Trinucleon asymptotic normalization constants: Comparison of ^{3}He and ^{3}H

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(Received 11 January 1988)

Calculations of the trinucleon S- and D-wave asymptotic normalization constants, with and without Coulomb effects, are extended to include all two-body partial waves up to $j \le 4$ (34 three-body channels). Wave functions were generated with configuration-space, Faddeev-type equations for Hamiltonians based upon the two-body forces of Reid and the Argonne group, plus the Tucson-Melbourne and Brazilian model three-body forces. Comparison with previously published results is made. Results for C_S, C_D, η, and D_2 are interpolated as a function of binding to extract best estimates for ^{3}H and ^{3}He. In agreement with our earlier ($j \le 1$) calculations, we find that Coulomb effects increase the S-wave asymptotic normalization of ^{3}He by less than 1% over that of ^{3}H and that Coulomb effects decrease the D-wave asymptotic normalization of ^{3}He relative to that of ^{3}H by about 6%. The distorted-wave Born approximation D-wave parameter D_2^C for ^{3}He is almost identical to D_2 for ^{3}H. Finally, we predict the ratio of the the D-wave to S-wave asymptotic normalization constants to be $\eta(^3\mathrm{H}) \simeq 0.046$ and $\eta^C(^3\mathrm{He}) \simeq 0.043$.

I. INTRODUCTION

Asymptotic normalization constants for the trinucleon bound states have become the subject of increased attention, in part because of the desire to utilize these physical observables to discriminate among trinucleon wave functions generated from various "realistic" models of the nucleon-nucleon (NN) interaction. This goal has not been achieved, because the experimental determination of these observables has not been of sufficient precision and the theoretical predictions were limited principally to models in which convergence of the calculation with respect to the number of two-body partial waves had not been proven. Recently, the convergence question for ^{3}H was addressed by Ishikawa and Sasakawa.[1,2] New measurements relating to the ratio of the D-wave to S-wave asymptotic normalization constants have recently been reported.[3-5] Our purpose here is to update the experimental and theoretical situation since Ref. 6. We extend our earlier calculations,[6] which first estimated Coulomb effects in all of the asymptotic normalization observables, to models which include two-body partial waves up to $j \le 4$ or 34 three-body channels[7] from the $^1S_0$,$^3S_1$-$^3D_1$, or five-channel, calculations possible at that time. We use the Reid soft core (RSC) (Ref. 8) and Argonne V_{14} (AV14) (Ref. 9) two-body forces in this investigation plus the two-pion-exchange three-nucleon force models of the Tucson-Melbourne (TM) (Ref. 10) and Brazilian (BR) (Ref. 11) groups. Special emphasis is given to the treatment of Coulomb effects.

Physically, an asymptotic normalization constant echoes the internal dynamics of the wave function through the overall normalization. The S-wave asymptotic normalization constant is defined such that its value is unity when the effective nuclear interaction in the asymptotic channel of interest is a zero-range interaction. This applies whether a Coulomb interaction is present or not. The presence of a Coulomb interaction changes the zero-range-comparison wave function from a simple exponential into an exponentially decreasing Whittaker function. Because the zero-range limit is never achieved in either case, due to the boundary condition on the wave function at the origin, asymptotic normalization constants differ from unity. The deuteron S-wave asymptotic normalization constant is greater than one, while the D-wave asymptotic normalization constant is less than one; the former is determined primarily by the ^{2}H binding energy and spin-triplet effective range, while the latter appears to follow from dispersion theory with only the ^{2}H binding energy and one-pion exchange as input.[12-14] We can offer no such fundamental explanation of the magnitudes of the trinucleon asymptotic normalization constants. Instead, we accept their basic definitions and their relation to the vertex constants which arise in dispersion theory[15] and ask whether the best estimates from theoretical model calculations agree with the available experimental data.

^{3}H and ^{3}He asymptotic normalization constants have been obtained from experiments by several different means: (1) forward dispersion relation analyses (FDR) (Ref. 16) and partial-wave dispersion relation analyses (PWDR) (Ref. 17); (2) FDR with Coulomb corrections (FDRC) (Refs. 16 and 18); (3) extrapolation of the ^{3}He(p,p)pd cross section to the pole (ECS) (Refs. 19 and 20); (4) distorted-wave Born approximation fits to tensor analyzing powers for ($\vec{d}$,^{3}H) and ($\vec{d}$,^{3}He) reactions

(DWBA):[3,4,21–26] and (5) extrapolation of tensor analyzing powers to the poles in the ($\vec{d}$,^{3}H) and ($\vec{d}$,^{3}He) reactions (ETAP).[5,27,28] The first two rely upon the relation between asymptotic normalization constants and the vertex constants of dispersion theory to extract absolute magnitudes of the S-wave constants C_S and C_S^C. The fourth procedure provides a direct measure of the distorted-wave Born approximation D-state parameter D_2, which is *approximately* related to the negative of the ratio of the D-wave constant C_D to the S-wave constant C_S. Finally, the empirical analytic continuation of the product of the differential cross section and the tensor polarization [to the neutron or proton transfer poles in the ($\vec{d}$,^{3}H) and ($\vec{d}$,^{3}He) reactions] can be directly related to the ratio of the D-wave to S-wave asymptotic normalization constants.

In Tables I and II we list what are considered to be the most reliable and latest values for ^{3}H and ^{3}He, respectively. Several points should be emphasized. First, the absolute values of C_S and C_S^C are not well determined experimentally. The existing values are not consistent, even when the quoted uncertainties are taken into account. It is perhaps time to rethink the determination of the S-wave normalization constants—the method of extraction and the means of measuring precision data. It is not even possible to check the theoretical prediction of Ref. 6, which is confirmed here, that $C_S^C \simeq C_S$. Second, the measured value of D_2 (and D_2^C), clearly establishes the existence of the D state in the triton wave function and that C_D is positive relative to C_S. The apparent lack of agreement among the experimental measurements deserves comment. The more recent analyses have included a complex tensor potential in the deuteron channel which improves the overall DWBA fit to the data.[29] The larger experimental uncertainties take into account all sources of error.[3] Third, the asymptotic D-wave to S-wave ratio $\eta = C_D/C_S$ (and $\eta^C = C_D^C/C_S^C$) is, in principle, directly measured by extrapolating the tensor polarization to the nucleon transfer pole. However, the accuracy of the technique has been called into question in the case of the deuteron. It is claimed by Londergan, Price, and Stephenson[30] to be limited by knowledge of the model amplitude required for the extrapolation. Furthermore, the Coulomb effects exhibited by the most recent data, from a simultaneous measurement using the ^{4}He($\vec{d}$,^{3}H)^{3}He [or ^{4}He($\vec{d}$,^{3}He)^{3}H] reaction,[5] are far larger than the theoretical model can accommodate.

