PHYSICAL REVIEW C VOLUME 37, NUMBER 6 JUNE 1988

Equation of state from nuclear and astrophysical evidence

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(Received 9 April 1987; revised manuscript received 1 December 1987)

Data on the nuclear equation of state from a number of different sources, from nuclei, high energy nuclear collisions, supernova, and neutron stars are analyzed. The current situation concerning supernova simulations is critically appraised. It is found that simulations that have achieved a prompt ejection do so with an equation of state that is too soft to support the measured masses of several known neutron stars. It is concluded that supernova explosions have not been proven to provide a significant constraint on the nuclear equation of state. Additionally it is concluded that the theoretical bias used to interpret data on neutron star masses as if they belonged to a population all having the same mass (of $1.4M_{\odot}$) is unjustified. Evidence from the various nuclear data and neutron star masses favor a high compression modulus, $K \approx 300$ MeV. No definitive statement can be made about the equation of state at higher density, save that the neutron star equation of state must be moderately stiff to accommodate neutron stars of mass $\approx 1.85M_{\odot}$.

I. INTRODUCTION

The equation of state of nuclear matter impinges on a number of areas of physics, such as the monopole resonance, high energy nuclear collisions, supernovae, and neutron stars. Generally, it is expressed in a form that provides the energy as a function of density, or pressure as a function of energy density. The function depends of course on the precise nature of the matter and the conditions under which it exists or is probed. Since nuclei are bound by the strong force, they are approximately isospin symmetric, and exactly so in the idealization of nuclear matter. In contrast, stars are bound by the gravitational force, and they must be charge neutral, since excess charge would be blown off. Given the starting condition of nuclei, the dense hot matter produced in nuclear collisions remains without net strangeness because the time scale of the collision is fast compared to weak interactions, although at sufficient temperature or compression it can develop other baryon and meson populations though the strong interaction.[1]

In contrast, the time scale is milliseconds in supernovae and millions of years in neutron stars so that the processes that produce strangeness are fast in comparison, and generalized beta equilibrium is achieved. The composition of all three systems is therefore different. In nuclear matter, $Z=\frac{1}{2}A$. Stars initially have large proton fraction, but it evolves, and in the intermediate density stage of supernovae collapse, $Z\approx\frac{1}{3}A$, because electron capture of relativistic electrons on nuclei gives a lower energy state for charge neutral matter. Although the electron capture rates do not keep pace with the collapse in the initial stages, at densities still far below nuclear density the weak interactions rates become fast on the hydrodynamic scale of the collapse, and the composition of matter follows the equilibrium path as the matter further compresses. At this stage it can become rich in composition, containing not only neutrons, protons, and leptons, but also hyperons. As the core further compresses to form the neutron star, the populations further evolve. Neither in the denser stages of collapsing matter in the supernova, nor in neutron stars, can the composition be characterized simply by Z/A, as for nuclear matter. The matter in these dense equilibrated systems is a multicomponent one.

The equation of state is therefore a many dimensional function, and it is concerning this that we would like ultimately to have knowledge. Of course all aspects of this function are related in the fundamental theory as well as in some effective theories that we have available. It is through the imperfect medium of the latter that we shall sometimes correlate information from diverse sources.

Properties at saturation are most accessible and best known, the binding energy, saturation density, and symmetry energy. Until recently, it has been assumed that the compression modulus, also, was reasonably well known through analysis of the giant monopole resonance in nuclei. Two types of analysis have been performed. In one, the random phase approximation (RPA) and a variety of phenomenological two-nucleon interactions are employed to reproduce the position of the observed resonance in various nuclei, and then the nuclear bulk properties are calculated in Hartree-Fock approximation to find the corresponding K that best reproduces that position of the resonance. It is $K\approx 210\pm 30$ MeV.[2] In the other, the asymptotic behavior of RPA sum rules is studied to ascertain the coefficients in an expansion of the compression of finite systems and determines in this way that $K\approx 220\pm 20$ MeV.[3] These are in satisfactory agreement. However, some difficulties have been raised in connection with these conclusions, which motivated a reexamination based on the Landau Fermi-liquid theory.[4] The conclusion of that work was that K is small, of the order 100 MeV. It has also been widely quoted that the fact of supernovae provides evidence that the equation of state must be soft, especially at higher density, in order to release sufficient energy for the prompt-bounce mechanism to work.[5] On the other hand, some analyses of high energy nuclear collisions have indicated a moderately stiff to very stiff equation of state,[6–8] although serious ambiguities have also been noted.[9,10]

The purpose of this paper is threefold. First, we wish to report on our own research that impinges on the equation of state. This concerns evidence that can be obtained from (1) Landau sum rule, (2) nuclear masses, (3) pion yields from high energy nuclear collisions, (4) neutron star masses. Second, we critically appraise the situation with respect to what supernovae can tell about the equation of state. Third, we summarize recent evidence from the work of others on the equation of state.

II. LANDAU SUM RULE

In the Landau theory of Fermi liquids, the compression modulus is given by

$$K=\frac{3\hbar^2k_F^2}{m^*}(1+F_0)\ , \tag{1}$$

where m^* is the effective mass and F_0 is one of the Landau parameters according to which the properties of the liquid can be characterized. Ordinarily, K, as a supposed observable, would be used to determine the Landau parameter F_0 from this relation. However, Brown and Osnes[4] proposed to determine K from this relation by finding F_0 from the Landau sum rule

$$\sum_{l=0}^{\infty}\frac{F_l}{l+F_l/(2l+1)}=-3\sum_{l=0}^{\infty}\frac{G_l'}{1+G_l'/(2l+1)}+\delta_t\ , \tag{2}$$

with specific choices of the remaining Landau parameters. The choices made in Ref. 4 were as follows:

$$\begin{aligned}&F_1=3(m^*/m-1)\ ,\\&m^*=0.9m\ ,\\&F_l=0,\quad l>1\\&G_0'=1.6\ ,\\&G_l'=0,\quad l>0\\&\delta_t=0.15\ .\end{aligned} \tag{3}$$

With these assumptions it was found that $K=106$ MeV.[4] It has been subsequently realized that m^* should be smaller than the above choice, and this increases K. A very recent determination of m^* has been made through application of dispersion relations to a study of the nuclear mean field from energies between -20 to 165 MeV, and a value $m^*/m=0.83$ was found.[11] However, neither this, nor the Landau parameters are known with perfect precision. Of course high precision can be obtained for direct observables, like masses, but other quantities, including effective force parameters, can be determined only approximately through the intermediary of a model or theory. In this case, we believe that it is optimistic to assert that the Landau parameters are known to better than 30%. Even at that, there remains the fact that the higher l Landau parameters are set to zero for lack of information. Even granted that the series should converge, this is a drastic assumption. But let us accept it for orientation and be optimistic that $m^*/m=0.83\pm20\,\%$ and that the Landau parameters are known to 30%. The results are summarized in Fig. 1 which shows the band in which K falls when up to a 30% uncertainty is acknowledged for the Landau parameters. The range of m^* corresponds to the assumed 20% uncertainty in this parameter. What we learn from this is that K is very poorly constrained by the Landau sum rule, a conclusion quite at variance with Ref. 4, where the dependence on uncertainty in the Landau parameters was not explored in this way. The range on K is 74 MeV $<K<$ 371 MeV. The range is even large if a larger error is admitted for the Landau parameters. We conclude therefore that the Landau sum rule as used by Brown and Osnes does not provide a small upper bound on K.

