Laser-assisted internal conversion

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A general formula for the coefficient of internal conversion in an intense laser field, which is valid for all atomic electron shells, is deduced in a simple nonrelativistic model. For special shells, for which internal conversion without the laser is energetically forbidden but which have an energy defect relatively small compared to the laser photon energy, the presence of the laser radiation is essential. Cases such as the $4s\frac{1}{2}$ shell internal conversion of the $E1$ transition of the isomeric state $^{183}W^m$ ($E_\gamma=544$ eV, $\tau=5.3$ s, $J=\frac{11}{2}^+$) and the $5d\frac{3}{2}$, $5d\frac{5}{2}$ shell internal conversions of the $E3$ transition of the isomeric state $^{235}U^m$ ($E_\gamma=73.5$ eV, $\tau=26$ min, $J=\frac{1}{2}^+$) are numerically investigated in the small momentum approximation.

I. INTRODUCTION

In a recent paper[1] (hereafter referred to as paper I) we dealt with the internal conversion process (ICP) that takes place in the presence of intense radiation (laser) field. In paper I a very simple model[2] was used, as the interaction Hamiltonian H_I, which causes internal conversion, was supposed to be of Coulomb type

$$H_I=-\sum_{p=1}^{Z}\frac{e^2}{|\mathbf{R}-\mathbf{x}_p|}+\frac{Ze^2}{R}\ , \tag{1}$$

where $\mathbf{R}$ and $\mathbf{x}_p$ denote electron and proton coordinates, respectively, and Ze is the nuclear charge. Furthermore, in paper I, only the K shell ICP in the small momentum approximation (see below) was investigated.

The exact treatment of laser free ICP needs a very complex calculation. Besides the above simple interaction Hamiltonian, it is also necessary to take into account the effect of photon exchanges. In such a calculation one has to use relativistic (Dirac-type) wave functions and also has to take into account the effect of the nuclear size and of the shielding of the Coulomb potential of the nucleus by inner-shell electrons.[3]

The treatment of the laser assisted ICP (LA-ICP) also needs electronic wave functions which are the solutions of the Dirac equation, but with the Hamiltonian[4] $H_E^D=H_{OE}+H_{AE}$ with $H_{OE}=-i\hbar c\boldsymbol{\alpha}\nabla+\beta mc^2-Ze^2/R$ and $H_{AE}=e\boldsymbol{\alpha}\mathbf{A}$ (for the notation see Ref. 4). Here $\mathbf{A}$ is the vector potential describing the laser radiation.

The main problem of the formulation of the LA-ICP is that there is no exact solution of the Dirac equation with the above Hamiltonian and the same holds for the nonrelativistic Hamiltonian

$$H_E=-\frac{Ze^2}{R}+\frac{1}{2m}\left[\mathbf{p}-\frac{e}{c}\mathbf{A}\right]^2\ , \tag{2}$$

which was used in paper I. Thus at present we have to take approximate solutions for the in and out electron states.

Our final aim is to construct a general theory of LA-ICP, but because of the above-mentioned numerous problems we are going to do it step by step. In the present paper we are extending the calculation in two directions. We restrict ourselves to the simple model of paper I but we investigate not only the K shell case; furthermore, at the beginning we do not use the $\hat{p}=a_0p/\hbar\ll1$ approximation [small momentum approximation (SMA)].

With these extensions we generalize the calculation of LA-ICP in two essential respects, we work out some of the mathematical grounds for LA-ICP which will also be useful in the case of a general relativistic calculation, and we account for the LA-ICP of all electronic shells making possible the discussion of special interesting cases.

The vector potential $\mathbf{A}$ of a circularly polarized external radiation field in the dipole approximation is

$$\mathbf{A}=a\left[\hat{\mathbf{e}}_1\cos\omega t-\hat{\mathbf{e}}_2\sin\omega t\right]\ , \tag{3}$$

where ω is the angular frequency, and a is the amplitude of the vector potential. $\hat{\mathbf{e}}_1$ and $\hat{\mathbf{e}}_2$ are the unit vectors perpendicular to each other and $\hat{\mathbf{e}}_3=\hat{\mathbf{e}}_1\times\hat{\mathbf{e}}_2$. We use the coordinate system $\hat{\mathbf{e}}_1,\hat{\mathbf{e}}_2,\hat{\mathbf{e}}_3$ further on.

