Configuration mixing and electromagnetic properties of odd-even nuclei

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The electromagnetic properties of ground and excited states of nuclei with an odd number of particles and holes outside the closed shells are described in the *j-j* coupling shell model. The residual interaction between the valence particles and holes causes two different configuration mixing: (a) one where the valence particles and holes are scattered to higher shell model states, and (b) ones where the valence particles and holes excite particles out of the core. The electromagnetic properties of odd-even nuclei are sensitive to the correct treatment of both types of configuration mixing. In this paper the matrix elements of the nuclear Hamiltonian evaluated using the basis of configuration mixing states of types (a) and (b), and the resulting eigenvalue equations, have been exactly calculated. The matrix elements of the electromagnetic multipole operators calculated with the resulting eigenvectors of type (b) are then related to the deviation observed between the experimental and valence expectation values.

I. INTRODUCTION

The importance of configuration mixing in studying the electromagnetic properties of odd-even nuclei with one valence particle (hole) has been pointed out by several authors.[1] The residual interactions between particles in different orbits are taken as the cause of configuration mixing and the correction to the single particle matrix elements of the electromagnetic operators has been calculated introducing selected particle-hole configurations.

The importance of introducing configuration mixing diagonalizing the nuclear Hamiltonian, in a nonrestricted $\Phi^1_{JM}(\alpha_1 J_1)$ and $\tilde{\Psi}^1_{JM}(\rho_1 J_1)$ basis, has been pointed out in Refs. 2 and 3. In the present paper we generalize the calculations, including in the model configuration mixing modes where the single particle (hole) excite out of the closed *j-j* core *n* particles. The calculation is then extended to odd-even nuclei with valence particles and holes interacting with the core via particle excitations.

In Sec. II we calculate the commutators of the nuclear Hamiltonian with the configuration mixing modes, retaining only the Tamm-Dancoff (TDA) diagrams. The matrix elements of the nuclear Hamiltonian calculated with the configuration mixing wave functions (CMWF) of the nth kind are then grouped in terms of the matrix elements calculated with the CMWF of the $(n-1)$ kind.

In Sec. III we expand the $\Phi^2_{JM}(\alpha_2 J_1 J_2)$ (three particle–two hole) CMWF in linear combinations of state vectors coupled

$$[\Phi^1_{J_r}(\alpha_1 J_1)\otimes\Phi^0_{J_s}(\bar{\alpha}_1 J_s)]^J ,$$

$$[\tilde{\Psi}^1_{J_r}(\rho_1 J_1)\otimes\Lambda^0_{J_s}(\bar{\rho}_1 J_s)]^J ,$$

and

$$[\Lambda^1_{J_r}(\epsilon_1 J_1)\otimes M^0_{J_s}(\bar{\epsilon}_1 J_s)]^J .$$

In Sec. IV we derive an iterative expression for the transformation coefficients. In Secs. V and VI we generalize the method expanding the $\Phi^n_{JM}(\alpha_n J_1 J_2 \cdots J_n)$ and $\tilde{\Psi}^n_{JM}(\rho_n J_1 J_2 \cdots J_n)$ CMWF and deriving the transformation coefficients iteratively. The general expressions for the matrix elements of the nuclear Hamiltonian in the basis of the $\Phi^n_{JM}(\alpha_n J_1 J_2 \cdots J_n)$ and $\tilde{\Psi}^n_{JM}(\rho_n J_1 J_2 \cdots J_n)$ CMWF are given in Secs. VII and VIII. And finally in Sec. IX we calculate the matrix elements of the electromagnetic operators.

The coupling of the valence particles and holes to complicated doorway states is exactly taken into consideration in the present paper. The nuclear Hamiltonian is then calculated in the full basis of CMWF. The ground and excited state wave functions are therefore showing contributions of perturbation terms to all order. As shown in Refs. 2 and 3 these contributions play an important role in studying the electromagnetic properties of odd-even nuclei.

II. CONFIGURATION MIXING AND LINEARIZATION METHOD

Odd-even nuclei with particles and holes outside the closed shells, interacting with the core via particle excitations, are characterized by wave functions of the following forms:

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A. CMWF for n particles–$(n-1)$ holes

$$|\Phi^n_{JM}(\alpha_n J_1 J_2\cdots J_n)\rangle = N\left[\sum_{\alpha_n^1 J_1^1 J_2^1\cdots J_n^1} X^n_{\alpha_n^1 J_1^1 J_2^1\cdots J_n^1 J} N^n_{\alpha_n^1 J_1^1 J_2^1\cdots J_n^1 J}|\Phi^n_{JM}(\alpha_n^1 J_1^1 J_2^1\cdots J_n^1)\rangle + \sum_{\alpha^2_{n_1} J_1^2 J_2^2\cdots J^2_{n_1}} X^{n_1}_{\alpha^2_{n_1} J_1^2 J_2^2\cdots J^2_{n_1} J} N^{n_1}_{\alpha^2_{n_1} J_1^2 J_2^2\cdots J^2_{n_1} J}|\Phi^{n_1}_{JM}(\alpha^2_{n_1} J_1^2 J_2^2\cdots J^2_{n_1})\rangle + \cdots + \sum_{\alpha^n_{n_n} J_1^n J_2^n\cdots J^n_{n_n}} X^{n_n}_{\alpha^n_{n_n} J_1^n J_2^n\cdots J^n_{n_n} J} N^{n_n}_{\alpha^n_{n_n} J_1^n J_2^n\cdots J^n_{n_n} J}|\Phi^{n_n}_{JM}(\alpha^n_{n_n} J_1^n J_2^n\cdots J^n_{n_n})\rangle\right] . \tag{2.1}$$

B. CMWF for $(n-1)$ particles–n holes

$$|\tilde\Psi^n_{JM}(\rho_n J_1 J_2\cdots J_n)\rangle = N\left[\sum_{\rho_n^1 J_1^1 J_2^1\cdots J_n^1} Y^n_{\rho_n^1 J_1^1 J_2^1\cdots J_n^1 J} N^n_{\rho_n^1 J_1^1 J_2^1\cdots J_n^1 J}|\tilde\Psi^n_{JM}(\rho_n^1 J_1^1 J_2^1\cdots J_n^1)\rangle + \sum_{\rho^2_{n_1} J_1^2 J_2^2\cdots J^2_{n_1}} Y^{n_1}_{\rho^2_{n_1} J_1^2 J_2^2\cdots J^2_{n_1} J} N^{n_1}_{\rho^2_{n_1} J_1^2 J_2^2\cdots J^2_{n_1} J}|\tilde\Psi^{n_1}_{JM}(\rho^2_{n_1} J_1^2 J_2^2\cdots J^2_{n_1})\rangle + \cdots + \sum_{\rho^n_{n_n} J_1^n J_2^n\cdots J^n_{n_n}} Y^{n_n}_{\rho^n_{n_n} J_1^n J_2^n\cdots J^n_{n_n} J} N^{n_n}_{\rho^n_{n_n} J_1^n J_2^n\cdots J^n_{n_n} J}|\tilde\Psi^{n_n}_{JM}(\rho^n_{n_n} J_1^n J_2^n\cdots J^n_{n_n})\rangle\right] , \tag{2.2}$$

with

$n_1 \equiv n$ +one particle-hole pair ,

$n_2 \equiv n$ +two particle-hole pairs ,

and

$n_n \equiv n+(n)$ particle-hole pairs .

Let us recall the results of Refs. 2 and 3, neglecting the single particle component and the off-diagonal terms that will be treated in Sec. VIII. The one-particle-hole–one-particle-hole pair ($n=1$, $n_1=2$) or the two-particle (-hole)–one-hole (-particle) ($n=2$) components of the (2.1) and (2.2) CMWF, both characterized by one particle-hole pair, are

$$|\Phi^1_{JM}(\alpha_1 J_1)\rangle = \sum_{\alpha_1' J_1'} X^1_{\alpha_1' J_1' J} N^1_{\alpha_1' J_1' J}|\Phi^1_{JM}(\alpha_1' J_1')\rangle = \sum_{\alpha_1' J_1'} X^1_{\alpha_1' J_1' J} N^1_{\alpha_1' J_1' J} A_1^\dagger(\alpha_1' J_1'; JM)|0\rangle \tag{2.3}$$

and

$$|\tilde\Psi^1_{JM}(\rho_1 J_1)\rangle = \sum_{\rho_1' J_1'} Y^1_{\rho_1' J_1' J} N^1_{\rho_1' J_1' J}|\tilde\Psi^1_{JM}(\rho_1' J_1')\rangle = \sum_{\rho_1' J_1'} Y^1_{\rho_1' J_1' J} N^1_{\rho_1' J_1' J} A_1(\rho_1' J_1'; J-M)|0\rangle , \tag{2.4}$$

where the operators $A_1^\dagger(\alpha_1' J_1'; JM)$ and $A_1(\rho_1' J_1'; J-M)$ create, respectively, a particle (hole) in a shell model state with quantum number $\{j_1\}$ ($\{j_1^{-1}\}$) and coupled then with a particle-hole pair with quantum numbers $\{j_2\},\{j_3^{-1}\}$ and total spin J_1' to a final spin $JM(J-M)$.

To simplify the notation we use the following conventions:

(a) $\alpha_1 \equiv \rho_1 \equiv \{n_1 l_1 j_1; n_2 l_2 j_2; n_3 l_3 j_3\}$.

(b) The particle-hole pair $\{j_2 - j_3^{-1}\}$ is coupled to the intermediate spin $J_1' M_1'$.

(c) The sum over all the projection quantum numbers is not included explicitly in the formulas.

(d) The subindex $\{1\}$ indicates the number of particle-hole pairs we are considering.

(e) $N^1_{\binom{\alpha_1'}{\rho_1'} J_1' J}$ are the normalization factors different from 1 if the coordinates of the single particle $\{j_1\}$ or of the single hole $\{j_1^{-1}\}$ are equivalent to the coordinates of the $\{j_2\}$ particle or of the $\{j_3^{-1}\}$ hole.

Calculating the commutators of (2.3) and (2.4) with the nuclear Hamiltonian

$$H=\sum \epsilon_\alpha a_\alpha^\dagger a_\alpha + \tfrac{1}{2}\sum_{\alpha\beta\gamma\delta} V_{\alpha\beta\gamma\delta} a_\alpha^\dagger a_\beta^\dagger a_\delta a_\gamma ,$$

where $V_{\alpha\beta\gamma\delta}$ are the matrix elements of the effective residual interaction

$$V_{\alpha\beta\gamma\delta} = (\alpha\beta|v^{\text{eff}}|\gamma\delta) ,$$

and linearizing we obtain in the TDA approximation

$$[H, A_1^\dagger(\alpha_1' J_1'; JM)] = \sum_{\beta_1 J_1''} \Omega_1^1(\alpha_1' J_1' J; \beta_1 J_1'' J) A_1^\dagger(\beta_1 J_1''; JM) \tag{2.5}$$

and

$$[H, A_1(\rho_1' J_1'; J-M)] = \sum_{\sigma_1 J_1''} \Omega_2^1(\rho_1' J_1' J; \sigma_1 J_1'' J) A_1(\sigma_1 J_1''; J-M) , \tag{2.6}$$

with

$$\Omega_1^1(\alpha_1'J_1'J;\beta_1J_1''J)\equiv\langle\Phi_J^1(\alpha_1'J_1')\|H\|\Phi_J^1(\beta_1J_1'')\rangle \quad (2.7)$$

and

$$\Omega_2^1(\rho_1'J_1'J;\sigma_1J_1''J)\equiv\langle\Psi_J^1(\rho_1'J_1')\|H\|\Psi_J^1(\sigma_1J_1'')\rangle\ . \quad (2.8)$$

The single particle and single hole energies, the particle-particle, the hole-hole, and the particle-hole matrix elements are contained in the $\Omega_1^1(\alpha_1'J_1'J;\beta_1J_1''J)$ and $\Omega_2^1(\rho_1'J_1'J;\sigma_1J_1''J)$ matrices.

