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Band structure in ^{133,135}Pr

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It is shown that the "normal" and "decoupled" band-like structures recently observed in the nuclei ^{133}Pr and ^{135}Pr can be reproduced within the framework of a standard version of the particle-rotor model. In addition, several positive parity levels in ^{135}Pr, to which no band assignment could be given earlier, are shown to be the members of two positive parity bands with opposite signatures based on the $\frac{3}{2}^+[411]$ Nilsson orbital. The most interesting observation of the present work is the near total attenuation of the Coriolis interaction in the positive parity bands in ^{135}Pr. It is suggested that this nucleus can be a very good testing ground for different versions of the particle-rotor models currently used by different workers.

I. INTRODUCTION

The ^{133,135}Pr isotopes ($Z=59$) belong to transitional nuclei below the $N=82$ neutron shell closure where the shape of the nucleus strongly depends on the neutron number. Recently,[1–3] the level structures of these two isotopes have been studied extensively through ^{118,120}Sn(^{19}F,4n) and ^{136}Ce(p,2n) reactions.[1–3] Earlier theoretical and experimental studies[2] indicate a slightly oblate ($\gamma > 30°$) deformation for $N > 78$ and a slightly prolate ($\gamma < 30°$) deformation for $N < 78$ odd-proton and odd-neutron nuclei in this mass region. A recent experimental study by Hildingsson *et al.*[1] has established, in ^{133}Pr, a negative parity decoupled band based on $1h_{11/2}$ proton orbitals, and two positive parity bands with opposite signatures based most probably on the $\frac{5}{2}^+[413]$ proton orbital. Similarly, in ^{135}Pr, recent experimental studies by Kortelahti *et al.*[2] and Semkow *et al.*[3] have shown the existence of a negative parity decoupled band based on the $1h_{11/2}$ proton orbital and two positive parity bands with opposite signatures based on the $1g_{7/2}$ proton orbital. It is well known that in the case of rotating nucleus, for some rotational frequency, the Coriolis interaction becomes strong enough to overcome the pairing interaction, resulting in a decoupled band. High-j orbitals with a small spin projection on the nuclear symmetry axis are most strongly influenced in this manner. In general, the Coriolis force introduces significant band mixing, resulting in complicated excitation spectra. Several earlier works[2,4] attempted to reproduce the negative parity band structure in these isotopes using particle-rotor formalism. Semkow *et al.*[3] have performed cranked shell model calculations in order to understand the γ-deformation degree of freedom in ^{135}Pr. However, no attempt has been made by these workers to compare in detail the observed level properties with their model predictions. We have shown earlier that a particle-rotor model,[5] which incorporates both pairing and the variable moment of inertia formalisms, gave good agreement with the experimental level properties of odd-proton $N=88$ transitional nuclei. In the present work, we have applied the same formalism to calculate the structures of the excited states below first backbending in ^{133,135}Pr. Since the formalism has been discussed in detail in earlier works,[5,6] we shall discuss only the results of the calculation.

