$E1$ polarizability of ^{7}Li and astrophysical S factor for ^{4}He(t,γ)^{7}Li

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The electric dipole polarizability of ^{7}Li is studied theoretically. A consistent interpretation is obtained among the electric dipole polarizability, the quadrupole moment, the tensor analyzing powers for aligned ^{7}Li scattering at sub-Coulomb energies, and the radiative capture (or photodisintegration) cross section for the $^3\mathrm{H}+\alpha\rightarrow{}^7\mathrm{Li}+\gamma$ reaction.

I. INTRODUCTION

The light elements ^{2}H, ^{3}He, ^{4}He, and ^{7}Li are produced in the Big Bang expansion.[1] Recent observation[2] of the ^{7}Li abundance in very old stars enables us to directly compare it with calculated ^{7}Li abundance in a standard Big Bang model. This realizes a precise determination of the universal baryon density.[3–5] The standard Big Bang model[1] requires knowledge of the number of (light) neutrino species, the axial vector coupling constant determined from the neutron half-life or angular correlation measurements, and many reaction rates in the energy range 10–200 keV (i.e., $T<10^9$ K). However, since this energy range is not accessible by direct measurement, reaction rates at these energies have been extrapolated from the data at higher energies. The production of ^{7}Li is mediated by ^{4}He(t,γ)^{7}Li at the lower baryon densities and ^{4}He(^{3}He,γ)^{7}Be followed by electron capture at the higher baryon densities. A theoretical ^{7}Li abundance in the Big Bang model is therefore very sensitive to the above two reaction rates.[6–8] As for the ^{4}He(t,γ)^{7}Li reaction, a predicted[9] remarkable increase of the S factor at very low energies has been verified by a new measurement[7] and by several recent theoretical calculations.[10–12] However, the two different experiments[7,8] of this reaction rate still show a 20–80% difference in magnitude. Another independent measurement is required to determine the astrophysical S factor for this reaction.

In a recent Letter,[13] a precise measurement of tensor analyzing powers T_{20} for aligned ^{7}Li scattering from heavy nuclei was reported by Weller *et al.* Although the tensor analyzing powers are affected strongly by the quadrupole deformation of ^{7}Li, the electric dipole polarizability also has a non-negligible effect by more than 10%. Investigating their highly accurate measurement of T_{20}, we have found a new theoretical method to determine the absolute strength of the astrophysical S factor for the ^{4}He(t,γ)^{7}Li reaction. This finding originates from the fact that the radiative alpha-triton capture is dominated by the electric dipole transition, which is related to the electric dipole polarizability. The ultimate purpose of this article is to determine the astrophysical S factor in this new method and compare it with the previous values that have been applied to several astrophysical problems.

An admixture of virtual $E1$ transitions in the Coulomb excitation of ^{7}Li has attracted much attention[13–22] in connection with giant dipole resonance. However, there is no clear giant dipole state observed in ^{7}Li, leaving a question as to where the strength of $E1$ polarizability is scattered. There is another question concerning ^{7}Li: The quadrupole moment -3.70 ± 0.08 e fm^2 which was determined from the Coulomb scattering experiment[13] is incompatible with the precise value -4.06 e fm^2 determined by atomic means.[23] In addition, the inferred $E1$ polarizabilities τ_{if} differ from one another in several different experiments. Weller *et al.*[13] have obtained $|\tau_{11}|=|\tau_{12}|=0.23\pm0.06$ fm^3, but the other groups[18–20] have, respectively, reported different values: $|\tau_{12}|=0.21\pm0.03$, 0.1, and 0.15 ± 0.01 fm^3. There are at least two reasons for this discrepancy. The first reason is that different quadrupole moments have been obtained (or used in Ref. 19) in each analysis, -3.70 ± 0.08, -3.66, -1.0 ± 2.0, or -4.0 ± 1.1 e fm^2. The second reason is the lack of knowledge of the sign of τ_{if}: Weller *et al.* reported only the absolute values of τ_{if} in their paper, and it is unclear as to which sign was adopted in the practical analysis. Hausser *et al.*[18] have adopted a positive sign for τ_{12} [or, equivalently, $S(E1)$ in their notation] in the analysis of the Coulomb excitation probability of ^{7}Li, based on the cluster model calculation done by Smilansky, Povh, and Traxel.[24] However, the LS-coupling cluster model leads to the same negative sign for τ_{11} and τ_{12}.

In this article we look for a consistency among all different kinds of observables on ^{7}Li. They are the quadrupole moment, tensor analyzing powers, $E1$ polarizability, and astrophysical S factor for the ^{4}H(t,γ)^{7}Li reaction. The consistency between the $E1$ polarizability and the inverse energy-weighted dipole sum also is discussed. The present analysis is twofold; first, in the next section, we set up the polarization potential for aligned ^{7}Li scattering in order to calculate the tensor analyzing powers T_{20}. There are a number of parameters in the reaction amplitude. We have applied several theoretical constraints upon the electromagnetic moments of ^{7}Li, assuming an LS-coupling scheme. Many theoretical calculations in this scheme have been successful for the reactions $\alpha + \mathrm{t} \to {}^7\mathrm{Li} + \gamma$ and $\alpha + \mathrm{t} \to \alpha + \mathrm{t}$ as well as the nuclear structure of ^{7}Li. We can then determine the $E1$ polarizability by fitting the observed T_{20} data in Sec. III. Second, in Sec. IV we extract the astrophysical S factor for the ^{4}He(t,γ)^{7}Li reaction from the determined $E1$ polarizability. The knowledge of the observed inverse energy-weighted dipole sum is used in the estimate of the S factor. We extend it to the mirror reaction ^{4}He(^{3}He,γ)^{7}Be. Finally, in Sec. V we summarize the discussion.

