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Estimate of the triton asymptotic D to S ratio

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The role of the deuteron asymptotic D to S normalization ratio η^d in the calculation of triton observables is emphasized both within a simple model and in an exact numerical calculation. We suggest a new correlation among η^t/η^d and E_t in dynamical three nucleon calculations, where η^t is the triton asymptotic D to S ratio and E_t is the triton binding energy. Studying this correlation we obtained $\eta^t/\eta^d=1.68\pm0.04$.

I. INTRODUCTION

A strong correlation between the three-nucleon low-energy observables—especially between the spin doublet neutron deuteron (n-d) scattering length $^2a_{nd}$ and the triton binding energy E_t—was first emphasized by Phillips,[1] although the pioneering calculation by Aaron, Amado, and Yam[2] had already indicated such a trend. Such correlations have great practical importance in that they have proved to be very useful in predicting experimental values of as yet unmeasured observables or in confirming or refining known experimental quantities. For example, Phillips[1] suggested a value of $^2a_{nd}$, by studying such correlations, which was later confirmed experimentally.[3] Recently, Friar and co-workers[4] suggested a value of the spin doublet proton deuteron (p-d) scattering length $^2a_{pd}$, which contradicts the existing experimental values.[5] The value of the S wave asymptotic normalization C_S of the triton was suggested by Girard and Fuda[6] from a study of its variation with E_t using the partial wave dispersion relations. More recently, Ishikawa and Sasakawa[7] predicted the value of the triton asymptotic D to S normalization ratio η^t from a study of its variation with E_t.

It has been suggested by Kim and Tubis[8] that the triton asymptotic normalization parameters C_S and C_D should be given the same importance as other triton observables, such as, the binding energy E_t, the magnetic moment, etc., in any trinucleon calculation. The important role played by the asymptotic normalization parameter has been clearly demonstrated in the case of the deuteron.[9] We shall see that the triton asymptotic D to S normalization ratio η^t is extremely sensitive to the asymptotic D to S normalization ratio η^d of the deuteron; the specification of E_t is not enough to determine η^t uniquely.

Ishikawa and Sasakawa[7] did not worry about having the correct information for the D wave asymptotic normalization built into their model calculation. In this paper we show that, in addition to other few nucleon observables, a correct representation of the deuteron asymptotic normalization is important for a precise evaluation of η^t. In particular, we suggest a new correlation among the ratio η^t/η^d and E_t. Both in a simple analytic model and in a numerical separable potential model we find that η^t is linearly correlated with η^d at a fixed triton energy E_t, when the other two nucleon properties are fixed. Thus η^d plays a more important role in any calculation of η^t than had been realized. Using the correlation between η^t/η^d vs E_t we make our theoretical estimate of η^t/η^d consistent with $E_t=8.48$ MeV: $\eta^t/\eta^d=1.68\pm0.04$.

The paper is organized as follows. In Sec. II we develop a simple model for the ratio η^t/η^d which exhibits its explicit dependence on E_t. In Sec. III we present our numerical results based on a separable potential three body model calculation. Finally, in Sec. IV several concluding remarks are presented. A short version of this paper has already appeared.[10]

II. A SIMPLE MODEL FOR η^t/η^d

The nucleon-nucleon tensor force and, correspondingly, the deuteron D state have been essential in producing the electric quadrupole moment and refining the value of the magnetic dipole moment which are considered to be basic deuteron properties. Any nucleon-nucleon interaction to be used in a trinucleon calculation must have the correct deuteron properties built in, in addition to the on-shell properties of the singlet state. Will the trinucleon observables, especially η^t, be sensitive to any specific property of the deuteron D state? The study of Gibson and Lehman[11] suggests that the deuteron D wave probability P_D should be important in a triton calculation. They studied the correlation among C_S (and C_D) and P_D which should be considered important for predicting a precise value of C_S and C_D of the triton.