III. NUCLEAR MASSES

The coefficient of various terms in the droplet model of nuclear masses have such significance as the volume energy, symmetry energy, compression modulus, etc., and the dozen or so such parameters in the expansion are able to represent the masses of thousands of nuclei to very high accuracy. In Fig. 2 we show a section of the surface of the rms deviation in mass about the minimum as a function of the compression modulus, the calculations for which were kindly provided by Möller.[12,13] The region of the minimum is very broad, but suggests a value $K\approx310\pm100$ MeV. However, one should note that the behavior of the rms deviation as a function of K depends on the precise formulation of the model. The macroscopic model that is studied here is the finite-range droplet model. This model combines the droplet model with the folding model surface and Coulomb energy integrals. It also incorporates a new exponential term that has a large influence on how the model describes nuclear compressi-

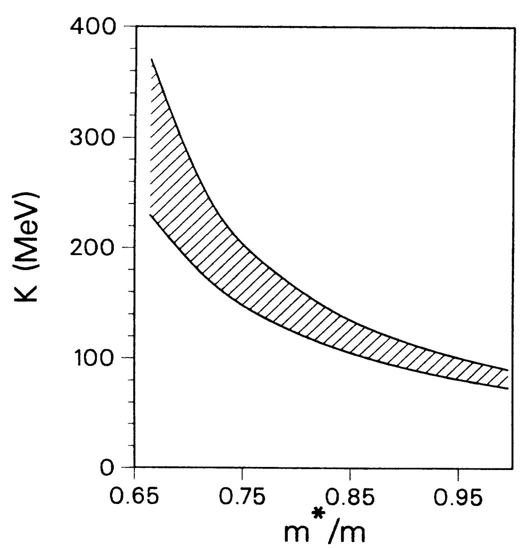

FIG. 1. Range in which the compression modulus falls assuming up to a 30% error in the Landau parameters, as a function of effective mass within a 20% range of the value $m^*/m=0.83$ established in Ref. 11.

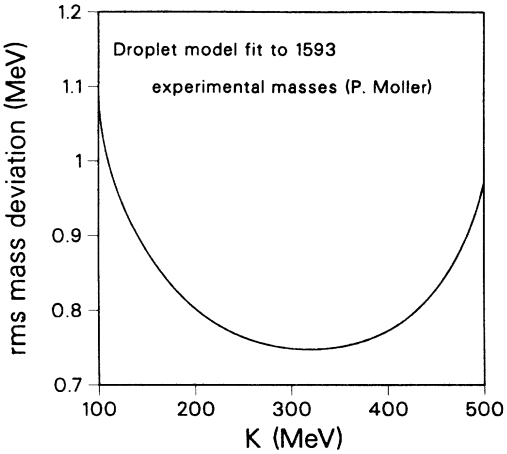

FIG. 2. Section of the rms mass deviation of the droplet model fit to atomic mass data as a function of nuclear compression modulus (Refs. 12 and 13).

bility. The error is assigned rather arbitrarily from these considerations and the flatness of the curve. A combined fit to masses and radii should improve on the determination, and we mention in Sec. VII A such a determination based on an early version of the droplet model. We mention also the earlier work on a compressible nuclear mass formula, for which $K=267\pm52$ MeV was found from a mass fit to heavy nuclei.[14]

IV. RELATIVISTIC NUCLEAR COLLISIONS

Since the seminal experiments of Stock *et al.*,[15] there has been a concerted effort to determine the nuclear equation of state. Many researchers have contributed to this work (cf. references in Refs. 16, 7, and 17). One approach that has not been fully exploited is a field theoretic description of the nuclear fireball, although it was first proposed in a more general context almost a decade ago.[1] Nuclear field theory has been extensively studied for finite nuclei. Once the coupling constants have been fitted to bulk nuclear properties, it is able to account for a growing body of data on finite nuclei.[18–20,21] This may be interpreted as attesting to the general correctness of its form, as an effective theory. Depending on how well the coupling constants are determined, it provides a more or less unique way of extrapolating to the domain of densities that are believed to be probed in the experiments, within the assumed validity of the theory. We emphasize that unlike the frequently used parametrizations of the equation of state, the high density behavior of the theory is determine, as is its saturation properties, by the coupling constants. These are fixed by the binding energy of nuclear matter, its saturation density, the effective nucleon mass *at* saturation, the symmetry energy, and the compression modulus, which we shall vary to obtain agreement with experiment.

A. Field theory of hot compressed matter

We shall describe the region of hot dense matter that is produced in a relativistic nuclear collision in the framework of relativistic nuclear field theory. We formulated the description of hot dense matter, sometimes referred to as a nuclear fireball, in this theory almost a decade ago[1] and refer to that and more recent work for discussion and details.[22–25] Within the theory, the compression and temperature effects are explicitly included. We draw attention to some recent related work in Refs. 26–29. The pion yields are computed from the primordial populations of pions and deltas in the manner described later.

The Lagrangian that we employ is

$$\mathcal{L}=\sum_B \bar{\psi}_B(i\gamma_\mu\partial^\mu - m_B + g_{\sigma B}\sigma - g_{\omega B}\gamma_\mu\omega^\mu - \tfrac{1}{2}g_{\rho B}\gamma_\mu\tau_3\rho_3^\mu)\psi_B + \tfrac{1}{2}(\partial_\mu\sigma\partial^\mu\sigma - m_\sigma^2\sigma^2)$$
$$-\tfrac{1}{4}\omega_{\mu\nu}\omega^{\mu\nu} + \tfrac{1}{2}m_\omega^2\omega_\mu\omega^\mu - \tfrac{1}{4}\rho_{\mu\nu}\cdot\rho^{\mu\nu} + \tfrac{1}{2}m_\rho^2\rho_\mu\cdot\rho^\mu - \tfrac{1}{3}bm_n(g_\sigma\sigma)^3 - \tfrac{1}{4}c(g_\sigma\sigma)^4 + \cdots \; . \tag{4}$$

Here $\omega_{\mu\nu}=\partial_\mu\omega_\nu-\partial_\nu\omega_\mu$, ψ_B denotes a baryon spinor, and the sum is over all charge states of N and Δ, and, in principle, over higher resonances as would be appropriate for very high energies.[1] The σ and ω mesons are Yukawa coupled to the baryons and the ρ meson is coupled to the isospin current. The ellipsis represent the Lagrangians of the mesons that are treated as thermal populations; in this case, pions and kaons. The cubic and quartic terms are scalar self-interactions whose strength can be exploited to adjust the compression modulus.[30] For symmetric matter, the expectation of the ρ field vanishes. The finite temperature solutions of the theory can be computed as in Ref. 1. We chose the coupling constants of the theory to yield the saturation density $\rho_0=0.145$ fm^{-3}, binding energy, 15.95 MeV, and effective nucleon mass at saturation $m^*/m=0.8$. The latter is close to the value of 0.78 for the scalar effective mass that corresponds to the Landau effective mass of 0.83 obtained in a recent analysis of scattering and bound state data.[11]

The initial state of the hot compressed matter can be estimated in several ways. One way is to assume perfect stopping of the interpenetrating matter, in which case the initial density of the compressed fireball is at least,

$$\rho_i=2\gamma\rho_0 \; . \tag{5}$$

This is the overlap density of the Lorentz contracted fireball, ρ_0 being normal nuclear density. Assumption of thermalization of the input energy at that density then specifies the state of the system.