II. PARAMETERS OF THE CALCULATION

The calculation involves several parameters, e.g., Nilsson parameters μ,k, the deformation δ, pairing strength G, and the parameter C used to simulate the variable moment of inertia[7] like behavior in the excitation spectrum. Initially, a reasonable choice of different parameter values was made from several considerations, described in detail in an earlier publication.[5] Then some of the parameters were allowed to vary to a small extent to obtain a good fit to the observed spectra. For ^{133}Pr, the calculations were done with $\mu=0.58$ (0.47) for $N=4$ (5) shell, $k=0.06$, $\delta=0.24$, and $C=1.0\times10^7$ keV^3. Similarly, for ^{135}Pr, the values $\mu=0.50$ for $N=4$, (5) shell, $k=0.06$, $\delta=0.18$, $C=1.0\ (3.0)\times10^7$ keV^3 for positive (negative) parity states were used. The pairing gap (Δ) and Fermi level (λ) were determined through a solution of the gap equation using a strength parameter $G=27/A$ MeV. The Fermi level was found to lie near the $\frac{3}{2}^-[541]$ orbital in both the isotopes. The calculated values of $\Delta=1.15$ MeV (^{135}Pr) and 0.90 MeV (^{133}Pr) were found to be close to that deduced from odd-even mass difference. Finally, the Coriolis matrix elements (including the decoupling term) had to be reduced to 80–90 % of their theoretical values for the calculation of positive and negative parity states in ^{133}Pr and for the negative parity states in ^{135}Pr. In the calculation of positive parity states in ^{135}Pr, a peculiar feature was observed. Good agreement with the experimental spectrum as well as the decay modes of the excited band members could only be reproduced if the Coriolis matrix elements were reduced to at least 40% of their theoretical values. This abnormal behavior will be discussed in more detail in Sec. V. The transition probabilities and the branching ratios of the decay modes were calculated with $g_1=1.0$, $(g_s)_{\rm eff}=3.5$, $g_R=Z/A$, and $e_{\rm eff}=1.0$.

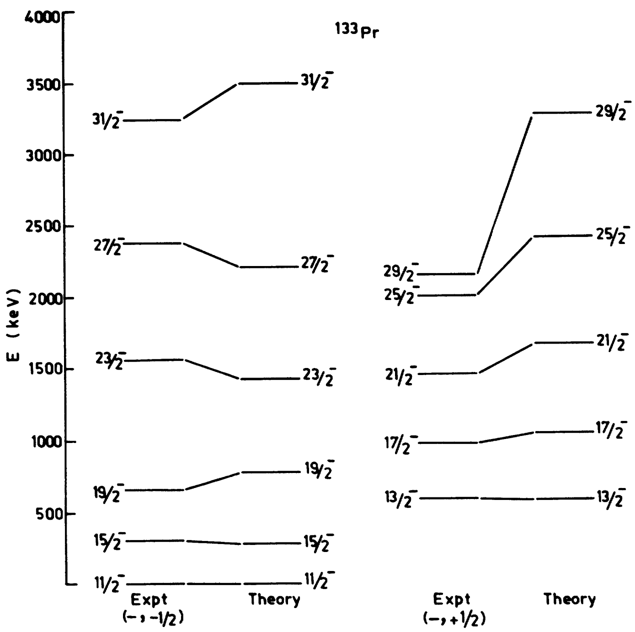

FIG. 1. Experimental (Ref. 1) and calculated negative-parity levels in ^{133}Pr. The excitation energies are relative to the $\frac{11}{2}^-$ level at 130 keV. The Coriolis attenuation factor is 0.9.

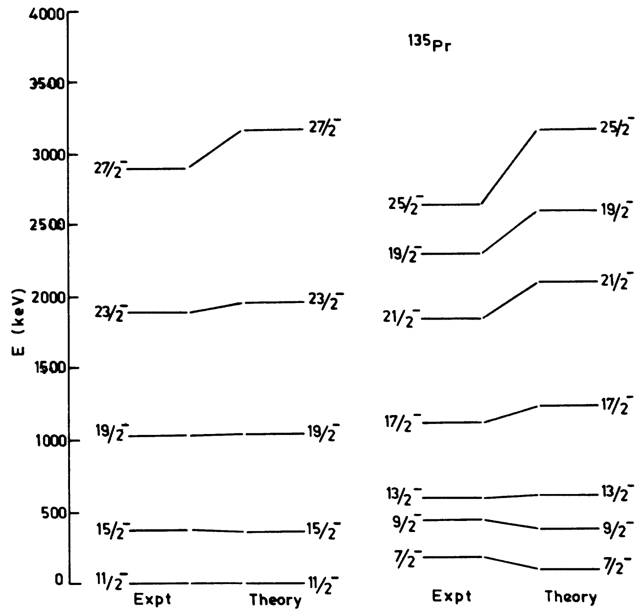

FIG. 2. Experimental (Refs. 2 and 3) and calculated negative-parity levels in ^{135}Pr. The excitation energies are relative to the $\frac{11}{2}^-$ level at 358 keV. The states forming the decoupled band are shown separately. The Coriolis attenuation factor is 0.9.