The deuteron D state probability P_D had been considered to be an important property of the deuteron ever since the D state was introduced. Recently, Amado and co-workers[12] commented that P_D should not be considered an observable of the deuteron. Because of theoretical difficulties,[12] no experiment, however accurate, will ever be able to fix the value of P_D. A theoretical calculation of P_D will be model dependent, as it will be sensitive to the deuteron wave function in the interior region. Thus a classification of triton properties as a function of P_D will be model dependent even if the "correct" value of P_D is known. To make matters worse there is no hope of determining P_D correctly. Amado and co-workers have suggested that P_D should not be given the status it has enjoyed, and have suggested that it is the deuteron asymptotic D to S normalization ratio η^{d} which should be given the "experimental" status of a single quantity to "measure" the D state. An analysis of the experimental data leads to a precise value of $\eta^{\mathrm{d}}=0.0271(4)$.[13] Therefore if a certain triton observable is sensitive to the value of η^{d}, apart from other few nucleon observables, a model must incorporate the correct value of η^{d} in order to make a model independent estimate of the particular triton observable. We shall see that the triton asymptotic normalization ratio η^{t} is sensitive to η^{d}, and this fact should be remembered while making a model independent estimate for η^{t}.

The correlation between η^{t} and η^{d} can be understood using some very general assumptions. The S and D wave triton asymptotic normalizations C_S and C_D are defined by

$$C_S=-\frac{2}{3}\left[\frac{2\pi}{\mu}\right]^{1/2}\lim_{k\to i\mu}\langle k,\phi_{\mathrm{d}}^{(23)},J_{\mathrm{t}}=\tfrac{1}{2},l=0,\hat{S}=\tfrac{1}{2}\,|\,V_2+V_3\,|\,\psi_{\mathrm{t}}\rangle\ , \tag{1a}$$

$$C_D=-\frac{2}{3}\left[\frac{2\pi}{\mu}\right]^{1/2}\lim_{k\to i\mu}\langle k,\phi_{\mathrm{d}}^{(23)},J_{\mathrm{t}}=\tfrac{1}{2},l=2,\hat{S}=\tfrac{3}{2}\,|\,V_2+V_3\,|\,\psi_{\mathrm{t}}\rangle \tag{1b}$$

in units of $\hbar=m=1$, where m is the nucleon mass, $\phi_{\mathrm{d}}^{(23)}$ is the deuteron wave function between particles 2 and 3, ψ_{t} is the triton wave function, and V_i is the nucleon-nucleon interaction between particles j and n where $i\neq j\neq n\neq i$. In Eqs. (1), k is the relative momentum between particle 1 and the deuteron (23), J_{t} is the total spin of the triton, l is the orbital angular momentum of neutron 1 with respect to the deuteron (23), $\hat{S}$ is obtained by coupling the total spin of the deuteron (23) and the spin of the neutron 1, and $\mu=\sqrt{4(E_{\mathrm{t}}-E_{\mathrm{d}})/3}$ where E_{t} is the triton binding energy and E_{d} is the deuteron binding energy.

It is easy to see that (1) can be written in terms of the Faddeev components ψ_i ($\psi_{\mathrm{t}}=\sum_{i=1}^{3}\psi_i$) of the triton wave function as

$$C_S=\frac{2}{3}\left[\frac{2\pi}{\mu}\right]^{1/2}\lim_{k\to i\mu}\langle k,\phi_{\mathrm{d}}^{(23)},J_{\mathrm{t}}=\tfrac{1}{2},l=0,\hat{S}=\tfrac{1}{2}\,|\,E_{\mathrm{d}}+p_1^2\,|\,\psi_2+\psi_3\rangle\ , \tag{2a}$$

$$C_D=\frac{2}{3}\left[\frac{2\pi}{\mu}\right]^{1/2}\lim_{k\to i\mu}\langle k,\phi_{\mathrm{d}}^{(23)},J_{\mathrm{t}}=\tfrac{1}{2},l=2,\hat{S}=\tfrac{3}{2}\,|\,E_{\mathrm{d}}+p_1^2\,|\,\psi_2+\psi_3\rangle\ , \tag{2b}$$

where p_1 is the relative momentum between the particles 2 and 3 forming the deuteron (23).

As expression (2) does not explicitly contain the nucleon-nucleon interactions, it is easy to see that this expression contains information about the asymptotic properties of the triton wave function ψ_{t}, or its Faddeev components ψ_i, because the integrations in this expression will extend beyond the range of the interaction V_i.