TABLE I. Experimental values for the ^{3}H asymptotic normalization parameters.

Quantity extracted	Value	Method	Reference
C_S^2	2.6 ±0.3	FDR	16
	3.3 ±0.1	PWDR	17
η	0.048±0.007	ETAP	27
	0.051±0.005	ETAP	28
	0.050±0.006	ETAP	5
D_2 (fm^2)	−0.279±0.012	DWBA	24
	−0.288±0.011	DWBA	25
	−0.259±0.014	DWBA	25
	−0.16 ±0.03	DWBA	26
	−0.22 ±0.03	DWBA	26
	−0.20 ±0.04	DWBA	3
	−0.25 ±0.05	DWBA	4

In Table III we give a synopsis of the theoretical situation with respect to Faddeev calculations based upon realistic models of the NN interaction.[2,6,31–34] (We restrict the table to entries for which at least five three-body channels were included, so that the binding energies are close to those of the full Hamiltonian.) The last two entries are variational results for the full potential models (no partial wave expansion).[34] Previous studies have explored the question of convergence of the binding energy with respect to the number of two-body partial waves included.[35] Thirty-four channels (all two-body partial waves for $j \leq 4$) are needed when three-body forces are included.[7] A recent study has shown that the omitted higher partial waves ($j > 4$) would contribute at most about 10 keV to the binding energy in the absence of a three-body force.[36] It is clear from the results in the table that there is a strong dependence upon the binding energy. We have pointed out this previously for other trinucleon observables.[37,38] It was noted explicitly for η in Ref. 1. We will use this relation to interpolate, from the numerous model calculations that we have performed, a best estimate of the theoretical values for each of the asymptotic normalization parameters. All previous model calculations comparable to those in Table III which addressed the question of Coulomb effects were reported in Ref. 6 and are not repeated here as Coulomb effects will be explored in detail below.[39]

With the above background in mind, our objectives in this paper are twofold: (1) To present complete results for the RSC and AV14 models of the NN interaction plus the TM and BR three-body force models, and (2) to make a best estimate of the theoretical values of the trinucleon asymptotic normalization constants and compare with the experimental data. To achieve this, we have structured the paper as follows. Section II contains a review of the formalism, Sec. III comprises our numerical results, Sec. IV covers our comparison with experiment, and Sec. V concludes with a brief discussion and summary.

II. FORMALISM

Integral relations for calculation of the triton asymptotic normalization constants were first derived in Ref. 40 (see also Ref. 41). This was generalized in Ref. 6 to include the Coulomb interaction[39] and therefore the asymptotic normalization of ^{3}He. The relationship to the distorted-wave Born approximation D-wave parameter D_2 (see Refs. 42–45) was explored numerically in Ref. 45. We briefly summarize the relevant relations[6] here, emphasizing the modifications that arise due to the presence of the Coulomb force acting between the two protons in ^{3}He.

We begin by defining the S- and D-wave asymptotic normalization constants for ^{3}H and ^{3}He. Recall that the trinucleon bound states have $J^\pi = \frac{1}{2}^+$ and the deuteron has $J^\pi = 1^+$. The ^{3}H asymptotic normalization constants are defined by

TABLE II. Experimental values for the ^{3}He asymptotic normalization parameters.

Quantity extracted	Value	Method	Reference
$(C_S^C)^2$	3.24 ±0.19	FDRC	16,18
	(average of 3.3±0.4 and 3.19±0.24)		
	2.40 ±0.18	ECS	19
	2.50 ±0.18	ECS	19
	3.5 ±0.2	ECS	20
	2.8 ±0.4	(reanalysis of	20
	2.8 ±0.4	published data)	20
η^C	0.036±0.006	ETAP	5
D_2^C (fm^2)	−0.37	DWBA	21
	−0.22	DWBA	22
	−0.339	DWBA	23
	−0.17 ±0.04	DWBA	26
	−0.21 ±0.04	DWBA	26
	−0.25 ±0.05	DWBA	3
	−0.24 ±0.04	DWBA	4

$$\lim_{y_1\to\infty} \Psi^{[1/2]}_{^3\mathrm{H}}(\mathbf{y}_1,\mathbf{x}_1) \to C_S N_{ZR}\frac{e^{-\beta y_1}}{y_1}[[Y_0(\hat{\mathbf{y}}_1)\times\chi^{1/2}(1)]^{[1/2]}\times\Phi^{[1]}(\mathbf{x}_1)]^{[1/2]}\frac{\eta'}{\sqrt{2}}$$
$$+C_D N_{ZR}\frac{e^{-\beta y_1}}{y_1}\left[1+\frac{3}{\beta y_1}+\frac{3}{\beta^2 y_1^2}\right][[Y_2(\hat{\mathbf{y}}_1)\times\chi^{(1)}(1)]^{[3/2]}\times\Phi^{[1]}(\mathbf{x}_1)]^{[1/2]}\frac{\eta'}{\sqrt{2}}\ , \tag{1}$$

where C_S and C_D are the triton S- and D-wave asymptotic normalization constants, respectively. The $Y_l(\hat{\mathbf{y}})$ is a spherical harmonic (m suppressed because of the coupling $[\times]^{[J]}$), $\chi^{1/2}$ is a spin-$\frac{1}{2}$ function, $\Phi^{[1]}$ is the deuteron wave function, and η' is the isospin-$\frac{1}{2}$ function with projection $-\frac{1}{2}$ (^{3}H) for three nucleons in which nucleons 2 and 3 (of the deuteron) are coupled to isospin 0. The Jacobi coordinates $\mathbf{x}_i=(\mathbf{r}_j-\mathbf{r}_k)$ and $\mathbf{y}_i=\mathbf{r}_i-\frac{1}{2}(\mathbf{r}_j+\mathbf{r}_k)$, which delineate the distance between the interacting pair (j,k) and between the pair and the spectator nucleon with (i,j,k) taken in cyclic order, are illustrated in Fig. 1. The N_{ZR} is the normalization constant for the zero-range function describing the spectator relative to the interacting pair and is defined by

$$N_{\mathrm{ZR}}^{-2}=\int_0^\infty dy_1 e^{-2\beta y_1} \tag{2a}$$

or

$$N_{\mathrm{ZR}}=\sqrt{2\beta}\ , \tag{2b}$$

where $\beta^2=4M[B(^3\mathrm{H})\text{-}B(^2\mathrm{H})]/3$ is 0.2012 fm^{-2} for the experimental binding energies.

