Systematics of isotope production rates

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The systematics of the production rates of stable and unstable nuclei from the nuclear collision are studied so as to unify different methods of analyses under a simple basic physical treatment. It is found that the isotope production rates are well expressed by the binding energy and the mass excess of the produced nucleus under two different thermalization processes: Fast high energy and so-called evaporation processes.

The production of stable and unstable nuclei (simply called "isotopes" in this paper) with nuclear collision is regarded as the scientific realization of modern alchemy. Wide and various research areas and subjects in nuclear physics and astrophysics are expected to be accessible through its study. The experiments on the isotope production have been undertaken with many different combinations of the projectile and target nuclei for a wide range of the incident energy. The isotope production rates so obtained have been studied with several different methods each under appropriate experimental conditions.[1-3] However, thus far no unified basic treatment for those methods has been proposed. In this paper, therefore, we show that the production mechanisms undertaken in those methods are essentially the same, and also that those different methods can be unified under a simple basic physical treatment.

The isotope production rate has been studied mainly through the following three methods.

(1) The empirical formula for simple parametrization. The spallation cross sections of the proton-nucleus collision are found to be well expressed by a simple analytic function of the incident energy and the charges and masses of the target and produced nuclei.[4] This analytic formula has been extended so as to cover a wide range of combinations of target and produced nuclei up to uranium, and made available to astrophysical research as basic data for the mass distribution yielded by a primary cosmic ray.[5] (The formula has also been applied to the case of high-energy heavy-ion collisions.[6]) While this method has a great advantage in the calculation of absolute cross sections and the average multiplicity, we do not know, so far, the physical implications and significance of the various parameters in the formula.[3]

(2) The Q_{gg} rule which has been found in low-energy heavy-ion collisions.[1,2,7] On the assumption of a binary reaction process for a combination of the projectile X, target A, ejectile Y, and residue B, the relative isotope production cross section of Y is found to be expressed by

$$\sigma(Y)\propto\exp(Q_{gg}/T) \text{ with } Q_{gg}=M_X+M_A-M_Y-M_B , \quad (1)$$

where $M_{X(A,Y,B)}$ is the mass of particle X (A, Y, and B, respectively). T is the nuclear temperature. To obtain better agreement with experimental data, however, we need corrections and modifications in the choice of Q_{gg}.[1,2,8]

(3) The two-step model[9] (typically the abrasion-ablation model).[10,11] The idea of this method has been extensively employed in the study of medium and high-energy heavy-ion collisions. Campi and Hüfner have studied the isotope production mechanism by the first (fast) step with the Glauber model and by the second (decay chain) step with the thermodynamical model,[12] and have derived an analytic formula for the decay chain. However, their formula does not work in the neutron-rich and neutron-poor regions.

In this paper, we generalize the first method so as to unify these three methods under a simple basic treatment, and discuss the physical meanings of the parameters in these methods. To show this, we start with a brief review of the first method. Rudstam has shown that the spallation cross section of the nuclear (Z, A) from the target (Z_t, A_t) is nicely expressed by a smooth analytic function such that[4]

$$\sigma(Z,A)=\sigma_0\exp(PA-R\,|\,Z-SA+TA^2\,|^{\nu}) . \quad (2)$$

Here ν is taken to be 2 or $\frac{3}{2}$, and S and T are adjusted so as to give the most stable proton number Z_P for a given A such that

$$Z_P(A)=SA-TA^2 \text{ with } S=0.49 \text{ and } T=0.000\,33 . \quad (3)$$

P and R are constants rather insensitive to Z and A. For $\nu=\frac{3}{2}$, and the incident energy E, P and R are given by

$$P=\begin{cases} a'E^{-b'} \text{ for } E<E_0, \quad a'=20\pm7, \quad b'=0.77\pm0.06 , \\ c' \text{ for } E\geq E_0, \quad c'=0.056\pm0.003 \text{ and } E_0=2.1 \text{ GeV} , \end{cases} \quad (4)$$

and

$$R = d' A^{-e'}, \quad d' = 11.8 \pm 3.0, \text{ and } e' = 0.45 \pm 0.07 , \tag{5}$$

for targets of $Z_t = 23-47$. (Note that P has target mass number dependence and is small for the $Z_t \simeq 82$ region.)