II. THEORY

The tensor analyzing powers T_{20} are affected strongly only by the tensor part of the internuclear potential. The tensor part of the polarization potential is defined in perturbation theory[14] by

$$\langle I_i M_i \mid V_{\mathrm{pol}}^{(2)} \mid I_f M_f \rangle = \left[\frac{4\pi}{5} \frac{Z_t e}{r^3} \langle I_i \| M(E2) \| I_f \rangle - \left[\frac{9\pi}{5} \right]^{1/2} \frac{Z_t^2 e^2}{r^4} \tau_{if} \right] Y_{2\mu}^*(\hat{\mathbf{r}} \cdot \hat{\mathbf{s}}) g \ , \tag{1}$$

$$\tau_{if} = \frac{8\pi}{9} \left[\frac{10}{3} \right]^{1/2} \sum_n W(1\,1\,I_i\,I_f;\ 2\,I_n) \frac{\langle I_i \| M(E1) \| I_n \rangle \langle I_n \| M(E1) \| I_f \rangle}{E_n - E_i} \ , \tag{2}$$

with

$$M(E\lambda,\mu) = e \sum_{\mathrm{p}} r_{\mathrm{p}}^{\lambda} Y_{\lambda\mu}(\hat{\mathbf{r}}_{\mathrm{p}}) \ , \tag{3}$$

where τ_{if} is the tensor moment of the $E1$ polarizability, and $M(E\lambda,\mu)$ is the electric λ-pole operator of nucleons. $|IM\rangle$ is the nuclear substate of ^{7}Li with total spin I and its projection M,

$$W(1\,1\,I_i\,I_f;\ 2\,I_n)$$

is the Racah coefficient,

$$(I_f\, M_f\, 2\,\mu \mid I_i\, M_i)$$

is the Clebsch-Gordan coefficient, and

$$g = (I_f\, M_f\, 2\,\mu \mid I_i\, M_i) / \sqrt{2I_i + 1} \ .$$

Only two channels of ^{7}Li, i.e., the ground ($\frac{3}{2}^-$) and first excited ($\frac{1}{2}^-$) states, are taken into account. At sub-Coulomb energies the first order effect of the electric quadrupole couplings of ^{7}Li modified by the second order effect of virtual dipole transitions dominates the ^{7}Li scattering from heavy nuclei. We have therefore used the potential (1) and the Coulomb force in the coupled-channels calculations.

In a recent study of the tensor analyzing powers, Weller *et al.*[13] have concluded that the nuclear force does not have a considerable effect on T_{20} for the $^7\vec{\mathrm{Li}} + {}^{58}$Ni and $^7\vec{\mathrm{Li}} + {}^{120}$Sn scatterings at the energies below 10.0 and 15.3 MeV, respectively. The Coulomb barriers for the respective systems are $V_C = 18.0$ and 26.0 MeV in the lab system. We have therefore not included the nuclear force in the potential in the present study at sub-Coulomb energies.

Among many possible multipole operators in the electromagnetic interaction, the $M1$ and $M3$ (or $M1$) operators also contribute to the elastic (or inelastic) scattering of ^{7}Li by nearly 1% of the total cross section (at backward angles).[18] These multipoles do not, however, contribute to the tensor analyzing powers. There is no other multipole operator having the first order effect on the ^{7}Li scattering, except for the $E2$ operator, which was taken into account in the present calculation. The second order effect arising from the couplings between the ground spin doublets and excited states (for example, the multipoles $E2$, $E4$, $M3$, and $M5$ for the transition $\frac{3}{2}^- \to \frac{7}{2}^-$ coupled to $E4$ or $M3$ for $\frac{7}{2}^- \to \frac{1}{2}^-$) might contribute weakly to the $\frac{3}{2}^- \to \frac{1}{2}^-$ transition amplitude. However, such a contribution is neglected completely at sub-Coulomb energies because the electromagnetic λ-pole operator depends on the internuclear distance r as $r^{-\lambda-1}$ and it has a vanishingly small effect with increasing multipolarity λ in the low energy region.

It has been reported[18,25] that even at sub-Coulomb energies the observed deviation of the cross section from Rutherford scattering requires several correction terms at the 0.1% level, such as atomic screening, relativistic effects, and vacuum polarization as well as the nuclear polarizability. Although the nuclear $E1$ polarizability has a major contribution among them by about 10% to the excitation probability of the first excited state, the other correction terms have a contribution with less than 1% for the ^{7}Li scattering on ^{208}Pb at 22 MeV ($< V_C = 36.0$ MeV).[18] These effects are usually included in the renormalization potential as the $E1$ polarizability in Eq. (1). Most of the other correction terms modify the scalar part of the potential, leaving the tensor part unchanged. We have hence neglected these effects, except for the nuclear $E1$ polarizability.

There are four parameters to be determined by calculating T_{20} with the potential (1). They are the $E2$ moment Q_s of the ground state, the $B(E2)$ strength for the transition between the ground spin doublets, and the associated tensor moments of the $E1$ polarizability, τ_{11} and τ_{12}, of ^{7}Li.

We now discuss theoretical constraints upon the electromagnetic moments. Let us first recall the fact that the LS-coupling scheme provides a good approximation to ^{7}Li in shell model, Hartree-Fock model, and microscopic cluster model calculations of the electromagnetic moments. We write the relation between the $B(E2)$ and quadrupole moment as follows:

$$B(E2;\tfrac{3}{2}^-\rightarrow\tfrac{1}{2}^-)=\frac{25}{16\pi}(eQ_s)^2(1+\delta)\ . \qquad (4)$$

In the LS scheme, δ is zero, but, in practice, there are threshold energy effects[9,26] that disturb the simple proportionality. Theoretical calculations[12,27] predict $\delta=0.085$, as shown by the solid curve in Fig. 1, independently of the assumed effective nuclear force. We use this value $\delta=0.085$ in our constraint.