The ^{3}He asymptotic normalization constants are defined analogously by

$$\lim_{y_1\to\infty} \Psi^{[1/2]}_{^3\mathrm{He}}(\mathbf{y}_1,\mathbf{x}_1) \to C_S^C N_W\frac{W_{-\kappa,1/2}(2\beta y_1)}{y_1}[[Y_0(\hat{\mathbf{y}}_1)\times\chi^{1/2}(1)]^{[1/2]}\times\Phi^{[1]}(\mathbf{x}_1)]^{[1/2]}\frac{\bar{\eta}'}{\sqrt{2}}$$
$$+C_D^C N_W\frac{W_{-\kappa,5/2}(2\beta y_1)}{y_1}[[Y_2(\hat{\mathbf{y}}_1)\times\chi^{1/2}(1)]^{[3/2]}\times\Phi^{[1]}(\mathbf{x}_1)]^{[1/2]}\frac{\bar{\eta}'}{\sqrt{2}}\ , \tag{3}$$

TABLE III. Theoretical values for the ^{3}H asymptotic normalization constants.

Model	B_3 (MeV)	C_S	C_D	η	$\tilde{D}_2$ (fm^2)	Reference
RSC5	6.96	1.776	0.065			31
OBE	7.38	1.706				32
RSC5	7.1	1.76	0.065			33
RSC5	7.02	1.758	0.0658		−0.243	6
RSC18(TM)$_{700}$	8.21	1.83	0.0769	0.0421	−0.219	2
AV34(TM)$_{700}$	8.42	1.87	0.0829	0.0444	−0.223	2
P34(TM)$_{700}$	8.32	1.84	0.0794	0.0431	−0.220	2
TRS34(TM)$_{700}$	8.47	1.85	0.0812	0.0439	−0.219	2
AV(UR)	8.37				−0.23 D_2	34
UR(UR)	8.48				−0.27 D_2	34

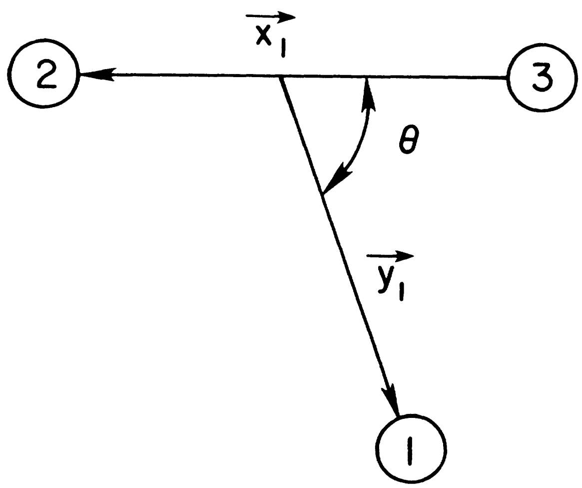

FIG. 1. Example of Jacobi coordinates as used in the text.

where C_S^C and C_D^C are the ^{3}He S- and D-wave asymptotic normalization constants, respectively. Here, $\bar{\eta}'$ is the isospin-$\frac{1}{2}$ function with projection $+\frac{1}{2}$(^{3}He). The $W_{-\kappa,J}(z)$ is a Whittaker function that behaves irregularly at the origin and decays exponentially for $z \to \infty$, and κ is defined in terms of the fine structure constant ($\alpha = \frac{1}{137}$) by

$$\kappa = \frac{2M}{3\beta}\alpha \ , \tag{4}$$

where β is given by

$$\beta^2 = 4M[B(^3\text{He}) - B(^2\text{H})]/3 \simeq 0.1766 \text{ fm}^{-2} \ .$$

The normalization constant N_W is defined by

$$N_W^{-2} = \int_0^\infty dy_1 [W_{-\kappa,1/2}(2\beta y_1)]^2 \tag{5}$$

and is related to N_{ZR} by

$$N_W = N_{\text{ZR}} \left[\frac{\Gamma(3+\kappa)\Gamma(2+\kappa)}{2\,{}_3F_2(\kappa,2,1+\kappa;3+\kappa,2+\kappa;1)} \right]^{1/2} \tag{6a}$$

$$= N_{\text{ZR}}\Gamma(2+\kappa)/[{}_3F_2(1,\kappa,\kappa;2+\kappa,2+\kappa;1)]^{1/2} \ . \tag{6b}$$

Clearly, when $\kappa \to 0$ (as the Coulomb interaction is turned off) one finds $N_W \to N_{\text{ZR}}$. Also, we have

$$\lim_{y_1 \to \infty} W_{-\kappa,1/2}(2\beta y_1) \to \frac{e^{-\beta y_1}}{(2\beta y_1)^\kappa} \underset{\kappa \to 0}{\longrightarrow} e^{-\beta y_1} \ . \tag{7}$$

We emphasize that it is not correct, nor is it a valid approximation, to use the asymptotic form of the Whittaker function in Eq. (3).[46]

The Faddeev equations are exactly equivalent to the Schrödinger equation. There are, however, alternative ways to include the Coulomb interaction and three-body forces along with the usual short-range two-body forces in the former.[6,7] Although these alternative formulations are all equivalent in principle, they differ in practice when the Faddeev equations are truncated in terms of the number of partial waves or three-body channels. Because the Coulomb force is long ranged, care should be exercised in treating it. We use here a prescription corresponding to $p = 1$ in our previous study.[6] That is, the term corresponding to the interaction of one proton with the center of mass of the remaining neutron-proton pair in ^{3}He enters only the asymptotic normalization integral, not the Faddeev equations *per se*. We ignore the coupling to the isoquartet channel due to the Coulomb interaction and the small charge dependence of the strong interaction. We employ a point Coulomb interaction in calculating our ^{3}He wave functions.[47] In treating the three-body force, we use the W_1 prescription of Ref. 7. That is, we decompose the three-body force in a manner analogous to the natural decomposition of the sum of the two-body forces.