III. NEGATIVE PARITY STATES

Both the nuclei under consideration show a $\Delta I = 2$ band of "favored" states based on an $\frac{11}{2}^-$ orbital. Present calculations give very good agreement with the observed decoupled band structure in ^{133}Pr up to the spin value $I^\pi = \frac{27}{2}^-$ (Fig. 1). For higher spin values, the calculated level energies are systematically higher than their experimental counterparts. The main problem lies in the fact that the two parameter variable moment of inertia (VMI) model cannot reproduce correctly the core energies of the high spin states in this mass region. Therefore, the experimental excitation energies of the high spin states cannot be reproduced correctly even after inclusion of the VMI approach in the model. Recently it was shown[8,9] that better agreement with the experimental energies of the ground state bands below backbending can be obtained by using an expression which contains at least three adjustable parameters. In ^{135}Pr also, the present calculation is able to reproduce the negative parity decoupled bands up to the spin value (Fig. 2) $\frac{27}{2}^-$. As far as the electromagnetic properties of these states are concerned, the present calculation predicts an enhancement of the $B(E2)$ rates by a factor of ≈ 15 times the single particle estimates. This enhancement seems to be consistent with the decay modes of these levels (Table I).

TABLE I. Calculated and experimental decay modes of the negative parity levels in ^{135}Pr. The numbers listed under $T(E2)$ and $T(M1)$ are to be multiplied by powers of ten given in the square brackets. Experimental transition energies are used in the calculation of transition probabilities.

Transition $I_i^\pi \to I_f^\pi$	$T(E2)$ sec^{-1}	$T(M1)$ sec^{-1}	Branching ratio Calc.	Branching ratio Expt.[a]
$\frac{27}{2}{}^-_1 \to \frac{23}{2}{}^-_1$	5.41[12]		1.0	1.0
$\frac{27}{2}{}^-_1 \to \frac{25}{2}{}^-_1$	2.99[8]	1.32[11]	0.024	
$\frac{23}{2}{}^-_1 \to \frac{19}{2}{}^-_1$	2.39[12]		1.0	1.0
$\frac{23}{2}{}^-_1 \to \frac{21}{2}{}^-_1$	4.92[4]	5.29[8]	0.002	
$\frac{25}{2}{}^-_1 \to \frac{21}{2}{}^-_1$	1.62[12]		1.0	1.0
$\frac{25}{2}{}^-_1 \to \frac{23}{2}{}^-_1$	1.09[11]	2.14[10]	0.080	0.418
$\frac{21}{2}{}^-_1 \to \frac{17}{2}{}^-_1$	9.65[11]		1.0	1.0
$\frac{21}{2}{}^-_1 \to \frac{19}{2}{}^-_1$	2.65[11]	6.93[10]	0.346	1.390
$\frac{17}{2}{}^-_1 \to \frac{13}{2}{}^-_1$	1.75[11]		1.0	1.0
$\frac{17}{2}{}^-_1 \to \frac{15}{2}{}^-_1$	2.24[11]	1.37[11]	2.060	16.67

[a]Reference 3.

IV. POSITIVE PARITY STATES

The positive parity calculation in ^{133}Pr were performed including in the basis, the Nilsson states $\frac{1}{2}[420]$, $\frac{3}{2}[422]$, $\frac{3}{2}[411]$, $\frac{5}{2}[413]$, $\frac{7}{2}[404]$, and $\frac{9}{2}[404]$, which lie near the Fermi level. For ^{135}Pr, an additional level $\frac{1}{2}[431]$ was also included in the basis states.