Taking the expectation value of (2.5) and (2.6) between ground and excited states we obtain the following equations for the amplitudes $X^1_{\alpha_1'J_1'J}$ and $Y^1_{\rho_1'J_1'J}$:

$$\sum_{\beta_1J_1''}\Omega_1^1(\alpha_1'J_1'J;\beta_1J_1''J)X^1_{\beta_1J_1''J}=E_{\alpha_1'J_1'J}X^1_{\alpha_1'J_1'J}\ ,$$
$$\sum_{\sigma_1J_1''}\Omega_2^1(\rho_1'J_1'J;\sigma_1J_1''J)Y^1_{\sigma_1J_1''J}=E_{\rho_1'J_1'J}Y^1_{\rho_1'J_1'J}\ . \quad (2.9)$$

We write then for the one-particle (-hole) and two particle-hole pairs ($n=1$, $n_2=3$) or the two-particle (-hole) −one-hole (-particle) and one particle-hole pair ($n=2$, $n_1=3$) or the three-particle (-hole) −two-hole (-particle) ($n=3$) components of the (2.1) and (2.2) CMWF, characterized by two particle-hole pairs, the following:

$$|\Phi^2_{JM}(\alpha_2J_1J_2)\rangle=\sum_{\alpha_2'J_1'J_2'}X^2_{\alpha_2'J_1'J_2'J}N^2_{\alpha_2'J_1'J_2'J}\,|\Phi^2_{JM}(\alpha_2'J_1'J_2')\rangle=\sum_{\alpha_2'J_1'J_2'}X^2_{\alpha_2'J_1'J_2'}N^2_{\alpha_2'J_1'J_2'J}A_2^\dagger(\alpha_2'J_1'J_2';JM)\,|0\rangle \quad (2.10)$$

and

$$|\tilde\Psi^2_{JM}(\rho_2J_1J_2)\rangle=\sum_{\rho_2'J_1'J_2'}Y^2_{\rho_2'J_1'J_2'J}N^2_{\rho_2'J_1'J_2'J}\,|\tilde\Psi^2_{JM}(\rho_2'J_1'J_2')\rangle=\sum_{\rho_2'J_1'J_2'}Y^2_{\rho_2'J_1'J_2'J}N^2_{\rho_2'J_1'J_2'J}A_2(\rho_2'J_1'J_2';J-M)\,|0\rangle\ , \quad (2.11)$$

where the operators $A_2^\dagger(\alpha_2'J_1'J_2';JM)$ and $A_2(\rho_2'J_1'J_2';J-M)$ create, respectively, a particle (hole) in a shell model state with quantum numbers $\{j_1\}$ ($\{j_1^{-1}\}$) coupled with two particle-hole pairs and where we use the following additional conventions.

(a) $\alpha_2\equiv\rho_2\equiv\{n_1l_1j_1;n_2l_2j_2;n_3l_3j_3;n_4l_4j_4;\ n_5l_5j_5\}$.

(b) The particle-hole pair $\{j_2-j_3^{-1}\}$ is coupled to the intermediate spin $J_1'M_1'$; the particle-hole pair $\{j_4-j_5^{-1}\}$ is coupled to the intermediate spin $J_2'M_2'$; the coupling between J_1' and J_2' is not included explicitly in the formulas.

(c) The subindex (2) indicates the number of particle-hole pairs.

(d) $N^2_{(\alpha_2'/\rho_2')J_1'J_2'J}$ are the normalization factors different from 1 if the quantum numbers of the particles (holes) are all (or in pair) equal, and (or) the quantum numbers of the two holes (particles) are equal.

Taking the commutators of the nuclear Hamiltonian with the $A_2^\dagger(\alpha_2'J_1'J_2';JM)$ and $A_2(\rho_2'J_1'J_2';J-M)$ operators between ground and excited states, and diagonalizing the resulting eigenvalue equations, we obtain the $X^2_{\alpha_2'J_1'J_2'J}$ and $Y^2_{\rho_2'J_1'J_2'J}$ amplitudes and the corresponding energies.

We are therefore confronted with the calculation of the following matrix elements:

$$\Omega_1^2(\alpha_2'J_1'J_2'J;\beta_2J_1''J_2''J)\equiv\langle\Phi_J^2(\alpha_2'J_1'J_2')\|H\|\Phi_J^2(\beta_2J_1''J_2'')\rangle \quad (2.12)$$

and

$$\Omega_2^2(\rho_2'J_1'J_2'J;\sigma_2J_1''J_2''J)\equiv\langle\tilde\Psi_J^2(\rho_2'J_1'J_2')\|H\|\tilde\Psi_J^2(\sigma_2J_1''J_2'')\rangle. \quad (2.13)$$

The treatment of the off-diagonal terms, connecting the particle (hole) with the two-particle (-hole) −one-hole (-particle), and the three-particle (-hole) −two-hole (-particle) CMWF, or the three-particle (-hole) −two-hole (-particle) with the two-particle (-hole) −one-hole (-particle) CMWF, is given explicitly in Sec. VIII.

We simplify the calculation grouping the resulting two body matrix elements accordingly to the following expressions:

$$\Omega_1^2(\alpha_2'J_1'J_2'J;\beta_2J_1''J_2''J)=\sum_{\alpha_1'\beta_1'J_iJ_i'J_r}P_1^2(\alpha_1'J_iJ_rJ;\beta_1'J_i'J_rJ)\Omega_1^1(\alpha_1'J_iJ_r;\beta_1'J_i'J_r)$$
$$+\sum_{\rho_1'\sigma_1'J_iJ_i'J_r}R_1^2(\rho_1'J_iJ_rJ;\sigma_1'J_i'J_rJ)\Omega_2^1(\rho_1'J_iJ_r;\sigma_1'J_i'J_r)$$
$$+\sum_{\epsilon_1'\eta_1'J_iJ_i'J_r}Q_1^2(\epsilon_1'J_iJ_rJ;\eta_1'J_i'J_rJ)\Omega_3^1(\epsilon_1'J_iJ_r;\eta_1'J_i'J_r) \quad (2.14)$$

and

$$\Omega_2^2(\rho_2'J_1'J_2'J;\sigma_2J_1''J_2''J)$$
$$= \sum_{\rho_1'\sigma_1'J_iJ_i'J_r} R_1^2(\rho_1'J_iJ_rJ;\sigma_1'J_i'J_rJ)\Omega_2^1(\rho_1'J_iJ_r;\sigma_1'J_i'J_r)$$
$$+ \sum_{\alpha_1'\beta_1'J_iJ_i'J_r} P_1^2(\alpha_1'J_iJ_rJ;\beta_1'J_i'J_rJ)\Omega_1^1(\alpha_1'J_iJ_r;\beta_1'J_i'J_r)$$
$$+ \sum_{\nu_1'\mu_1'J_iJ_i'J_r} S_1^2(\nu_1'J_iJ_rJ;\mu_1'J_i'J_rJ)\Omega_4^1(\nu_1'J_iJ_r;\mu_1'J_i'J_r) \ . \tag{2.15}$$

The sums in Eqs. (2.14) and (2.15) are extended over all possible two-particle–one-hole, two-hole–one-particle, three-particle and three-hole combinations that we can form out of the $\{\alpha_2\}$ and $\{\rho_2\}$ coordinates. The $P_1^2(\alpha_1'J_iJ_rJ;\beta_1'J_i'J_rJ)$, $R_1^2(\rho_1'J_iJ_rJ;\sigma_1'J_i'J_rJ)$, $Q_1^2(\epsilon_1'J_iJ_rJ;\eta_1'J_i'J_rJ)$, and $S_1^2(\nu_1'J_iJ_rJ;\mu_1'J_i'J_rJ)$ are recoupling coefficients, time δ functions in the s.p. coordinates in $\{\alpha_2',\beta_2,\rho_2',\sigma_2\}$ not active in $\{\alpha_1',\beta_1',\rho_1',\sigma_1',\epsilon_1',\eta_1',\nu_1',\mu_1'\}$. In Eqs. (2.14) and (2.15) we have introduced the following additional matrix elements:

$$\Omega_3^1(\epsilon_1'J_iJ_r;\eta_1'J_i'J_r) \equiv \langle \Lambda_{J_r}^1(\epsilon_1'J_i)\|H\|\Lambda_{J_r}^1(\eta_1'J_i')\rangle \tag{2.16}$$

and

$$\Omega_4^1(\nu_1'J_iJ_r;\mu_1'J_i'J_r) \equiv \langle M_{J_r}^1(\nu_1'J_i)\|H\|M_{J_r}^1(\mu_1'J_i')\rangle \ , \tag{2.17}$$

where

$$|\Lambda_{JM}^1(\epsilon_1J_1)\rangle = \sum_{\epsilon_1'J_1'} W^1_{\epsilon_1'J_1'J} N^1_{\epsilon_1'J_1'J} |\Lambda_{JM}^1(\epsilon_1'J_1')\rangle$$
$$= \sum_{\epsilon_1'J_1'} W^1_{\epsilon_1'J_1'J} N^1_{\epsilon_1'J_1'J} B_1^\dagger(\epsilon_1'J_1';JM)|0\rangle \ , \tag{2.18}$$

$$|M_{JM}^1(\nu_1J_1)\rangle = \sum_{\nu_1'J_1'} Z^1_{\nu_1'J_1'J} N^1_{\nu_1'J_1'J} |M_{JM}^1(\nu_1'J_1')\rangle$$
$$= \sum_{\nu_1'J_1'} Z^1_{\nu_1'J_1'J} N^1_{\nu_1'J_1'J} B_1(\nu_1'J_1';JM)|0\rangle \tag{2.19}$$

are the CMWF for three-particle and three-hole excitations. Let us generalize the calculation to the nth components of the (2.1) and (2.2) CMWF.

The solutions of the eigenvalue equations for the amplitudes and the energies depend upon the knowledge of the matrix elements:

$$\Omega_1^n(\alpha_n'J_1'J_2'\cdots J_n'J;\beta_nJ_1''J_2''\cdots J_n''J)$$
$$\equiv \langle \Phi_J^n(\alpha_n'J_1'J_2'\cdots J_n')\|H\|\Phi_J^n(\beta_nJ_1''J_2''\cdots J_n'')\rangle \tag{2.20}$$

and

$$\Omega_2^n(\rho_n'J_1'J_2'\cdots J_n'J;\sigma_nJ_1''J_2''\cdots J_n''J)$$
$$\equiv \langle \tilde\Psi_J^n(\rho_n'J_1'J_2'\cdots J_n')\|H\|\tilde\Psi_J^n(\sigma_nJ_1''J_2''\cdots J_n'')\rangle \ . \tag{2.21}$$

According to (2.14) and (2.15) we write

$$\Omega_1^n(\alpha_n'J_1'J_2'\cdots J_n'J;\beta_nJ_1''J_2''\cdots J_n''J) = f_J(\Omega_1^{n-1}(\alpha_{n-1}J_{k_1}J_{k_2}\cdots J_{k_{n-1}}J_r;\beta_{n-1}J_{k_1}'J_{k_2}'\cdots J_{k_{n-1}}'J_r))$$
$$+g_J(\Omega_2^{n-1}(\rho_{n-1}J_{k_1}J_{k_2}\cdots J_{k_{n-1}}J_r;\sigma_{n-1}J_{k_1}'J_{k_2}'\cdots J_{k_{n-1}}'J_r))$$
$$+h_J(\Omega_3^{n-1}(\epsilon_{n-1}J_{k_1}J_{k_2}\cdots J_{k_{n-1}}J_r;\eta_{n-1}J_{k_1}'J_{k_2}'\cdots J_{k_{n-1}}'J_r)) \tag{2.22}$$

and

$$\Omega_2^n(\rho_n'J_1'J_2'\cdots J_n'J;\sigma_nJ_1''J_2''\cdots J_n''J) = f_J(\Omega_2^{n-1}(\rho_{n-1}J_{k_1}J_{k_2}\cdots J_{k_{n-1}}J_r;\sigma_{n-1}J_{k_1}'J_{k_2}'\cdots J_{k_{n-1}}'J_r))$$
$$+g_J(\Omega_1^{n-1}(\alpha_{n-1}J_{k_1}J_{k_2}\cdots J_{k_{n-1}}J_r;\beta_{n-1}J_{k_1}'J_{k_2}'\cdots J_{k_{n-1}}'J_r))$$
$$+k_J(\Omega_4^{n-1}(\nu_{n-1}J_{k_1}J_{k_2}\cdots J_{k_{n-1}}J_r;\mu_{n-1}J_{k_1}'J_{k_2}'\cdots J_{k_{n-1}}'J_r)) \ , \tag{2.23}$$

where f_J, g_J, h_J, and k_J denote linear combinations.