The quadrupole moment $-4.06\ e\,\mathrm{fm}^2$, which was determined by atomic means,[23] is adopted to fix Q_s. This value $-4.06\ e\,\mathrm{fm}^2$ is the most precise among many different observed data of Q_s for the following two reasons: First, it was determined from accurate measurement of the quadrupole coupling constant eqQ_s of LiH in molecular-beam electric resonance spectroscopy. The experimental error bar of this quantity eqQ_s is less than 0.2%. Although the determination of Q_s from eqQ_s is subject to possible theoretical uncertainties in the calculation of the electric field gradient q, Sundholm *et al.*[23] estimate the effects of various truncations of the model space and find that the predicted Q_s moment changes by less than 1%. Second, the atomic measurement of Q_s is free from nuclear dynamics. This second point is an advantage of the static atomic measurement, in contrast with the nuclear measurement using heavy ion collisions. Q_s is a very important parameter in the present analysis because small changes here will have a large effect on the inferred value of τ_{if}. The Q_s dependence of inferred $E1$ polarizability τ_{if} is discussed in the next section.

One could also use the measured $B(E2)$ value instead of Q_s in Eq. (4), but it has a larger uncertainty than the quadrupole moment determined from the molecular beam measurement. We feel that our theoretical constraint (4) is as reliable as the quoted error[18–20,28] on the $B(E2)$.

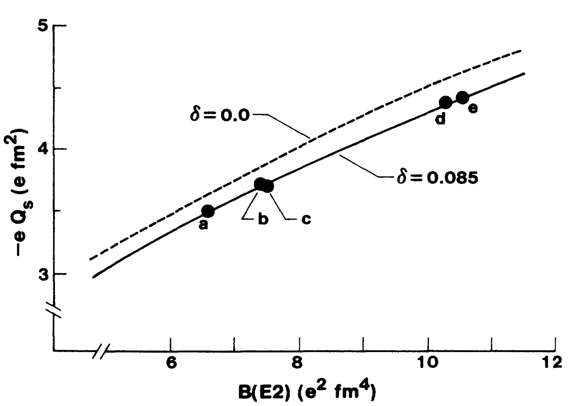

FIG. 1. Relation between the quadrupole moment eQ_s and $B(E2;\tfrac{3}{2}^-\rightarrow\tfrac{1}{2}^-)$ strength of ^{7}Li calculated in the microscopic cluster model. Solid and dashed curves denote the relation with ($\delta=0.085$) and without ($\delta=0.0$) the threshold energy effects. a–e are the results obtained by using different effective N-N interactions from one another. See text and Ref. 12 for details.

The second theoretical constraint relates τ_{11} and τ_{12}. The giant dipole state is a candidate for the intermediate states $|I_n\rangle$ in Eq. (2), as discussed by many authors.[14–18] However, a clear dipole resonance is not observed in any photodisintegration measurements[29–31] of ^{7}Li up to 50 MeV. Even if such a state exists at high excitation energy around 20 MeV, τ_{if} vanishes if the giant dipole mode couples weakly to the nuclear spin of the ground state of ^{7}Li and if the single-particle wave functions are approximated by harmonic oscillator functions. On the other hand, several shell model calculations,[15,17] which take into account the giant dipole mode, give an overly small $E1$ polarizability in the strong coupling limit.

Therefore, the virtual breakups of ^{7}Li to several continuum states of the $\alpha+$t, n$+\,^6$Li, p$+\,^6$He, and d$+\,^5$He channels, etc., are presumed to make the major contribution to the $E1$ polarizability. These breakup channels have low thresholds (2.5–10.0 MeV), and strong $E1$ transitions to the fragment channels are observed in the photodisintegrations of ^{7}Li (Table I). We show how strong they are, based on the observed data. The molecular dipole sum rule[32] for the energy weighted photodisintegration cross section

$$\sigma_0(\text{molecular})=\frac{2\pi^2e^2\hbar}{M_{\rm N}c}\frac{(Z_1A_2-Z_2A_1)^2}{(A_1+A_2)A_1A_2}\ ,$$

is a useful measure for a strength of the $E1$ transition. This equation predicts 2.86, 12.9, 22.9, and 0.86 MeV mb for the ^{7}Li(γ,t)^{4}He, ^{7}Li(γ,n)^{6}Li, ^{7}Li(γ,p)^{6}He, and ^{7}Li(γ,d)^{5}He breakup cross sections, respectively. These theoretical values are comparable to the observed data in Table I, showing that the $E1$ transition to these continuum channels is so strong as to exhaust the molecular dipole sum rule. It is notable that the observed photodisintegration cross section indicates a broad maximum around 3–5 MeV above the threshold in every breakup channel.

Taking account of these breakup states for $|I_n\rangle$ and assuming the LS-coupling scheme just as in the $E2$ matrix elements, we find the relation

$$\tau_{11}=\tau_{12}\ , \qquad (5)$$

and also that these quantities are negative. Several microscopic calculations[11,33] of the ^{4}He$\,+\,$t system give $\tau_{12}=(1+\eta)\tau_{11}$ with $\eta=0.073$, again reflecting the different threshold effect for the ground spin doublets of ^{7}Li. For our analysis, this small difference is insignificant.