In Sec. II the electronic wave functions in the initial and final states are given. In Sec. III the transition probability per unit time of the process is deduced. Section IV is devoted to the general form of LA-ICP and in Sec. V the result is discussed in the SMA. In Sec. VI numerical results and their discussions are given. The mathematical formulae necessary for the computation, in general, are listed in Appendix A. Appendix B contains the method of computation of the quantities denoted by $i_{J,B}^{(A,lm)}$ and $T_{l_1m_1,l_2m_2}^{(C,D)}$ in the paper. Appendix C gives the way the quantity T (depicted in Fig. 1) was numerically computed in. Finally, in Appendix D we deal with the approximations used at the deduction of the initial and final states. As the initial state looks like a wave function of MTA type[5] the relation to the MTA is also discussed there.

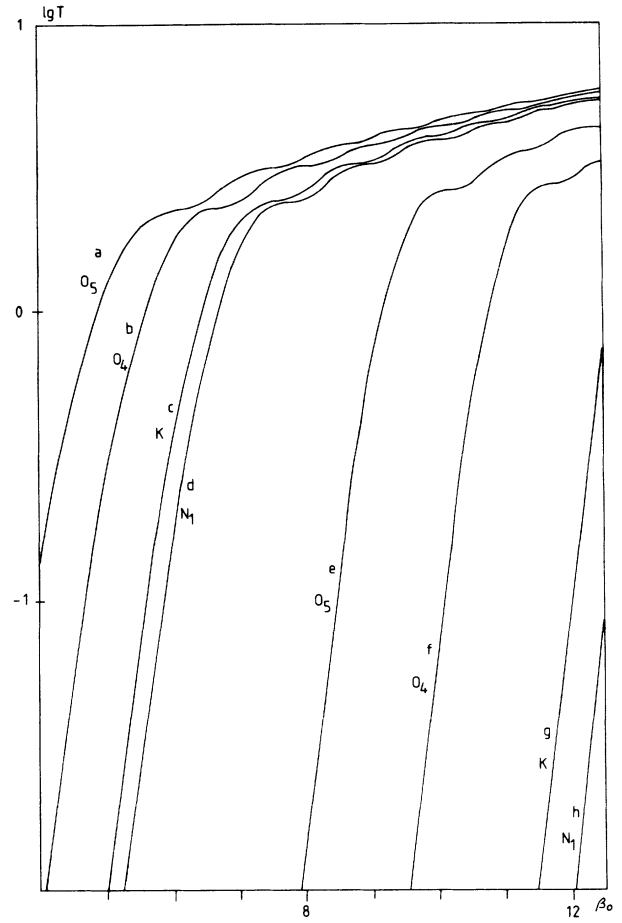

FIG. 1. The quantity $T(\beta_0,\Delta/\hbar\omega)$ defined by (30) for different materials and lasers. $I=8.73\times10^{11}\beta_0^2(\hbar\omega)^3$ is the laser intensity, $\Delta=E_\gamma-E_B$, where E_γ is the nuclear transition energy, and E_B is the electronic binding energy. The curves a–d are obtained with $\hbar\omega=5$ eV and e–h with $\hbar\omega=1.16$ eV. The corresponding material is denoted by its electronic shell (see Tables I and II). The curves c and g denoted by K correspond to the ^{105m}Ag, K shell case discussed in paper I. (lg$T$ means the logarithm of T to the base 10.)

II. INITIAL AND FINAL ELECTRONIC STATES IN THE RADIATION GAUGE

The initial and final electronic states are two different, approximate solutions of H_E (see Appendix D). The initial states are H-like bound states[6]

$$v_i=\sum_{L=-\infty}^{\infty} v_i^{(L)} , \tag{4a}$$

$$v_i^{(L)}=\phi_{n\lambda\mu}(\xi)i^L J_L(b\xi\sin\vartheta)e^{iL\varphi}e^{-i(-K/\hbar-L\omega)t} . \tag{4b}$$