We shall present very plausible arguments for an approximate calculation of (2), which will clearly yield the desired correlation between η^{t} and η^{d}. For our purpose we evaluate expression (2b) approximately by maintaining only those parts of ψ_2 and ψ_3 in which the nucleon pairs, (13) and (12), are in the spin triplet state. Assuming that particle 3 is a neutron and that only the S wave nucleon-nucleon interaction is important, only ψ_3 can contain two nucleons (12) in the spin triplet state, because in ψ_2 the two nucleons (13) are neutrons which can exist only in the spin singlet state. Thus only ψ_3 will contribute. The Faddeev component ψ_3 outside the range of V_3 satisfies a free Schrödinger equation. Assuming that the S wave nucleon-nucleon interaction is the only interaction responsible for triton binding, the form of ψ_3 must be

$$\psi_3(\mathbf{r}_3,\mathbf{R}_3)=\frac{[2(E_{\mathrm{d}})^{1/2}]^{1/2}}{16\pi^{5/2}}\int d^3q_3C_S^{\mathrm{d}}\frac{e^{-[E_{\mathrm{t}}+(3/4)q_3^2]^{1/2}R_3}}{R_3}\times e^{i\mathbf{q}_3\cdot\mathbf{r}_3}\chi(\mathbf{q}_3)\ . \tag{3}$$

This is the exact form of ψ_3 outside the range of V_3, independent of the form of the nucleon-nucleon interaction. We assume the form (3) to be valid over all space.

Here $C_S^{\rm d}$ is the S wave asymptotic normalization of the deuteron. In Eq. (3), $(\mathbf{r}_3,\mathbf{R}_3)$ is the usual set of Jacobi coordinates: $\mathbf{R}_3$ is the relative separation between particles 1 and 2 and $\mathbf{r}_3$ that between particle 3 and pair (12). All the information about the interaction of the nucleons inside the triton is contained in the spectator function $\mathcal{X}(\mathbf{q}_3)$. Asymptotically, as $r_3\to\infty$, ψ_3 has the form

$$\psi_3(\mathbf{r}_3,\mathbf{R}_3)\underset{r_3\to\infty}{\longrightarrow}[2(E_{\rm d})^{1/2}]^{1/2}C_S^{\rm d}\frac{e^{-\sqrt{E_{\rm d}}R_3}}{R_3}Y_{00}(\hat{R}_3)\frac{C_S}{\sqrt{2}}\times\sqrt{2\mu}\frac{e^{-\mu r_3}}{r_3}Y_{00}(\hat{r}_3)\ . \tag{4}$$

Condition (4) requires that the spectator function $\mathcal{X}(\mathbf{q}_3)$ of (3) has a pole at $q_3=i\mu$, which is taken as

$$\mathcal{X}(\mathbf{q}_3)=\left[\frac{4\mu}{\pi}\right]^{1/2}\frac{C_S}{\mu^2+q_3^2}\ . \tag{5}$$

The Fourier transform of (3) yields the momentum space wave function

$$\langle \mathbf{p}_3,\mathbf{q}_3\,|\,\psi_3\rangle=\frac{2}{\pi}[2\mu(E_{\rm d})^{1/2}]^{1/2}\frac{C_S}{(\mu^2+q_3^2)}\frac{C_S^{\rm d}}{(E_{\rm t}+\frac{3}{4}q_3^2+p_3^2)}\times Y_{00}(\hat{q}_3)Y_{00}(\hat{p}_3)\ , \tag{6}$$

where $\mathbf{p}_3$ is the relative momentum between particles 1 and 2, and $\mathbf{q}_3$ is the relative momentum between particle 3 and the deuteron (12). In Eq. (6) the deuteron is assumed to be in the S state only.

In order to evaluate expression (2) we employ the minimal (zero range) deuteron wave function given in momentum space by[12]

$$\phi_{\rm d}(\mathbf{p})=\frac{1}{E_{\rm d}+p^2}\left[\frac{4(E_{\rm d})^{1/2}}{\pi}\right]^{1/2}C_S^{\rm d}\left[\mathcal{Y}_{01}^{1M}(\hat{p})-\eta^{\rm d}\frac{p^2}{E_{\rm d}}\mathcal{Y}_{21}^{1M}(\hat{p})\right], \tag{7}$$

where $\mathcal{Y}_{LS}^{JM}$ is the spin-angular momentum function defined by

$$\mathcal{Y}_{LS}^{JM}(\hat{p})=\sum_{M_S,M_L}C_{M\,M_L\,M_S}^{J\,L\,S}Y_{LM_L}(\hat{p})\,|\,SM_S\rangle\ .$$