Practical calculations of the tensor analyzing powers have been done by using the program ECIS79.[34] We have used definitely negative sign for τ_{if}. A negative τ_{11} value has a destructive effect on the $E2$ moment, which is negative for ^{7}Li, as clearly seen in the polarization potential (1). In Weller's analysis, τ_{11} seems to have been adopted as a positive value because the effect of $E1$ polarizability has a constructive effect on the $E2$ moment in their calculation of T_{20}. We have also found that their τ_{12} value was set to be negative, as in the present calculation.

TABLE I. Observed energy-weighted moments of the photodisintegration cross section of ^{7}Li; $\sigma_n = \int \sigma_{\rm dis}(E)E^n dE$.

	^{7}Li(γ,t)[a]	^{7}Li(γ,n)[b]	^{7}Li(γ,p)[c]	^{7}Li(γ,d)[d]
σ_0 (MeV mb)	5.53	20.1	15.4	4.03
σ_{-1} (mb)	0.611	1.15	0.964	0.307
σ_{-2} (MeV^{-1} mb)	0.0898	0.071	0.0905	0.0245

[a]Reference 29. $E_{\rm th} < E < 28.0$ MeV. The low energy data are supplemented by the radiative capture cross section from Ref. 8. The experimental error is $\pm 15\%$. Reference 30 is another set of independent data, but is inconsistent with Refs. 29 and 31. (See text.)
[b]Reference 39. $E_{\rm th} < E < 30.5$ MeV.
[c]Reference 29. $E_{\rm th} < E < 28.0$ MeV, extrapolated smoothly from the higher energy (12.0 MeV $< E$) data. The experimental error is $\pm 15\%$.
[d]Reference 29. The same as in footnote c.

III. TENSOR ANALYZING POWERS AND $E1$ POLARIZABILITY

The calculated tensor analyzing powers for ^{58}Ni($^7\vec{\rm Li}$,^{7}Li) and ^{120}Sn($^7\vec{\rm Li}$,^{7}Li) are compared with experimental data in Figs. 2 and 3. First, we calculated T_{20} by turning off the effect of the $E1$ polarizability $\tau_{11}=\tau_{12}=0$ (dashed curves). Then, we introduce negative τ_{if} values with the relationship $\tau_{12}=(1+\eta)\tau_{11}$ to get the best fit to the observed T_{20} values (solid curves). The inferred τ_{if} values are

$$\tau_{11} = -0.269 \text{ fm}^3 , \tag{6a}$$

$$\tau_{12} = -0.289 \text{ fm}^3 . \tag{6b}$$

It has long been a mystery[13–20] that the $E2$ matrix elements of ^{7}Li are too large to explain the excitation probability of the first excited state. The negative and, hence, the destructive effect of the virtual $E1$ excitations of ^{7}Li on the continuum states is likely to solve this problem. We compare the present result with previous experiments in Table II. Although the observed $|\tau_{12}|$ values scatter greatly, the present result is close to Weller's $-|\tau_{12}|$ value.

We would like to briefly discuss the Q_s dependence of the $E1$ polarizability τ_{if}. As shown in the preceding paragraph, $\tau_{11}=-0.269$ fm^3 and $\tau_{12}=-0.289$ fm^3 were obtained by setting $eQ_s=-4.06\ e$ fm^2. When we use $eQ_s=-3.89\ e$ fm^2 instead of $-4.06\ e$ fm^2, which is the mean average of four independent nuclear and atomic measurements[20,23,35,36] of Q_s, $\tau_{11}=-0.165$ fm^3 and $\tau_{12}=-0.177$ fm^3 are obtained. Let us change the quadrupole moment by 7% (i.e., $-3.77\ e$ fm^2): Similarly, a good fit of T_{20} is fulfilled with $\tau_{11}=-0.106$ fm^3 and $\tau_{12}=-0.114$ fm^3 in this case. The inferred $E1$ polarizability τ_{if} depends on Q_s. However, we believe that the adopted quadrupole moment $-4.06\ e$ fm^2 is the most precise value, as discussed in Sec. II. Hence, we proceed to a discussion of an astrophysical S factor with τ_{if} values of (6a) and (6b), which were determined by setting $eQ_s=-4.06\ e$ fm^2.

IV. ASTROPHYSICAL S FACTOR

Let us first derive the mathematical relation between the astrophysical S factor for ^{4}He(t,γ)^{7}Li and the $E1$ polarizability of ^{7}Li.

The astrophysical S factor $S_{\alpha t}(E_{\rm c.m.})$ is defined by

$$S_{\alpha t}(E_{\rm c.m.}) = E_{\rm c.m.} \exp(2\pi\eta_C)\sigma^{\alpha t}_{\rm cap}(E_{\rm c.m.}) \tag{7a}$$

$$= \sum_{I_f} E_{\rm c.m.} \exp(2\pi\eta_C) \sum_{\lambda} \frac{8\pi(\lambda+1)}{\lambda[(2\lambda+1)!!]^2} \frac{1}{\hbar} \left[\frac{\omega}{c} \right]^{2\lambda+1} \frac{1}{2} \sum_{M_f,\nu} |\langle I_f M_f | M^{E(M)\lambda} | S=\tfrac{1}{2}\nu \rangle|^2 , \tag{7b}$$

where $\sigma^{\alpha t}_{\rm cap}$ is the capture cross section, $E_{\rm c.m.}$ is the incident energy between ^{4}He and t in the c.m. system, $\eta_C = Z_1 Z_2 e^2/\hbar v$, ω is the emitted photon energy, and $M^{E(M)\lambda}$ is the electromagnetic λ-pole operator. In the low energy region $E_{\rm c.m.} < 5$ MeV more than 98% of the total capture cross section is dominated by the electric dipole ($E1$) transition. Even at the higher energies $5 < E_{\rm c.m.} < 20$ MeV at least 95% of the cross section is still exhausted by the $E1$ transition. Hence, we take an approximation of the $E1$ dominance as