The calculation of the recoupling coefficients, due to the high number of terms occurring in Eqs. (2.22) and (2.23), is, however, still too laborious. We apply, therefore, the expansions directly to the CMWF and we calculate the expansion coefficients looking at the transformation properties of the CMWF under the group of following substitutions:

$$|\Phi_{JM}^n(\alpha_nJ_1J_2\cdots J_n)\rangle$$
$$\Longrightarrow \sum_{M'} (U)_{MM'} |\Phi_{JM'}^n(\alpha_nJ_1J_2\cdots J_n)\rangle \tag{2.24}$$

and

$$|\tilde\Psi_{JM}^n(\rho_nJ_1J_2\cdots J_n)\rangle$$
$$\Longrightarrow \sum_{M'} (H)_{MM'} |\tilde\Psi_{JM'}^n(\rho_nJ_1J_2\cdots J_n)\rangle \ , \tag{2.25}$$

where the matrices $(U)_{MM'}$ and $(H)_{MM'}$ are infinitesimal generators of the $SU_{2J+1}(n)$ group.

III. EXPANSION OF THE CONFIGURATION MIXING WAVE FUNCTIONS

The matrix elements of the nuclear Hamiltonian calculated with the $\Phi^2_{JM}(\alpha_2 J_1 J_2)$ CMWF have been expressed in Eq. (2.14) in terms of the matrix elements calculated with the $\Phi^1_{JM}(\alpha_1 J_1)$, $\tilde{\Psi}^1_{JM}(\rho_1 J_1)$, and $\Lambda^1_{JM}(\epsilon_1 J_1)$ CMWF. Therefore, we write the $\Phi^2_{JM}(\alpha_2 J_1 J_2)$ CMWF in form of linear combinations of the $\Phi^1_{JM}(\alpha_1 J_1)$ coupled to a particle-hole, of the $\tilde{\Psi}^1_{JM}(\rho_1 J_1)$ coupled to a particle-particle, and of the $\Lambda^1_{JM}(\epsilon_1 J_1)$ coupled to the hole-hole CMWF; we write

$$\begin{aligned} |\Phi^2_{JM}(\alpha_2 J_1 J_2)\rangle = &\sum_{\alpha_1 J_k J_r J_s}^{\vartheta_1^2} {}^5T_J(\alpha_2 J_1 J_2 \,|\, \}\alpha_1 J_r \bar{\alpha}_1 J_s)[\,|\Phi^1_{J_r}(\alpha_1 J_k)\rangle \otimes |\Phi^0_{J_s}(\bar{\alpha}_1 J_s)\rangle]^J_M \\ &+ \sum_{\rho_1 J_k J_r J_s}^{\vartheta_2^2} {}^5Z_J(\alpha_2 J_1 J_2 \,|\, \}\rho_1 J_r \bar{\rho}_1 J_s)[\,|\tilde{\Psi}^1_{J_r}(\rho_1 J_k)\rangle \otimes |\Lambda^0_{J_s}(\bar{\rho}_1 J_s)\rangle]^J_M \\ &+ \sum_{\epsilon_1 J_k J_r J_s}^{\vartheta_3^2} {}^5V_J(\alpha_2 J_1 J_2 \,|\, \}\epsilon_1 J_r \bar{\epsilon}_1 J_s)[\,|\Lambda^1_{J_r}(\epsilon_1 J_k)\rangle \otimes |M^0_{J_s}(\bar{\epsilon}_1 J_s)\rangle]^J_M \ , \end{aligned} \quad (3.1)$$

where ϑ_1^2, ϑ_2^2, and ϑ_3^2 give the number of different $\{\alpha_1\}$, $\{\rho_1\}$, and $\{\epsilon_1\}$ combinations we can form with the $\{\alpha_2\}$ coordinates, where we have introduced the following transformation coefficients ${}^5T_J(\alpha_2 J_1 J_2 \,|\, \}\alpha_1 J_r \bar{\alpha}_1 J_s)$, ${}^5Z_J(\alpha_2 J_1 J_2 \,|\, \}\rho_1 J_r \bar{\rho}_1 J_s)$, and ${}^5V_J(\alpha_2 J_1 J_2 \,|\, \}\epsilon_1 J_r \bar{\epsilon}_1 J_s)$, and where $\Phi^0_{J_s}(\bar{\alpha}_1 J_s)$, $\Lambda^0_{J_s}(\bar{\rho}_1 J_s)$, and $M^0_{J_s}(\bar{\epsilon}_1 J_s)$ denote the particle-hole, particle-particle, and hole-hole wave functions, respectively. The indices $\{\bar{\alpha}_1\}$, $\{\bar{\rho}_1\}$, and $\{\bar{\epsilon}_1\}$ are the indices complementary to the $\{\alpha_1\}$, $\{\rho_1\}$, and $\{\epsilon_1\}$, so that $\{\alpha_1,\bar{\alpha}_1\}\equiv\{\alpha_2\}$, $\{\rho_1,\bar{\rho}_1\}\equiv\{\rho_2\}$, and $\{\epsilon_1,\bar{\epsilon}_1\}\equiv\{\epsilon_2\}$.

The coefficients

$${}^5T_J(\alpha_2 J_1 J_2 \,|\, \}\alpha_1 J_r \bar{\alpha}_1 J_s) \ ,$$

$${}^5Z_J(\alpha_2 J_1 J_2 \,|\, \}\rho_1 J_r \bar{\rho}_1 J_s) \ ,$$

and

$${}^5V_J(\alpha_2 J_1 J_2 \,|\, \}\epsilon_1 J_r \bar{\epsilon}_1 J_s)$$

are called "transformation coefficients of the second kind." To calculate the transformation coefficients we introduce unit tensor operators in the space spanned by the $\Phi^2_{JM}(\alpha_2 J_1 J_2)$ CMWF. The algebra of unit tensor operators and their utility in calculating the coefficients of fractional parentage for n particles (holes) in the same $\{j\}$ shell is well understood.[4] We will define the following sets of unit tensor operators: $u^k_{m_k}(n)$, $\tilde{h}^k_{m_k}(n)$, and $p^k_{m_k}(n)$ with the following properties.

(a) They are generators of the $SU_{2J+1}(n)$ group acting on the basis of the (2.1) and (2.2) CMWF.

(b) The generators of the group of unitary substitutions on the CMWF of the nth kind are

$$u^k_{m_k}(n) = \sum_{\alpha_{n-1}} u^k_{m_k}(n-1,\alpha_{n-1}) \ ,$$

$$\tilde{h}^k_{m_k}(n) = \sum_{\rho_{n-1}} \tilde{h}_{m_k}(n-1,\rho_{n-1}) \ ,$$

$$p^k_{m_k}(n) = \sum_{\epsilon_{n-1}} p^k_{m_k}(n-1,\epsilon_{n-1}) \ .$$

(c) Because we are interested in the group of unitary substitutions (2.24), we regard the $u^k_{m_k}(n)$, $\tilde{h}^k_{m_k}(n)$, and $p^k_{m_k}(n)$ as unit tensor operators defined by the following equations:

$$u^k_{m_k}(n)\,|\Phi^n_{JM}(\alpha_n J_1 J_2 \cdots J_n)\rangle = \sum_{M'} (u^k_{m_k}(n))_{MM'}\,|\Phi^n_{JM'}(\alpha_n J_1 J_2 \cdots J_n)\rangle \ ,$$

$$\tilde{h}^k_{m_k}(n)\,|\Phi^n_{JM}(\alpha_n J_1 J_2 \cdots J_n)\rangle = \sum_{M'} (\tilde{h}^k_{m_k}(n))_{MM'}\,|\Phi^n_{JM'}(\alpha_n J_1 J_2 \cdots J_n)\rangle \ ,$$

and

$$p^k_{m_k}(n)\,|\Phi^n_{JM}(\alpha_n J_1 J_2 \cdots J_n)\rangle = \sum_{M'} (p^k_{m_k}(n))_{MM'}\,|\Phi^n_{JM'}(\alpha_n J_1 J_2 \cdots J_n)\rangle \ .$$

Let us begin with the introduction of the following tensor operators for the $n=1$, $n_1=2$, or $n=2$ components:

$$\bar{u}^k_{m_k}(1) = [\bar{U}^\dagger_{\bar{\alpha}_1}(J_i) \otimes \bar{U}_{\bar{\alpha}_1}(J_i')]^k_{m_k} \quad (3.2)$$

where the

$$\bar{U}_{\bar{\alpha}_1}(J_i') = \sum_{m_i m_j'} (-1)^{j_j'-m_j'} \begin{bmatrix} j_i' & j_j' & J_j' \\ m_i' & -m_j' & M_i' \end{bmatrix} a^\dagger_{j_j'} a_{j_i'}$$

is an operator that destroys a particle-hole pair with quantum numbers $\{\bar{\alpha}_1\}$ and

$$\bar{\bar{h}}\,^k_{m_k}(1)=(-1)^{k-m_k}\bar{h}\,^k_{-m_k}(1)$$
$$=[\bar{H}\,^\dagger_{\bar{\rho}_1}(J_i)\otimes\bar{H}_{\bar{\rho}_1}(J_i')]^k_{-m_k} \tag{3.3}$$

where the

$$\bar{H}_{\bar{\rho}_1}(J_i')=\frac{1}{(1+\delta_{j_j j_j'})^{1/2}}\sum_{m_j m_j'}\begin{bmatrix} j_j' & j_j & J_i' \\ m_j' & m_j & M_i'\end{bmatrix} a^\dagger_{j_j} a^\dagger_{j_j'}$$

is an operator that destroys two holes with quantum numbers $\{\bar{\rho}_1\}$. Applying the operators (3.2) and (3.3) on the $\Phi^1_{JM}(\alpha_1 J_1)$ and $\tilde{\Psi}\,^1_{JM}(\rho_1 J_1)$ CMWF we get

$$\langle \Phi^1_{JM}(\alpha_1 J_1')\,|\,\bar{u}\,^k_{m_k}(1)\,|\,\Phi^1_{JM'}(\alpha_1 J_1)\rangle$$
$$=\delta_{J_1' J_1} M^{\alpha_1 J_1 Jk}(1)\begin{bmatrix} J & k & J \\ M & m_k & M'\end{bmatrix} \tag{3.4}$$

and

$$\langle \tilde{\Psi}\,^1_{JM}(\rho_1 J_1')\,|\,\bar{\bar{h}}\,^k_{m_k}(1)\,|\,\tilde{\Psi}\,^1_{JM'}(\rho_1 J_1)\rangle$$
$$=\delta_{J_1' J_1} L^{\rho_1 J_1 Jk}(1)\begin{bmatrix} J & k & J \\ M & m_k & M'\end{bmatrix}\,. \tag{3.5}$$

The coefficients $M^{\alpha_1 J_1 Jk}(1)$ and $L^{\rho_1 J_1 Jk}(1)$ are explicitly calculable in terms of recoupling coefficients and are given in the Appendix.