Here J_L is the Bessel function of the first kind of order L, $b=eaa_0/\hbar c$, K is the binding energy of the electron on shell of quantum numbers n,λ,μ and $\phi_{n\lambda\mu}(\xi)$ is a spinless, unperturbed H-like wave function[7] of this shell:

$$\phi_{n\lambda\mu}(\xi)=a_0^{-3/2}\pi^{-1/2}e^{-\xi}f_{n\lambda}(\xi)\hat{Y}_{\lambda\mu} , \tag{4c}$$

with

$$f_{n\lambda}(\xi)=\sum_{q=0}^{n-1} d_q(n\lambda)\xi^q \tag{4d}$$

and

$$\hat{Y}_{\lambda\mu}=(4\pi)^{1/2}Y_{\lambda\mu},\quad \xi=R/a_0,\quad a_0=na_B/Z_{\rm eff}(j) ,$$

where R,ϑ,φ are the spherical coordinates of the electron in the coordinate system defined by $\hat{\mathbf{e}}_1,\hat{\mathbf{e}}_2,\hat{\mathbf{e}}_3$, $Y_{\lambda\mu}$ is spherical harmonics, a_B is the Bohr radius, $Z_{\rm eff}$ is the effective charge, i.e., the charge of a H-like ion with an electron having the same binding energy in a state of quantum numbers n,j,λ as in the subshell in question. Here j is the total angular momentum of the state. Thus the differences in ICC's of subshells caused by the different total angular momenta mainly included in $Z_{\rm eff}$. The quantities $d_0(n\lambda),\ldots,d_{n-1}(n\lambda)$ are constants which determine the radial part of the electronic wave function $\phi_{n\lambda\mu}$. Their values are given for two states in Sec. VI. The shielding of the nuclear Coulomb potential is partly accounted for by the quantity $Z_{\rm eff}$.[8]

The final states are nonrelativistic Volkov solutions[9] given by formulae similar to (5a) and (5b) of paper I.

$$v_f=\sum_{N=-\infty}^{\infty} v_f^{(N)} , \tag{5a}$$

with

$$v_f^{(N)}=\frac{1}{\sqrt{V}}e^{i\hat{\mathbf{p}}\boldsymbol{\xi}}J_N(\beta\sin\theta)e^{i[N\chi+(N\omega-E/\hbar)t]} \tag{5b}$$

and

$$\beta=\frac{eap}{mc\hbar\omega},\quad \hat{p}=a_0p/\hbar , \tag{5c}$$

where $V,\mathbf{p},m$ stand for the normalization volume, electron momentum, and rest mass, respectively, $E=p^2/2m$ and θ,χ are polar angles of $\mathbf{p}$ in the above $(\hat{\mathbf{e}}_1,\hat{\mathbf{e}}_2,\hat{\mathbf{e}}_3)$ coordinate system. We expand $\exp(i\hat{\mathbf{p}}\boldsymbol{\xi})$ according to (A1) in terms of partial waves, which leads to the form of the final state

$$v_f=\sum_{N=-\infty}^{\infty}\sum_{l=0}^{\infty}\sum_{m=-l}^{l} v_{flm}^{(N)} , \tag{6a}$$

where

$$v_{f,lm}^{(N)}=\frac{1}{\sqrt{V}}g_{lm}(\xi,\vartheta,\varphi)G_{lm}^{(N)}(\theta,\chi)e^{i(N\omega-E/\hbar)t} , \tag{6b}$$

with

$$g_{lm}=i^l j_l(\hat{p}\xi)\hat{Y}_{lm}(\vartheta,\varphi) \tag{6c}$$

and

$$G_{lm}^{(N)}=J_N(\beta\sin\theta)\hat{Y}^*_{lm}(\theta,\chi)e^{iN\chi} . \tag{6d}$$

Here j_l denotes the spherical Bessel function.

The SMA ($\hat{p}\ll1$) gives $j_l=0$ if $l\neq0$ and $j_l=1$ if $l=0$ which results in only $v_{f,00}^{(N)}$ remaining the sum. As $g_{00}=1$

and $G_{00}^{(N)}=J_N(\beta\sin\theta)\exp(iN\chi)$ we have for the final state in this approximation the form of (5d) of paper I.

