it is important to give not only the ratio of the transverse and longitudinal response functions, but also the absolute values of each response function at various kinematics. It is too naive to draw a conclusion about the size of the nucleon inside the nuclear medium just from deviations of the observed ratios from a simple free value of G_M/G_E.

Our result on the effect of the spin-orbit potential is different from that by Boffi *et al.*[10] Our calculation predicts larger effects than those obtained in Ref. 1, where they found negligible effects from the spin-orbit term for the ratio.[11]

Finally, we briefly comment on the longitudinal spin operator $\boldsymbol{\sigma}\cdot\mathbf{q}$. Similar to Eq. (9), we get

$$\boldsymbol{\sigma}\cdot\hat{\mathbf{q}}\begin{bmatrix} D & N \\ -N & D\end{bmatrix}\boldsymbol{\sigma}\cdot\hat{\mathbf{q}}=\begin{bmatrix} D & -N \\ N & D\end{bmatrix} \tag{13}$$

in the coordinate system $q\parallel z$. The nondiagonal spin-flip component N contributes opposite in sign to the longitudinal spin response compared with the Coulomb response. The following remarks are in order: (1) the longitudinal spin response is more sensitive to spin flips of the outgoing proton than the transverse spin response; (2) the ratio of transverse spin to longitudinal spin responses would be reduced if the ratio of transverse spin to Coulomb responses is enhanced. An intermediate energy (p,p′p″) reaction is one candidate to investigate the longitudinal spin response as the reaction is dominated by $\boldsymbol{\sigma}\cdot\mathbf{q}$ operator. The low energy $(\gamma,\pi^{\pm}p)$ reaction, which is dominated by Kroll-Ruderman operator $\boldsymbol{\sigma}\cdot\boldsymbol{\varepsilon}$, is another candidate suited for investigation on both longitudinal and transverse spin responses.

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