

Relativistic deuteron-nucleus scattering in the Kemmer-Duffin-Petiau formalism

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We present a description of the spin-1 Kemmer-Duffin-Petiau equations and make an application to deuteron-nucleus scattering. Comparison with $d + {}^{58}\text{Ni}$ elastic scattering data at 400 MeV shows that the Kemmer-Duffin-Petiau model reproduces experimental spin observables quite well.

The success of the Dirac equation in describing intermediate-energy proton-nucleus scattering¹⁻⁴ encourages the use of relativistic wave equations in treating other nuclear probes. Recently, the Kemmer-Duffin-Petiau (KDP) formalism,⁵⁻⁷ which yields first-order equations for both spin-0 and spin-1 particles, has been applied to pion-nucleus and kaon-nucleus scattering.⁸ The KDP approach has so far yielded results which are very similar to those obtained from standard treatments of meson-nucleus scattering. It should be remembered, however, that the most dramatic differences between the Dirac and Schrödinger approaches to proton-nucleus scattering occur in the spin observables, which do not exist for the scattering of spin-0 particles from spinless nuclei. For this reason, and for those mentioned in Ref. 8, we now apply the KDP formalism to the scattering of spin-1 probes. In this paper we will give a brief description of the spin-1 KDP formalism, followed by an application to deuteron-nucleus scattering. We find that the KDP-based deuteron-nucleus optical potentials are in close agreement with those obtained by Yahiro *et al.*⁹ using the usual Watanabe¹⁰ approach to d - A scattering. The model is also used to calculate $d + {}^{58}\text{Ni}$ scattering observables at 400 MeV.

The free-particle KDP equation⁵ is

$$(i\beta^\mu \partial_\mu - m)\psi = 0, \quad (1)$$

where the β^μ obey

$$\beta^\mu \beta^\nu \beta^\lambda + \beta^\lambda \beta^\nu \beta^\mu = g^{\mu\nu} \beta^\lambda + g^{\lambda\nu} \beta^\mu. \quad (2)$$

A 16×16 representation for β^μ which satisfies Eq. (2) is

$$\beta^\mu = \frac{1}{2}(I_1 \otimes \gamma_2^\mu + \gamma_1^\mu \otimes I_2), \quad (3)$$

where I is the 4×4 identity matrix, the γ^μ are the Dirac matrices, and $\otimes$ indicates an outer product. The numerical subscript indicates the space in which a given matrix operates. The representation given by Eq. (3) is reducible

and can be decomposed into three irreducible representations: a one-dimensional representation in which all $\beta^\mu = 0$, a five-dimensional one which results in a set of spin-0 equations, and a ten-dimensional representation which gives a set of spin-1 equations. For the free-particle spin-1 case, ψ is a ten-component vector given by $(A^\mu, F^{\mu\nu})$, where the A^μ satisfy the free-particle Proca equation and

$$F^{\mu\nu} = \partial^\mu A^\nu - \partial^\nu A^\mu.$$

As with the Dirac equation an interaction U can be introduced into the KDP formalism yielding

$$(i\beta^\mu \partial_\mu - m - U)\psi = 0. \quad (4)$$

Like the spin-0 case, the most general form of U contains two scalars and two vectors.¹¹ In addition, there exists a pseudoscalar, which is omitted because parity invariance is assumed, and tensor terms, which are not considered because they may produce noncausal behavior.¹¹ The two scalars are given by the 10×10 identity matrix I and the ten-dimensional projection matrix, $P = \text{diag}(1, 1, 1, 1, 0, \dots, 0)$. The two vectors are β^μ and $\tilde{\beta}^\mu = P\beta^\mu - \beta^\mu P$. The form for U , assuming rotational invariance, is

$$U = S(r) + S_p(r)P + \beta^0 V(r) + \beta^0 V_p(r)P. \quad (5)$$

The spacelike parts of the vectors do not contribute to elastic scattering and have been left out of Eq. (5).¹² Using a specific representation¹² of the β^μ , Eqs. (4) and (5) can be combined and written in time-independent form as

$$i\nabla \cdot \mathbf{E} = m_1 \phi, \quad (6a)$$

$$\omega_2 \mathbf{E} - i\nabla \times \mathbf{B} = m_1 \mathbf{A}, \quad (6b)$$

$$\omega_1 \mathbf{A} - i\nabla \phi = m_2 \mathbf{E}, \quad (6c)$$

$$-i\nabla \times \mathbf{A} = m_2 \mathbf{B}, \quad (6d)$$

where

$$m_1(r) = m + S(r) + S_p(r), \quad (7a)$$

$$m_2(r) = m + S(r), \quad (7b)$$

$$\omega_1(r) = E - V(r) - V_p(r), \quad (7c)$$

$$\omega_2(r) = E - V(r), \quad (7d)$$

and $\psi = (\phi, \mathbf{A}, \mathbf{E}, -\mathbf{B})$ in obvious analogy with electrodynamics.

