Alpha particle cluster states in ^{40}Ca

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A coupled channels calculation of the lowest lying positive parity bands of alpha particle cluster states in ^{40}Ca based on an ^{36}Ar core in its 0^+ ground or 2^+ first excited states is presented. The calculated energies and electromagnetic transition rates are in good agreement with the available experimental data, and it is suggested that analogous cluster states should be present in neighboring nuclei.

The existence of alpha particle clustering in light nuclei is now well documented and has been treated in a variety of theoretical approaches.[1,2] There is, in addition, strong evidence that it is also present in a number of actinide nuclei.[3] These observations lead one to ask whether alpha particle clustering also occurs in the mass range between these two extremes (and if so, to what extent). There have been several recent studies in intermediate mass nuclei[4–6] which give an affirmative answer to this question. In particular, the application in Ref. 4 of the Buck, Dover, and Vary cluster model[7] to ^{40}Ca, gave a successful description of the energies and electromagnetic transitions between half a dozen states in the lowest lying alpha cluster band. This model has recently been extended to include excited states of the core in a coupled channels formalism, thus predicting several higher lying bands of cluster states. In this Brief Report we apply this extension to the ^{36}Ar-α system and seek to identify the corresponding "new" states in the spectrum of ^{40}Ca.

The original, single channel form of the model describes cluster and core as interacting via a local potential obtained by folding their densities with an effective nucleon-nucleon interaction. This local potential can be conveniently and accurately parametrized in the form

$$V(r)=\frac{-V_0\left[1+\cosh\frac{R}{a}\right]}{\cosh\frac{r}{a}+\cosh\frac{R}{a}}=-V_0 f(r) . \quad (1)$$

Pal and Lovas[4] have used a radius of $R=2.9$ fm, a diffuseness of $a=1.4$ fm, and a depth of $V_0=160.2$ MeV for the $^{36}\text{Ar}+\alpha$ system. The main exigencies of the Pauli principle are satisfied by treating the cluster as a single particle whose relative motion about the core is characterized by the principal quantum number N and orbital angular momentum L, with the restriction that N and L correspond to the microscopic situation in which the cluster nucleons all occupy a different major shell from the core nucleons. In the present study where the alpha cluster nucleons will be placed in the (fp) shell this means that

$$N=2n+L\geq 12 \quad (2)$$

(n is the number of interior nodes in the radial wave function). Any remaining effects of antisymmetrization may be absorbed into the effective potential of Eq. (1). The cluster state energies and wave functions are then identified as the bound states and resonances obtained by solving the single-particle Schrödinger equation with this potential (plus Coulomb and centrifugal terms). The Coulomb potential may be taken as that appropriate to a pointlike cluster interacting with a uniformly charged spherical core (of radius $R_c=4.0$ fm for ^{36}Ar).

To include excited states of the core, the model is extended as follows. The wave function for a given total angular momentum J (projection M) is written as a sum over the channels i of a product of relative radial coordinates and relative angular/intrinsic functions

$$\psi_{JM}=\sum_i \frac{1}{r}U_i^J(r)\phi_i^{JM}(\hat{\mathbf{r}},\xi) , \quad (3)$$

where each channel i is defined by the core spin I_i, relative orbital angular momentum L_i ($J=L_i\otimes I_i$), internal energy e_i, and any other necessary quantum numbers. In Eq. (3), r is the magnitude and $\hat{\mathbf{r}}$ the angular coordinates of the vector $\mathbf{r}$ between the centers of mass of cluster and core, and ξ represents the internal coordinates. Substitution of this into the single-particle Schrödinger equation and projection onto the channel states yields, for each channel i,

$$\left[-\frac{\hbar^2}{2\mu}\left[\frac{d^2}{dr^2}-\frac{L_i(L_i+1)}{r^2}\right]+e_i-E\right]U_i^J(r)+\sum_k V_{ik}^J(r)U_k^J(r)=0 , \quad (4)$$

with μ the reduced mass of the cluster-core system. The potential matrix is given by

$$V_{ik}^{J}(r)=(\phi_i^{JM}(\hat{\mathbf{r}},\xi)\,|\,V(\mathbf{r},\xi)\,|\,\phi_k^{JM}(\hat{\mathbf{r}},\xi))\ , \qquad (5)$$

where the parentheses imply integration over all internal coordinates and over the relative angular coordinates $\hat{\mathbf{r}}$.

The interaction term $V(\mathbf{r},\xi)$ contains a central diagonal term and a quadrupole coupling term. It is assumed that the 0^+ ground state and the 2^+ first excited state of ^{36}Ar are the first two members of a $K^\pi=0^+$ rotational band, and that the only internal coordinate required is the orientation $\hat{\mathbf{s}}$ of the symmetry axis. The potential may thus be written as

$$V(\mathbf{r},\xi)=V_0 f(r)+V_0\left[-\frac{r df}{dr}\right]\beta\sum_q D_{q0}^{2}(\hat{\mathbf{s}})Y_{2q}^{*}(\hat{\mathbf{r}}) \qquad (6)$$

for small deformations (where the usual derivative form has been taken for the radial part of the coupling potential).