$$S_{\alpha t}(E_{\rm c.m.}) = \sum_{I_f, I_n} S_{\alpha t}(E_{\rm c.m.}, E1; I_n \to I_f) \tag{8a}$$

$$= \sum_{I_f} \frac{8\pi}{9} \frac{\pi^2}{(\hbar c)(\mu c^2)} \exp(2\pi\eta_C)(E_{\rm c.m.} + E^{\alpha t}_{\rm th} - E_f)^3 |\langle (L_f \tfrac{1}{2}) I_f \| M^{E1} \| (L_n \tfrac{1}{2}) I_n \rangle|^2 , \tag{8b}$$

where μ is the reduced mass of the ^{4}He + t system, $E_{\rm th}^{\alpha t}=2.467$ MeV, $E_f=0$ MeV (or 0.478 MeV) for $I_f=\frac{3}{2}$ (or $\frac{1}{2}$), and $\hbar\omega=E_{\rm c.m.}+E_{\rm th}^{\alpha t}-E_f$. In Eq. (7) the incident scattering wave function is normalized to be of unit flux. We here changed the wave function normalization to the δ function in energy in order to incorporate it into the definition of the $E1$ polarizability [Eq. (2)]. We have used the LS-coupling scheme for the wave functions in transforming Eq. (7) into Eq. (8).

The tensor moment of the $E1$ polarizability τ_{11} is rewritten in terms of the astrophysical $E1$ S factor (8) as

$$\tau_{11}^{\alpha t}=\left(\tfrac{10}{3}\right)^{1/2}\frac{(\hbar c)(\mu c^2)}{\pi^2}\sum_{I_n}(-1)^{I_n-3/2}W(1\,1\,\tfrac{3}{2}\,\tfrac{3}{2};\,2\,I_n)\int_0^\infty\frac{S_{\alpha t}(E_{\rm c.m.},E1;\,I_n^+\to\frac{3}{2}^-)\exp(-2\pi\eta_C)}{(E_{\rm c.m.}+E_{\rm th}^{\alpha t})^4}dE_{\rm c.m.}\ . \qquad (9)$$

We would like to comment on the sign of τ_{11} again. The negative sign of τ_{11} is quite physical because it leads to a positive astrophysical S factor[37] for ^{3}H$(\alpha,\gamma)^7$Li.

One should recall that the inferred value of $\tau_{11}=-0.269$ fm^3 in the preceding section is the sum of many contributions coming from different channels of ^{7}Li,

$$\tau_{11}=\sum_c \tau_{11}^c\ , \qquad (10)$$

where c spans over all breakup channels, and this is a natural extension of the sum over all intermediate states in Eq. (2) that connect to the ground spin doublets by the electric dipole operator. Only the adiabatic condition $\xi>1$, where $\xi=\eta_C(E_n-E_0)/2E_{\rm c.m.}$ is the adiabaticity, was assumed[14] in the perturbative derivation of the polarization potential (2). The reactions considered here satisfy the condition. In addition, the intermediate states are not restricted only to the bound or quasibound states of ^{7}Li. They should form a complete set of the

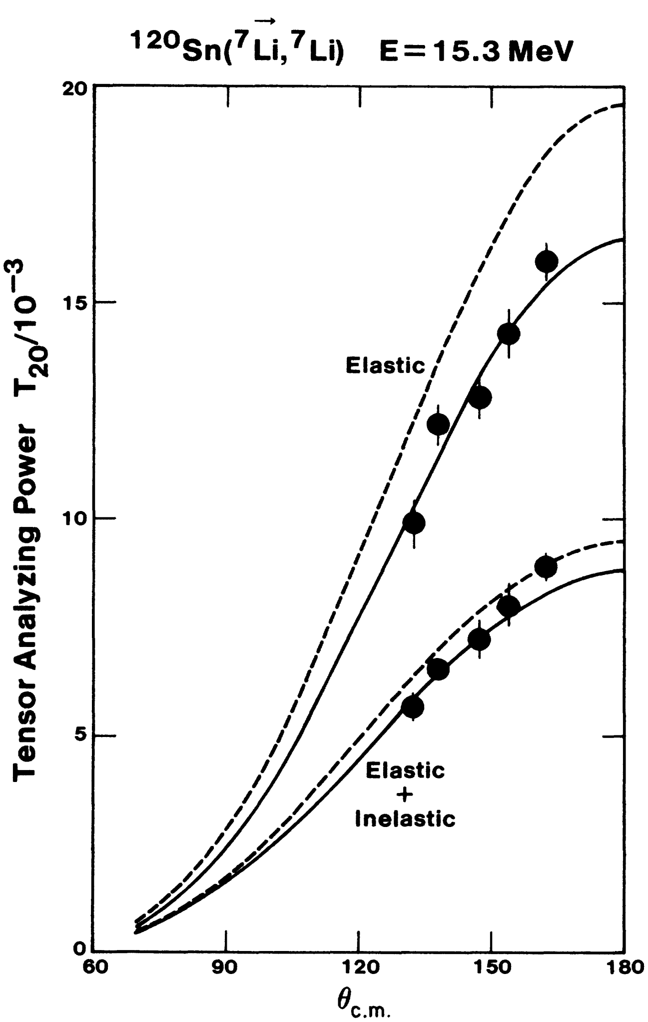

FIG. 2. Tensor analyzing powers T_{20} for the scattering of ^{7}Li from ^{120}Sn. Experimental data are from Ref. 13. Solid and dashed curves are the calculated results with and without the effects of the $E1$ polarizability of ^{7}Li, respectively.