The expressions given in Eqs. (3.4) and (3.5) are then the reduced matrix elements of the irreducible tensor operators

$$\langle \Phi^1_J(\alpha_1 J_1')\|\bar{u}\,^k(1)\|\Phi^1_J(\alpha_1 J_1)\rangle = M^{\alpha_1 J_1 Jk}(1)\delta_{J_1' J_1}\,, \tag{3.6}$$

$$\langle \tilde{\Psi}\,^1_J(\rho_1 J_1')\|\bar{\bar{h}}\,^k(1)\|\tilde{\Psi}\,^1_J(\rho_1 J_1)\rangle = L^{\rho_1 J_1 Jk}(1)\delta_{J_1' J_1}\,, \tag{3.7}$$

and the operators

$$u^k_{m_k}(1)=\frac{\bar{u}\,^k_{m_k}(1)}{M^{\alpha_1 J_1 Jk}(1)}\,, \tag{3.8}$$

$$\tilde{h}\,^k_{m_k}(1)=\frac{\bar{\bar{h}}\,^k_{m_k}(1)}{L^{\rho_1 J_1 Jk}(1)} \tag{3.9}$$

are unit tensor operators in the sense of Ref. 4. The commutation relations of two $u^k_{m_k}(1)$'s and two $\tilde{h}\,^k_{m_k}(1)$'s, calculated in the Appendix, are

$$[u^{k_1}_{m_{k_1}}(1),u^{k_2}_{m_{k_2}}(1)]=(-1)^{k_1+k_2+1}\begin{Bmatrix} k_1 & k_2 & k \\ J & J & J\end{Bmatrix}[1-(-1)^{k_1+k_2-k}]\begin{bmatrix} k_1 & k_2 & k \\ m_{k_1} & m_{k_2} & m_k\end{bmatrix}u^k_{m_k}(1) \tag{3.10}$$

and

$$[\tilde{h}\,^{k_1}_{m_{k_1}}(1),\tilde{h}^{k_2}_{m_{k_2}}(1)]=(-1)^{k_1+k_2+1}\begin{Bmatrix} k_1 & k_2 & k \\ J & J & J\end{Bmatrix}[1-(-1)^{k_1+k_2-k}]\begin{bmatrix} k_1 & k_2 & k \\ m_{k_1} & m_{k_2} & m_k\end{bmatrix}\tilde{h}\,^k_{m_k}(1)\,. \tag{3.11}$$

The sets of all $u^k_{m_k}(1)$'s and $\tilde{h}\,^k_{m_k}(1)$'s, except for the $u^0_0(1)$'s and $\tilde{h}\,^0_0(1)$'s, are therefore infinitesimal generators of the $SU_{2J+1}(1)$ group and the wave functions $\Phi^1_J(\alpha_1 J_1)$ and $\tilde{\Psi}\,^1_J(\rho_1 J_1)$ carry irreducible representations of this group.

Going now a step further let us define unit tensor operators for the $n=1$, $n_2=3$ or $n=2$, $n_1=3$ or $n=3$ components:

$$\langle \Phi^2_J(\alpha_2 J_1' J_2')\|u^k(2)\|\Phi^2_J(\alpha_2 J_1 J_2)\rangle=\delta_{J_1' J_1}\delta_{J_2' J_2}\,, \tag{3.12}$$

$$\langle \Phi^2_J(\alpha_2 J_1' J_2')\|\tilde{h}\,^k(2)\|\Phi^2_J(\alpha_2 J_1 J_2)\rangle=\delta_{J_1' J_1}\delta_{J_2' J_2}\,, \tag{3.13}$$

and due to the three particle CMWF we have introduced

$$\langle \Phi^2_J(\alpha_2 J_1' J_2')\|p^k(2)\|\Phi^2_J(\alpha_2 J_1 J_2)\rangle=\delta_{J_1' J_1}\delta_{J_2' J_2} \tag{3.14}$$

with unit tensor operators

$$p^k_{m_k}(2)=\sum_{\epsilon_1}^{\vartheta_3^2} p^k_{m_k}(1,\epsilon_1)\,,$$

where

$$\bar{p}\,^k_{m_k}(1)=[\bar{P}\,^\dagger_{\bar{\epsilon}_1}(J_i)\otimes\bar{P}_{\bar{\epsilon}_1}(J_i')]^k_{m_k} \tag{3.15}$$

and where the

$$\bar{P}_{\bar{\epsilon}_1}(J_i')=\frac{1}{(1+\delta_{j_i j_i'})^{1/2}}\sum_{m_{j_i} m_{j_i}'}\begin{bmatrix} j_i' & j_i & J_i' \\ m_i' & m_i & M_i'\end{bmatrix} a_{j_i} a_{j_i'}$$

is an operator that destroys two particles with quantum numbers $\{\bar{\epsilon}_1\}$. The matrix elements of the $\bar{p}\,^k_{m_k}(1)$ with the three-particle CMWF are

$$\langle \Lambda^1_{JM}(\epsilon_1 J_1')\,|\,\bar{p}\,^k_{m_k}(1)\,|\,\Lambda^1_{JM'}(\epsilon_1 J_1)\rangle$$
$$=\delta_{J_1' J_1} O^{\epsilon_1 J_1 Jk}(1)\begin{bmatrix} J & k & J \\ M & m_k & M'\end{bmatrix} \tag{3.16}$$

and the reduced matrix elements are

$$\langle \Lambda_J^1(\epsilon_1 J_1')\|\bar{p}^{\,k}(1)\|\Lambda_J^1(\epsilon_1 J_1)\rangle = O^{\epsilon_1 J_1 Jk}(1)\delta_{J_1' J_1} \tag{3.17}$$

so that the

$$p^k_{m_k}(1)=\frac{\bar{p}^{\,k}_{\ m_k}(1)}{O^{\epsilon_1 J_1 Jk}(1)} \tag{3.18}$$

are unit tensor operators in the sense of Ref. 4. The matrix elements $O^{\epsilon_1 J_1 Jk}(1)$ are calculated in the Appendix and two of the $p^k_{m_k}(1)$'s obey the following commutation relations:

$$[p^{k_1}_{m_{k_1}}(1),p^{k_2}_{m_{k_2}}(1)]=(-1)^{k_1+k_2+1}\begin{Bmatrix} k_1 & k_2 & k \\ J & J & J\end{Bmatrix}[1-(-1)^{k_1+k_2-k}]\begin{bmatrix} k_1 & k_2 & k \\ m_{k_1} & m_{k_2} & m_k\end{bmatrix}u^k_{m_k}(1) \; . \tag{3.19}$$

The commutators of two $u^k_{m_k}(2)$'s, of two $\tilde{h}^{\,k}_{\ m_k}(2)$'s, and of two $p^k_{m_k}(2)$'s are then expressed in terms of the $u^k_{m_k}(2)$'s, $\tilde{h}^{\,k}_{\ m_k}(2)$'s, and $p^k_{m_k}(2)$'s by relations similar to the ones given in Eqs. (3.10), (3.11), and (3.19). We conclude, therefore, that the sets of all $u^k_{m_k}(2)$'s, $\tilde{h}^{\,k}_{\ m_k}(2)$'s, and $p^k_{m_k}(2)$'s, except for the $u^0_0(2)$'s, $\tilde{h}^{\,0}_{\ 0}(2)$'s, and $p^0_0(2)$'s, are unitary generators of the $SU_{2J+1}(2)$ group and that the $\Phi^2_{JM}(\alpha_2 J_1 J_2)$ CMWF carry irreducible representations of this group.

IV. TRANSFORMATION COEFFICIENTS OF THE SECOND KIND

To calculate the transformation coefficients of the $\Phi^2_{JM}(\alpha_2 J_1 J_2)$ CMWF we write for the unit tensor operators $u^k_{m_k}(2)$, $\tilde{h}^{\,k}_{\ m_k}(2)$, and $p^k_{m_k}(2)$ the following relations:

$$u^k_{m_k}(2)=\sum_{\alpha_1}^{\vartheta_1^2} u^k_{m_k}(1,\alpha_1) \; , \tag{4.1}$$

$$\tilde{h}^{\,k}_{\ m_k}(2)=\sum_{\rho_1}^{\vartheta_2^2} \tilde{h}^{\,k}_{\ m_k}(1,\rho_1) \; , \tag{4.2}$$

$$p^k_{m_k}(2)=\sum_{\epsilon_1}^{\vartheta_3^2} p^k_{m_k}(1,\epsilon_1) \; , \tag{4.3}$$

or equivalently, using the definitions of complementary indices:

$$u^k_{m_k}(2)=\sum_{\alpha_1}^{\vartheta_1^2} [u^k_{m_k}(1,\alpha_1)\otimes \mathbf{1}(1,\bar{\alpha}_1)]^k_{m_k} \; , \tag{4.4}$$

$$\tilde{h}^{\,k}_{\ m_k}(2)=\sum_{\rho_1}^{\vartheta_2^2} [\tilde{h}^{\,k}_{\ m_k}(1,\rho_1)\otimes \mathbf{1}(1,\bar{\rho}_1)]^k_{m_k} \; , \tag{4.5}$$

$$p^k_{m_k}(2)=\sum_{\epsilon_1}^{\vartheta_3^2} [p^k_{m_k}(1,\epsilon_1)\otimes \mathbf{1}(1,\bar{\epsilon}_1)]^k_{m_k} \; . \tag{4.6}$$

The transformation coefficients of the second kind are then defined in terms of the matrix elements of the $u^k_{m_k}(2)$, $\tilde{h}^{\,k}_{\ m_k}(2)$, and $p^k_{m_k}(2)$ unit tensor operators by the following expressions:

$$({}^5T_J(\alpha_2 J_1 J_2 \mid \}\alpha_1 J_r \bar{\alpha}_1 J_s))^{\dagger}\, {}^5T_J(\alpha_2 J_1 J_2 \mid \}\alpha_1 J_r \bar{\alpha}_1 J_s)$$
$$=\langle \Phi^2_J(\alpha_2 J_1 J_2)\|U^{\dagger}_{\bar{\alpha}_1}(J_s)\|\Phi^1_{J_r}(\alpha_1 J_k)\rangle\langle \Phi^1_{J_r}(\alpha_1 J_k)\|U_{\bar{\alpha}_1}(J_s)\|\Phi^2_J(\alpha_2 J_1 J_2)\rangle \; , \tag{4.7}$$

$$({}^5Z_J(\alpha_2 J_1 J_2 \mid \}\rho_1 J_r \bar{\rho}_1 J_s))^{\dagger}\, {}^5Z_J(\alpha_2 J_1 J_2 \mid \}\rho_1 J_r \bar{\rho}_1 J_s)$$
$$=\langle \Phi^2_J(\alpha_2 J_1 J_2)\|H_{\bar{\rho}_1}(J_s)\|\tilde{\Psi}^{\,1}_{\ J_r}(\rho_1 J_k)\rangle\langle \tilde{\Psi}^{\,1}_{\ J_r}(\rho_1 J_k)\|H^{\dagger}_{\bar{\rho}_1}(J_s)\|\Phi^2_{J_r}(\bar{\alpha}_2 J_1 J_2)\rangle \; , \tag{4.8}$$

and

$$({}^5V_J(\alpha_2 J_1 J_2 \mid \}\epsilon_1 J_r \bar{\epsilon}_1 J_s))^{\dagger}\, {}^5V_J(\alpha_2 J_1 J_2 \mid \}\epsilon_1 J_r \bar{\epsilon}_1 J_s)$$
$$=\langle \Phi^2_J(\alpha_2 J_1 J_2)\|P_{\bar{\epsilon}_1}(J_s)\|\Lambda^1_{J_r}(\epsilon_1 J_k)\rangle\langle \Lambda^1_{J_r}(\epsilon_1 J_k)\|P^{\dagger}_{\bar{\epsilon}_1}(J_s)\|\Phi^2_J(\alpha_2 J_1 J_2)\rangle \; . \tag{4.9}$$