A. The nucleus ^{133}Pr

The ^{133}Pr isotope has been studied mainly through ^{118}Sn(^{19}F,4n) reactions.[1] The experimental data suggest the existence of two positive parity bands with opposite signatures based most probably on the $\frac{5}{2}^+[413]$ proton orbital. If the Coriolis interaction strength is reduced to 90% of their calculated values, these bands can be reproduced quite accurately up to the spin value $I^\pi = \frac{19}{2}^+$ (Fig. 3). For higher spin values, the calculated energies are higher than the corresponding experimental excitation energies. In addition, a positive parity band based mainly on the $\frac{3}{2}^+[411]$ orbital with spin values $I^\pi = \frac{3}{2}^+, \frac{7}{2}^+, \frac{11}{2}^+, \frac{15}{2}^+$, etc. is also found in the calculated spectrum. Excitation energies of the members of this band are somewhat higher (200–400 keV) than those of the corresponding members of the band based on the $\frac{5}{2}^+[413]$ orbital. The other members of this band with opposite signatures, having spin values $I^\pi = \frac{5}{2}^+, \frac{9}{2}^+, \frac{13}{2}^+$, etc., are found to be highly mixed in character. It would be interesting if the existence of these bands can be established through a suitable reaction study.

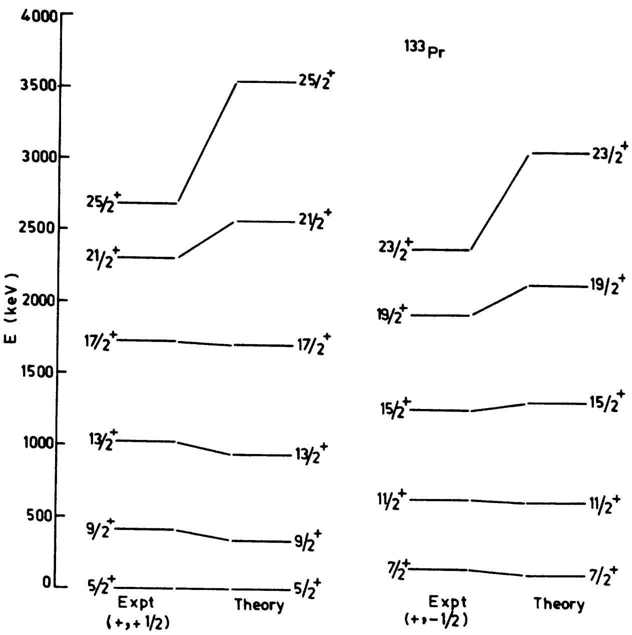

FIG. 3. Experimental (Ref. 1) and calculated positive-parity levels based on the $\frac{5}{2}[413]$ orbital in ^{133}Pr. The Coriolis attenuation factor is 0.8.