First, we examine the relationship between the $R(Z - Z_P)^{\nu}$ term and the M_Y term in the Q_{gg}. Employing the experimental mass excesses M^{ex}, we can find the following relationship for the R term numerically:

$$[M^{ex}(Z) - M^{ex}(Z_P)]/R(Z - Z_P)^{\nu} \simeq \text{const } (1.5 \sim 2.0) . \tag{6}$$

This relationship suggests that the R term can be rewritten as follows:

$$R(Z - Z_P)^{\nu} = [M^{ex}(Z) - M^{ex}(Z_P)]/T_{ev} (T_{ev} = 1.5 \sim 2.0 \text{ MeV}) . \tag{7}$$

In this sense, Rudstam's formula is very similar to the Q_{gg} rule except for the contribution (M_B) from the residue B. However, the exclusion of the M_B term looks much more realistic because the residual nucleus B is not uniquely determined in the actual situation even in the low-energy heavy-ion collision. That is, since the residue B would involve all the possible nuclear states (not only one fragment but also multifragments), Eq. (1) should be integrated over all possible M_B to obtain the correct production rate of Y. Hence, we obtain the following expression as a reasonable modification of the Q_{gg} rule:

$$\sigma(Y) \sim \exp[-M^{ex}(Z, A)/T_{ev}] . \tag{8}$$

To examine this relationship, we study the mass excess dependence of the experimental isotope production cross sections of ^{16}O-^{208}Pb reactions at the incident energies of 140 MeV and 33.6 GeV (Ref. 1), and summarize those in Figs. 1 and 2. We find that almost all the isotope production rates of the ^{16}O-^{208}Pb reaction at 140 MeV are well reproduced by only one parameter $T_{ev} = 4.9$ MeV. We also find that the replacement of Eq. (1) by Eq. (8) can nicely reproduce the systematics of isotope production rates for the ^{16}O-^{208}Pb reaction at 33.6 GeV, while the Q_{gg} rule makes it difficult to reproduce them.[1] Thus, we can say that Rudstam's formula includes the Q_{gg} rule because Eq. (8) is a reasonable generalization of the Q_{gg} rule. Here we can make the following remark: We do not know, so far, whether we should employ the mass excess $[M^{ex}(Z) - M^{ex}(Z_P)]$, or the binding energy $-[E_B(Z) - E_B(Z_P)]$, in Eq. (7) because both values are close to each other and the difference between them is technically just due to the proton-neutron mass difference (including the contribution of electrons). We, however, choose expression (7) for the following two reasons: (1) The inclusion of mass excesses in Eq. (7) [i.e., Eq. (8)] gives better agreement with experimental data in detailed comparison. (Especially at very neutron excess nuclei.) (2) The R term is chosen to be the difference from the most stable (that is, the most β stable) nucleus. The second reason stems from the question of whether it is really possible to assume that precisely no electrons are captured after the isotope production. While we need more detailed discussion on this choice, we leave it for future study.

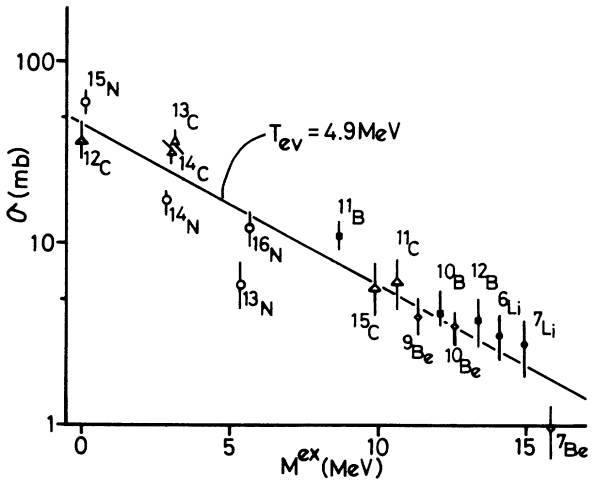

FIG. 1. The mass excess dependence of isotope production cross sections of the ^{16}O-^{208}Pb reaction at 140 MeV incident energy. The experimental data (marks) are taken from Ref. 1.