The Schrödinger wave function for ^{3}He can be written in terms of the Faddeev amplitudes as

$$\Psi_{^3\text{He}}(\mathbf{y}_1,\mathbf{x}_1) = (1 + P^- + P^+)\psi^C(\mathbf{y}_1,\mathbf{x}_1) \ , \tag{8}$$

where P^- and P^+ are the permutation operators that yield

$$P^-\psi^C(\mathbf{y}_1,\mathbf{x}_1) = \psi^C(\mathbf{y}_2,\mathbf{x}_2) \tag{9a}$$

and

$$P^+\psi^C(\mathbf{y}_1,\mathbf{x}_1) = \psi^C(\mathbf{y}_3,\mathbf{x}_3) \ . \tag{9b}$$

The Faddeev amplitude is written as a sum

$$\psi^C(\mathbf{y}_1,\mathbf{x}_1) = \sum_{\nu=1}^{N} \psi_\nu^C(y_1,x_1)\,|\,\nu\rangle \tag{10}$$

over $|\,\nu\rangle$ states containing spin, isospin, and orbital angular momentum.

The integer N numbers the three-body channels comprising a given model: $N = 5$ corresponds to including all two-body partial waves with

$$j^\pi \le 1^+ \ ({}^1S_0, {}^3S_1 - {}^3D_1) \ ,$$

$N = 9$ corresponds to $j^\pi \le 2^+$, $N = 18$ corresponds to $j \le 2$, and $N = 34$ corresponds to $j \le 4$. Convergence for various physical observables as a function of N was explored in Refs. 35 and 7. The convergence study of Ishikawa and Sasakawa for the triton asymptotic normalization constants[1,2] and the results below for ^{3}He demonstrate that $N = 34$ provides a reliable result, even when three-nucleon forces are included.

Our Faddeev amplitudes satisfy equations of the form

$$[H_0 + V(\mathbf{x}_1) + V_C(x_1,x_2,x_3) + W_1(\mathbf{y}_1,\mathbf{x}_1) - E]\psi^C(\mathbf{y}_1,\mathbf{x}_1) = -[V(\mathbf{x}_1) + W_1(\mathbf{y}_1,\mathbf{x}_1)][\psi^C(\mathbf{y}_2,\mathbf{x}_2) + \psi^C(\mathbf{y}_3,\mathbf{x}_3)] \ , \tag{11}$$

where

$$V_C(x_1,x_2,x_3) = P_C^{1,2}\frac{\alpha}{x_3} + P_C^{2,3}\frac{\alpha}{x_1} + P_C^{3,1}\frac{\alpha}{x_2} \ , \tag{12a}$$

$$P_C^{i,j}=\tfrac{1}{4}(1+\tau_3^i)(1+\tau_3^j)\ , \tag{12b}$$

$$V(\mathbf{x}_i)=\sum_{\substack{\nu'=1\\ \nu=1}}^{N} |\nu\rangle V_{\nu\nu'}(x_i)\langle\nu'|\ , \tag{13a}$$

$$V_{2BF}=\sum_{i=1}^{3} V(\mathbf{x}_i)\ , \tag{13b}$$

$$W_i(\mathbf{y}_1,\mathbf{x}_1)=\sum_{\substack{\nu'=1\\ \nu=1}}^{N} |\nu\rangle W_{i\nu\nu'}(y_1,x_1)\langle\nu'|\ , \tag{14a}$$

and

$$W_{3BF}=\sum_{i=1}^{3} W_i(\mathbf{y}_1,\mathbf{x}_1)\ . \tag{14b}$$

Following the procedure outlined from Eq. (41) of Ref. 6, the integral relations that yield C_L^C $(L=S,D)$ in terms of projections involving the deuteron S-state and D-state functions coupled to the spectator nucleon $\langle d,L;\mathbf{y}_1|$ are

$$\begin{aligned}C_L^C=&-\frac{4M}{3}\frac{\left[\prod_{n=1}^{L}(1+\kappa/n)\right]\Gamma(1+\kappa)}{N_W 2^{L+1}\beta(2L+1)!!}\\ &\times\int_0^\infty y_1 dy_1 M_{-\kappa,L+1/2}(2\beta y_1)\\ &\times\Bigg[\langle d,L;\mathbf{y}_1|V(\mathbf{x}_1)|\psi^C(\mathbf{y}_2,\mathbf{x}_2)+\psi^C(\mathbf{y}_3,\mathbf{x}_3)\rangle\\ &+\langle d,L;\mathbf{y}_1|W_1(\mathbf{y}_1,\mathbf{x}_1)|\psi^C(\mathbf{y}_1,\mathbf{x}_1)+\psi^C(\mathbf{y}_2,\mathbf{x}_2)+\psi^C(\mathbf{y}_3,\mathbf{x}_3)\rangle+\left\langle d,L;\mathbf{y}_1\left|V_C-\frac{\alpha}{y_1}\right|\psi^C(\mathbf{y}_1,\mathbf{x}_1)\right\rangle\Bigg]\ ,\end{aligned} \tag{15}$$

where $M_{-\kappa,J}(z)$ is the Whittaker function that vanishes at the origin and diverges exponentially as $z\to\infty$. The terms that appear under the integral are the terms that remain on the right-hand side of Eq. (11) after converting the left-hand side to the form required to obtain the needed Green's function. The non-Coulomb or ^{3}H integral relations follow immediately by taking the limit $\kappa\to 0$. Because