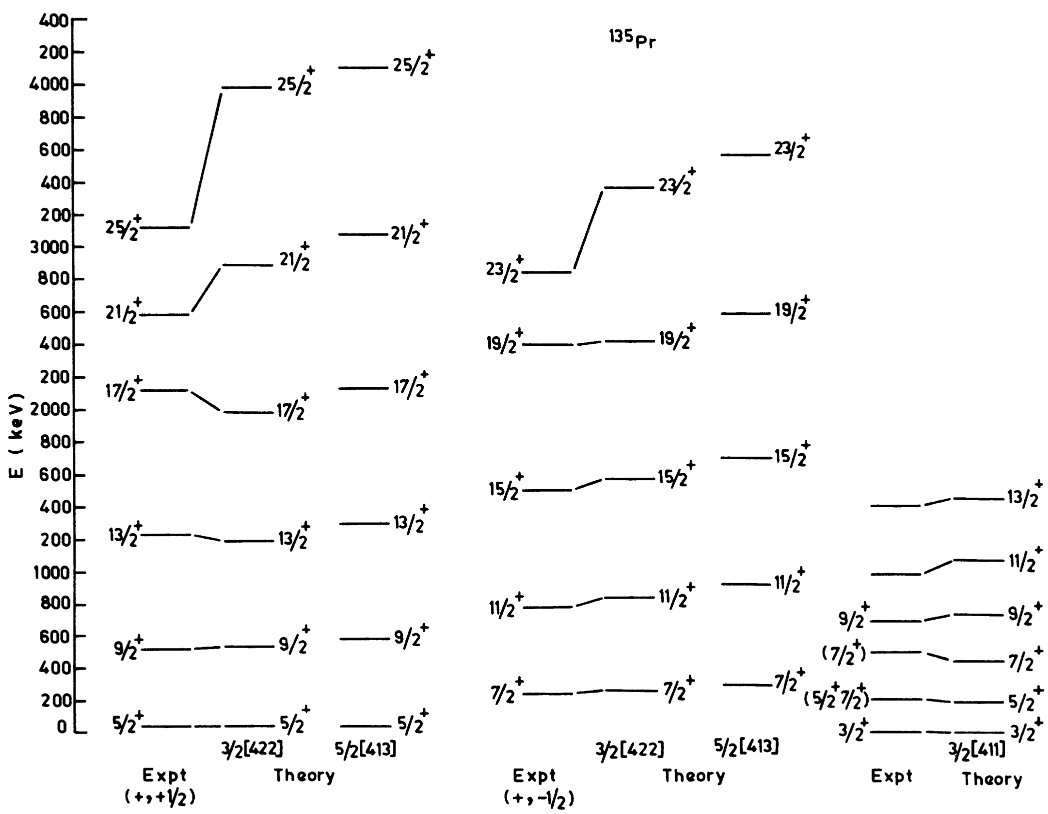

FIG. 4. Experimental (Refs. 2, 3, and 12) and calculated positive-parity levels based on the $\frac{3}{2}[422]$, $\frac{5}{2}[413]$, and $\frac{3}{2}[411]$ orbitals in ^{135}Pr when the Coriolis attenuation factor is zero.

TABLE II. Calculated decay modes of a few positive parity levels based on the $\frac{3}{2}$[422] orbital in ^{135}Pr for different Coriolis attenuation factors. The numbers listed under $T(E2+M1)$ are to be multiplied by powers of ten given in the square brackets.

Transition $I_i^\pi \rightarrow I_f^\pi$	$T(E2+M1)$ sec^{-1} Attn. Factor 0.1	0.4	0.9	Branching ratio Attn. Factor 0.1	0.4	0.9	Expt.[a]
$\frac{15}{2}^+ \rightarrow \frac{11}{2}^+$	8.93[11]	8.96[11]	8.88[11]	1.0	1.0	1.0	1.0
$\frac{15}{2}^+ \rightarrow \frac{13}{2}^+$	4.35[9]	3.66[9]	4.15[9]	0.005	0.004	0.005	0.086
$\frac{13}{2}^+ \rightarrow \frac{9}{2}^+$	7.22[11]	7.47[11]	7.47[11]	1.0	1.0	1.0	1.0
$\frac{13}{2}^+ \rightarrow \frac{11}{2}^+$	3.97[10]	2.99[10]	2.57[10]	0.055	0.040	0.034	0.308
$\frac{11}{2}^+ \rightarrow \frac{7}{2}^+$	1.36[11]	1.48[11]	1.49[11]	1.0	1.0	1.0	1.0
$\frac{11}{2}^+ \rightarrow \frac{9}{2}^+$	4.10[9]	3.12[9]	3.17[9]	0.030	0.021	0.021	0.077
$\frac{9}{2}^+ \rightarrow \frac{5}{2}^+$	4.83[10]	6.48[10]	6.75[10]	1.0	1.0	1.0	1.0
$\frac{9}{2}^+ \rightarrow \frac{7}{2}^+$	6.38[9]	4.29[9]	3.72[9]	0.132	0.066	0.055	0.18
$\frac{7}{2}^+ \rightarrow \frac{3}{2}^+$	1.45[8]	7.51[8]	1.00[9]	1.0	1.0	1.0	1.0
$\frac{7}{2}^+ \rightarrow \frac{5}{2}^+$	1.93[9]	1.24[9]	1.04[9]	13.310	1.651	1.04	11.045

[a]Reference 3.