III. CALCULATION OF THE TRANSITION PROBABILITY PER UNIT TIME

In this section we follow the steps of Sec. III of paper I. We use the scattering matrix formalism, and the matrix element to be determined is the same as (6) of I. Using the expressions (4a) and (6a) it can be written as

$$S_{fi}=\sum_{N,L,l,m} S_{fi,lm}^{(N,L)} , \tag{7}$$

where

$$S_{fi,lm}^{(N,L)}=\frac{1}{i\hbar}\int v_{f,lm}^{(N)*}\psi_f^{b*}H_I\psi_i^a v_i^{(L)}d^3R\,d\tau\,dt . \tag{8}$$

Here ψ_i^a and ψ_f^b are nuclear wave functions for the initial (a) and final (b) states having energies E_a and E_b, respectively, and

$$d\tau=\sum_{\text{proton spin}} d^3x_1\cdots d^3x_z ,$$

with $x_1,\ldots,x_z$ denoting proton coordinates. After carrying out time integration in (8) we have

$$S_{fi,lm}^{(N,L)}=-iH_{I,ab}^{(L,lm)}\frac{1}{\sqrt{V}}G_{lm}^{(N)*}\cdot 2\pi \times\delta[K+E-\hbar\omega_{ab}+(L-N)\hbar\omega] , \tag{9}$$

with the notation

$$H_{I,ab}^{(L,lm)}=\int\hat{\psi}_f^{b*}g_{lm}^*H_I\hat{\psi}_i^a\hat{v}_i^{(L)}d^3R\,d\tau , \tag{10}$$

and $\hbar\omega_{ab}=E_a-E_b$. The caret denotes the space dependent part of the wave functions.

The above modifications alter formula (11) of paper I

$$|S_{fi}|^2\frac{Vp^2dp\,d\Omega_p}{(2\pi\hbar)^3}=\sum_{N,L,N',L'}\sum_{l_1,m_1,l_2,m_2}4\pi^2H_{I,ab}^{(L,l_1,m_1)}H_{I,ab}^{(L',l_2m_2)*}G_{l_1m_1}^{(N)*}G_{l_2m_2}^{(N')}\delta(N,L)\delta(N',L')\frac{mp}{(2\pi\hbar)^3}dE\,d\Omega_p , \tag{11}$$

where $\delta(N,L)$ denotes the Dirac delta function in (9) and $\delta(N',L')$ has the same meaning but N' and L' stand in the argument instead of N and L. Using the identity

$$\delta(N,L)\delta(N',L')=\delta(\omega(L-N-L'+N'))\delta(N,L)/\hbar ,$$

we can obtain the following result in the usual way[4]

$$dW_{fi}(m_a,\mu\to m_b)=\sum_{N,L,N',L'}n_0(jn\lambda)\frac{2\pi}{\hbar}H_{I,ab}^{(L,l_1m_1)}H_{I,ab}^{(L',l_2m_2)*}G_{l_1m_1}^{(N)*}G_{l_2m_2}^{(N')}\frac{mp(N,L)}{(2\pi\hbar)^3}\delta_{L-L',N-N'}d\Omega_p , \tag{12}$$

where the symbol $p(N,L)$ denotes that p has to take a value determined by the argument of $\delta(N,L)$, $n_0(jn\lambda)$ is the density of the initial electronic states, i.e., the number of electrons on the given subshell of quantum numbers j,n,λ, μ is the magnetic quantum number of the initial electronic state, and m_a,m_b are the magnetic quantum numbers of nuclear states (a) and (b), respectively, $\delta_{L-L',N-N'}$ is the Kronecker delta.

We work in the point nucleus approximation, i.e., we neglect the nuclear size beside the size of the electronic shell so we use the expansion (A2) of $1/|\mathbf{R}-\mathbf{x}_p|$ in terms of spherical harmonics valid if $x_p<R$. Thus we obtain

$$H_{I,ab}^{(L,l_sm_s)}=\sum_{l=0}^{\infty}\sum_{m=-l}^{l}\frac{4\pi}{2l+1}Q_{lm}(a,b)I_{lm,\lambda\mu}^{(L,l_sm_s)} , \tag{13}$$

where $s=1,2$ and $Q_{lm}(a,b)$ is the matrix element of the electric multipole moment of order l,m [see (14) in paper I] between nuclear states (a) and (b) which can be written with the use of the Wigner-Eckart theorem[7] through its reduced matrix element $\langle b\|Q_l\|a\rangle$ as

$$Q_{lm}(a,b)=(-1)^{j_b-m_b}\langle b\|Q_l\|a\rangle\begin{pmatrix} j_b & l & j_a \\ -m_b & m & m_a\end{pmatrix} . \tag{14}$$