If we now allow the deuteron (12) also to be in a D state, then the modified version of Eq. (6) can be easily written, using Eq. (7), as (now including the spin functions)

$$\langle \mathbf{p}_3,\mathbf{q}_3\,|\,\psi_3\rangle=\frac{2}{\pi}[2\mu(E_{\rm d})^{1/2}]^{1/2}\frac{C_S}{(\mu^2+q_3^2)}\frac{C_S^{\rm d}}{(E_{\rm t}+\frac{3}{4}q_3^2+p_3^2)}Y_{00}(\hat{q}_3)\times\left\{\sum_{m,m_S}\left[C_{M_{\rm t}\,m\,m_S}^{1/2\;1\;1/2}\mathcal{Y}_{01}^{1m}(\hat{p}_3)\,|\,\tfrac{1}{2}m_S\rangle-\eta^{\rm d}\frac{(p_3)^2}{E_{\rm d}}C_{M_{\rm t}\,m\,m_S}^{1/2\;1\;1/2}\mathcal{Y}_{21}^{1m}(\hat{p}_3)\,|\,\tfrac{1}{2}m_S\rangle\right]\right\}. \tag{8}$$

Next, substituting Eqs. (7) and (8) in expression (2b), and maintaining only terms linear in $\eta^{\rm d}$, we obtain, after some straightforward angular momentum algebra,

$$\frac{\eta^{\rm t}}{\eta^{\rm d}}=\frac{16}{3\pi}(C_S^{\rm d})^2\int_0^\infty\frac{q^2dq}{[(\mu^2/E_{\rm d})+q^2]}\int_{-1}^{+1}\frac{dx}{1-\frac{1}{4}(\mu^2/E_{\rm d})+q^2+(i\mu/E_{\rm d}^{1/2})qx}\times\left[-\frac{q^2}{8}P_2(x)+\frac{i\mu}{4\sqrt{E_{\rm d}}}qP_1(x)-\frac{7}{16}\frac{\mu^2}{E_{\rm d}}P_0(x)\right]. \tag{9}$$

It is expected from Eq. (9) that if $E_{\rm t}$ is held constant in a dynamical calculation, $\eta^{\rm t}$ varies linearly with $\eta^{\rm d}$, assuming that $C_S^{\rm d}$ varies very slowly in the process. In the limit $E_{\rm t}\to E_{\rm d}$, from Eq. (9) we find that $\eta^{\rm t}/\eta^{\rm d}\sim\phi^2$, where $\phi^2=(E_{\rm t}-E_{\rm d})/E_{\rm d}$. In the other extreme, $E_{\rm t}\gg E_{\rm d}$, $\mu\to\infty$, Eq. (9) yields $\eta^{\rm t}/\eta^{\rm d}\sim\phi$. We can easily combine these analytic behaviors in the following simple form:

$$\frac{\eta^{\rm t}}{\eta^{\rm d}}\sim(C_S^{\rm d})^2\frac{\phi^2}{\phi+\gamma}\ , \tag{10}$$

where γ is a constant which can be obtained by fitting the result of Eq. (9).

Equation (10) is the principal result of this section, which clearly demonstrates two interesting properties. First, if the ratio of the triton and deuteron binding energies and the parameter $C_S^{\rm d}$ are held fixed in dynamical calculations one clearly sees a linear correlation between $\eta^{\rm t}$ and $\eta^{\rm d}$, which we shall confirm in a separable potential model in Sec. III. Second, it is obvious from Eq. (10) that as $E_{\rm t}/E_{\rm d}$ increases in such calculations, the ratio $\eta^{\rm t}/\eta^{\rm d}$ increases, which will also be verified in Sec. III. The linear correlation between $\eta^{\rm t}$ and $\eta^{\rm d}$ was observed recently by Ericson and Rosa-Clot.[9] They did not give any explanation of the behavior, however.

III. NUMERICAL RESULTS

To test the above ideas we performed a Faddeev calculation using a separable $^1S_0$ and $^3S_1$-$^3D_1$ interaction

TABLE I. Values of η^t for some choices of η^d and ${}^1r_0$, with ${}^1a_0=-23.7$ fm (-17 fm) which yield $E_t=8.48$ MeV. Also shown are the parameters of the Yamaguchi tensor interaction, $\langle pJLM \mid V \mid p'JL'M \rangle = -\lambda_{\rm tens} g_L(p) g_{L'}(p')$, where $g_0(p)=1/(\alpha_0^2+p^2)$ and $g_2(p)=tp^2/(\alpha_2^2+p^2)^2$, for the N-N triplet parameters: ${}^3a_1=5.424$ fm, $E_d=2.225$ MeV, and a deuteron quadrupole moment of 0.2859 fm^2.