$$M_{0,L+1/2}(2z)=\frac{(2L+1)!}{L!}\sqrt{2\pi z}\,I_{L+1/2}(z)\ , \tag{16}$$

where $I_J(z)$ is the modified Bessel function, we obtain

$$\begin{aligned}C_L=&-\frac{2M}{3}\frac{\sqrt{\pi}}{\beta}\int_0^\infty y_1^{3/2}dy_1 I_{L+1/2}(\beta y_1)\\ &\times[\langle d,L;\mathbf{y}_1|V(\mathbf{x}_1)|\psi(\mathbf{y}_2,\mathbf{x}_2)+\psi(\mathbf{y}_3,\mathbf{x}_3)\rangle\\ &+\langle d,L;\mathbf{y}_1|W_1(\mathbf{y}_1,\mathbf{x}_1)|\psi(\mathbf{y}_1,\mathbf{x}_1)+\psi(\mathbf{y}_2,\mathbf{x}_2)+\psi(\mathbf{y}_3,\mathbf{x}_3)\rangle]\ .\end{aligned} \tag{17}$$

The quantity η is the ratio of the D-wave to S-wave asymptotic normalization constants:

$$\eta=C_D/C_S \tag{18a}$$

and

$$\eta^C=C_D^C/C_S^C\ . \tag{18b}$$

The η's may also be thought of as the ratios of coupling constants for ^{3}H$\to$nd(^{3}He$\to$pd) vertices. Experimentally they can be determined from an analytic continuation of T_{20} measurements in ^{4}He(d,^{3}He)^{3}H and ^{2}H(d,p)^{3}H experiments to the appropriate pole.[5,27] Because of the difficulty in making these extrapolations, more attention has been given the distorted-wave Born approximation D-wave parameters D_2 and D_2^C, which are both defined by[48]

$$D_2=-\frac{\int_0^\infty dy_1 y_1^4 u_2(y_1)}{15\int_0^\infty dy_1 y_1^2 u_0(y_1)}\ . \tag{19}$$

The $u_l(y_1)$ are the effective wave functions in the spectator coordinate of the *N-d* system, the Fourier transformations of the N-d momentum distribution amplitudes. They are given by the projections[45]

$$\langle \mathbf{y}_1 \chi^{1/2}_{m_N}; \Phi^{[1]}_{m_d} \mid \Psi^{[1/2]}_m(i) \rangle = u_0(y_1) \langle \tfrac{1}{2} m_N 1 m_d \mid \tfrac{1}{2} m \rangle Y_{00}(\hat{\mathbf{y}}_1) + u_2(y_1) \sum_M \langle \tfrac{1}{2} m_N 1 m_d \mid \tfrac{3}{2} M \rangle \sum_{m_l} \langle 2 m_l \tfrac{3}{2} M \mid \tfrac{1}{2} m \rangle Y_{2m_l}(\hat{\mathbf{y}}_1) \ . \tag{20}$$

These definitions hold for both ^{3}H and ^{3}He and are consistent with the asymptotic normalization constant definitions given in Eqs. (1) and (3). The two cases are distinguished by $i = {}^3$H,

$$u_0(y_1) \underset{y_1 \to \infty}{\longrightarrow} C_S N_{ZR} \frac{e^{-\beta y_1}}{y_1} \ , \tag{21a}$$

$$u_2(y_1) \underset{y_1 \to \infty}{\longrightarrow} C_D N_{ZR} \frac{e^{-\beta y_1}}{y_1} \left[1 + \frac{3}{\beta y_1} + \frac{3}{(\beta y_1)^2} \right] \tag{21b}$$

and $i = {}^3$He,

$$u_0(y_1) \underset{y_1 \to \infty}{\longrightarrow} C_S^C N_W \frac{W_{-\kappa,1/2}(2\beta y_1)}{y_1} \ , \tag{22a}$$

$$u_2(y_1) \underset{y_1 \to \infty}{\longrightarrow} C_D^C N_W \frac{W_{-\kappa,5/2}(2\beta y_1)}{y_1} \ . \tag{22b}$$

To the extent that the y_1^4 in the integrand of the numerator and the y_1^2 in the integrand of the denominator justify replacing the $u_l(y_1)$ functions by their respective asymptotic forms, one can derive

$$\tilde{D}_2 = -\frac{C_D}{\beta^2 C_S} \tag{23a}$$

and

$$\tilde{D}_2^C = -\frac{C_D^C}{\beta^2 C_S^C} f(\kappa) \tag{23b}$$

where

$$f(\kappa) = 6 \frac{{}_2F_1(2, \kappa-2; 5+\kappa; -1)}{(4+\kappa)(3+\kappa)\,{}_2F_1(2, \kappa; 3+\kappa; -1)} \tag{24a}$$

$$= 1 - \tfrac{17}{30}\kappa + \frac{89 - 112\ln(2)}{60}\kappa^2 + \cdots \ . \tag{24b}$$

The D_2 and D_2^C are proportional to the experimental spectroscopic factors extracted from (d,t) and $(d,{}^3\mathrm{He})$ transfer reactions. They are also the ratio of the amplitudes in momentum space (Ref. 45), $-u_2(q)/q^2 u_0(q)$ evaluated at $q^2=0$. We emphasize the sign difference between η and D_2. We also note that the approximation

$$-\eta = \beta^2 \tilde{D}_2 \simeq \beta^2 D_2 \tag{25}$$

has been demonstrated to be excellent for the deuteron.[49] It was found to be acceptable for the triton in a separable potential study[45] and in Ref. 2. We also recall that we demonstrated previously[6] that

$$\tilde{D}_2^C \simeq \tilde{D}_2 \ , \tag{26}$$

because the decrease in the trinucleon binding energy between ^{3}H and ^{3}He is almost exactly compensated for by the decrease in C_D^C compared to C_D and the factor $f(\kappa)$.

III. NUMERICAL RESULTS

Results of our numerical calculations are summarized in Tables IV and V for the RSC and AV14 models. The third decimal is retained in order that one can examine the convergence with respect to the number of three-body channels included. Comparison of the RSC five-channel results quoted here with those in Ref. 6 shows a slight difference in the third decimal due to the use of different meshes in the numerical solution of the configuration-space differential equations. Here we have used the smaller mesh (14 ρ points and 14 θ points), so that differences among the various entries in Tables IV and V are not due to the use of different meshes. The parameter Λ in the three-body force models was selected to be 5.8 m_π; see Ref. 7.