Another estimate of initial conditions is obtained by assuming relativistic fluid dynamics. The Rankine-

Hugoniot relations then specify the conditions of matter in the shock zone, the temperature, compression, and through the theory of matter, the composition, etc.,

$$\gamma=\frac{\epsilon/\rho}{\epsilon_0/\rho_0}, \quad \left[\frac{\rho}{\rho_0}\right]^2=\frac{\epsilon(p+\epsilon)}{\epsilon_0(p+\epsilon_0)} . \tag{6}$$

Here γ is the Lorentz factor, p and ϵ refer to pressure and energy density in the shock zone, and ϵ_0 to the energy density of matter in the ground state of the incoming nuclei (represented in all calculations that employ these equations as semi-infinite slabs).

We shall use both of the above estimates. The first gives a lower estimate of the initial density when stopping occurs, and the latter gives an upper estimate, since it assumes planar geometry.[31] We then assume, as in the original work,[15,32] that the pion yield equals the thermal population of the pions and deltas, at the initial condition. We shall also relax this assumption by considering a scenario in which the dense matter expands adiabatically to a prescribed freeze-out density.

In either case the pion yield is computed from the particle densities as

$$\frac{n_\pi}{A}=\frac{\rho_\pi+\rho_\Delta+\bar{\rho}_\Delta}{\rho_N-\bar{\rho}_N+\rho_\Delta-\bar{\rho}_\Delta} , \tag{7}$$

where the bars indicate antiparticles.

For three assumed values of the compression modulus *of the corresponding cold matter,* the calculated yields of pions and deltas are compared in Fig. 3 for the case that the initial conditions are given by the solution of the Rankine-Hugoniot relations. A similar comparison could be made for the overlap condition, Eq. (2). However, the results are not substantially different[29] and we do not show them. In both cases the pion yield computed from Eq. (4) (namely the sum of thermal pions and deltas in the figures), is too large compared to the observations, as is shown for the shock initial condition in Fig. 4. Even for the very large compression modulus, the computed yield

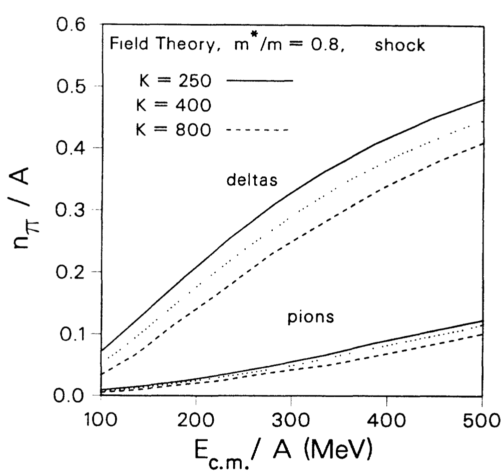

FIG. 3. Pion and delta populations in a nuclear fireball described in nuclear field theory, as a function of c.m. energy, at the shock density. Results for three values for K of the corresponding cold nuclear matter are shown.

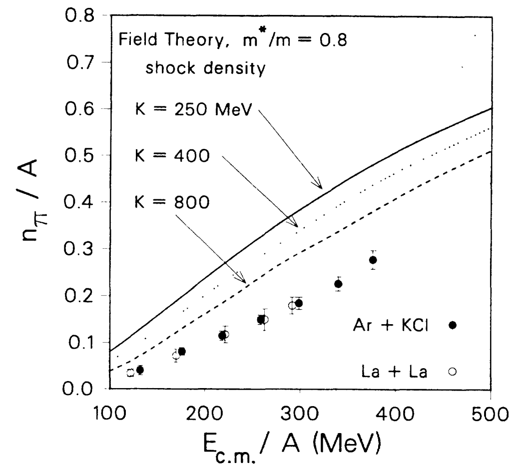

FIG. 4. Total pion yields computed in a relativistic nuclear field theory of the fireball at the shock density prescribed by the Rankine-Hugoniot conditions are compared with the data (Ref. 32).

is too large. We apparently must conclude that the conceptual division of the energy into two parts, the compression and thermal, the second of which is supposed to be responsible for pion and delta production according to Stock's suggestion, is not even approximately realized. Indeed, an examination of the expression for the energy density in the theory,[1] and a realization that the field variables appearing in it must be obtained self-consistently at each temperature and density, reveals that the division into compression and thermal energy as functions of only density and temperature, *respectively, is not possible. Both energies depend* on the other variable.[29] Moreover, compression alone can produce a Δ population when the Fermi energy of the nucleons exceeds the threshold energy of the Δ.

Clearly the assumption that the pion yield is frozen at the initial thermal population of pions and deltas may be an overestimate, since the reabsorption of both in a subsequent expansion and cooling will reduce the ultimate yield. We estimate the reabsorption by supposing that, once formed at the initial conditions prescribed by Eqs. (2) or (3), each fluid element of the fireball evolves at constant entropy, equal to the initial entropy of the fireball, and that the populations remain in equilibrium until a prescribed freeze-out density.[33] This is a quasi-hydrodynamic expansion, inasmuch as the internal conditions match those of hydrodynamics, but the space-time structure is absent. The internal energy decreases, the missing energy being accounted for by the collective fluid energy, and of course the temperature drops. In Fig. 5 the resulting pion yields are shown for an adiabatic expansion from initial conditions prescribed by the shock equations to a freeze-out density. We can anticipate that the smaller the freeze-out density, the smaller will be *both* the yield *and* the slope of the yield as a function of energy. The reasons for this are quite clear. For the slope, the higher the bombarding energy, the higher the initial energy density, and hence, the greater will be the cooling and reabsorption during expansion to freeze out. Of

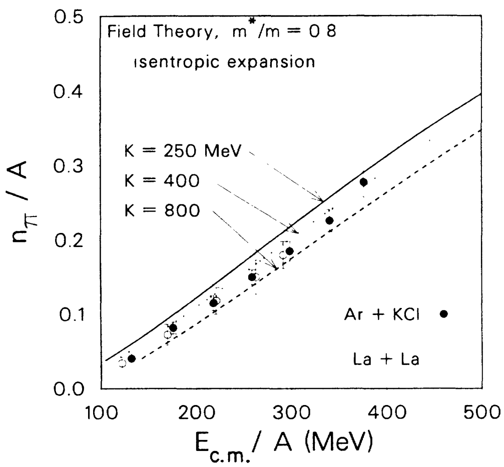

FIG. 5. Total pion yields computed in a relativistic nuclear field theory of the fireball following an adiabatic expansion to a freeze-out density of $\rho_f = 2\rho_0$ from an initial density prescribed by the Rankine-Hugoniot conditions. Data are from Ref. 32.

course we are not assured that the yield and the slope will correspond *simultaneously* to observation, but in Fig. 5 we see that that is actually achieved for a pion freeze out of $\rho_f = 2\rho_0$. This gives some confidence in the scenario described above.