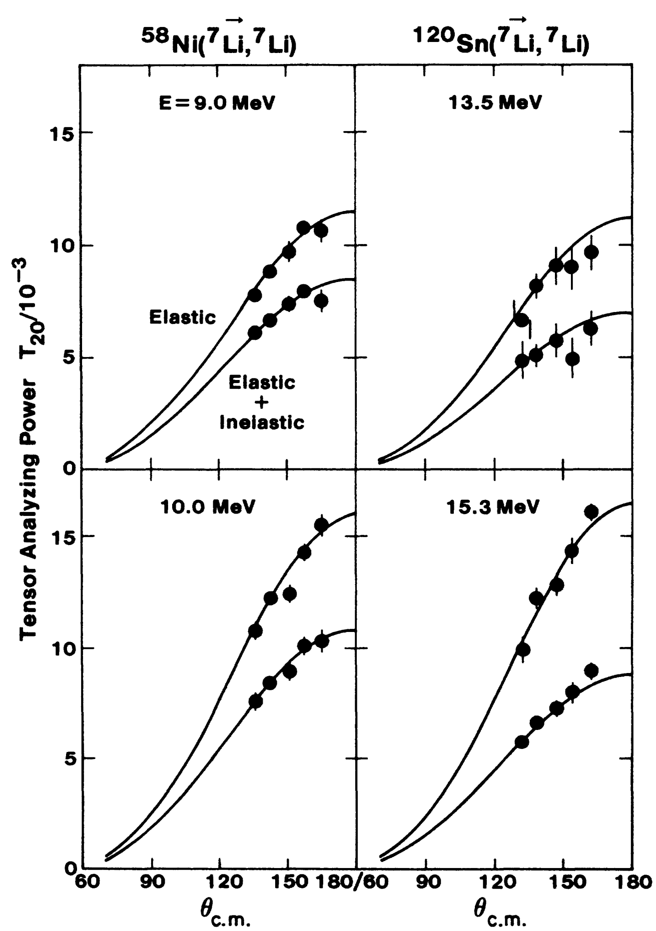

FIG. 3. The same as in Fig. 2 for the ^{7}Li scattering from ^{58}Ni and ^{120}Sn.

$A=7$ nuclear system including the continuum states—only the problem is an orthogonality among these channels. As stressed in Sec. II, without an existence of quasibound state as the giant dipole resonance in ^{7}Li, the continuum breakup channels at the energies 3–5 MeV above the threshold make a main contribution to $E1$ polarizability. At these energies the $E1$ matrix elements are dominated by the external region where the different particle channels are orthogonal to each other. We, therefore, believe that Eq. (10) is correct in the present analysis.

Let us assume that the entire $E1$ polarizability (6a) comes from the ^{3}H$(\alpha,\gamma)^7$Li channel alone, namely $\tau_{11}=\tau_{11}^{\alpha t}$. Then, one obtains a very large S factor, $S_{\alpha t}(0)=0.297$ keV b, for ^{3}H$(\alpha,\gamma)^7$Li from Eq. (9), assuming a theoretical[5,12] energy dependence of the S factor. This result is too large, compared with the values of prompt γ-ray measurements $S_{\alpha t}(0)=0.100\pm0.025$ keV b (Refs. 5 and 8) and $S_{\alpha t}(0)=0.162\pm0.024$ or 0.134 ± 0.020 keV b (Ref. 7), although these two measurements differ from each other.

In order to estimate the partial strength $\tau_{11}^{\alpha t}$ of the ^{3}H$(\alpha,\gamma)^7$Li channel from Eq. (6a), the observed inverse energy-weighted photodisintegration cross section is a useful observable,

$$\sigma_{-2}^{\alpha t}=\int_{E_{\rm th}^{\alpha t}}[\sigma_{\rm dis}^{\alpha t}(\tfrac{1}{2}^+)/E^2+\sigma_{\rm dis}^{\alpha t}(\tfrac{3}{2}^+)/E^2+\sigma_{\rm dis}^{\alpha t}(\tfrac{5}{2}^+)/E^2]dE\ , \tag{11}$$

where $\sigma_{\rm dis}^{\alpha t}$ is the photodisintegration cross section defined by

$$\sigma_{\rm dis}^{\alpha t}(I_n^+)=\frac{4\pi^3}{9}\frac{E}{\hbar c}\,|\langle\tfrac{3}{2}^-\|M(E1)\|I_n^+\rangle|^2\ . \tag{12}$$

Here, $E=E_{\rm c.m.}+E_{\rm th}^{\alpha t}$ is the photon energy, and we use a notation

$$\sigma_{\rm dis}^{\alpha t}(I_n^+)=\sigma_{\rm dis}^{\alpha t}(E,E1;\ \tfrac{3}{2}^-\to I_n^+)$$

for simplicity, assuming the $E1$ dominance. Using Eqs. (7a) and (9) and relating[38] the radiative capture cross section to $\sigma_{\rm dis}^{\alpha t}$, $\tau_{11}^{\alpha t}$ is expressed as

$$\tau_{11}^{\alpha t}=(-1)\frac{\sqrt{5}}{3}\frac{\hbar c}{\pi^2}\int_{E_{\rm th}^{\alpha t}}[\sigma_{\rm dis}^{\alpha t}(\tfrac{1}{2}^+)/E^2-\tfrac{4}{5}\sigma_{\rm dis}^{\alpha t}(\tfrac{3}{2}^+)/E^2+\tfrac{1}{5}\sigma_{\rm dis}^{\alpha t}(\tfrac{5}{2}^+)/E^2]dE\ . \tag{13}$$