In the course of investigating the convergence properties of the Faddeev equation solutions we used diverse combinations of two-body and three-body potentials as well as numbers of channels, which resulted in a wide range of eigenvalues. The corresponding wave functions were used to calculate physical observables and form a type of theoretical data base. Many of these observables, when plotted versus the corresponding binding energy, exhibit scaling with that energy. That is, the observables exhibit a simple behavior as a function of the trinucleon binding energy B_3, with only a small spread of values for a given B_3. Such is the case for the asymptotic normalization parameters. Fits to these distributions, extrapolated to the experimental binding energy, probably provide the best theoretical estimates of the trinucleon observables, providing that scaling is valid. (Some caution is warranted, however, because data sets limited to models which do not include the correct physics can lead to false conclusions.) We have employed this technique previously to estimate charge radii, Coulomb energies, etc.[37,38] Sasakawa and Ishikawa[1] have used it to estimate the triton asymptotic normalization ratio η. We shall use it here to predict asymptotic normalization parameters for ^{3}H and ^{3}He.

In Figs. 1–4 we present plots of our calculated $C_S(C_S^C)$, $C_D(C_D^C)$, $\eta(\eta^C)$, and $\tilde{D}_2(\tilde{D}_2^C)$ versus the ^{3}H(^{3}He) binding energy. It is from the best fits to these results that we make our estimate of the values that these parameters should exhibit for the physical trinucleon systems. From our numerical results shown in Fig. 2 we find best-fit values at the experimental binding energies of

$$C_S = 1.850 \text{ and } C_S^C = 1.854 \ .$$

TABLE IV. Asymptotic normalization results for the RSC model.

	$B(^3\text{H})$	$B(^3\text{He})$	C_S	C_S^C	C_D	C_D^C	η	η^C	$\tilde{D}_2$ (fm^2)	$\tilde{D}_2^C$ (fm^2)
RSC										
5	7.02	6.37	1.752	1.757	0.0670	0.0628	0.0382	0.0357	−0.248	−0.259
9	7.21	6.55	1.768	1.770	0.0694	0.0647	0.0393	0.0367	−0.242	−0.254
18	7.23	6.58	1.766	1.778	0.0689	0.0652	0.0390	0.0367	−0.242	−0.253
34	7.35	6.70	1.774	1.787	0.0707	0.0672	0.0398	0.0376	−0.242	−0.253
RSC-TM ($\Lambda=5.8$ m_π)										
5	7.55	6.88	1.791	1.796	0.0728	0.0688	0.0406	0.0383	−0.237	−0.247
9	8.33	7.64	1.833	1.837	0.0818	0.0772	0.0446	0.0420	−0.227	−0.234
18	8.92	8.22	1.860	1.875	0.0881	0.0848	0.0474	0.0452	−0.220	−0.228
34	8.86	8.16	1.862	1.877	0.0895	0.0862	0.0480	0.0460	−0.225	−0.234
RSC-BR ($\Lambda=5.8$ m_π)										
5	7.66	6.99	1.795	1.800	0.0756	0.0714	0.0421	0.0396	−0.241	−0.250
9	8.77	8.06	1.848	1.852	0.0879	0.0832	0.0476	0.0449	−0.226	−0.232
18	8.70	8.00	1.847	1.862	0.0867	0.0833	0.0469	0.0447	−0.226	−0.234
34	8.89	8.20	1.861	1.876	0.0904	0.0871	0.0486	0.0464	−0.226	−0.235

Thus, Coulomb effects increase the S-wave asymptotic normalization constant by less than 1%. To a very good approximation, the ^{3}H and ^{3}He S-wave asymptotic normalization constants should be the same, as was found earlier. (For a detailed discussion of the Coulomb effects in C_S^C, see Refs. 6 and 50.) Our values of C_S agree with those of Ishikawa and Sasakawa in the sense that even their values for potential models other than those which we have investigated lie close to our best fit curve. [Note that their three-body force cutoff (see Table III) is 700 MeV whereas ours is 5.8 m_π ($\simeq 810$ MeV).] Also, our C_S calculations for the RSC and AV14 NN force models alone (with no three-body force) appear to agree with theirs (not shown in Table III) at the level of ± 0.01. Of the models quoted in Table III, only the OBE calculation of Ref. 32 appears to disagree with our results; from our best-fit curve we would predict $C_S=1.786$ based upon that model's binding energy instead of the quoted 1.706.

Unfortunately, it is not possible to make a definitive comparison with experiment for either ^{3}H or ^{3}He, because the experimental values for C_S and C_S^C disagree among themselves. Our value of $C_S^2=3.42$ would favor the early PWDR evaluation of Ref. 17. Likewise, our value of $(C_S^C)^2=3.44$ would favor the FDRC value of Ref. 18 as well as the larger of the value from Ref. 20. However, the group using the former procedure (FDRC) disagree with our prediction for C_S^2 and the latter group offer three values in their reanalysis of older data, two of which agree with one another and disagree with our prediction. Thus, we urge that a definitive measurement of the S-wave asymptotic normalization constants for ^{3}H and ^{3}He be made, to test the model prediction from these benchmark Faddeev solutions of the trinucleon bound-state problem.

From our numerical results plotted in Fig. 3, we find best-fit values at the experimental binding energies of

$$C_D=0.0849 \text{ and } C_D^C=0.0803 .$$

For the D-wave asymptotic normalization constant, Coulomb effects are not insignificant, being some 5.5%.

TABLE V. Asymptotic normalization results for the AV14 model.