However, we note two negative points. The first is that, as in Fig. 4, the yields corresponding to vastly different values of K are not much different from each other. Second, the coupling of the delta to the meson fields was here assumed to be the same as for the nucleon. Uncertainty in these couplings introduces additional uncertainty in the yields. This has been the subject of detailed study in Ref. 27.

Thus, although the data is rather well reproduced in the above model, the sensitivity to the equation of state is low, and it cannot be defined within very broad limits.

V. SUPERNOVA EXPLOSIONS

In the late stages of the evolution of a star, thermonuclear combustion burns to the end point or minimum mass possible for the number of baryons present. At this time a dynamic instability sets in and gravitational collapse commences.[34] However, numerical simulations have not, until recently, produced a successful scenario in which most of the imploding material from the collapse of a massive star is ejected. Failure to eject means that the stellar material will once more be accreted by gravity, and the massive remnant will subside into a black hole rather than a neutron star whenever the mass exceeds a critical value of several solar masses.

A. Prompt bounce

This scenario in which mass ejection occurs on the time scale of a few hydrodynamical crossings of the iron core (≈ 10 ms) is a tenuous one. On the one hand, stellar evolution calculations of the precollapse configuration of the star find that the iron core mass is an increasing function of progenitor mass with a lower bound of $\approx 1.3 M_\odot$ for the core mass of the lightest progenitors of type II supernovae $\approx 10 M_\odot$), while numerous simulations of the subsequent evolution find that the mass of the iron core cannot exceed $\approx 1.35 M_\odot$ and still allow a successful prompt explosion.[35] Otherwise the shock is dissipated by neutrino losses and photodisintegration, and stalls at the order of 100 km and the star does not explode. Within this narrow window, Baron *et al.*[5] found that if they choose an equation of state that is sufficiently soft at *high* densities, a successful prompt ejection can occur. If this were the whole story, then a tentative conclusion could be reached that the equation of state must be sufficiently soft at high density to produce type II supernovae.

Very recently[36] it has been discovered by Nomoto and by Woosley and collaborators, that a very small correction to the Coulomb energy in the presupernova, corresponding to the lower energy of a lattice compared to a free electron gas, lowers the iron core mass by about $\frac{1}{10} M_\odot$. This seemingly small effect significantly reduces the dissipation of the shock as it traverses the iron core, leaving a greater energy for ejection. Its effect should be to moderately improve the chances for prompt explosions. At the same time it can be remarked that when uncertainties so small as this ($\approx 10\%$) become of vital concern as to whether the simulation of the collapse leads to successful mass ejection, it must be concluded, in view of the exceedingly complex physics of the entire scenario, from the evolution of the presupernova to the collapse, that there are other uncertainties at least as large as this. It has been found, for example, that for each $\frac{1}{10} M_\odot$ of iron core that the shock traverses, about 2×10^{51} ergs is dissipated in dissociation energy.[37] This is comparable to the entire explosion energy of typical supernova.

However, taking advantage of the smaller iron cores it is still found, in very recent work, that a soft equation of state is necessary to achieve the prompt ejection *with* the desired explosion energy.[37] In particular, the authors seek to account for SN1987a, the supernova event of early 1987, with the prompt bounce mechanism using a nuclear equation of state based on the parametrization known as BCK. It is characterized by the compression modulus $K(x)$ at the relevant proton fraction $x = Z/A$ and by the index γ, which is the power dependence of the pressure on the baryon density. The particular parameters used give $K(\frac{1}{2}) = 180$ MeV for nuclear matter and $K(\frac{1}{3}) = 138$ MeV for the neutron rich matter near the rebound density, which is about four times nuclear density.[5] The value of the index is $\gamma = 2.5$. With such a parametrization, and using precisely the form described in Ref. 5, we have solved the Oppenheimer-Volkoff equations of star structure. Our results are shown in Fig. 6 for the value of $x = Z/A = \frac{1}{3}$ employed in the supernova simulation, and for a somewhat smaller one, as might be more pertinent to the more highly evolved material of a neutron star. As is characteristic, there exists a maximum star mass as a function of the central density. This maximum mass for the equation of state used in the paper of Baron *et al.*,[37] is seen to be smaller than the masses of two known neutron stars. One is the very accurately measured mass of PSR1913 + 16,[38] and the other is the

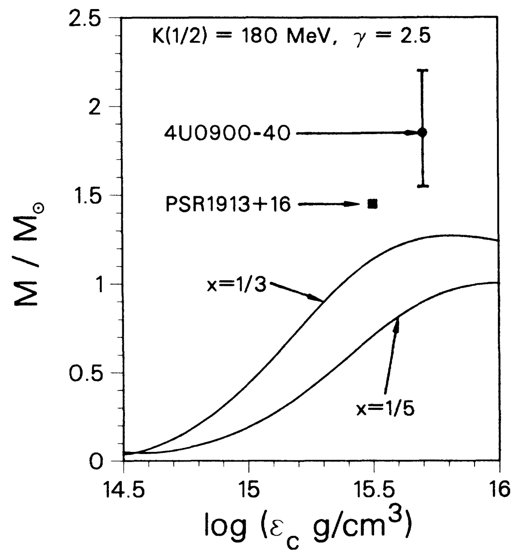

FIG. 6. Gravitational mass of neutron stars as a function of central density for the BCK equation of state used in Ref. 37 for the value of $x=Z/A=\frac{1}{3}$ employed in the reference, and a smaller value, as might be reached in the further evolution of the collapsing matter to the neutron star. The x coordinate of the two measured masses is not significant since the central density is not measured. The error on the mass of PSR-1913 + 16 measurements is smaller than the data point.

less well known mass of 4U0900-40.[39]

Therefore, a shock energy large enough to survive the dissipation due to nuclear dissociation as the shock traverses the core, and still eject the mantle promptly with the energy estimated from the light curve of SN1987a, is bought, in the simulation, at the price of an equation of state that is too soft to support the measured masses of several known neutron stars.

We have also tested those equations of state employed in Ref. 5 that are cited as producing successful first bounce supernovae. We find that of the five cases listed as successful, four of them are incompatible with neutron star masses and the remaining one $[K(\frac{1}{2})=180$ MeV, $\gamma=3]$, yields a low explosion energy, too low to account for the energy inferred from the light curve of SN1987a. It can account for 4U0900-40 only at the lower bound on the error in the mass measurement.