${}^1r_0$ (fm)	$\lambda_{\rm tens}$ (fm^{-3})	α_0 (fm^{-1})	α_2 (fm^{-1})	t	η^d	η^t
2.010 (1.820)	1.2799	1.1881	2.1079	−6.6142	0.0250	0.0419 (0.0427)
2.780 (2.628)	2.3950	1.2619	1.6938	−2.6413	0.0275	0.0462 (0.0473)
3.059 (2.917)	3.0135	1.2925	1.4659	−1.5188	0.0300	0.0502 (0.0514)

with Yamaguchi[14] form factors. In our calculation we always keep the triplet parameters—deuteron binding, triplet scattering length ${}^3a_1$, and electric quadrupole moment—fixed and vary η^d. The parameter C_S^d remains practically constant in our analysis. We keep the provision of varying the singlet effective range ${}^1r_0$ and scattering length ${}^1a_0$. We use ${}^1a_0=-17$ and -23.7 fm to simulate the neutron-neutron and the proton-neutron systems, respectively. The variation of ${}^1r_0$ is motivated by an attempt to better understand the origin of certain low energy correlations. We shall comment on this variation later. Apart from this flexibility our model is very similar to that of Ref. 11. Our model uses a deuteron binding energy of $E_d=2.225$ MeV, ${}^3a_1=5.424$ fm, a deuteron quadrupole moment of 0.2859 fm^2, and an η^d from 0.025 to 0.030. From these quantities we obtain the four parameters of the Yamaguchi ${}^3S_1$-${}^3D_1$ separable interaction. The triton D state appears in our model only when we include the tensor ${}^3S_1$-${}^3D_1$ nucleon-nucleon interaction, which produces a D wave component in the deuteron. The single quantity which measures the deuteron D state is the asymptotic quantity η^d. In order to study the correlation between η^t and η^d in our model we chose ${}^1r_0$ at each value of η^d such that our model produces the experimental triton binding energy of 8.48 MeV, with the other deuteron parameters and ${}^1a_0$ fixed, as we show in Table I. We plotted the resulting η^t vs η^d in Fig. 1, which yields a linear curve passing through the origin $\eta^d=\eta^t=0$. This result is consistent with Eq. (10).

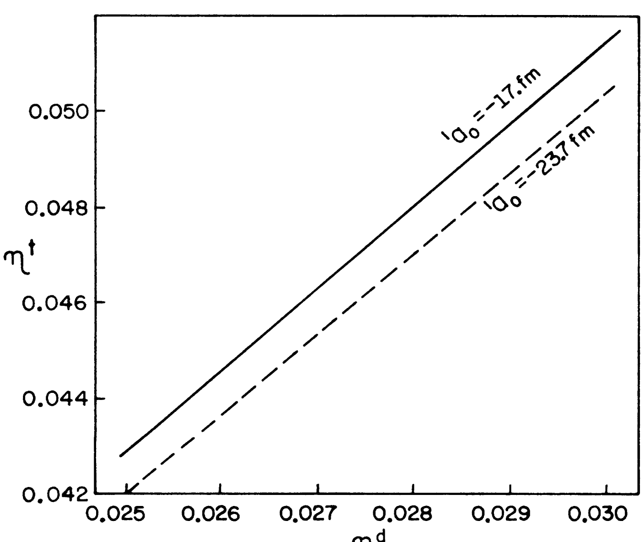

FIG. 1. η^t-η^d correlation at a fixed E_t ($=8.48$ MeV) using the present separable potential model with ${}^1a_0=-17$ and -23.7 fm. We vary ${}^1r_0$ to generate the points which lie on the straight lines passing through the origin.

We also calculated the neutron-deuteron scattering length ${}^2a_{\rm nd}$ for these points. The variation of ${}^1r_0$ in the range 1.8 to 3.6 fm generated a small variation in the value of ${}^2a_{\rm nd}$, which contributes to the width of the Phillips plot at a fixed energy. Therefore we observed from Fig. 1 that in our model, when we keep the deuteron properties ${}^3a_1$ and E_d fixed and vary η^d, then η^t varies linearly with η^d and is insensitive to small variations of ${}^2a_{\rm nd}$. This conclusion will be confirmed in the plot of Fig. 3, which is in part motivated by this conclusion. Therefore any attempt at a theoretical evaluation of η^t must take into consideration the correct value of η^d. The parameter η^t, being an asymptotic observable, is expected to be insensitive to the interior part of the triton wave function, and, hence, to ${}^2a_{\rm nd}$, except for the information about ${}^2a_{\rm nd}$ which is indirectly contained in E_t.