Taking the pure LS-coupling limit,

$$\langle\tfrac{3}{2}^-\|M(E1)\|\tfrac{5}{2}^+\rangle=3\langle\tfrac{3}{2}^-\|M(E1)\|\tfrac{3}{2}^+\rangle$$

should hold for the transitions to the spin-doublet states $I_n^+=\frac{5}{2}^+$ and $\frac{3}{2}^+$. In this limit $\tau_{11}^{\alpha t}$ has the form

$$\tau_{11}^{\alpha t}=(-1)\frac{\sqrt{5}}{3}\frac{\hbar c}{\pi^2}\int_{E_{\rm th}^{\alpha t}}[\sigma_{\rm dis}^{\alpha t}(\tfrac{1}{2}^+)/E^2+\sigma_{\rm dis}^{\alpha t}(\tfrac{3}{2}^+)/E^2]dE\ . \tag{14}$$

Two quantities $\tau_{11}^{\alpha t}$ [Eq. (14)] and $\sigma_{-2}^{\alpha t}$ [Eq. (11)] have a strong similarity to each other, except for an overall normalization and sign. Essentially the same similarity relation between τ_{11}^c and σ_{-2}^c is derived theoretically for any channel c, provided that the LS-coupling scheme is a good approximation to the ground state of ^{7}Li. Hence, we assume a proportionality between the two observables,

$$\tau_{11}^c\Big/\sum_c\tau_{11}^c=\left[\sigma_{-2}^c\Big/\sum_c\sigma_{-2}^c\right]_{\rm exp}\ . \tag{15}$$

Fortunately, photodisintegration cross sections have been observed[29–31] for all fragmentation channels of ^{7}Li (Table I), and we use the observed σ_{-2}^c values here.

Let us look at the observed data on σ_{-2}^c. Figure 4 shows an energy dependence of σ_n^c (with $n=0$, -1, and -2) for the ^{7}Li$(\gamma,t)^4$He reaction as an example. The photodisintegration cross section σ_0^c has a broad maximum in relatively low energy region $E=E_{\rm th}\sim10$ MeV, as discussed in Sec. II on the molecular dipole sum rule. This tendency is similarly observed[29–31] in any photodisintegration channels of ^{7}Li. The position of the maximum σ_{-2}^c is shifted to much lower energy due to the inverse energy weight $1/E^2$. Much larger (-2)th moments than the estimates of Levinger[21] and Migdal[22] are observed experimentally in ^{7}Li, as summarized in Ref. 18. Seeing this together with the fact that the contribution from the low energy region just above the threshold dominates σ_{-2}^c as shown in Fig. 4, we believe that Eq. (15) is a good approximation.

TABLE II. Static and dynamical moments of ^{7}Li in heavy ion reactions. † denotes values that were assumed in the analysis. ‡ denotes an assumed value (referred from Ref. 40), proved to be wrong in the careful atomic calculation of Sundholm *et al.* (Ref. 23). An * denotes that the sign of τ_{if} is not reported. A * * means refer to Sec. II for the sign of τ_{11} and τ_{12}.

eQ_s (e fm^2)	$B(E2)$ (e^2fm^4)	τ_{11} (fm^3)	τ_{12} (fm^3)	
-4.06†	8.90†	-0.269	-0.289	present work
		$\lvert\tau_{11}\rvert$* (fm^3)	$\lvert\tau_{12}\rvert$* (fm^3)	Ref.
-4.0 ± 1.1	7.42 ± 0.14		0.16 ± 0.01	20
-3.66‡	8.3 ± 0.6		0.21 ± 0.03	18
-1.0 ± 2.0	7.4 ± 0.1		0.1†	19
-3.70 ± 0.08	8.3 ± 0.5	0.23 ± 0.06**	0.23 ± 0.06**	13

Equation (15) and Table I predict

$$\tau_{11}^{\alpha t}=-0.088\pm0.035\ \text{fm}^3 \tag{16}$$

as the partial strength of τ_{11} for the ^{7}Li(γ,t)^{4}He channel. In this estimate we have included the typical experimental error $\pm15\%$ of the data σ_{-2}^{c} in Table I. Leung *et al.*[30] have reported systematically larger cross sections for the ^{7}Li(γ,t) reaction in electrodisintegration experiment; the difference between the data of Leung *et al.* and those of Junghans *et al.* is a factor 1.7 at low energies and 1.5 at higher energies. Scaling the data of Junghans *et al.* for ^{7}Li(γ,t)^{4}He by the factor 1.7, one obtains a 38% larger partial strength $\tau_{11}^{\alpha t}$. Although Leung *et al.* assumed a flat angular distribution of the cross section and hence their total σ_0 is less accurate, we have also included this change in the error. Theoretical calculations[11,33] using the microscopic cluster model give values of $\tau_{if}^{\alpha t}=-(0.08-0.13)$ fm^3, which are consistent with the present result.