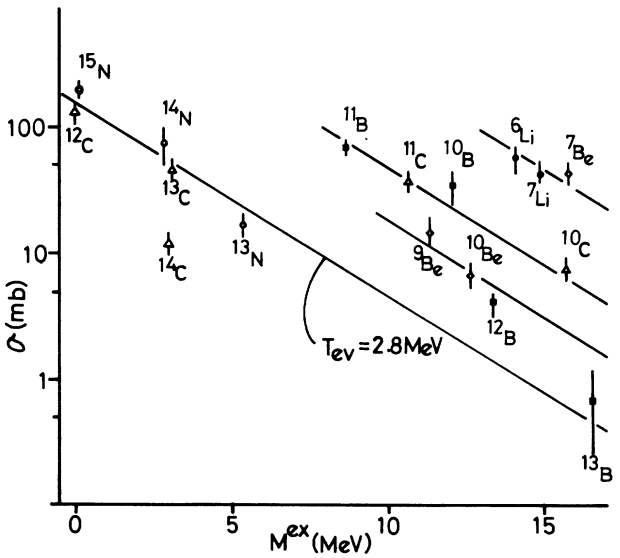

FIG. 2. The mass excess dependence of isotope production cross sections of the ^{16}O-^{208}Pb reaction at 33.6 GeV incident energy. The experimental data (marks) are taken from Ref. 1.

Next, we can make the following remarks for the PA term. Since the average nucleon binding energy of the nucleus is about 8 MeV, we can numerically replace the mass number A by $\frac{1}{8}E_B$ for the binding energy E_B in units of MeV. We then find the following approximate relationships for the PA term:

$$PA = \begin{cases} E_B/T \text{ for } E < E_0 \\ \dfrac{0.056}{8} E_B \simeq E_B/T_0 \text{ for } E \gtrsim E_0 \text{ with } T_0 \simeq 140 \text{ MeV} . \end{cases} \tag{9}$$

Here the nuclear temperature T is assumed to be given by the relationship $E = aT^2$, in which the parameter a is given by the rule $A/13$ (Ref. 13) $\sim A/15$ as a high-temperature value,[14] and the approximation of mass number $A \simeq 75$ is taken for this $Z_t = 23-47$ region. Furthermore, since the value of T is consistent with the usual definition of the nuclear temperature, T_0 may be considered to be a saturation temperature, while 140 MeV of T_0 is almost twice as large as the slope parameter observed in the usual proton-nucleus collisions.[15] Thus, we say that the PA term is related to the temperature given in the one-particle inclusive cross section for the high-energy nuclear collisions. The existence of two temperature parameters T and T_{ev} in the formula suggests that the isotope production mechanism consists of two-step processes; the first process is the fast high-energy process which is consistent with the (thermal) many particle emission process. The second one is the decay chain which is related to the so-called evaporation process. In this sense, the underlying physics in Rudstam's formula is consistent with that of the two-step method. In fact, the parametrization of the decay chain [Eq. (4.3) in Ref. 12] by Campi and Hüfner is very similar to Eq. (7). While their decay chain formula does not work in the neutron-rich and neutron-poor regions, part of this difficulty is resolved if we choose the mass excesses instead of binding energies and chemical energies μ_N and μ_Z in their formula (4.3). Thus, we can say that Rudstam's formula stems from the isotope production mechanism which is parametrized with the binding energy and the mass excess under two different thermalizations: fast high-energy, and so-called evaporation processes, and also that the so-called evaporation process is a common process for these three methods.