Therefore, it is unproven that the prompt-bounce mechanism can be made to work in supernova simulations with equations of state that are soft enough to release enough energy for prompt ejection of the mantle, and still enough to be compatible with certain known neutron star masses and with the preponderance of evidence from nuclear physics that is summarized in Sec. VIII. This conclusion is reached with the *same* equation of state as used by the authors of the supernova simulations. Consequently, it is premature to conclude, as has been done, that supernovae, by their fact, imply a soft equation of state. This is all the more reinforced by the observation that there exists an alternative mechanism that does not impose this restriction on the equation of state. This we now discuss.

B. Late-time shock revival

A different scenario has been recently discovered by Wilson[40] which was reported to lead to successful ejection for a wide range of precollapse cores arising from the wide range of star masses that occur in nature. Except for light mass progenitors, $8 \lesssim M/M_{\odot} \lesssim 15$ and under, the restriction of a soft equation of state as described above, and in all cases for progenitors with $M \gtrsim 15 M_{\odot}$, the shock typically stalls at about 100 km. Wilson found that it can be revived on a long time scale by reheating due to absorption of a neutrino shower emitted by the cooling neutron star. This mechanism has been reported to yield successful explosions for a wide range of progenitors and may , in fact, describe all type II supernova. It produces neutron stars in a wide mass range from 1.2 to 2 solar masses. However, this mechanism, like the prompt one, is sensitive to small effects; in this case to accumulation of numerical inaccuracies because of the extremely long evolution time.[41]

C. Softening due to hyperons

Elsewhere we have shown that the hyperon threshold in neutron star matter is around $\rho=3\rho_0$, and that the equation of state is very much softened by the opening of these degrees of freedom.[42] This density is below that which is attained at the time of the shock formation in supernovae. Since the weak interaction time in dense matter is short on the hydrodynamic scale of milliseconds that governs the collapse, this softening will play a role in supernovae. Its significance in gravitational problems can be gauged by the fact that hyperons reduce the limiting mass of neutron stars by up to $\frac{3}{4}M_{\odot}$, which is to be measured on a scale of several solar masses. It would appear therefore that the nuclear equation of state can be stiff at low density, and still be sufficiently soft at high density due to nucleon conversion to hyperons, as to release sufficient energy to the shock so that it can promptly expel the mantle. This suggestion[42] has so far not been explored in supernovae simulations, but appears to be a possible resolution to the problem of the prompt bounce.

D. Conclusions

We note the following conclusions.

(1) In 7 cases out of 8 in which the simulation of the prompt-bounce mechanism of supernovae explosions has been cited to work in Refs. 5 and 37, the equation of state employed is too soft to support known neutron star masses. In the one case for which it worked for a somewhat stiffer equation of state, the explosion energy was too small to account for SN1987a. Therefore, the burden of proof, that the prompt mechanism can be made compatible with neutron star masses and other evidence on the equation of state, rests on the proponents of the prompt mechanism.

(2) Consequently, it has not been established that the occurrence in nature of supernova explosions provides a constraint on the equation of state, as has been frequently quoted in the literature.

(3) This is further reinforced by the fact that an alternative mechanism, the late-time one, works without the restrictive condition.

(4) Both the prompt and the late-time neutrino reheating mechanism are very sensitive to physical effects and numerical inaccuracies at the 5–10% level. This can lead one to suspect that there may be a major physical effect that has not yet been realized or implemented in the simulations.

(5) The narrow window or iron core mass for which the prompt bounce mechanism was thought to work has led to a bias in the way in which observations on neutron star masses are interpreted, namely that they have an almost unique mass which is to be discovered by finding the mass that is compatible with the overlapping errors in the measurements. In this interpretation the probable mass is $1.4 \pm 0.2 M_\odot$.[39] As shown elsewhere, this would place a lower bound on the nuclear compression modulus of symmetric matter of ≈ 200 MeV.[43] However, in view of the success of the late-time explosion mechanism for a wide range of progenitor star masses, and hence of neutron star masses, we should accept the dispersion in mass determinations as representative of neutron stars of *different* masses. These range from $1.05 M_\odot$ to $1.87 M_\odot$ generally with large errors except for PSR1913-16 which is known very accurately as $M = 1.451 \pm 0.007 M_\odot$.[38] The most probable mass in the case of one of the largest mass determinations is $1.85^{+0.35}_{-0.30} M_\odot$ for 4U0900-40. *This is the mass that theory must account for, and not the lower value of* $1.4 M_\odot$ *generally employed as the critical value.*

VI. NEUTRON STARS

We analyze neutron stars in the framework of relativistic nuclear field theory[44] generalized to beta-stable charge neutral neutron star matter, *including* all baryon species that are required to achieve equilibrium over the relevant density range.[45] The hadronic part of the Lagrangian for this theory is given in Eq. (4), and the full Lagrangian including leptons is given in Ref. 45 together with the equation of state. When the field equations are solved subject to the subsidiary condition of isospin symmetry, the solution corresponds to symmetric nuclear matter. When this same theory is solved with subsidiary conditions of charge neutrality and beta equilibrium, we get the solution for neutron star matter. These solutions, by convention, will be denoted always by the properties of the corresponding solutions of symmetric matter.

In this analysis of neutron stars, we vary the stiffness of the equation of state of neutron star matter at high density. This is accomplished through a variation of the coupling constants of the theory which leaves the bulk properties of cold symmetric matter fixed at saturation, with the exception of K. In this way we are able to place a lower bound on K. The reason why a lower bound is imposed by neutron star masses is that, for a given equation of state, there is a maximum or limiting mass that a neutron star can attain. The limiting mass is an increasing function of the stiffness of the equation of state. An acceptable equation of state must have a limiting mass at least as large as the largest known neutron star mass. Hence, the lower bound. Since there is uncertainty in the effective nucleon mass at saturation, which is one of the saturation properties used to fix the coupling constant, and because it also effects the behavior of the equation of state at higher density, we shall use a range of values for this quantity. We note that, corresponding to the Landau effective mass 0.83 found in Ref. 11, the scalar effective mass of this theory at saturation is 0.78, the two being related by $m_L^* = (m_{\rm sat}^{*2} + k_F^2)^{1/2}$.

Several authors have suggested that neutron star masses do not depend sensitively on K or the properties of matter near saturation because their central densities are high. We disagree with this for several reasons. First, for any theory of matter and for nature too, the equation of state is everywhere specified by its coupling constants, both its high density behavior as well as its behavior near saturation. This factor establishes the link, albeit in practice through an imperfect theory. Second, the central density does not contribute much to the mass because of the three dimensional geometry. We have computed that half of the mass of a neutron star, even at the limiting mass, is contributed by matter at densities not too much above saturation density. In Fig. 7 we show the fraction of mass $M(\rho)/M$ of the limiting mass star that is composed of matter is excess of ρ, and what it reveals in the particular case is that half the mass lies at densities less than three times nuclear density. Thus, the mass of a neutron star, *at the limiting mass,* is dominated neither by very dense matter, nor by matter near nuclear density. The balance between these domains in lighter stars shifts to the latter.