	$B(^3\text{H})$	$B(^3\text{He})$	C_S	C_S^C	C_D	C_D^C	η	η^C	$\tilde{D}_2$ (fm^2)	$\tilde{D}_2^C$ (fm^2)
AV14										
5	7.44	6.78	1.813	1.816	0.0723	0.0681	0.0399	0.0375	−0.238	−0.248
9	7.57	6.90	1.825	1.826	0.0741	0.0692	0.0406	0.0379	−0.236	−0.244
18	7.57	6.92	1.815	1.827	0.0735	0.0698	0.0405	0.0389	−0.235	−0.245
34	7.67	7.01	1.821	1.833	0.0750	0.0715	0.0412	0.0390	−0.235	−0.245
AV14-TM ($\Lambda=5.8$ m_π)										
5	8.26	7.57	1.859	1.866	0.0824	0.0785	0.0444	0.0421	−0.229	−0.237
9	8.94	8.24	1.892	1.898	0.0901	0.0857	0.0476	0.0452	−0.221	−0.227
18	9.49	8.78	1.909	1.925	0.0961	0.0929	0.0503	0.0483	−0.215	−0.222
34	9.36	8.65	1.908	1.924	0.0969	0.0938	0.0508	0.0487	−0.221	−0.229
AV-14-BR ($\Lambda=5.8$ m_π)										
5	8.32	7.62	1.859	1.866	0.0849	0.0808	0.0457	0.0433	−0.233	−0.242
9	8.27	8.55	1.901	1.906	0.0955	0.0907	0.0502	0.0476	−0.222	−0.227
18	9.06	8.36	1.888	1.904	0.0930	0.0896	0.0492	0.0470	−0.224	−0.232
34	9.22	8.51	1.899	1.915	0.0963	0.0930	0.0507	0.0485	−0.225	−0.233

Thus, C_D^C is smaller than C_D, as was discussed previously in Ref. 6. In this case, our values of C_D appear to be some 3–5 % larger than those of Ref. 2, where direct comparison can be made for the NN force Hamiltonian without a three-body force. At most 1% of this can be attributed to our use of the smaller mesh here as compared to our previous study. The difference is larger in the case of the RSC potential model, where short-range repulsion is greater, than for the AV14 model. So far, the trinucleon D-wave asymptotic normalization constants have not been directly extracted from experiment. Only their ratios to the S-wave constants have been measured, which we discuss next.

From our numerical results shown in Fig. 4, we find best-fit values for the ratios of the S-wave to D-wave asymptotic normalization constants at the experimental binding energies of

$$\eta=0.0458 \text{ and } \eta^C=0.0432 .$$

The difference reflects the measurable Coulomb effect in the D-wave asymptotic normalization constants discussed above. Thus, η^C is smaller than η by about 6%. Our best estimate of η differs from the value of 0.0432 quoted by Ishikawa and Sasakawa[1] by some 6%. In fact, the slope of our fit (0.0471) differs significantly from theirs (0.0416), and results primarily from differences in the predictions of C_D. (We noted above that our predictions for C_S agree very well.) Thus, the two model calculations disagree slightly on the predictions of C_D and η for unknown reasons, since the model values for the binding energies and C_S agree within expectation. The experimental values for the η parameter are in reasonable agreement with one another (see Table I), and they agree within errors with our prediction. Such is not the case for η^C. Here there is a single measured value, which indicates a much larger Coulomb effect than our model predicts. The experimental value of $\eta^C=0.035\pm0.006$ lies as far below our theoretical prediction of 0.043 as the corresponding experimental value of $\eta=0.051\pm0.004$ (see Ref. 5 for this average) lies above our theoretical prediction of 0.046. Theory and experiment agree upon the sign of the Coulomb effect, but there is a clear question about the magnitude.

Based upon the investigations of Refs. 2 and 45, we concluded that it is reasonable to use $\tilde{D}_2$ as an approximation to D_2. From our numerical results shown in Fig. 5, we find best fit values for the distorted-wave Born approximation parameters at the experimental binding energies of

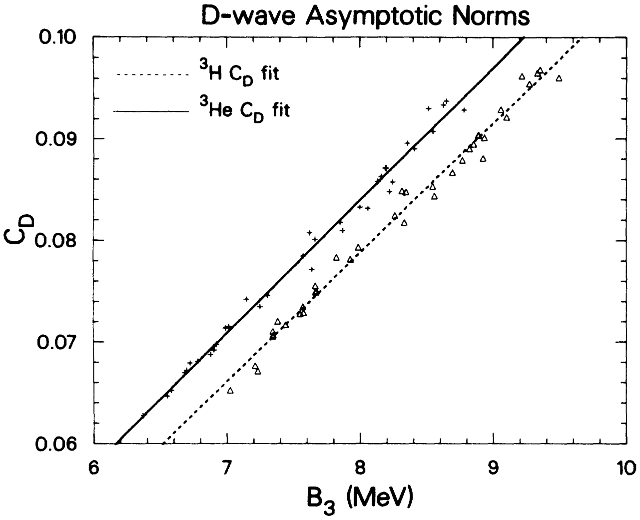

FIG. 3. Trinucleon D-wave asymptotic normalization constants calculated for diverse two-body and three-body force models plotted vs binding energy.

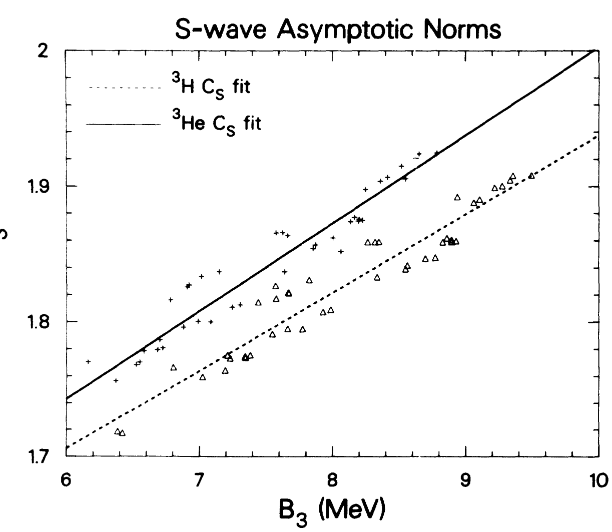

FIG. 2. Trinucleon S-wave asymptotic normalization constants calculated for diverse two-body and three-body force models plotted vs binding energy.