Here, as a generalization of these three methods, we examine the following isotope production rate under the two-step processes:

$$\sigma \propto C \exp[\beta E_B - M^{ex}(Z, A)/T_{ev}] . \tag{10}$$

While the $M^{ex}(Z_P)$ term does not show up in the above expression, it is considered to be included in the normalization constant C. Employing expression (10), we study the sodium isotope production cross sections for the p-U collision at the incident energy of 28 GeV,[16] and summarize those in Fig. 3. We obtain the values of $T_{ev} = 2.4$ MeV, $1/\beta = -85$ MeV, and $C = 2.0$ mb by fitting data. These absolute values of T_{ev} and $1/\beta$ obtained are, respectively, consistent with the slope parameters of evaporation and the "slow moving-source" found in the proton-nucleus collisions.[15] Especially, the agreement in the β parameter is quite satisfactory, while its sign is negative. It is also interesting to note that if both the ground state of ^{24}Na and its isomer at 0.472 MeV excitation energy are equally produced with Eq. (10), the total production rate of ^{24}Na agrees with the experimental one. It seems almost impossible to reproduce the rate of ^{21}Na. On the other hand, by applying Eq. (10) to the isotope production rates of ^{16}O-^{208}Pb reactions, we obtain the following values: $\beta = 0.0$ (MeV)$^{-1}$ and $T_{ev} = 4.9$ MeV at 140 MeV incident energy, and $\beta = -0.063$ (MeV)$^{-1}$ and $T_{ev} = 2.8$ MeV at 33.6 GeV.

While negative values of β are obtained with Eq. (10) in contrast with Rudstam's formula (2), this change in the sign of β is considered to be caused by neglect of the $M^{ex}(Z_p, A)$ term in Eq. (10). In fact, if we employ the isotope production rate

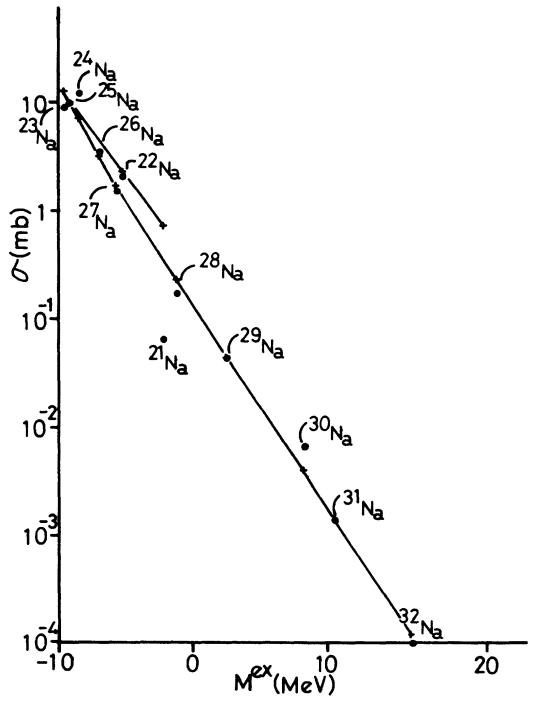

FIG. 3. The mass excess dependence of sodium isotope production cross sections of the p-U reaction at 28 GeV incident energy. The experimental data (solid circles) are taken from Ref. 16. The solid line with + marks is obtained with Eq. (10).

$$\sigma \propto C \exp\{\beta E_B - [M^{ex}(Z, A) - M^{ex}(Z_p, A)]/T_{ev}\} \quad (11)$$

instead of Eq. (10), we can obtain the values of $T_{ev}=8.2$ MeV and $\beta=0.039$ $(\text{MeV})^{-1}$ for Fig. 1, $T_{ev}=4.0$ MeV and $\beta=0.0084$ $(\text{MeV})^{-1}$ for Fig. 2, and $T_{ev}=2.7$ MeV and $\beta=0.11$ $(\text{MeV})^{-1}$ for Fig. 3. Thus, we find that the change in the sign of β is related to the choice of expressions [either Eq. (10) or (11)]. While Eq. (10) reproduces a little bit better values than Eq. (11), the difference in the production rates reproduced is small as far as the data in Figs. 1–3 are concerned. Here we note that the negative sign of β is not inconsistent with the coalescence-type isotope formation process in the nuclear canonical ensemble.[17] Since the βE_B term is related to the isotope production mechanism at the first fast step, the systematic study of the βE_B term may give an answer to the incident energy and momentum, and the target and projectile dependences of the isotope production mechanism.

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