B. The nucleus ^{135}Pr

The ground state spin of this isotope is $\frac{3}{2}^+$ and the spin of the first excited state at 41.5 keV is known to be $\frac{5}{2}^+$. In the decay work,[10] two $\frac{7}{2}^+$ states at 206.2 and 245.6 keV have been found. The $\frac{7}{2}^+$ state at 245 keV state decays predominantly to the first excited $\frac{5}{2}^+$ state through $M1$ transition, whereas the other state at 206.2 keV, tentatively assigned spin parity $\frac{7}{2}^+$, shows almost equal branching for decays to the ground ($\frac{3}{2}^+$) and the first excited ($\frac{5}{2}^+$) states. This shows that the $E2$ decay rate to the ground state must be considerably enhanced (if the spin is $\frac{7}{2}$) or the $M1$ transition rate to the first excited state to be considerably hindered. On the basis of the decay modes of the excited levels as found in the decay work, the excited states can be approximately classified into two groups. The ground state ($\frac{3}{2}^+$) and the excited states at 206.2 ($\frac{5}{2}^+$, $\frac{7}{2}^+$), 493.0 ($\frac{7}{2}^+$), 688.1 ($\frac{7}{2}^+$, $\frac{9}{2}^+$), and 984.0 keV etc. form one group, whereas the excited states at 41.5 ($\frac{5}{2}^+$), 245.6 ($\frac{7}{2}^+$), 517.3 ($\frac{7}{2}^+$, $\frac{9}{2}^+$), 777.4 keV, etc. form anther group. Recent experimental investigations[2,3] through ^{136}Ce(p,2n) and ^{120}Sn(^{19}F,4n) reactions have firmly established two positive parity bands of opposite signatures comprising of the states at 41.5 ($\frac{5}{2}^+$), 245.6 ($\frac{7}{2}^+$), 517.3 ($\frac{9}{2}^+$), 777.4 ($\frac{11}{2}^+$), 1232.1 ($\frac{13}{2}^+$), and 1505.9 ($\frac{15}{2}^+$) keV, etc. Several other excited levels have also been found, e.g., states at 206.0, 688.1, and 1409.7 keV, to which no definite band structure could be assigned.

An interesting feature is observed in the calculated spectrum. If the Coriolis interaction is artificially switched off (by putting the attenuation factor equal to zero), then it is found that the calculated positive parity band based on the $\frac{3}{2}^+$[422] orbital reproduces quite accurately the observed positive parity band structure (Fig. 4). Even the decay modes of the members of the band agree qualitatively with those observed experimentally (Table II). The calculated band based on the $\frac{5}{2}^+$[413] orbit is also shown in Fig. 4. It is found that of these two calculated bands based on $\frac{3}{2}^+$[422] and $\frac{5}{2}^+$[413] orbitals, the former is in better agreement with the experiment. Similarly, the observed states at 206, 493, 684, 984, and 1409 keV seem to form another band based on

TABLE III. Calculated decay modes of a few positive parity states based on the $\frac{3}{2}$[411] orbital in ^{135}Pr for the Coriolis attenuation factor equal to zero. The numbers listed under $T(E2)$ and $T(M1)$ are to be multiplied by powers of ten given in the square brackets.