To see the importance of η^d in a model three nucleon calculation of η^t, we plot in Fig. 2 the values of $\eta^t/\eta^d_{\rm expt}$ as a function of the triton binding energy, where $\eta^d_{\rm expt}$ ($=0.0271$) is η^d from Ref. 13. This plot is equivalent to that of η^t vs E_t as $\eta^d_{\rm expt}$ is just a constant.

In Fig. 2 we show the results for η_t and E_t for several

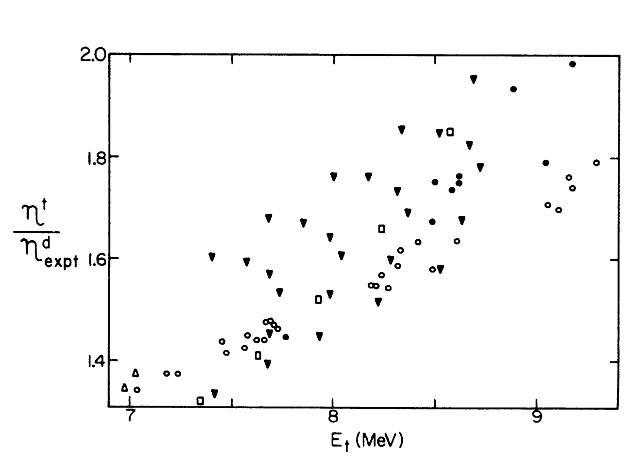

FIG. 2. $\eta^t/\eta^d_{\rm expt}$-E_t. The open circles are from Ref. 7, the solid circles from Ref. 16, the open triangles from Ref. 15, the open squares from Ref. 11, and the solid inverted triangles represent our separable potential calculations with ${}^1a_0=-17$ fm for various values of ${}^1r_0$ and η^d.

realistic potential models with and without three-nucleon forces,[7,15] separable potential models,[11,16] and our calculations for ${}^1a_0=-17$ fm which we obtained by varying both ${}^1r_0$ and η^d. As the various models have different values of η^d, the width of the band for η^t vs E_t is enhanced when compared with that of Ref. 7, which makes a model independent estimative for η^t difficult. We must stress that the differences in $(C_S^d)^2$ between the realistic model and our model are less than 0.5%.

Motivated by the linear correlation of η^t vs η^d of Fig. 1, we plot in Fig. 3 η^t/η^d vs E_t. In Fig. 3 we show results corresponding to all the calculations of Fig. 2. In addition, we exhibit results of our separable potential calculations for ${}^1a_0=-23.7$ fm, which we generated by varying both ${}^1r_0$ and η^d. Our results of ${}^1a_0=-17$ and -23.7 fm fall on two straight lines and the calculation of Gibson and Lehman[11] for ${}^1a_0=-16.85$ fm coincides with our calculation of ${}^1a_0=-17$ fm. We see that all the calculations fall within a very narrow band, in sharp contrast to the results of Fig. 2. In our calculation we varied ${}^1r_0$ and η^d. (The variation of ${}^1r_0$ has been shown in a different context to simulate[18] the off-shell variation of two-nucleon interactions in three-nucleon calculations.) We verify that the parameter η^t/η^d does not contain much information about the off-shell behavior of nucleon-nucleon interaction except that contained in the value of E_t.

In order to do complete justice to Eq. (10), we utilize it for extrapolating each of the calculated values of η^t/η^d to the experimental energy $E_t=8.48$ MeV. Equation (10) can be rewritten as

$$\frac{\eta^t}{\eta^d}\equiv f(\phi)=K\frac{\phi^2}{\phi+\gamma}, \quad \gamma=1 . \tag{11}$$