In order to obtain elastic scattering amplitudes it is sufficient to consider the solutions for the three-component equation for $\mathbf{A}$. This equation is

$$\omega_1 \mathbf{A} + \nabla \left[\frac{1}{m_1} \nabla \cdot \left[\frac{m_1}{\omega_2} \mathbf{A} \right] \right] + \nabla \left\{ \frac{1}{m_1} \nabla \cdot \left[\frac{1}{\omega_2} \nabla \times \left[\frac{1}{m_2} \nabla \times \mathbf{A} \right] \right] \right\} - \frac{m_1 m_2}{\omega_2} \mathbf{A} - \frac{m_2}{\omega_2} \nabla \times \left[\frac{1}{m_2} (\nabla \times \mathbf{A}) \right] = 0. \quad (8)$$

It is clearly preferable to use the linear equation. However, at this point we use Eq. (8) to investigate deuteron-nucleus elastic scattering in order to use existing computer codes. The form of this equation depends on the interaction chosen in Eq. (4). In the following we argue that the physics of the problem dictates a particular form for this interaction U .

It is possible to use the procedure of Ref. 8 to formulate a relativistic impulse approximation for the spin-1 KDP equation. However, the loose structure of the deuteron makes it likely that its constituents interact with the nucleus nearly independently of each other. This makes a straightforward application of the impulse approximation questionable for this case. Physically, it is more appropriate to relate deuteron-nucleus scattering to the scattering of two independent nucleons from a nucleus.

To implement the above-mentioned physical picture, we go back to the 16-dimensional reducible KDP equation and note that the structure of the β^μ is suggestive of

a system of two noninteracting spin- $\frac{1}{2}$ particles. We can carry this analogy further by introducing an interaction V of the form

$$V = U_1 \otimes I_2 + I_1 \otimes U_2 \quad (9)$$

which describes two noninteracting particles in the external fields U_1 and U_2 . In our model of deuteron-nucleus scattering the U_i are taken to be nucleon-nucleus optical potentials at half the incident deuteron energy. This treatment is motivated by the usual nonrelativistic model.^{10,13} For the usual scalar-vector Dirac optical potential we write

$$U_i(E) = I_i S^N(E/2) + \gamma_i^\mu V_\mu^N(E/2), \quad (10)$$

where the superscript N denotes a nucleon-nucleus potential and $S^N(V^N)$ is the scalar (vector) part of that potential. Assuming $U_1 = U_2$, the 16×16 matrix equation for ψ is

$$\left\{ \frac{1}{2} (I_1 \otimes \gamma_2^\mu + \gamma_1^\mu \otimes I_2) [p_\mu - 2V_\mu^N(E/2)] - (I_1 \otimes I_2) [m + 2S^N(E/2)] \right\} \psi = 0. \quad (11)$$

Here, twice the nucleon vector potential is subtracted from the derivative term and $2S^N(E/2)$ appears as a "correction" to the mass. The preceding argument suggests that an appropriate choice for the deuteron-nucleus optical potential for use in Eq. (4) is

$$U^D(E) = 2S^N(E/2) + 2V^N(E/2)\beta^0, \quad (12)$$

and that the terms in Eq. (5) containing the projection operator P do not occur. We stress, however, that our approach is one which treats the deuteron-nucleus many-body interaction as an effective one-body framework. The above argument simply makes use of the KDP algebra to give some physical reason for the choice made in Eq. (12). We note that this choice is of the same form as the usual Watanabe model when zero-range deuteron wave functions are assumed.