To prove the expressions (4.7), (4.8), and (4.9) let us calculate the matrix elements (3.12) using for the $\Phi^2_{JM}(\alpha_2 J_1 J_2)$

CMWF the expansion (3.1); we get

$$\langle \Phi_J^2(\alpha_2 J_1 J_2)\|u^k(2)\|\Phi_J^2(\alpha_2 J_1 J_2)\rangle = \sum_{\alpha_1 J_r J_s} (-1)^{J_r+J_s+J+k}(\hat{k})^{1/2} \begin{Bmatrix} J_r & J & J_s \\ J & J_r & k \end{Bmatrix} ({}^5T_J(\alpha_2 J_1 J_2 \mid \}\alpha_1 J_r \bar{\alpha}_1 J_s))^\dagger\, {}^5T_J(\alpha_2 J_1 J_2 \mid \}\alpha_1 J_r \bar{\alpha}_1 J_s) . \quad (4.10)$$

On the other hand, using Eq. (4.1) and the definition (3.2) of the $u^k_{m_k}(1,\alpha_1)$'s, we calculate

$$\langle \Phi_J^2(\alpha_2 J_1 J_2)\|u^k(2)\|\Phi_J^2(\alpha_2 J_1 J_2)\rangle = \sum_{\alpha_1 J_r J_s} (-1)^{J_r+J_s+J+k}(\hat{k})^{1/2} \begin{Bmatrix} J_r & J & J_s \\ J & J_r & k \end{Bmatrix} \times \langle \Phi_J^2(\alpha_2 J_1 J_2)\|U^\dagger_{\bar{\alpha}_1}(J_s)\|\Phi^1_{J_r}(\alpha_1 J_k)\rangle \langle \Phi^1_{J_r}(\alpha_1 J_k)\|U_{\bar{\alpha}_1}(J_s)\|\Phi_J^2(\alpha_2 J_1 J_2)\rangle . \quad (4.11)$$

From (4.10) and (4.11) we obtain the expression (4.7). Analogously we prove the expressions (4.8) and (4.9), taking the matrix elements of the $\tilde{h}^k_{m_k}(2)$'s and $p^k_{m_k}(2)$'s in the $\Phi^2_{JM}(\alpha_2 J_1 J_2)$ CMWF.

The problem of finding transformation coefficients of the second kind is now regarded as the problem of reducing the $SU_{2J+1}(2)$ representation carried by the

$$[\Phi^1_{J_r}(\alpha_1 J_k)\otimes\Phi^0_{J_s}(\bar{\alpha}_1 J_s)]^J_M \quad (4.12)$$

wave functions. To effect this reduction we calculate the matrix of the Casimir operator of $SU_{2J+1}(2)$ in the basis states (4.12) and then diagonalize it. According to Ref. 4 we can prove that

$$\sum_k (u^k(2)\cdot u^k(2))$$

commutes with all the $u^k(2)$, $k=1,\ldots,2J$, and therefore is the Casimir operator for $SU_{2J+1}(2)$. We calculate then the following matrix elements:

$$\langle {}^J[\Phi^1_{J_r}(\alpha_1 J_k)\otimes\Phi^0_{J_s}(\bar{\alpha}_1 J_s)]\|(u^k(2)\cdot u^k(2))\|[\Phi^1_{J'_r}(\beta_1 J'_k)\otimes\Phi^0_{J'_s}(\bar{\beta}_1 J'_s)]^J\rangle = ({}^5T_J(\alpha_2 J_1 J_2 \mid \}\alpha_1 J_r \bar{\alpha}_1 J_s))^\dagger({}^5T_J(\alpha_2 J_1 J_2 \mid \}\alpha_1 J_r \bar{\alpha}_1 J_s)) . \quad (4.13)$$

On the other hand, using Eq. (4.4), we have

$$\langle {}^J[\Phi^1_{J_r}(\alpha_1 J_k)\otimes\Phi^0_{J_s}(\bar{\alpha}_1 J_s)]\|(u^k(1,\alpha)\cdot 1(1,\bar{\alpha})\cdot u^k(1,\beta)\cdot 1(1,\bar{\beta}))\|[\Phi^1_{J'_r}(\beta_1 J'_k)\otimes\Phi^0_{J'_s}(\bar{\beta}_1 J'_s)]^J\rangle$$
$$= \sum_k (-1)^{J_r+J'_r+1}(\hat{k})^{-3/2} \begin{Bmatrix} J_r & J & J_s \\ J & J_r & k \end{Bmatrix} \begin{Bmatrix} J'_r & J & J_s \\ J & J'_r & k \end{Bmatrix} \langle \Phi^1_{J_r}(\alpha_1 J_k)\|u^k(1)\|\Phi^1_{J_r}(\alpha_1 J_k)\rangle \langle \Phi^1_{J'_r}(\beta_1 J'_k)\|u^k(1)\|\Phi^1_{J'_r}(\beta_1 J'_k)\rangle . \quad (4.14)$$

Comparing the result obtained with the one of Eq. (4.13), we can write the following iterative formula for the transformation coefficients:

$$({}^5T_J(\alpha_2 J_1 J_2 \mid \}\alpha_1 J_r \bar{\alpha}_1 J_s))^\dagger\, {}^5T_J(\alpha_2 J_1 J_2 \mid \}\alpha_1 J_r \bar{\alpha}_1 J_s)$$
$$= \sum_{\substack{k J_i J'_i \\ J^2_r J^3_r}} (-1)^{J_1+J'_i+J^2_r+J^3_r+1}(\hat{k})^{-3/2}\hat{J}_r\hat{J}'_r \begin{Bmatrix} J_r & J & J_s \\ J & J_r & k \end{Bmatrix} \begin{Bmatrix} J'_r & J & J_s \\ J & J'_r & k \end{Bmatrix} \begin{Bmatrix} J_i & J_r & J^2_r \\ J_r & J_i & k \end{Bmatrix} \begin{Bmatrix} J'_i & J'_r & J^3_r \\ J'_r & J'_i & k \end{Bmatrix}$$
$$\times ({}^3T^k_{J_r}(\alpha_1 J_k \mid \}\alpha_0 J_i \bar{\alpha}_0 J^2_r))^\dagger\, {}^3T^k_{J_r}(\alpha_1 J_k \mid \}\alpha_0 J_i \bar{\alpha}_0 J^2_r)({}^3T^k_{J'_r}(\beta_1 J'_k \mid \}\beta_0 J'_i \bar{\beta}_0 J^3_r))^\dagger\, {}^3T^k_{J'_r}(\beta_1 J'_k \mid \}\beta_0 J'_i \bar{\beta}_0 J^3_r) . \quad (4.15)$$

The transformation coefficients depend upon the ${}^3T^k_{J_r}(\alpha_1 J_k \mid \}\alpha_0 J_i \bar{\alpha}_0 J^2_r)$, i.e., the k components of the transformation coefficients of the first kind we need to expand the $\Phi^1_{J_r M_r}(\alpha_1 J_k)$ CMWF in terms of a particle coupled to a particle-hole pair, which are given by the following equations:

$$\sum_{\alpha_0 J_i J_r^2} (-1)^{J_i+J_r^2+J_r+K}(\hat{k})^{1/2}\begin{Bmatrix} J_i & J_r & J_r^2 \\ J_r & J_i & k \end{Bmatrix}({}^3T^k_{J_r}(\alpha_1 J_k \mid \}\alpha_0 J_i \bar{\alpha}_0 J_r^2))^\dagger\, {}^3T^k_{J_r}(\alpha_1 J_k \mid \}\alpha_0 J_i \alpha_0 J_r^2) = M^{\alpha_1 J_k J_r k}(1) \;. \tag{4.16}$$

The eigenvalues of the matrix defined in Eq. (4.15) uniquely identify the $SU_{2J+1}(2)$ transformation properties. The procedure is thus an iterative one defining the ${}^5T_J(\alpha_2 J_1 J_2 \mid \}\alpha_1 J_r \bar{\alpha}_1 J_s)$ transformation coefficients in terms of the known ${}^3T^k_{J_r}(\alpha_1 J_k \mid \}\alpha_0 J_i \bar{\alpha}_0 J_r^2)$ transformation coefficients. To calculate the ${}^5Z_J(\alpha_2 J_1 J_2 \mid \}\rho_1 J_r \bar{\rho}_1 J_s)$ transformation coefficients we have to diagonalize the Casimir operator:

$$\sum_k (\tilde{h}^{\,k}(2)\cdot\tilde{h}^{\,k}(2))$$

in the basis states

$$[\tilde{\Psi}^{\,1}_{J_r}(\rho_1 J_k)\otimes \Lambda^0_{J_s}(\bar{\rho}_1 J_s)]^J_M \;.$$

The expression we get is not given explicitly, but defines the ${}^5Z_J(\alpha_2 J_1 J_2 \mid \}\rho_1 J_r \bar{\rho}_1 J_s)$ transformation coefficients in terms of the ${}^3Z^k_{J_r}(\rho_1 J_k \mid \}\rho_0 J_i' \bar{\rho}_0 J_r^2)$.

The ${}^5V_J(\alpha_2 J_1 J_2 \mid \}\epsilon_1 J_r \bar{\epsilon}_1 J_s)$ transformation coefficients have to be calculated using the following relations:

$$\sum_{\epsilon_1 J_r J_s} (-1)^{J_r+J_s+J+k}(\hat{k})^{1/2}\begin{Bmatrix} J_r & J & J_s \\ J & J_r & k \end{Bmatrix}({}^5V^k_J(\alpha_2 J_1 J_2 \mid \}\epsilon_1 J_r \bar{\epsilon}_1 J_s))^\dagger\, {}^5V^k_J(\alpha_2 J_1 J_2 \mid \}\epsilon_1 J_r \bar{\epsilon}_1 J_s)$$
$$= O^{\alpha_2 J_1 J_2 J k}(2) \Longrightarrow \Sigma f(k)\cdot O^{\epsilon_1 J_k J_r k}(1) \;. \tag{4.17}$$

V. GENERALIZATION OF THE METHOD

In this section we generalize the method for the calculation of the transformation coefficients for the CMWF of the nth kind. We write the following linear combination:

$$\begin{aligned}
|\Phi^n_{JM}(\alpha_n J_1 J_2 \cdots J_n)\rangle = & \sum_{\alpha_{n-1} J_{k_1} J_{k_2} \cdots J_{k_{n-1}} J_r J_s}^{\vartheta_1^n} {}^{2n+1}T_J(\alpha_n J_1 J_2 \cdots J_n \mid \}\alpha_{n-1} J_r \bar{\alpha}_{n-1} J_s) \\
& \times [\,|\Phi^{n-1}_{J_r}(\alpha_{n-1} J_{k_1} J_{k_2} \cdots J_{k_{n-1}})\rangle \otimes |\Phi^0_{J_s}(\bar{\alpha}_{n-1} J_s)\rangle]^J_M \\
& + \sum_{\rho_{n-1} J_{k_1} J_{k_2} \cdots J_{k_{n-1}} J_r J_s}^{\vartheta_2^n} {}^{2n+1}Z_J(\alpha_n J_1 J_2 \cdots J_n \mid \}\rho_{n-1} J_r \bar{\rho}_{n-1} J_s) \\
& \times [\,|\tilde{\Psi}^{n-1}_{J_r}(\rho_{n-1} J_{k_1} J_{k_2} \cdots J_{k_{n-1}})\rangle \otimes |\Lambda^0_{J_s}(\bar{\rho}_{n-1} J_s)\rangle]^J_M \\
& + \sum_{\epsilon_{n-1} J_{k_1} J_{k_2} \cdots J_{k_{n-1}} J_r J_s}^{\vartheta_3^n} {}^{2n+1}V_J(\alpha_n J_1 J_2 \cdots J_n \mid \}\epsilon_{n-1} J_r \bar{\epsilon}_{n-1} J_s) \\
& \times [\,|\Lambda^{n-1}_{J_r}(\epsilon_{n-1} J_{k_1} J_{k_2} \cdots J_{k_{n-1}})\rangle \otimes |M^0_{J_s}(\bar{\epsilon}_{n-1} J_s)\rangle]^J_M \;,
\end{aligned} \tag{5.1}$$

where we have introduced the nth kind transformation coefficients and where the ϑ_1^n, ϑ_2^n, and ϑ_3^n give the number of different $\{\alpha_{n-1}\}$, $\{\rho_{n-1}\}$, and $\{\epsilon_{n-1}\}$ combinations we can form with the $\{\alpha_n\}$ coordinates. The transformation coefficients are then given by the matrix elements of the unit tensor operators:

$$u^k_{m_k}(n) = \frac{\bar{u}^{\,k}_{m_k}(n)}{L^{\alpha_n J_1 J_2 \cdots J_n J k}(n)} \;, \tag{5.2}$$

$$\tilde{h}^{\,k}_{m_k}(n) = \frac{\bar{\tilde{h}}^{\,k}_{m_k}(n)}{M^{\alpha_n J_1 J_2 \cdots J_n J k}(n)} \;, \tag{5.3}$$

$$p^k_{m_k}(n) = \frac{\bar{p}^{\,k}_{m_k}(n)}{O^{\alpha_n J_1 J_2 \cdots J_n J k}(n)} \;, \tag{5.4}$$

calculated in the basis of the $\Phi^n_{JM}(\alpha_n J_1 J_2, \ldots, J_n)$ CMWF.