So far only half a dozen or so neutron star mass determinations have been made The most probable mass in the case of one of the largest mass determinations is $1.85^{+0.35}_{-0.30} M_\odot$ for 4U0900-40. In Fig. 8 we show our calculation of limiting neutron star mass as a function of the stiffness at high density of the equation of state, as con-

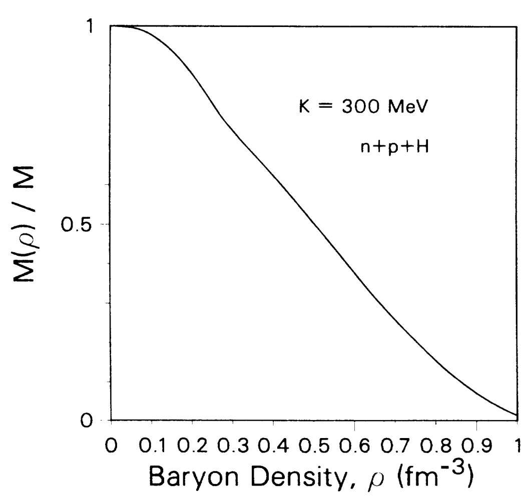

FIG. 7. The fraction of mass $M(\rho)/M$ contained in matter at density greater than ρ as a function of baryon density, ρ, for a neutron star at the limiting mass, $1.85 M_\odot$, in the case that $K = 300$ MeV and the nucleon effective mass in nuclear matter with the same coupling constants is $m^*/m = 0.75$ at saturation.

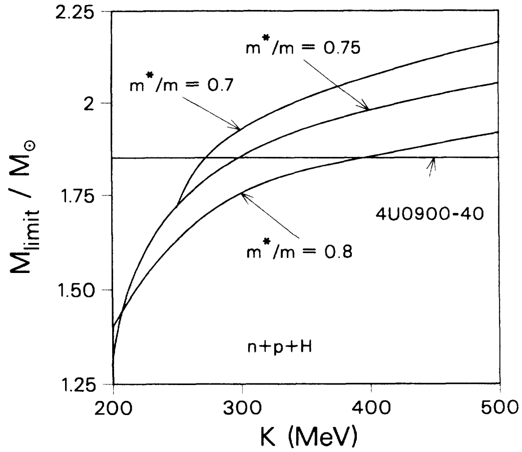

FIG. 8. Limiting neutron star mass as a function of compression modulus, of the corresponding cold symmetric matter, are shown for several nucleon effective masses (at saturation density) computed in the relativistic nuclear field theory (Ref. 45). Horizontal line represents the most probable mass of neutron star 4U0900-40.

trolled through the coupling constants of the theory, and characterized by the nuclear compression modulus of symmetric nuclear matter and the nucleon effective mass at saturation. The most probable value of the mass of the above star is found to place a lower bound of $K \geq 335$ MeV. If the lower bound on the mass measurement of 4U0900-40 is used, then the lower bound on K becomes about 225 MeV, while corresponding to the upper bound, $K \approx 700$ MeV. We note that the sequence of limiting masses as a function of m^* at fixed K changes at $K \approx 260$ so that lower m^* than used here will not effect the conclusion on the range of K that is needed to account for the mass of this star.

The authors of some recent neutron star calculations find limiting masses for given K that are larger than what we find. These calculations in some cases are for pure neutron matter (cf. Ref. 46), and in another include only neutrons, protons, and leptons.[47] However, above a certain density, of the order of two or three times nuclear density, the ground state of charge neutral matter contains also hyperons.[45,48–50] We have shown elsewhere that hyperons significantly soften the equation of state beginning at the thresholds for these particles and reduce the limiting neutron star mass by an amount ranging from $\frac{1}{2}$ to $\frac{3}{4}M_\odot$ depending on the intrinsic stiffness of the equation of state, with the effect being largest, the smaller K.[42] When account is taken of this, the quoted results appear to be in accord with ours.

We should remark that such an important role for hyperons is supported by the nonrelativistic calculations of Pandharipande[49] but not by Bethe and Johnson.[51] However, since these early works, it has been realized that when such calculations as those are carried to convergence, nuclear matter saturates at twice the empirical density.[52] Moreover, even though neutron stars have dense interiors, and are the most isospin asymmetric objects known, the compression modulus and symmetry energy were not listed among the seven constraints on the early work.[51] As we showed earlier, about half the mass of the heaviest neutron stars is composed of matter in the lower density domain below $3\rho_0$ so that such uncertainties as those mentioned are quite important for neutron star structure. Moreover, such uncertainties propagate, by continuity, into the high density domain. In addition, the ordering of thresholds for the higher baryon states in both of the above works[49,51] suggests that the symmetry energy at higher density becomes small in comparison with that expected from the coupling of baryons to the rho-meson, as was discussed elsewhere.[45]

VII. REVIEW OF OTHER RESULTS

A. Masses and radii in the droplet model

We saw earlier that the droplet model fit to masses as a function of K is very flat. An improved determination could be achieved through a combined fit to masses and radii. This has not been done yet with the improved droplet model of Ref. 12. However, an older version applied to a combined fit of masses and charge radii gives $K = 280 \pm 65$ MeV,[53] in agreement within the errors with the new result presented in Sec. III.

B. Flow angle in high energy nuclear collisions

The measurement of momentum flow in high energy nuclear collisions carries information on the equation of state but, as recently emphasized, a momentum dependence of the mean field of a nucleon in a nucleus introduces a large ambiguity in the interpretation.[9,10] The presence of a momentum dependence in the mean field has not been in doubt since Weisskopf pointed out this consequence of nuclear saturation and the approximate independent-particle structure of nuclei, but its form is by no means known.[54] In Ref. 9 it was found that the flow angle could be accounted for by either a soft momentum-dependent potential, or a stiff momentum-independent potential. We conjecture that these are but two of a continuum of representations of the flow according to the method of analysis used in Ref. 9. The "proof" is as follows: The particular momentum dependent mean field employed in Ref. 9 is

$$U(\rho,\mathbf{p}) = a\frac{\rho}{\rho_0} + b\left[\frac{\rho}{\rho_0}\right]^\sigma + c\frac{\rho}{\rho_0}\{[1+(\mathbf{p}-\langle\mathbf{p}'\rangle)^2/\Lambda^2]^{-1} + \langle[1+(\mathbf{p}'/\Lambda)^2]^{-1}\rangle\} , \tag{8}$$

which depends on five parameters a, b, σ, c, and Λ. The last is arbitrarily fixed throughout while c is fixed by the value of the nucleon effective mass. The first three can be determined by the saturation density and binding of nuclear matter, and the momentum flow angle. The compression modulus is then fully determined. In this way we can deduce that there is a continuum of momentum dependent mean fields that can reproduce the flow data, each yielding a different value of K. The first and last entries of Table I of Ref. 9 are two members of this continuum. Other representations of the flow data correspond, for example, to σ varying between $\frac{7}{6}$ and 2, and m^*/m varying between 0.7 and 1, with corresponding values of K between 215 MeV and 377 MeV, the momentum dependence being largest at the first extreme, and zero at the other. Since these correspond to m^*/m equal to 0.7 and 1, respectively, they may effectively be regarded as the range in which this method determines K. If we assume that the dependence of K on m^* determined in this way is linear, and choose $m^*/m = 0.83$ in accord with its recent determination in Ref. 11, then $K = 285$ MeV gives the correct flow angle and saturation properties.