The free parameter V_0 is chosen to fit the ^{36}Ar-α band head in ^{40}Ca at 3.35 MeV (we thus use $V_0=158.1$ MeV) and the deformation parameter β is chosen to adjust the excited state energies as well as possible (although it should be consistent with the quadrupole deformation of ^{36}Ar). In this region of the periodic table (just before a double shell closure), we expect an oblate deformation and accordingly take $\beta=-0.06$. We retain the potential geometry of Pal and Lovas,[4] and proceed to solve the coupled equations to obtain cluster state energies and wave functions. (Calculational details for bound states, $E<0$, may be found in Ref. 1 and for resonances, $E>0$, in Ref. 8.)

The calculated energies of some low lying states (with experimental counterparts) are given in Table I. All states generated by the model for $N=12$ are shown in Fig. 1. The band of states based on the ^{36}Ar core in its ground state is very similar to that calculated in Ref. 4, although some interband mixing does shift the energies a little. The experimental states corresponding to the members of this lowest lying band are readily identified by their strong, selective excitation in the ^{36}Ar(^{16}O,^{12}C)^{40}Ca alpha transfer reaction.[9] The experimental states corresponding to members of the higher lying bands (based on an excited ^{36}Ar core) are suggested by comparing their predicted energies and/or $E2$ transition strengths with the data compilations of Endt and van der Leun.[10] It is possible that this identification could be confirmed by resonant particle spectroscopy experiments[11] in which ^{40}Ca breaks up into ^{36}Ar and alpha particle fragments whose excitation energies are measured.

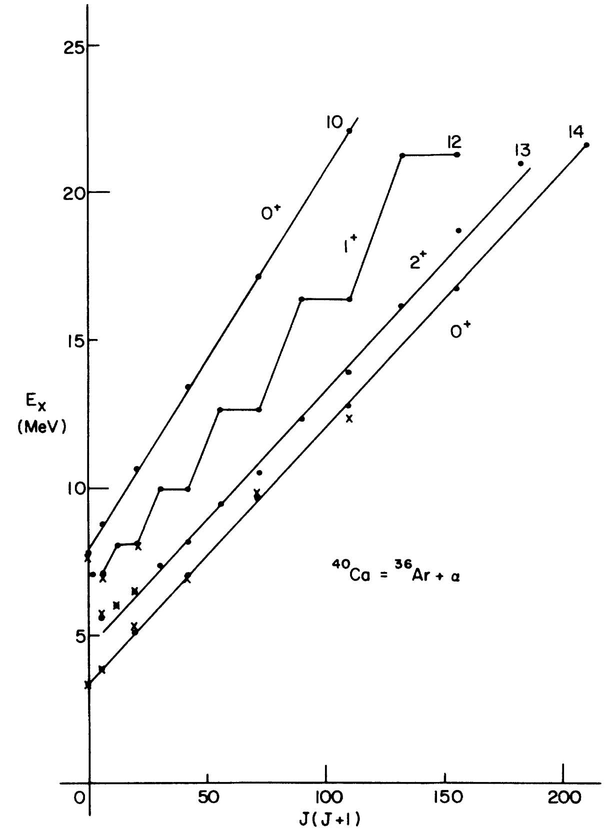

FIG. 1. The calculated energies of the lowest lying positive parity bands of ^{36}Ar-α cluster states in ^{40}Ca plotted vs $J(J+1)$ (full circles) compared with some suggested experimental counterparts (crosses). The lines are drawn to make the band structure apparent, and are labeled by the spins of the initial and final states in each band.

An ^{36}Ar 2^+ excitation energy of 2.4 MeV has been used in these calculations (compared with the measured value of 1.97 MeV in free ^{36}Ar) to place the 2^+ state in ^{40}Ca formed by coupling $I=2\otimes L=0$ at 5.63 MeV. This state is so identified because of its small $E2$ transition

TABLE I. Calculated spectrum of some low lying ^{36}Ar-α cluster states in ^{40}Ca and their suggested experimental counterparts. Predominant core spins and cluster-core relative angular momenta are indicated.

$J^\pi=I^\pi\otimes L$	$E_{\rm cal}$ (MeV)	$E_{\rm exp}$ (MeV)	$J^\pi=I^\pi\otimes L$	$E_{\rm cal}$ (MeV)	$E_{\rm exp}$ (MeV)
0^+ 0^+ 0	3.34	3.35	0^+ 2^+ 2	7.77	7.70
2^+ 0^+ 2	3.86	3.90	2^+ 2^+ 0	5.60	5.63
4^+ 0^+ 4	5.07	5.28	2^+ 2^+ 2	7.06	6.91
6^+ 0^+ 6	7.00	6.93	3^+ 2^+ 2	6.04	6.03
8^+ 0^+ 8	9.62	9.9	4^+ 2^+ 2	6.56	6.54
10^+ 0^+10	12.86	12.4	4^+ 2^+ 4	8.04	7.93

TABLE II. Calculated and measured (Ref. 10) $E2$ transition strengths $B(E2\downarrow)$ between some of the suggested alpha particle cluster states in ^{40}Ca. Effective charges of 0 and 0.1 are used.