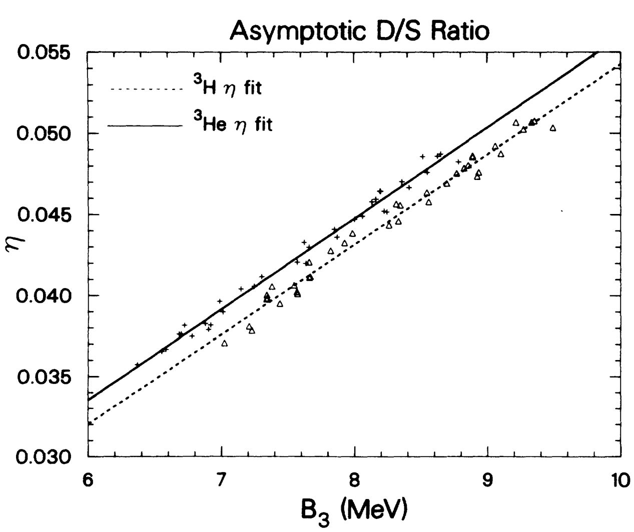

FIG. 4. Ratio of D-wave to S-wave asymptotic normalization constants calculated for diverse two-body and three-body force models plotted vs binding energy.

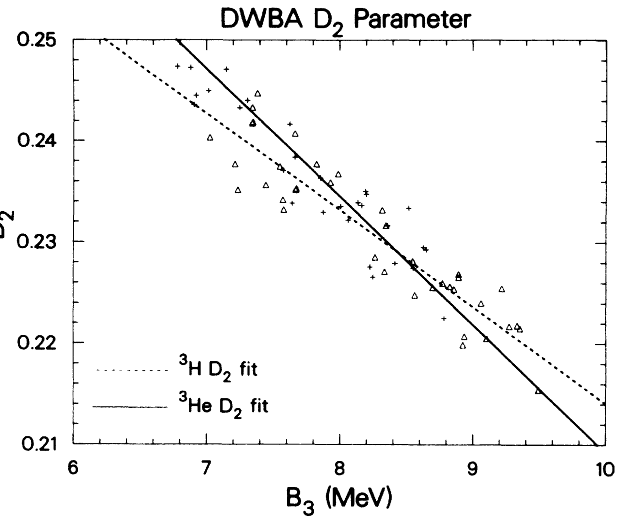

FIG. 5. Distorted-wave Born approximation parameters $\tilde{D}_2$ and $\tilde{D}_2^C$ calculated for diverse two-body and three-body force models plotted vs binding energy. Units of D_2 are fm^2.

$$\tilde{D}_2 = -0.229 \text{ fm}^2 \text{ and } \tilde{D}_2^C = -0.238 \text{ fm}^2 .$$

That is, we predict $\tilde{D}_2^C \simeq \tilde{D}_2$, as we did in our previous investigation. Our estimates of $\tilde{D}_2$ appear to be about 2–3 % larger than those of Ref. 2. At the same time, the variational wave function result of Ref. 34 for the AV14 (plus three-body force) model appears to agree exactly with our best-fit curve while the result for the UR (plus three-body force) model disagrees with our prediction by about 15% Unfortunately, the scatter in the experimental data makes a comparison between theory and experiment difficult. Most of the more recent measurements, which quote larger uncertainties, are consistent with the theoretical predictions. However, one would like to see a more definitive test of these detailed model calculations.

IV. CONCLUSIONS

We have confirmed the conclusions of our original investigation[6] of Coulomb effects in the asymptotic normalization constants of the trinucleon systems. We find here the following. (1) Coulomb effects lead to less than a 1% increase in C_S^C over C_S. (2) Coulomb effects lead to an approximately 6% reduction in C_D^C relative to C_D and correspondingly in η^C relative to η. (3) Coulomb effects lead to a very small difference between $\tilde{D}_2^C$ and $\tilde{D}_2$. (Note that D_2 is negative while η is positive.) Our results here supercede those of Ref. 6 for absolute magnitudes, because we have been able to include a sufficient number of partial waves to establish convergence for each parameter.

Our results agree essentially with Refs. 1 and 2 concerning the triton values for C_S and $\tilde{D}_2$. However, our predictions for C_D and η appear to be about 6% larger near the experimental ^{3}H binding energy. The slope of our η versus B_3 differs significantly from that of Ref. 1. We agree with Ref. 34 on the value of $\tilde{D}_2$ for the AV14 model but predict a value for their UR model some 15% smaller than they obtain.

Our best estimate for the values of the asymptotic normalization parameters at the experimental binding energies are

$$C_S = 1.85 \pm 0.02, \quad C_S^C = 1.85 \pm 0.02 ,$$

$$C_D = 0.085 \pm 0.001, \quad C_D^C = 0.080 \pm 0.001 ,$$

$$\eta = 0.046 \pm 0.001, \quad \eta^C = 0.043 \pm 0.001 ,$$

$$\tilde{D}_2 = -0.229 \pm 0.005 \text{ fm}^2, \; \tilde{D}_2^C = -0.238 \pm 0.005 \text{ fm}^2 .$$

It is clear that improved experimental precision is needed in order to fully test these predictions. In the case of the S-wave asymptotic normalization constants, improved measurements are needed to resolve the present conflict among the various data. A direct measurement of the D-wave constants does not exist. While the Coulomb effect exhibited in the measured η^C compared with η can be said to be consistent with our prediction at the level of one standard deviation, the large spread in the nominal values of the measurements compared with the near equality of the theoretical values is disturbing. Although the two most recent measurements of D_2 and D_2^C appear to be consistent with the predicted values, comparison with the entire database is not so encouraging, and a clarification of the experimental situation is needed.

Finally, it was originally proposed that the trinucleon asymptotic normalization constants be accorded the same status as the binding energy and charge radii.[51] The strong correlations of these normalization parameters with binding energy has been established. They, therefore, have no greater status than the charge radii[37] and Nd doublet scattering lengths.[52,53] It remains to be seen whether precision experimental measurements of these parameters will be in agreement with the extrapolated values of contemporary models.

ACKNOWLEDGMENTS

We thank W. Grüebler for keeping us informed of the status of the analysis of the η and η^C measurements and A. M. Eiró for an explanation of the differences in D_2 and D_2^C extractions. The work of J. L. Friar and B. F. Gibson is performed under the auspices of the U. S. Department of Energy. The work of D. R. Lehman and G. L. Payne is supported in part by the U. S. Department of Energy.

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