It is easy to show that the unit tensor operators of the

nth kind, (5.2), (5.3), and (5.4), have commutation relations of the forms (3.10), (3.11), and (3.19), so that they are unitary generators of the $SU_{2J+1}(n)$ group. The transformation coefficients of the nth kind are then calculated reducing the $SU_{2J+1}(n)$ representations carried by the

$$[\Phi^{n-1}_{J_r}(\alpha_{n-1}J_{k_1}J_{k_2}\cdots J_{k_{n-1}})\otimes\Phi^0_{J_s}(\bar{\alpha}_{n-1}J_s)]^J_M\ , \qquad (5.5)$$

$$[\tilde{\Psi}^{n-1}_{J_r}(\rho_{n-1}J_{k_1}J_{k_2}\cdots J_{k_{n-1}})\otimes\Lambda^0_{J_s}(\bar{\rho}_{n-1}J_s)]^J_M\ , \qquad (5.6)$$

$$[\Lambda^{n-1}_{J_r}(\epsilon_{n-1}J_{k_1}J_{k_2}\cdots J_{k_{n-1}})\otimes M^0_{J_s}(\bar{\epsilon}_{n-1}J_s)]^J_M \qquad (5.7)$$

wave functions.

To effect these reductions we calculate the matrices of the Casimir operators of the $SU_{2J+1}(n)$,

$$\sum_k (u^k(n)\cdot u^k(n))\ , \qquad (5.8)$$

$$\sum_k (\tilde{h}^k(n)\cdot\tilde{h}^k(n))\ , \qquad (5.9)$$

and

$$\sum_k (p^k(n)\cdot p^k(n)) \qquad (5.10)$$

in the basis of (5.5), (5.6), and (5.7) CMWF. We obtain the following iterative expression for the ${}^{2n+1}T_J(\alpha_nJ_1J_2\cdots J_n\,|\,\}\alpha_{n-1}J_r\bar{\alpha}_{n-1}J_s)$ transformation coefficients:

$$({}^{2n+1}T_J(\alpha_nJ_1J_2\cdots J_n\,|\,\}\alpha_{n-1}J_r\bar{\alpha}_{n-1}J_s))^{\dagger}\,{}^{2n+1}T_J(\alpha_nJ_1J_2\cdots J_n\,|\,\}\alpha_{n-1}J_r\bar{\alpha}_{n-1}J_s)$$
$$=\sum_{\substack{kJ_iJ_i'\\ J_r^2J_r^3}}(-1)^{J_i+J_i'+J_r^2+J_r^3+1}(\hat{k})^{-3/2}\hat{J}_r\hat{J}'_r\begin{Bmatrix}J_r & J & J_s\\ J & J_r & k\end{Bmatrix}\begin{Bmatrix}J_r' & J & J_s\\ J & J_r' & k\end{Bmatrix}\begin{Bmatrix}J_i & J_r & J_r^2\\ J_r & J_i & k\end{Bmatrix}\begin{Bmatrix}J_i' & J_r' & J_r^3\\ J_r' & J_i' & k\end{Bmatrix}$$
$$\times[({}^{2n-1}T^k_{J_r}(\alpha_{n-1}J_{k_1}J_{k_2}\cdots J_{k_{n-1}}\,|\,\}\alpha_{n-2}J_i\bar{\alpha}_{n-2}J_r^2))^{\dagger}\,{}^{2n-1}T^k_{J_r}(\alpha_{n-1}J_{k_1}J_{k_2}\cdots J_{k_{n-1}}\,|\,\}\alpha_{n-2}J_i\bar{\alpha}_{n-2}J_r^2)]$$
$$\times[({}^{2n-1}T^k_{J_r'}(\beta_{n-1}J'_{k_1}J'_{k_2}\cdots J'_{k_{n-1}}\,|\,\}\beta_{n-2}J_i'\bar{\beta}_{n-2}J_r^3))^{\dagger}\,{}^{2n-1}T^k_{J_r'}(\beta_{n-1}J'_{k_1}J'_{k_2}\cdots J'_{k_{n-1}}\,|\,\}\beta_{n-2}J_i'\bar{\beta}_{n-2}J_r^3)]\ . \qquad (5.11)$$

The matrix elements

$$\langle\Phi^{n-1}_{J_r}(\alpha_{n-1}J_{k_1}J_{k_2}\cdots J_{k_{n-1}})\|u^k(n-1)\|\Phi^{n-1}_{J_r}(\alpha_{n-1}J_{k_1}J_{k_2}\cdots J_{k_{n-1}})\rangle$$
$$=\sum_{\substack{\alpha_{n-2}\\ J_iJ_r^2}}(-1)^{J_i+J_r^2+J_r+k}(\hat{k})^{1/2}\begin{Bmatrix}J_i & J_r & J_r^2\\ J_r & J_i & k\end{Bmatrix}({}^{2n-1}T^k_{J_r}(\alpha_{n-1}J_{k_1}J_{k_2}\cdots J_{k_{n-1}}\,|\,\}\alpha_{n-2}J_i\bar{\alpha}_{n-2}J_r^2))^{\dagger}$$
$$\times\,{}^{2n-1}T^k_{J_r}(\alpha_{n-1}J_{k_1}J_{k_2}\cdots J_{k_{n-1}}\,|\,\}\alpha_{n-2}J_i\bar{\alpha}_{n-2}J_r^2)$$

can now be expressed in terms of the already known matrix elements

$$\langle\Phi^{n-2}_{J_r''}(\alpha_{n-2}J_{k_1}J_{k_2}\cdots J_{k_{n-2}})\|u^k(n-2)\|\Phi^{n-2}_{J_r''}(\alpha_{n-2}J_{k_1}J_{k_2}\cdots J_{k_{n-2}})\rangle$$

until we reach the matrix elements of the first kind, defined in Eq. (4.16).

The eigenvalues of the matrix defined in Eq. (5.11) identify uniquely the $SU_{2J+1}(n)$ transformation properties, and therefore the transformation coefficients. Analogous expressions can be derived for the

$${}^{2n+1}Z_J(\alpha_nJ_1J_2\cdots J_n\,|\,\}\rho_{n-1}J_r\bar{\rho}_{n-1}J_s)$$

and

$${}^{2n+1}V_J(\alpha_nJ_1J_2\cdots J_n\,|\,\}\epsilon_{n-1}J_r\bar{\epsilon}_{n-1}J_s)$$

transformation coefficients, taking the matrix elements of the Casimir operators (5.9) and (5.10) with respect to the (5.6) and (5.7) CMWF, and diagonalizing the resulting matrices.

The formulas we obtain are iterative formulas fully defining the three sets of transformation coefficients in terms of the known $M^{\alpha_1 J_1 Jk}(1)$, $L^{\rho_1 J_1 Jk}(1)$, and $O^{\alpha_2 J_1 J_2 Jk}(2)$ matrix elements.

VI. ONE-HOLE CONJUGATION

The nth components of Eq. (2.2) for the one-hole conjugation states are expanded according to the formula

$$\begin{aligned}|\tilde{\Psi}^n_{JM}(\rho_n J_1 J_2 \cdots J_n)\rangle = &\sum_{\rho_{n-1} J_{k_1} J_{k_2} \cdots J_{k_{n-1}} J_r J_s}^{\eta_1^n} {}^{2n+1}Z_J(\rho_n J_1 J_2 \cdots J_n \mid \}\rho_{n-1} J_r \bar{\rho}_{n-1} J_s) \\ &\times [\,|\tilde{\Psi}^{n-1}_{J_r}(\rho_{n-1} J_{k_1} J_{k_2} \cdots J_{k_{n-1}})\rangle \otimes |\Phi^0_{J_s}(\bar{\rho}_{n-1} J_s)\rangle]^J_M \\ &+ \sum_{\alpha_{n-1} J_{k_1} J_{k_2} \cdots J_{k_{n-1}} J_r J_s}^{\eta_2^n} {}^{2n+1}T_J(\rho_n J_1 J_2 \cdots J_{n-1} \mid \}\alpha_{n-1} J_r \bar{\alpha}_{n-1} J_s) \\ &\times [\,|\Phi^{n-1}_{J_r}(\alpha_{n-1} J_{k_1} J_{k_2} \cdots J_{k_{n-1}})\rangle \otimes |M^0_{J_s}(\bar{\alpha}_{n-1} J_s)\rangle]^J_M \\ &+ \sum_{\nu_{n-1} J_{k_1} J_{k_2} \cdots J_{k_{n-1}} J_r J_s}^{\eta_3^n} {}^{2n+1}C_J(\rho_n J_1 J_2 \cdots J_n \mid \}\nu_{n-1} J_r \bar{\nu}_{n-1} J_s) \\ &\times [\,|M^{n-1}_{J_r}(\nu_{n-1} J_{k_1} J_{k_2} \cdots J_{k_{n-1}})\rangle \otimes |\Lambda^0_{J_s}(\bar{\nu}_{n-1} J_s)\rangle]^J_M ,\end{aligned} \tag{6.1}$$

where we have introduced the following additional transformation coefficients:

$$({}^{2n+1}C_J(\rho_n J_1 J_2 \cdots J_n \mid \}\nu_{n-1} J_r \bar{\nu}_{n-1} J_s))^\dagger = \langle \tilde{\Psi}^n_J(\rho_n J_1 J_2 \cdots J_n \| D_{\bar{\nu}_{n-1}}(J_s) \| M^{n-1}_{J_r}(\nu_{n-1} J_{k_1} J_{k_2} \cdots J_{k_{n-1}})\rangle , \tag{6.2}$$

and where η_1^n, η_2^n, and η_3^n give the number of different $\{\rho_{n-1}\}$, $\{\alpha_{n-1}\}$, and $\{\nu_{n-1}\}$ combinations that we can form with the $\{\rho_n\}$ coordinates and where we have introduced the

$$s^k_{m_k}(n) = \sum_{\nu_{n-1}} s^k_{m_k}(n-1, \nu_{n-1})$$

unit tensor operators defined by

$$\bar{s}^k_{m_k}(1) = [\bar{D}^\dagger_{\bar{\nu}_1}(J_i) \otimes \bar{D}_{\bar{\nu}_1}(J_i')]^k_{m_k} \tag{6.3}$$

where the

$$\bar{D}_{\bar{\nu}_1}(J_i') = \frac{1}{(1+\delta_{j_j j_j'})^{1/2}} \sum_{m_j m_j'} \begin{bmatrix} j_j' & j_j & J_i' \\ m_j' & m_j & M_i' \end{bmatrix} a^\dagger_{j_j} a^\dagger_{j_j'}$$

is an operator that destroys two holes with quantum numbers $\{\bar{\nu}_1\}$.