$J_i^\pi(E_i)$ (MeV)	$J_f^\pi(E_f)$ (MeV)	$B(E2\downarrow)$ Weisskopf units Calculation $\epsilon=0$	$\epsilon=0.1$	Expt.
$2^+(3.90)$	$0^+(3.35)$	21.8	30.5	30.0±4
$2^+(5.63)$	$0^+(3.35)$	1.2	1.7	2.1±1
$4^+(5.28)$	$2^+(3.90)$	29.0	40.6	59.5±9
$2^+(5.63)$	$2^+(3.90)$	0.3	0.4	<2.5
$3^+(6.03)$	$2^+(3.90)$	1.8	2.5	4.1±1
$4^+(6.54)$	$2^+(3.90)$	2.2	3.1	2.9±1
$6^+(6.93)$	$4^+(5.28)$	27.4	38.4	27±13
$4^+(6.54)$	$2^+(5.63)$	18.9	26.5	70±30

rates to the 0^+ (3.35) and 2^+ (3.90) states of ^{40}Ca which are the experimental counterparts of the states formed by coupling $I=0\otimes L=0$ and $I=0\otimes L=2$, respectively. Small $B(E2)$ values are predicted by the model because the cluster-core relative motion plays almost no part in these transitions, which are thus due almost entirely to the $2^+\rightarrow 0^+$ transition in the core. Although a 2^+ state exists in ^{40}Ca at 5.25 MeV (i.e., 1.9 MeV above the 0^+ cluster band head) it decays strongly to the 2^+ (3.90) state, and cannot therefore be the $I=2\otimes L=0$ state we seek.

The figure shows that four bands of alpha cluster states, labeled by the spins of their initial and final members, are produced in this extended version of the model. The mixing of states with equal J but different I and/or L is only substantial in cases of accidental near degeneracy. It is interesting to note that the 14^+ state formed by coupling $I=2\otimes L=12$ is a "perfect" rotationally spaced extension of the lowest band (although we do not expect to see it very strongly in alpha transfer). In all, an additional six states have been identified in the ^{40}Ca spectrum as having a predominant ^{36}Ar(2^+)-α cluster-core structure.

The details of the calculation of $E2$ transition rates between cluster states are given in Ref. 1. The basic form of the equation for a transition between a state of initial angular momentum J_i and one of final angular momentum J_f is

$$B(E2;J_i\rightarrow J_f)=\frac{1}{2J_i+1}\,|\,\langle J_f\|M_2(^{36}\mathrm{Ar})\|J_i\rangle+\langle J_f\|e\beta_2 r^2 Y_2(\hat{\mathbf{r}})\|J_i\rangle\,|^2\,, \qquad (7)$$

which contains contributions from the intrinsic quadrupole deformation of ^{36}Ar and from the cluster-core relative motion, respectively. The factor β_2 is related to the decomposition of the nucleus (Z,A) into cluster (Z_2,A_2) and core (Z_1,A_1) such that

$$\beta_2=\frac{Z_1A_2^2+Z_2A_1^2}{(A_1+A_2)^2}\,. \qquad (8)$$

Our results are presented in Table II. The ^{36}Ar quadrupole deformation parameter Q_0 was taken from the measured $B(E2)$ transition strength between its ground and first excited states, namely 13.6 W.u.[10] Transitions between cluster states based on the same core state are very strong because of the large contribution of the relative motion term. However, since there is not generally a great deal of interband mixing, this term is very much smaller between cluster states based on different core states. In these latter cases the main contribution is from the ^{36}Ar $2^+\rightarrow 0^+$ transition, and the $B(E2)$ value is corresponding smaller. Even with no effective charge, the agreement is impressive. It improves even more if a small effective charge of 0.1 is introduced.

In view of the excellent agreement achieved so far between the extended cluster model and the rather limited available experimental data, it would clearly be desirable to carry out improved gamma ray spectroscopy to try to identify (and measure the properties of) the remaining states predicted by the model. In addition, the measurement of alpha decay widths for those states above the ^{36}Ar-α threshold in ^{40}Ca (7.04 MeV) would be very useful, since they are readily calculated in the model[1] and would provide an even more convincing demonstration of its power. Finally, we note that this type of alpha clustering is known to persist from mass 16 to at least mass 20 in (sd)-shell nuclei and so it will be interesting to see if similar bands of alpha cluster states can be traced from ^{40}Ca to the neighboring (fp)-shell nuclei.

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