C. Recent results for the giant monopole resonance

New results for K have been reported by the Groningen group who made precision measurements on additional nuclei to those used in the analyses of the breathing mode of a decade ago. The new data is for isotopic chains of Sn and Sm isotopes,[55,56] and comprises a larger database than before. The errors on the location of the giant monopole resonance in the new experiments are typically $\frac{1}{2}$ to $\frac{1}{4}$ what they were in the old, and the experimenters report that the sum rule appears to be exhausted. The new value obtained for the nuclear matter compression modulus is $K = 299 \pm 25$ MeV.[56]

VIII. SUMMARY

We have analyzed evidence on the equation of state coming from a wide range of sources, from nuclear masses, the Landau sum rule, pion yields from high energy nuclear collisions, and neutron stars. Taken together, these analysis favor a large nuclear compression modulus and a stiff equation of state except for the Landau sum rule, which places only a very broad constraint from low to high K. Secondly, we critically appraised the situation on supernova simulations with regard to the equation of state. It has been previously reported that, to obtain a prompt ejection in supernova simulations, the equation of state must be soft at high density. We showed that with one exception (our of eight) the equation of state used in those simulations were so soft that they could not support the measured mass of several neutron stars. The explosion energy is bought at the expense of neutron star mass. In the exceptional case, the explosion energy was small and marginal. Beyond this there is an alternative mechanism, Wilson's late time neutrino reheating, that does work *without* the restriction on the eqution of state. However, it was noted that both mechanisms are sensitive to effects at the 5–10% level, and are therefore marginal scenarios as they stand. It may be that there is an, as yet, unrealized physical effect that has not been included in the simulations. Thirdly, we briefly discussed additional evidence from other work in Sec. VII. This included the momentum flow angle in nuclear collisions for which we showed that there is a continuous ambiguity in the determination of K unless the nucleon effective mass is well established. An exciting new development is the new high precision data and a broader data base for analysis of the giant monopole resonance (GMR) obtained by the Groningen group.[56]

In Fig. 9 we summarize results for K from the broad range of evidence studied or quoted in this work. All the evidence agrees within their errors except for the old and new monopole results which do not agree with each other, and the prompt-bounce supernova simulation, which however should not be regarded as a constraint for the reasons discussed in Sec. V. All of the evidence that we have considered is consistent, within the lower error bounds, with the original analysis of the (GMR) data. However the present evidence lives more comfortably with the new GMR data and its analysis. It would appear therefore that the compression modulus lies around 300 MeV, with considerable error.

Concerning the equation of state at higher than nuclear density, it is not possible at this stage to make very precise statements. Certainly the neutron star equation of state must be sufficiently stiff as to support the mass of known neutron stars. The original suggestion of Stock, that a rather direct measurement of the density dependence of the nuclear matter equation of state is possible from pion yields in nuclear collisions through a measurement of the "missing potential energy" is not a rigorous procedure as discussed in Sec. IV A and proven in Ref. 29. Unfortunately, it is not born out even qualitatively in the calculations of Sec. IV B. If thermodynamic equilibrium in the dense matter is not achieved, then an extraction of the equation of state becomes entangled with many uncertain issues concerning the treatment of the dynamics, a glimpse of which is seen in the discussion of Sec. VII B.

Finally, we wish to acknowledge that K is a less than perfect characterization of the stiffness of the equation of state. We have related different kinds of phenomena, some of which sample the equation of state over a range

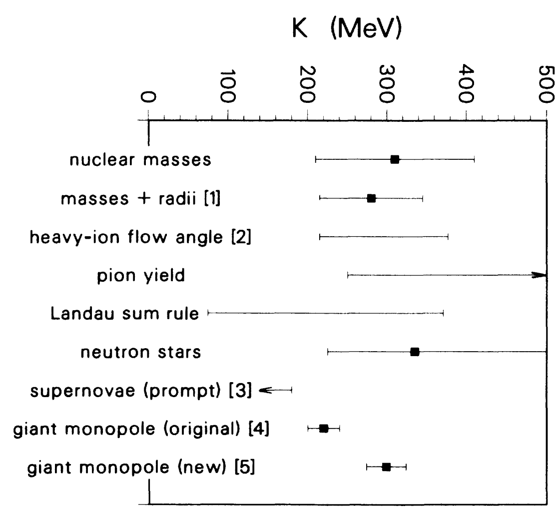

FIG. 9. Summary of results for K. The numbered items are taken from other sources referenced as follows. Item numbers 1 is from Ref. 53, item 2 from Ref. 9, item 3 and from Ref. 5, item 4 from Ref. 3 and item 5 from Ref. 56. Unnumbered items in the figure are from this work.

of densities, through relativistic nuclear field theory. This is model dependent but quite precise as to its meaning, as we have described the theory.

ACKNOWLEDGMENTS

I thank Peter Möller for kindly performing the calculations reported in Sec. III. I appreciate discussions of the physics of supernovae with J. Wilson, R. Mayle, and D. Hartmann. I also appreciate communication before publication of effects of Coulomb lattice energy on the iron core mass of presupernovae by S. Woosley. I thank J. H. Taylor for communicating his latest results on the mass measurement of PSR1913 + 16, before publication. I also appreciate communications from the researchers of the Groningen group concerning the giant monopole, and in particular A. van der Woude for a conversation on the subject at Orsay. A travel grant from the French Ministry of Foreign Affairs and the hospitality of the Institut de Physique Nucléiare, Orsay, where part of this work was completed, are gratefully acknowledged. This work was supported by the U. S. Department of Energy under Contract DE-AC03-76SF00098.

[1]S. I. A. Garpman, N. K. Glendenning, and Y. J. Karant, Nucl. Phys. **A322**, 382 (1979).

[2]J. P. Blaizot, D. Gogny, and B. Grammiticos, Nucl. Phys. **A265**, 315 (1976); J. P. Blaizot, Phys. Rep. **64**, 171 (1980).

[3]J. Treiner, H. Krevine, O. Bohigas, and J. Martorell, Nucl. Phys. **A317**, 253 (1981).

[4]G. E. Brown and E. Osnes, Phys. Rev. Lett. **159B**, 223 (1985).

[5]E. Baron, J. Cooperstein, and S. Kahana, Phys. Rev. Lett. **55**, 126 (1985).

[6]M. Sano, M. Gyulassy, M. Wakai, and Y. Kitazoe, Phys. Lett. **156B**, 27 (1985).