The transformation coefficients of the nth kind for the one-hole conjugation case are then calculated taking the matrix elements of the Casimir operators:

$$\sum_k (u^k(n) \cdot u^k(n)) , \tag{6.4}$$

$$\sum_k (\tilde{h}^k(n) \cdot \tilde{h}^k(n)) , \tag{6.5}$$

$$\sum_k (s^k(n) \cdot s^k(n)) , \tag{6.6}$$

on the basis of the

$$[\tilde{\Psi}^{n-1}_{J_r}(\rho_{n-1} J_{k_1} J_{k_2} \cdots J_{k_{n-1}}) \otimes \Phi^0_{J_s}(\bar{\rho}_{n-1} J_s)]^J_M , \tag{6.7}$$

$$[\Phi^{n-1}_{J_r}(\alpha_{n-1} J_{k_1} J_{k_2} \cdots J_{k_{n-1}}) \otimes M^0_{J_s}(\bar{\alpha}_{n-1} J_s)]^J_M , \tag{6.8}$$

$$[M^{n-1}_{J_r}(\nu_{n-1} J_{k_1} J_{k_2} \cdots J_{k_{n-1}}) \otimes \Lambda^0_{J_s}(\bar{\nu}_{n-1} J_s)]^J_M \tag{6.9}$$

wave functions, and diagonalizing the resulting matrices. The expressions we derive are similar to the ones we have for the matrix elements of the Casimir operators (5.8), (5.9), and (5.10). The transformation coefficients

$${}^{2n+1}Z_J(\rho_n J_1 J_2 \cdots J_n \mid \}\rho_{n-1} J_r \bar{\rho}_{n-1} J_s) ,$$

$${}^{2n+1}T_J(\rho_n J_1 J_2 \cdots J_n \mid \}\alpha_{n-1} J_r \bar{\alpha}_{n-1} J_s) ,$$

and

$${}^{2n+1}C_J(\rho_n J_1 J_2 \cdots J_n \mid \}\nu_{n-1} J_r \bar{\nu}_{n-1} J_s)$$

are therefore defined iteratively in terms of the $L^{\rho_1 J_1 Jk}(1)$, $M^{\alpha_1 J_1 Jk}(1)$, and $S^{\rho_2 J_1 J_2 Jk}(2)$ matrix elements.

VII. MATRIX ELEMENTS OF THE NUCLEAR HAMILTONIAN: DIAGONAL TERMS

Using the transformation coefficients introduced in the previous sections for the basis of the (5.1) and (6.1) CMWF, we write for the matrix elements (2.20) and (2.21) the following iterative expressions:

$$\Omega_1^n(\alpha_n J_1 J_2 \cdots J_n J;\beta_n J_1' J_2' \cdots J_n' J)=\sum_{\substack{\alpha_{n-1}\beta_{n-1}J_s J_r\\ J_{k_1}J_{k_2}\cdots J_{k_{n-1}}\\ J_{k_1}'J_{k_2}'\cdots J_{k_{n-1}}'}} {}^{2n+1}T_J(\alpha_n J_1 J_2 \cdots J_n \mid \}\alpha_{n-1}J_r\bar{\alpha}_{n-1}J_s)$$
$$\times {}^{2n+1}T_J(\beta_n J_1' J_2' \cdots J_n' \mid \}\beta_{n-1}J_r\bar{\alpha}_{n-1}J_s)$$
$$\times \Omega_1^{n-1}(\alpha_{n-1}J_{k_1}J_{k_2}\cdots J_{k_{n-1}}J_r;\beta_{n-1}J_{k_1}'J_{k_2}'\cdots J_{k_{n-1}}'J_r)$$
$$+\sum_{\substack{\rho_{n-1}\sigma_{n-1}J_s J_r\\ J_{k_1}J_{k_2}\cdots J_{k_{n-1}}\\ J_{k_1}'J_{k_2}'\cdots J_{k_{n-1}}'}} {}^{2n+1}Z_J(\alpha_n J_1 J_2 \cdots J_n \mid \}\rho_{n-1}J_r\bar{\rho}_{n-1}J_s)$$
$$\times {}^{2n+1}Z_J(\beta_n J_1' J_2' \cdots J_n' \mid \}\sigma_{n-1}J_r\bar{\rho}_{n-1}J_s)$$
$$\times \Omega_2^{n-1}(\rho_{n-1}J_{k_1}J_{k_2}\cdots J_{k_{n-1}}J_r;\sigma_{n-1}J_{k_1}'J_{k_2}'\cdots J_{k_{n-1}}'J_r)$$
$$+\sum_{\substack{\epsilon_{n-1}\eta_{n-1}J_s J_r\\ J_{k_1}J_{k_2}\cdots J_{k_{n-1}}\\ J_{k_1}'J_{k_2}'\cdots J_{k_{n-1}}'}} {}^{2n+1}V_J(\alpha_n J_1 J_2 \cdots J_n \mid \}\epsilon_{n-1}J_r\bar{\epsilon}_{n-1}J_s)$$
$$\times {}^{2n+1}V_J(\beta_n J_1' J_2' \cdots J_n' \mid \}\eta_{n-1}J_r\bar{\epsilon}_{n-1}J_s)$$
$$\times \Omega_2^{n-1}(\rho_{n-1}J_{k_1}J_{k_2}\cdots J_{k_{n-1}}J_r;\sigma_{n-1}J_{k_1}'J_{k_2}'\cdots J_{k_{n-1}}'J_r)$$

(7.1)

and

$$\Omega_2^n(\rho_n J_1 J_2 \cdots J_n J;\sigma_n J_1' J_2' \cdots J_n' J)=\sum_{\substack{\rho_{n-1}\sigma_{n-1}J_s J_r\\ J_{k_1}J_{k_2}\cdots J_{k_{n-1}}\\ J_{k_1}'J_{k_2}'\cdots J_{k_{n-1}}'}} {}^{2n+1}Z_J(\rho_n J_1 J_2 \cdots J_n \mid \}\rho_{n-1}J_r\bar{\rho}_{n-1}J_s)$$
$$\times {}^{2n+1}Z_J(\sigma_n J_1' J_2' \cdots J_n' \mid \}\sigma_{n-1}J_r\bar{\rho}_{n-1}J_s)$$
$$\times \Omega_2^{n-1}(\rho_{n-1}J_{k_1}J_{k_2}\cdots J_{k_{n-1}}J_r;\sigma_{n-1}J_{k_1}'J_{k_2}'\cdots J_{k_{n-1}}'J_r)$$
$$+\sum_{\substack{\alpha_{n-1}\beta_{n-1}J_s J_r\\ J_{k_1}J_{k_2}\cdots J_{k_{n-1}}\\ J_{k_1}'J_{k_2}'\cdots J_{k_{n-1}}'}} {}^{2n+1}T_J(\rho_n J_1 J_2 \cdots J_n \mid \}\alpha_{n-1}J_r\bar{\alpha}_{n-1}J_s)$$
$$\times {}^{2n+1}T_J(\sigma_n J_1' J_2' \cdots J_n' \mid \}\beta_{n-1}J_r\bar{\alpha}_{n-1}J_s)$$
$$\times \Omega_1^{n-1}(\alpha_{n-1}J_{k_1}J_{k_2}\cdots J_{k_{n-1}}J_r;\beta_{n-1}J_{k_1}'J_{k_2}'\cdots J_{k_{n-1}}'J_r)$$
$$+\sum_{\substack{\nu_{n-1}\mu_{n-1}J_s J_r\\ J_{k_1}J_{k_2}\cdots J_{k_{n-1}}\\ J_{k_1}'J_{k_2}'\cdots J_{k_{n-1}}'}} {}^{2n+1}C_J(\rho_n J_1 J_2 \cdots J_n \mid \}\nu_{n-1}J_r\bar{\nu}_{n-1}J_s)$$
$$\times {}^{2n+1}C_J(\sigma_n J_1' J_2' \cdots J_n' \mid \}\mu_{n-1}J_r\bar{\nu}_{n-1}J_s)$$
$$\times \Omega_4^{n-1}(\nu_{n-1}J_{k_1}J_{k_2}\cdots J_{k_{n-1}}J_r;\mu_{n-1}J_{k_1}'J_{k_2}'\cdots J_{k_{n-1}}'J_r)\ .$$

(7.2)

Comparing Eqs. (7.1) and (7.2) with Eqs. (2.22) and (2.23) we see that the coefficients of the linear expansions f_J, g_J, h_J, and k_J are uniquely determined as function of transformation coefficients.

VIII. MATRIX ELEMENTS OF THE NUCLEAR HAMILTONIAN: OFF-DIAGONAL TERMS

The off-diagonal matrix elements of the first kind, calculated between the one-particle (-hole) wave function and the two-particle (-hole) –one-hole (-particle) CMWF

$$\Xi_1^{0,1}(\beta_0 J;\alpha_1 J_1 J)\equiv\langle \Lambda_J^{-1}(\beta_0)\|H\|\Phi_J^1(\alpha_1 J_1)\rangle \tag{8.1}$$

and

$$\Xi_2^{0,1}(\rho_0 J;\rho_1 J_1 J)\equiv\langle M_J^{-1}(\rho_0)\|H\|\tilde{\Psi}_J^1(\rho_1 J_1)\rangle \tag{8.2}$$

are considered the starting point of the iterative expressions we derive for the off-diagonal matrix elements of the nth kind.

In general we write the following equations for the off-diagonal matrix elements of the nuclear Hamiltonian between CMWF with a different number of particle-hole pairs:

$$\begin{aligned}
&\Xi_1^{n-1,n}(\alpha_{n-1}J_1J_2\cdots J_{n-1}J;\beta_n J_1'J_2'\cdots J_n'J)\\
&\quad=\langle \Phi_J^{n-1}(\alpha_{n-1}J_1J_2\cdots J_{n-1})\|H\|\Phi_J^n(\beta_n J_1'J_2'\cdots J_n')\rangle\\
&\quad=\sum_{\alpha_{n-2}\beta_{n-1}} {}^{2n-1}T_J(\alpha_{n-1}J_1J_2\cdots J_{n-1}\,|\,\}\alpha_{n-2}J_r\bar{\alpha}_{n-2}J_s)^{2n+1}T_J(\beta_n J_1'J_2'\cdots J_n'\,|\,\}\beta_{n-1}J_r\bar{\alpha}_{n-2}J_s)\\
&\qquad\qquad\times\Xi_1^{n-2,n-1}(\alpha_{n-2}J_{k_1}J_{k_2}\cdots J_{k_{n-2}}J_r;\beta_{n-1}J_{k_1}'J_{k_2}'\cdots J_{k_{n-1}}'J_r)\\
&\qquad+\sum_{\rho_{n-2}\sigma_{n-1}} {}^{2n-1}Z_J(\alpha_{n-1}J_1J_2\cdots J_{n-1}\,|\,\}\rho_{n-2}J_r\bar{\rho}_{n-2}J_s)^{2n+1}Z_J(\beta_n J_1'J_2'\cdots J_n'\,|\,\}\sigma_{n-1}J_r\bar{\rho}_{n-2}J_s)\\
&\qquad\qquad\times\Xi_2^{n-2,n-1}(\rho_{n-2}J_{k_1}J_{k_2}\cdots J_{k_{n-2}}J_r;\sigma_{n-1}J_{k_1}'J_{k_2}'\cdots J_{k_{n-1}}'J_r)\\
&\qquad+\sum_{\epsilon_{n-2}\eta_{n-1}} {}^{2n-1}V_J(\alpha_{n-1}J_1J_2\cdots J_n\,|\,\}\epsilon_{n-2}J_r\bar{\epsilon}_{n-2}J_s)^{2n+1}V_J(\beta_n J_1'J_2'\cdots J_n'\,|\,\}\eta_{n-1}J_r\bar{\epsilon}_{n-2}J_s)\\
&\qquad\qquad\times\Xi_3^{n-2,n-1}(\epsilon_{n-2}J_{k_1}J_{k_2}\cdots J_{k_{n-2}}J_r;\eta_{n-1}J_{k_1}'J_{k_2}'\cdots J_{k_{n-1}}'J_r)\,.
\end{aligned} \tag{8.3}$$

In Eq. (8.3) and following the indices $\{J_sJ_r;J_{k_1}J_{k_2}\cdots J_{k_{n-1}};J_{k_1}'J_{k_2}'\cdots J_{k_{n-1}}'\}$ have been, for simplicity, omitted from the sums, and the terms

$$\Xi_3^{n-2,n-1}(\epsilon_{n-2}J_{k_1}J_{k_2}\cdots J_{k_{n-1}}J_r;\eta_{n-1}J_{k_1}'J_{k_2}'\cdots J_{k_{n-1}}'J_r)$$

are the matrix elements of the nuclear Hamiltonian between three-particle states coupled, respectively, with $(n-2)$ particle-hole and the $(n\text{-}1)$ particle-hole pairs.