With this choice, Eq. (8) can be used to derive an effective second-order equation of the form¹²

$$\{p^2 + 2E_d [U_{\text{cent}} + U_{\text{SO}}(\mathbf{L} \cdot \mathbf{S}) + U_{\text{Darwin}}(\mathbf{r} \cdot \mathbf{p}) + U_T^1 \mathbf{S}_2 \cdot \mathbf{R}_2(\mathbf{r}, \mathbf{r}) + U_T^2 \mathbf{S}_2 \cdot \mathbf{R}_2(\mathbf{r}, \mathbf{p})]\} \mathbf{A} \approx k_d^2 \mathbf{A}, \quad (13)$$

with $E_d^2 = k_d^2 + m_d^2$ where E_d is the deuteron energy in the c.m. frame, m_d is its mass, and $\mathbf{S}_2 \cdot \mathbf{R}_2(\mathbf{a}, \mathbf{b})$ are tensor of the form discussed by Satchler.¹⁴ The third term on the left-hand side of Eq. (8) has been neglected since it is of order m_d^{-3} . The expressions for the central and spin-orbit potentials are given by (suppressing energy dependence),

$$2E_d U_{\text{cent}}^d \approx 4[m_d S^N + (S^N)^2 + E_d V^N - (V^N)^2], \quad (14a)$$

$$2E_d U_{\text{SO}}^d = -\frac{1}{r} \frac{\partial}{\partial r} \ln(m_d + 2S^N), \quad (14b)$$

where small terms involving derivatives have been neglected in Eq. (14a). Those terms and the forms of the other potentials in Eq. (13) will be given in Ref. 12.

Relativistic deuteron-nucleus scattering has also been considered by Shepard, Rost, and Murdock¹⁵ employing the Breit equation, and Santos and collaborators, who use

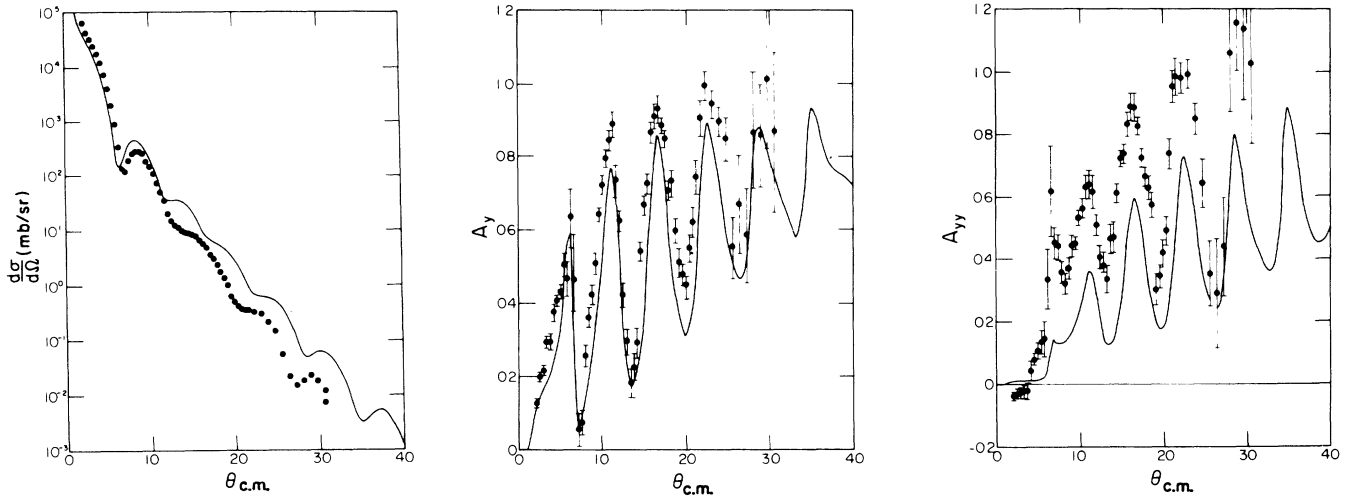


FIG. 1. Calculated elastic scattering observables for $d + {}^{58}\text{Ni}$ at 400 MeV using the effective KDP central and spin-orbit potentials of Eq. (14). The data are from Ref. 18.

the Proca and Weinberg equations.^{16,17} In each of these studies parameters were varied in order to fit the deuteron-nucleus data. Our KDP-based calculation, in contrast, is parameter-free in the sense that once the nucleon-nucleus potentials have been fixed by fitting proton-nucleus data, no other parameters are varied. This provides a clear test of the validity of a relativistic description of deuteron-nucleus scattering data and indicates that the model has predictive power.