[7]H. Stocker and W. Greiner, Phys. Rep. **137**, 277 (1986).

[8]J. J. Molitoris, D. Hahn, and H. Stocker, Nucl. Phys. **A447**, 13c (1985).

[9]C. Gale, G. Bertsch, and S. DasGupta, Phys. Rev. C **35**, 415 (1987).

[10]J. Aichelin, A. Rosenhauer, G. Peilert, H. Stocker, and W. Greiner, Phys. Rev. Lett. **58**, 1926 (1987).

[11]C. H. Johnson, D. J. Horen, and C. Mahaux, Phys. Rev. C **36**, 2252 (1987).

[12]P. Möller, W. D. Myers, W. J. Swiatecki, and J. Treiner, At. Data Nucl. Data Tables (to be published).

[13]P. Möller and J. R. Nix, submitted to At. Data Nucl. Data Tables.

[14]F. D. Mackie and G. Baym, Nucl. Phys. **A285**, 332 (1977).

[15]R. Stock *et al.*, Phys. Rev. Lett. **49**, 1236 (1982).

[16]H. Stocker, W. Greiner, and W. Schied, Z. Phys. A **286**, 121 (1978).

[17]R. Stock, Phys. Rep. **135**, 259 (1986).

[18]M. Jamion, C. Mahaux, and P. Rochus, Phys. Rev. Lett. **43**, 1097 (1979); Phys. Rev. C **22**, 2027 (1980).

[19]B. D. Serot and J. D. Walecka, Phys. Lett. **87B**, 172 (1980); C. J. Horowitz and B. D. Serot, Nucl. Phys. **A368**, 413 (1981); J. Boguta, *ibid.* **372**, 386 (1981).

[20]P.-G. Reinhard, M. Rufa, J. Marhun, W. Greiner, and J. Friedrich, Z. Phys. A **323**, 13 (1986).

[21]K. Wehrberger and F. Beck, Phys. Rev. C **35**, 298 (1987); *ibid.*, p. 2337.

[22]N. K. Glendenning, Gesellschaft für Schwerionenforschung, Report GSI-85-10, 1985.

[23]N. K. Glendenning, Phys. Lett. **144B**, 158 (1984).

[24]J. Theis, G. Grabner, G. Buchwald, J. Maruhn, W. Greiner, H. Stocker, and J. Polonyi, Phys. Rev. D **28**, 2286 (1983); T. Nakai and S. Takagi, Prog. Theor. Phys. **71**, 1118 (1984).

[25]N. K. Glendenning, Phys. Lett. **B185**, 275 (1987); N. K. Glendenning, Nucl. Phys. **A469**, 600 (1987).

[26]J. A. Maruhn and H. Stocker, Z. Phys. A **75**, 723 (1986).

[27]B. M. Waldhauser, J. A. Maruhn, H. Stocker, and W. Greiner, Z. Phys. A **328**, 19 (1987).

[28]M. Cubero, M. Schonhofen, H. Feldmeier, and W. Norenberg, Gesellschaft für Schwerionenforschung, Report GSI-87-45, 1987.

[29]N. K. Glendenning, Phys. Rev. C **37**, 1442 (1988).

[30]J. Boguta and A. R. Bodmer, Nucl. Phys. **A292**, 413 (1977).

[31]R. B. Clare, D. Strottman, and J. Kapusta, Phys. Rev. C **33**, 1288 (1986).

[32]J. W. Harris *et al.*, Phys. Rev. Lett. **58**, 463 (1987).

[33]N. K. Glendenning, in *Theoretical Methods in Medium-Energy and Heavy-Ion Physics,* edited by K. W. McVoy and W. A. Friedman (Plenum, New York, 1978).

[34]W. Hildebrandt, K. Nomoto, and R. G. Wolff, Astron. Astrophys. **133**, 175 (1984).

[35]S. E. Woosley and T. A. Weaver, Annu. Rev. Astron. Astrophys. **24**, 205 (1986).

[36]R. Mayle and S. Woosley (private communication).

[37]E. Baron, H. A. Bethe, G. E. Brown, J. Cooperstein, and S. Kahana, Phys. Rev. Lett. **59**, 736 (1987).

[38]J. M. Weisenberg and J. H. Taylor, Phys. Rev. Lett. **52**, 1348 (1984); (private communication).

[39]P. C. Joss and S. A. Rappaport, Annu. Rev. Astron. Astrophys. **22**, 537 (1984).

[40]J. R. Wilson, in *Numerical Astrophysics,* edited by J. Centrella, J. LeBlanc, and R. Bowers (Jones and Bartlett, Boston, 1985) p. 422; J. R. Wilson, R. Mayle, S. E. Woosley, and T. Weaver, Ann. N.Y. Acad. Sci. **470**, 267 (1986).

[41]J. R. Wilson (private communication).

[42]N. K. Glendenning, Z. Phys. A **327**, 295 (1987).

[43]N. K. Glendenning, Phys. Rev. Lett. **57**, 1120 (1986).

[44]M. H. Johnson and E. Teller, Phys. Rev. **98**, 783 (1955); H. P. Duerr, *ibid.* **103**, 469 (1956); J. D. Walecka, Ann. Phys. **83**, 491 (1974); B. Banerjee and N. K. Glendenning, and M. Gyulassy, Nucl. Phys. **A361**, 326 (1981).

[45]N. K. Glendenning, Astrophys. J. **293**, 470 (1985); N. K. Glendenning, Z. Phys. A **326**, 57 (1987); N. K. Glendenning, Phys. Lett. **114B**, 392 (1982).

[46]C. J. Horowitz and B. D. Serot, Nucl. Phys. **A464**, 613 (1987).

[47]M. Pakrash and T. L. Ainsworth, Phys. Rev. C **36**, 346 (1987).

[48]V. A. Ambartsumyan and G. S. Saakyan, Astron. Zh. **37**, 193 (1963) [Sov. Astron. AJ,4 187 (1960)].

[49]V. R. Pandharipande, Nucl. Phys. **A178**, 123 (1971).

[50]N. K. Glendenning, Nucl. Phys. **A480**, 597 (1988).

[51]H. A. Bethe and M. Johnson, Nucl. Phys. **A230**, 1974 (1974).

[52]B. D. Day and R. B. Wiringa, Phys. Rev. C **32**, 1057 (1985).

[53]J. Friedrich and N. Voelger, Phys. Rev. Lett. **47**, 1385 (1981).

[54]V. F. Weisskopf, Rev. Mod. Phys. **29**, 174 (1957).
[55]M. M. Sharma, W. T. A. Borhgols, S. Brandenberg, S. Crona, J. H. Meier, A. van der Woude, and M. N. Harakeh, Kernfysisch Versneller Instituut, Report KVI-689, 1987.
[56]M. M. Sharma, W. T. A. Borghols, S. Brandenberg, S. Crona, A. van der Woude, and M. N. Harakeh, Kernfysisch Versneller Instituut, Report KVI-691, 1988; submitted to Phys. Rev. C.