It is straightforward to estimate the astrophysical S factor for ^{3}H(α,γ)^{7}Li. Equation (9) explicitly relates $\tau_{11}^{\alpha t}$ and the $E1$ S factor. Assuming only the energy dependence of the astrophysical S factor and the relative ratio of the s-wave ($I_n^+=\frac{1}{2}^+$) to d-wave ($\frac{3}{2}^+$ and $\frac{5}{2}^+$) contributions given by the microscopic cluster model calculations,[5,12] the absolute strength of the S factor is extracted from the inferred $\tau_{11}^{\alpha t}$ value (16). The calculated S factor is shown by the solid curve in Fig. 5 and the $S_{\alpha t}(0)$ value at $E_{\text{c.m.}}=0$ MeV turns out to be

$$S_{\alpha t}(0)=0.097\pm0.038\ \text{keV b} \tag{17}$$

for the ^{4}He(t,γ)^{7}Li reaction. The inferred $S_{\alpha t}(0)$ value is consistent with the Griffiths data[5,8] 0.100±0.025 keV b, but smaller than a recent observation of 0.162±0.024 keV b or the 0.134±0.020 keV b of Schroder *et al.*[7]

There is also a theoretical finding[12] for the $S(0)$ values between ^{4}He(t,γ)^{7}Li and ^{4}He(^{3}He,γ)^{7}Be,

$$S_{\alpha\,^3\text{He}}(0)/S_{\alpha t}(0)=5.0\pm0.1\ . \tag{18}$$

Applying this scaling relation to (17), we obtain

$$S_{\alpha\,^3\text{He}}(0)=0.49\pm0.14\ \text{keV b} \tag{19}$$

for the ^{4}He(^{3}He,γ)^{7}Be reaction. Although this value is estimated in a completely different method from the previous ones[6] measuring prompt γ rays (Parker and Kavanagh, Nagatani, Dwarakanath, and Ashery, Osborne *et al.*, and Alexander *et al.* in Ref. 6) or activation (Osborne *et al.*, Robertson *et al.*, and Volk *et al.* in Ref. 6), their average[5] 0.56±0.03 keV b is consistent with the present result.

V. SUMMARY

We have analyzed the measured[13] tensor analyzing powers for aligned ^{7}Li scattering below the Coulomb barrier. Assuming several reasonable theoretical constraints on the electromagnetic moments of ^{7}Li, we found that the $E1$ polarizabilities $\tau_{11}=-0.269$ fm^3 and $\tau_{12}=-0.289$ fm^3 give a best fit to the observed tensor analyzing powers, together with the precise value of the $E2$ moment determined by atomic means. We found an approximate proportionality of the $E1$ polarizability τ_{11} to the inverse energy-weighted photodisintegration cross section σ_{-2} of ^{7}Li, and the partial strength of τ_{11} for the ^{4}He(t,γ)^{7}Li process was estimated by means of the observed σ_{-2} data. Using the mathematical relation between the $E1$ polarizability and the astrophysical S factor, we then calculated the absolute strength of the S factor for ^{4}He(t,γ)^{7}Li. The inferred $S(0)$ value 0.097±0.038 keV b is consistent with the Griffiths result

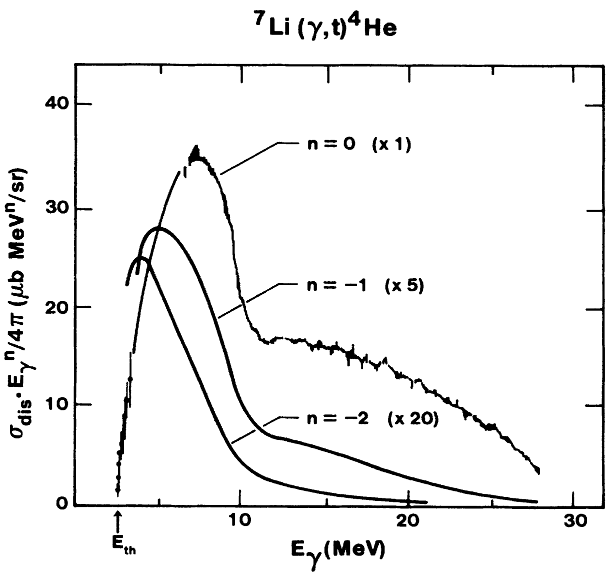

FIG. 4. Energy-weighted photodisintegration cross sections for ^{7}Li(γ,t)^{4}He. E_{th} is the breakup threshold 2.467 MeV for ^{7}Li$\rightarrow$ ^{4}He + t.

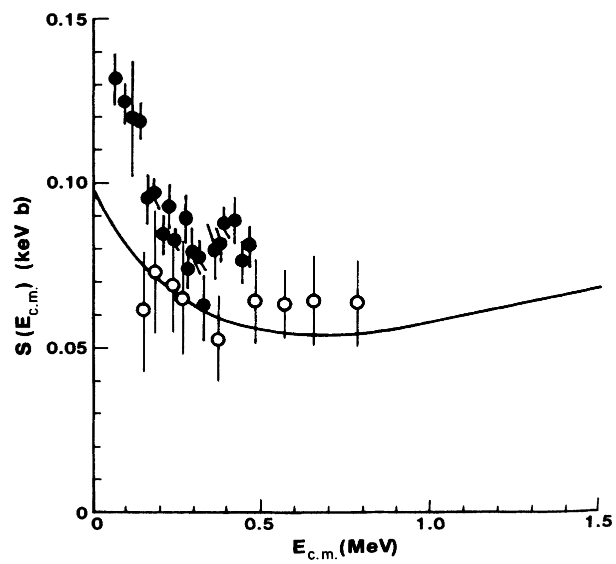

FIG. 5. Astrophysical S factor for ^{4}He(t,γ)^{7}Li. Closed and open circles are the experimental data from Refs. 7 and 8, respectively. Solid curve is the present calculation. See text for details.

but smaller than Schroder's data, both of which were determined in prompt γ-ray measurements.

The extracted τ_{if} values in the present method, and, of course, the S factors too, are very sensitive to the adopted $E2$ moment of ^{7}Li. If the quoted accuracy of the atomic determination of Q_s can be trusted, then the nuclear measurement[13] of the tensor analyzing powers for aligned ^{7}Li scattering are just on the border of providing a useful determination of the astrophysical S factor. An experiment[41] measuring the $E1$ polarizability τ_{if} independently of the $E2$ moment will provide one with more useful astrophysical information.

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