The scalar and vector nucleon-nucleus optical potentials, chosen to fit $p + {}^{40}\text{Ca}$ data at 200 MeV, are scaled to represent $p + {}^{58}\text{Ni}$ as was done in Ref. 9. The nuclear parts of the neutron and proton potentials are assumed to be identical and a Coulomb potential is added to the nuclear proton potential. The scalar and vector deuteron-nucleus potentials are large and opposite in sign, and as in the case of proton-nucleus scattering, these large potentials "cancel" to produce an effective central and spin-orbit potentials of reasonable size.

The effective KDP central and spin-orbit potentials are used in the Schrödinger equation with relativistic kinematics as described in Ref. 9, and the $d + {}^{58}\text{Ni}$ elastic

scattering observables are calculated. The results are shown in Fig. 1 along with the data of Ref. 18. The spin observables are reasonably well represented, and are quite similar to the results of Ref. 9. The deviation of the calculated cross sections from the data at larger angles is an expected feature of the model as it contains, for example, no contribution from breakup.

In this paper we have described the spin-1 KDP formalism and used it to calculate deuteron-nucleus optical potentials. We have also found good agreement with deuteron-nucleus data for ${}^{58}\text{Ni}$ at 400 MeV with the spin observables being well represented. It appears promising that the KDP formalism can provide a relativistic framework in which to analyze deuteron-nucleus scattering, and investigation of other cases is underway.¹²

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¹B. C. Clark, S. Hama, and R. L. Mercer, in *The Interaction Between Medium Energy Nucleons in Nuclei (Indiana Cyclotron Facility, Bloomington, Indiana)*, Proceedings of the Workshop on the Interactions Between Medium Energy Nucleons in Nuclei, AIP Conf. Proc. No. 97 edited by H. O. Meyer (AIP, New York, 1983), p. 290.

²J. A. McNeil, J. Shepard, and S. J. Wallace, Phys. Rev. Lett. **50**, 1439 (1983); J. Shepard, J. A. McNeil, and S. J. Wallace, *ibid.* **50**, 1443 (1983).

³B. C. Clark, S. Hama, R. L. Mercer, L. Ray, and B. D. Serot, Phys. Rev. Lett. **50**, 1644 (1983); B. C. Clark, S. Hama, R. L. Mercer, L. Ray, G. W. Hoffmann, and B. D. Serot, Phys.

Rev. C **28**, 1421 (1983).

⁴R. D. Amado, J. Pickarewicz, D. A. Sparrow, and J. A. McNeil, Phys. Rev. C **28**, 2180 (1983).

⁵N. Kemmer, Proc. R. Soc. London, Ser. A **173**, 91 (1939).

⁶R. J. Duffin, Phys. Rev. **54**, 1114 (1938).

⁷G. Petiau, Acad. R. Belg. Mem. Cl. Sci. Collect. **8** **16**, No. 2 (1936).

⁸B. C. Clark, S. Hama, G. R. Kälbermann, R. L. Mercer, and L. Ray, Phys. Rev. Lett. **55**, 592 (1985).

⁹M. Yahiro, H. Kameyama, Y. Iseri, M. Kamimura, and M. Kawai, in Proceedings of the Sixth International Symposium on Polarization Phenomena in Nuclear Physics, Osaka, 1985, [J. Phys. Soc. Jpn. Suppl. **55**, 322 (1986)].

¹⁰S. Watanabe, Nucl. Phys. **8**, 484 (1958).

- ¹¹B. Vijayalakshmi, M. Seetharaman, and P. M. Mathews, J. Phys. A **12**, 665 (1979).
- ¹²B. C. Clark, S. Hama, R. E. Kozack, V. K. Mishra, G. R. Kälbermann, R. L. Mercer, and L. Ray (unpublished).
- ¹³H. Amakawa and N. Austern, Phys. Rev. C **27**, 922 (1983), and references therein.
- ¹⁴G. R. Satchler, *Direct Nuclear Reactions* (Clarendon Press, Oxford, 1983).
- ¹⁵J. R. Shepard, E. Rost, and D. Murdock, Phys. Rev. Lett. **49**, 14 (1982).
- ¹⁶F. Santos and H. van Dam, Phys. Rev. C **34**, 250 (1986); F. Santos, Phys. Lett. B **175**, 110 (1986).
- ¹⁷F. Santos, A. Amorim, A. M. Eiro, and J. C. Fernandes, Proceedings of the Sixth International Symposium on Polarization Phenomena in Nuclear Physics, Osaka, 1985, [J. Phys. Soc. Jpn. Suppl. **55**, 942 (1986)].
- ¹⁸N. Van Sen *et al.*, Phys. Lett. **156B**, 185 (1985).