Transition $I_i^\pi \rightarrow I_f^\pi$	$T(E2)$ sec^{-1}	$T(M1)$ sec^{-1}	Branching ratio Calc.
$\frac{13}{2}^+ \rightarrow \frac{9}{2}^+$	8.26[11]		1.0
$\frac{13}{2}^+ \rightarrow \frac{11}{2}^+$	8.54[9]	2.75[11]	0.343
$\frac{11}{2}^+ \rightarrow \frac{7}{2}^+$	4.22[11]		1.0
$\frac{11}{2}^+ \rightarrow \frac{9}{2}^+$	6.93[9]	1.94[11]	0.476
$\frac{9}{2}^+ \rightarrow \frac{5}{2}^+$	1.64[11]		1.0
$\frac{9}{2}^+ \rightarrow \frac{7}{2}^+$	5.21[9]	1.24[11]	0.788
$\frac{7}{2}^+ \rightarrow \frac{3}{2}^+$	3.64[10]		1.0
$\frac{7}{2}^+ \rightarrow \frac{5}{2}^+$	3.18[9]	6.40[10]	1.766

the $\frac{3}{2}^{+}[411]$ orbital (Fig. 4 and Table III). If the calculations are made with any reasonable value of the attenuation factor (0.7–0.9), then the observed spectra cannot be reproduced. In fact, a reasonable agreement with the observed experimental data can only be obtained if the Coriolis interaction strength is reduced to at least 40% of their theoretical values. The spectra calculated with different attenuation factors ($=0.1$, 0.4, and 0.9) are shown in Fig. 5 along with the experimental spectrum. It is intersting to note that the best agreement with the experimental energies can be obtained for the band with the signature $\alpha=+\frac{1}{2}$, $(-\frac{1}{2})$ with a value of the attenuation factor of 0.10 (0.40).

V. ATTENUATION OF CORIOLIS INTERACTION

In the particle-rotor model, the Coriolis term represents a coupling between the rotational and intrinsic motion and it distorts the normal rotational bands. However, it has also been known for a long time that the effect of the Coriolis term seems to be far too strong compared with experiment. This problem is normally tackled by artificially reducing the effect of the Coriolis interaction through the introduction of an attenuation factor. This attenuation factor has been found[11] to lie in the range 1.0–0.5 depending on the microscopic structure of the intrinsic wave functions. However, in most of the calculations done in the particle-rotor model, it has been found necessary to use this factor in order to reproduce the experimental spectra; no satisfactory physical argument has been found within the framework of the model as practiced in its simple version. Engeland[11] has discussed in detail this problem, and has emphasized the importance of correctly calculating the intrinsic structure in order to overcome this *ad hoc* reduction of the Coriolis interaction. Engeland and coworkers[11,12] have suggested an alternative approach of the particle-rotor model, where instead of using only one valence particle outside the deformed core, several valence particles outside the core are considered. The exact number of valence particle to be considered cannot be predicted before hand; rather it should be determined through proper reproduction of the experimental spectra. In their approach, the recoil term takes the form of a two-body operator and should be explicitly calculated. They have shown that, in several cases, artificial reduction of the Coriolis interaction can be overcome if their formalism is judiciously applied. Almberger *et al.*[13] have also discussed several possible mechanisms responsible for this attenuation. In our present work it is found that, even in the same nucleus, the negative parity structure can be accurately reproduced by almost the full strength of the Coriolis interaction, whereas the positive parity spectrum can only be reproduced by a significant attenuation of the same interaction. Therefore, the nucleus ^{135}Pr seems to be a very good testing ground of this alternative approach suggested by Engeland and co-workers.[11,12]