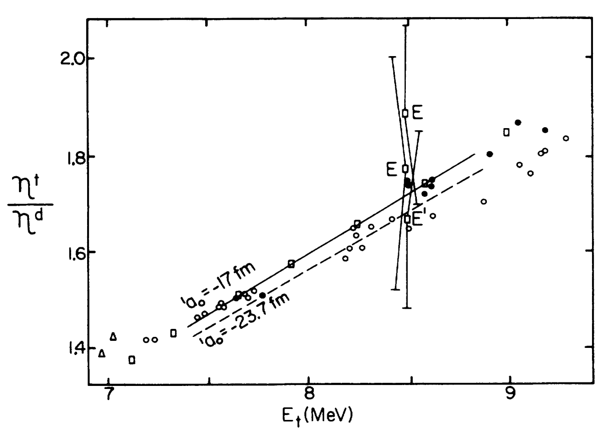

FIG. 3. η^t/η^d-E_t correlation. The straight lines are generated by varying both ${}^1r_0$ and η^d in the present separable potential model for ${}^1a_0=-17$ and -23.7 fm. The open circles are from Ref. 7, the solid circles from Ref. 16, the triangles from Ref. 15, and the open squares are from Ref. 11; the points marked E and E' are experimental results of Refs. 17 and 19 with $\eta^d=(0.0271\pm0.0004)$ of Ref. 13.

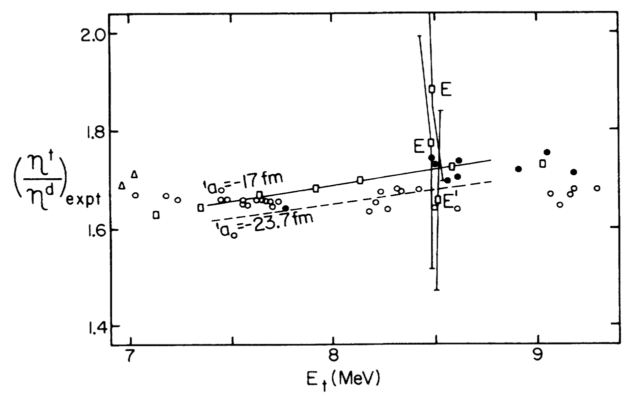

FIG. 4. $(\eta^t/\eta^d)_{expt}$-E_t. Values of η^t/η^d extrapolated to the experimental triton binding energy. For explanation see Fig. 3.

The constant K of Eq. (11) was determined by a least squares fit to all points of Fig. 2, and the value of $\gamma=1$ is obtained by fitting (11) to an exact numerical evaluation of (9) for a wide range of ϕ. The extrapolation of η^t/η^d to the experimental E_t was made in the following simple way, which is independent of K:

$$\left[\frac{\eta^t}{\eta^d}\right]_{expt}=\left[\frac{\eta^t}{\eta^d}\right]_{calc}\frac{f(\phi_{expt})}{f(\phi_{calc})}, \tag{12}$$

where calc and expt denote the theoretically calculated and extrapolated values of the quantity concerned. In Fig. 4 we present the extrapolated η^t/η^d and the calculated binding energies for all points of Fig. 3. The extrapolation of η^t/η^d to $E_t=8.48$ MeV by our procedure shows its reliability and the desired model independence is revealed to some extent. All the realistic calculations fall in a very narrow band, and the nonlocal separable model calculations have a large dispersion which represents, for η^t/η^d, general off-shell variations. Our theoretical prediction of η^t/η^d is calculated by taking the arithmetic mean of the points presented in Fig. 4. The arithmetic mean for the local potentials yields $\eta^t/\eta^d=1.66\pm0.02$ $(K=1.58\pm0.02)$. The overall average of all the points yields $\eta^t/\eta^d=1.68\pm0.04$ $(K=1.60\pm0.04)$.

IV. CONCLUSION

We have studied new correlations between the triton and deuteron asymptotic normalization parameters. These correlations are important for making theoretical estimates of the triton asymptotic D to S ratio. We showed within a simple model the origin of the strong correlation between η^t and η^d for a fixed triton binding energy and verified this in a separable potential model calculation. In view of our discussion, one must bear in mind that a good theoretical evaluation of η^t cannot be separated from the precise knowledge of η^d.

In addition, we obtained a model independent analytical formula which expresses the behavior of η^t/η^d as the triton binding energy is varied. We have used this expression to extrapolate the theoretical potential model values for η^t/η^d to the experimental value $E_t = 8.48$ MeV, and with that we have made an overall average to obtain this quantity. We found that the value of $\eta^t/\eta^d = 1.68 \pm 0.04$ is associated with a wide class of possible off-shell variations in triton calculations. This indicates that unless the experimental error for this observable can be lowered to less than 3%, we cannot choose the best model, due to the strong model independence which is broken only at the level of a very few percentage points of precision.

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