The matrix elements of the nuclear Hamiltonian calculated on the basis of (6.1) CMWF with a different number of particle-hole pairs are given by an expression similar to the one given in Eq. (8.3), where we introduce the extra terms

$$\Xi_4^{n-2,n-1}(\nu_{n-2}J_{k_1}J_{k_2}\cdots J_{k_{n-2}}J_r;\mu_{n-1}J_{k_1}'J_{k_2}'\cdots J_{k_{n-1}}'J_r)\,,$$

i.e., the matrix elements of the nuclear Hamiltonian calculated in the three-hole states coupled, respectively, with the $(n-2)$ particle-hole and the $(n-1)$ particle-hole pairs.

IX. MATRIX ELEMENTS OF THE ELECTROMAGNETIC OPERATORS

Using the $X^n_{\alpha_n J_1 J_2\cdots J_n J}$ amplitudes and the matrix elements of the electromagnetic operators calculated for the $n=1$, $n_1=2$ case, we write the matrix elements of the electromagnetic operators on the basis of the (5.1) CMWF. Two types of matrix elements occur.

(a) The diagonal

$$\langle \Phi_J^n(\alpha_n J_1 J_2 \cdots J_n) \| M_\lambda \| \Phi_{J'}^n(\beta_n J_1' J_2' \cdots J_n') \rangle$$
$$= \sum_{\alpha_n \beta_n} X^n_{\alpha_n J_1 J_2 \cdots J_n J} X^n_{\beta_n J_1' J_2' \cdots J_n' J'}$$
$$\times \Bigg[\sum_{\alpha_{n-1}\beta_{n-1}} {}^{2n+1}T_J(\alpha_n J_1 J_2 \cdots J_n \mid \} \alpha_{n-1} J_r \bar{\alpha}_{n-1} J_s)\, {}^{2n+1}T_{J'}(\beta_n J_1' J_2' \cdots J_n' \mid \} \beta_{n-1} J_r' \bar{\alpha}_{n-1} J_s)$$
$$\times M_1^{n-1}(\alpha_{n-1} J_{k_1} J_{k_2} \cdots J_{k_{n-1}} J_r; \beta_{n-1} J'_{k_1} J'_{k_2} \cdots J'_{k_{n-1}} J_r')$$
$$+ \sum_{\rho_{n-1}\sigma_{n-1}} {}^{2n+1}Z_J(\alpha_n J_1 J_2 \cdots J_n \mid \} \rho_{n-1} J_r \bar{\rho}_{n-1} J_s)\, {}^{2n+1}Z_{J'}(\beta_n J_1' J_2' \cdots J_n' \mid \} \sigma_{n-1} J_r' \bar{\rho}_{n-1} J_s)$$
$$\times M_2^{n-1}(\rho_{n-1} J_{k_1} J_{k_2} \cdots J_{k_{n-1}} J_r; \sigma_{n-1} J'_{k_1} J'_{k_2} \cdots J'_{k_{n-1}} J_r')$$
$$+ \sum_{\epsilon_{n-1}\eta_{n-1}} {}^{2n+1}V_J(\alpha_n J_1 J_2 \cdots J_n \mid \} \epsilon_{n-1} J_r \bar{\epsilon}_{n-1} J_s)\, {}^{2n+1}V_{J'}(\beta_n J_1' J_2' \cdots J_n' \mid \} \eta_{n-1} J_r' \bar{\epsilon}_{n-1} J_s)$$
$$\times M_3^{n-1}(\epsilon_{n-1} J_{k_1} J_{k_2} \cdots J_{k_{n-1}} J_r; \eta_{n-1} J'_{k_1} J'_{k_2} \cdots J'_{k_{n-1}} J_r') \Bigg] . \tag{9.1}$$

(b) The off-diagonal

$$\langle \Phi_J^{n-1}(\alpha_{n-1} J_1 J_2 \cdots J_n) \| M_\lambda \| \Phi_{J'}^n(\beta_n J_1' J_2' \cdots J_n') \rangle$$
$$= \sum_{\alpha_{n-1} \beta_n} X^{n-1}_{\alpha_{n-1} J_1 J_2 \cdots J_n J} X^n_{\beta_n J_1' J_2' \cdots J_n' J'}$$
$$\times \Bigg[\sum_{\alpha_{n-2}\beta_{n-1}} {}^{2n-1}T_J(\alpha_{n-1} J_1 J_2 \cdots J_{n-1} \mid \} \alpha_{n-2} J_r \bar{\alpha}_{n-2} J_s)\, {}^{2n+1}T_{J'}(\beta_n J_1' J_2' \cdots J_n' \mid \} \beta_{n-1} J_r' \bar{\alpha}_{n-2} J_s)$$
$$\times M_1^{n-2,n-1}(\alpha_{n-2} J_{k_1} J_{k_2} \cdots J_{k_{n-2}} J_r; \beta_{n-1} J'_{k_1} J'_{k_2} \cdots J'_{k_{n-1}} J_r')$$
$$+ \sum_{\rho_{n-2}\sigma_{n-1}} {}^{2n-1}Z_J(\alpha_{n-1} J_1 J_2 \cdots J_{n-1} \mid \} \rho_{n-2} J_r \bar{\rho}_{n-2} J_s)\, {}^{2n+1}Z_{J'}(\beta_n J_1' J_2' \cdots J_n' \mid \} \sigma_{n-1} J_r' \bar{\rho}_{n-2} J_s)$$
$$\times M_2^{n-2,n-1}(\rho_{n-2} J_{k_1} J_{k_2} \cdots J_{k_{n-2}} J_r; \sigma_{n-1} J'_{k_1} J'_{k_2} \cdots J'_{k_{n-1}} J_r')$$
$$+ \sum_{\epsilon_{n-2}\eta_{n-1}} {}^{2n-1}V_J(\alpha_{n-1} J_1 J_2 \cdots J_{n-1} \mid \} \epsilon_{n-2} J_r \bar{\epsilon}_{n-2} J_s)\, {}^{2n+1}V_{J'}(\beta_n J_1' J_2' \cdots J_n' \mid \} \eta_{n-1} J_r' \bar{\epsilon}_{n-2} J_s)$$
$$\times M_3^{n-2,n-1}(\epsilon_{n-2} J_{k_1} J_{k_2} \cdots J_{k_{n-2}} J_r; \eta_{n-1} J'_{k_1} J'_{k_2} \cdots J'_{k_{n-1}} J_r') \Bigg] . \tag{9.2}$$

Formulas (9.1) and (9.2) give the matrix elements of the electromagnetic operators iteratively in terms of the already calculated $M_1^{n-1}, M_2^{n-1}, M_3^{n-1}$ and $M_1^{n-2,n-1}$, $M_2^{n-2,n-1}, M_3^{n-2,n-1}$ matrix elements. For the hole conjugation case we write expressions analogous to the ones given in Eqs. (9.1) and (9.2) with the $Y^n_{\rho_n J_1 J_2 \cdots J_n J}$ amplitudes and the corresponding matrix elements of the electromagnetic operators.

Finally let us consider the following off-diagonal matrix element: $M_1^{1,0}(\alpha_1 J_1 J_r; \alpha_0 J_r')$ calculated between the two-particle–one-hole CMWF, and a single particle wave function. We have

$$M_1^{1,0}(\alpha_1 J_1 J_r; \alpha_0 J_r') \equiv f(J_r J_r') \langle j_2 \| M_\lambda \| j_3 \rangle , \tag{9.3}$$

where the recoupling coefficients have been included in the $\{f\}$ and where $\{j_2\}$ and $\{j_3^{-1}\}$ are the coordinates of the particle-hole pair interacting with the valence particle.

Equation (9.3), restricted to selected particle-hole pairs, gives the correction introduced by Arima *et al.*[5] to the single particle expectation values of the electromagnetic operators. The correction given by the terms of Eq. (9.2) not included in the perturbation theory gives important contributions to the electromagnetic properties of odd-even nuclei, as pointed out in Refs. 2 and 3.

X. CONCLUSION

The electromagnetic properties of the odd-even nuclei are considered mainly from the standpoint of configuration mixing of the shell model. Iterative expressions introduced for the matrix elements of the nuclear Hamiltonian and of the electromagnetic operators simplify the calculations. This new approach introduced for the matrix elements enables us to calculate exactly configuration mixing terms in all order.

The deviation of the valence particle dues of the electromagnetic operators from the experimental values is then explainable in terms of a correct treatment of the configuration mixing as we have proven in ^{17}O, ^{203}Tl, ^{205}Tl, and ^{209}Bi nuclei[2,3] and references quoted therein. The method will be generalized to include random-phase approximation diagrams and extended to even-even nuclei in a further paper. Numerical application to light and medium nuclei is presently under investigation.

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APPENDIX

The normalization constants for the $\Phi^1_{JM}(\alpha_1 J_1)$, $\tilde{\Psi}^1_{JM}(\rho_1 J_1)$, and $\Lambda^1_{JM}(\epsilon_1 J_1)$ CMWF are given by the following expressions:

$$M^{\alpha_1 J_1 Jk}(1)=\left[f(j_1 J_1 k)-\delta_{j_1 j_2}\sum_{J'_r} g(j_1 J_1 J'_r k)\right] , \tag{A1}$$

$$L^{\rho_1 J_1 Jk}(1)=\sum_{J'_r}\left[f^1(j_1 J_1 J'_r k) -\delta_{j_1 j_2}\sum_{J''_r} g^1(j_1 J_1 J'_r J''_r k)\right] , \tag{A2}$$

$$O^{\epsilon_1 J_1 Jk}(1)=\sum_{J'_r}\left[f^2(j_1 J_1 J'_r k)-\sum_{J''_r} g^2(j_1 J_1 J'_r J''_r k)\right] , \tag{A3}$$

where $f(j_1 J_1 k)$, $g_1(j_1 J_1 J'_r k)$, $f^1(j_1 J_1 J'_r k)$, $g^1(j_1 J_1 J'_r J''_r k)$, $f^2(j_1 J_1 J'_r k)$, and $g^2(j_1 J_1 J'_r J''_r k)$ include recoupling coefficients.

To compute the commutator relations for the unit tensor operators $u^k_{m_k}(n)$, $\tilde{h}^k_{m_k}(n)$, and $p^k_{m_k}(n)$, consider the following matrix element:

$$\langle \Phi^n_J(\alpha_n J_1 J_2 \cdots J_n)\|[u^{k_1}_{m_{k_1}}(n)\otimes u^{k_2}_{m_{k_2}}(n)]^k\|\Phi^n_J(\alpha_n J_1 J_2 \cdots J_n)\rangle \tag{A4}$$

and operate on the right two times with the $u^k_{m_k}$'s. The same holds for the $\tilde{h}^k_{m_k}$'s and $p^k_{m_k}$'s.

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