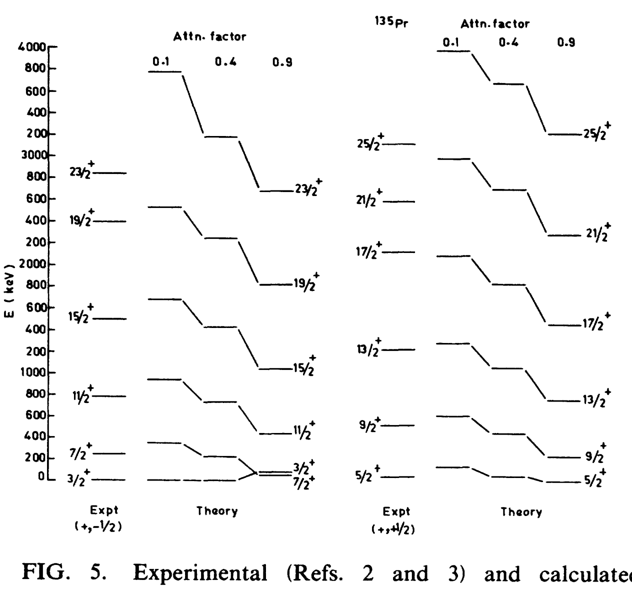

FIG. 5. Experimental (Refs. 2 and 3) and calculated positive-parity levels based on the $\frac{3}{2}[422]$ orbital for different values of the Coriolis attenuation factor.

VI. COMPARISON WITH OTHER THEORETICAL WORK

The band structure of the ^{135}Pr nucleus was calculated earlier by Wisshak *et al.*[4] and Kortelahti *et al.*[3] Wisshak *et al.* calculated the negative parity band structure based on the $1h_{11/2}$ orbital in a rotation alignment model incorporating the concept of variable moment of inertia and pairing formalism. The chemical potential λ was treated as an adjustable parameter to reproduce the negative parity band structure. Kortelahti *et al.* studied the positive and negative parity band structures in ^{135}Pr within the framework of a triaxial rotor-plus-particle model with $\beta=0.20$ and $\gamma=21°$. The model was successful in reproducing, qualitatively, the observed negative parity bands; however, for higher spin values the calculated energies were found to be somewhat higher than the corresponding experimental values. They also treated the chemical potential as an adjustable parameter. They did not make any comment about the decay modes of the excited states. The present model seems to be more successful in reproducing correctly the excitation energies as well as the decay modes of the band members, both positive and negative.

VII. CONCLUSIONS

A simple axially symmetric rotor model incorporating the ideas of the VMI approach was found to be successful in reproducing the properties of a number of transitional nuclei in the mass 70, 100, 120, and 150 regions.[5,6] The present work shows that the majority of the available experimental data on the transitional nuclei ^{133,135}Pr can also be reproduced within the framework of a similar approach. In fact, it is found that an axially symmetric description of the core gives better agreement with the experimental spectra than a triaxial model description with significant γ asymmetry. Several excited positive parity levels in ^{135}Pr, to which no definite band assignment could be given earlier, are suggested to form a band based on the $\frac{3}{2}^{+}[411]$ Nilsson orbital. Of course, there are several parameters in the calculation, but with reasonable choice of their values, the experimental spectra can be reproduced quite accurately. For higher spin values, the calculated energies are found to be systematically higher than their corresponding experimental values. It has already been pointed out that even a variable moment of inertia description of the core cannot reproduce correctly the energies of the high spin states. Moreover, in the present work, the moment of inertia for different spin values has been calculated using an expression[5] which has been derived on the basis of an assumption that $\mathcal{J}_0=0$, which is obviously not very realistic. If a nonzero value of $\mathcal{J}_0$ is assumed, then a similar expression with a different power $n<\frac{1}{3}$ has to be used; since it will involve the use of an additional parameter, we have not used it in the present work. However, the most interesting observation of the present work is the almost complete attenuation of the Coriolis interaction in the positive parity band structure in ^{135}Pr. No satisfactory explanation for this phenomenon can be given within the framework of the model used in the present work. It would be very interesting to calculate the band structure of these isotopes in the versions of the particle-rotor model suggested by several workers.[11–13] This work has already been undertaken and the results will be communicated later.

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