Furthermore,

$$I_{lm,\lambda\mu}^{(L,l_sm_s)}=Ci_{lm,\lambda\mu}^{(L,l_sm_s)}\ \text{with}\ C=\frac{e}{(\pi a_0^3)^{1/2}}\cdot a_0^{2-l} \tag{15}$$

and

$$i_{lm,\lambda\mu}^{(L,l_sm_s)}=\int e^{-\xi}f_{n\lambda}(\xi)\hat{Y}_{\lambda\mu}(\vartheta,\varphi)Y_{lm}(\vartheta,\varphi)J_L(b\xi\sin\vartheta)i^Le^{iL\varphi}g_{l_sm_s}(\xi,\vartheta,\varphi)\xi^{1-l}d\xi\,d\varphi\sin\vartheta\,d\vartheta . \tag{16}$$

Since $\langle b\|Q_l\|a\rangle$ decreases rapidly with increasing l, we retain from the sum for l only the term of lowest l which gives nonzero reduced matrix element. Thus l is determined by the multipolarity of the nuclear transition as usual.

We use the addition formula of the spherical harmonics[7] and write the product $\hat{Y}_{\lambda\mu}Y_{lm}$ in the form

$$\hat{Y}_{\lambda\mu}Y_{lm}=\sum_{J,M}\sqrt{\hat{J}\,\hat{l}\,\hat{\lambda}}\begin{bmatrix}\lambda & l & J\\ 0 & 0 & 0\end{bmatrix}\begin{bmatrix}\lambda & l & J\\ \mu & m & M\end{bmatrix}Y^{*}_{JM}\ . \tag{17}$$

Here $\hat{J}=2J+1$, $\hat{l}=2l+1$, and $\hat{\lambda}=2\lambda+1$. Using this in $H^{(L,l_1m_1)}_{I,ab}$ and in $H^{(L',l_2m_2)^*}_{I,ab}$, the formulae (13)–(16) and twice the condition of orthogonality of the $3j$ symbols (A3); furthermore, averaging over the magnetic quantum numbers of the initial nuclear state (a) and the initial electronic state, i,e., over m_a and μ, and summing over the magnetic quantum number m_b of the final nuclear state (b) as usual, we obtain

$$dW_{fi}=\sum_{N,L,N',L'}\sum_{l_1,m_1,l_2,m_2}4n_0(jn\lambda)\frac{me^2p(N,L)}{\pi\hat{l}^2\hbar^4}G^{(N)*}_{l_1m_1}G^{(N')}_{l_2m_2}$$
$$\times\delta_{L-L',\mathrm{N}-\mathrm{N}'}\sum_{J,M}\begin{bmatrix}\lambda & l & J\\ 0 & 0 & 0\end{bmatrix}^2 i^{(L,l_1m_1)}_{JM}i^{(L',l_2m_2)^*}_{JM}\frac{|\langle b\|Q_l\|a\rangle|^2}{(2j_a+1)a_0^{2l-1}}d\Omega_p\ , \tag{18}$$

where

$$i^{(L,l_sm_s)}_{JM}=\int e^{-\xi}f_{n\lambda}(\xi)Y_{JM}(\vartheta,\varphi)J_L(b\xi\sin\vartheta)e^{iL\varphi}i^L g_{l_sm_s}(\xi,\vartheta,\varphi)\cdot\xi^{1-l}d\xi\,d\varphi\sin\vartheta\,d\vartheta,\quad s=1,2\ . \tag{19}$$

Finally, the total transition probability per unit time can be obtained as

$$W_{fi}=\sum_{N,L,N',L'}\sum_{l_1,m_1,l_2,m_2}\frac{8n_0(jn\lambda)me^2}{\hbar^3\hat{l}^2}\frac{p(N,L)}{\hbar}$$
$$\times\sum_{JM}\begin{bmatrix}\lambda & l & J\\ 0 & 0 & 0\end{bmatrix}^2 i^{(L,l_2m_1)}_{JM}i^{(L',l_2m_2)^*}_{JM}\delta_{L-L',\mathrm{N}-\mathrm{N}'}T^{(\mathrm{N},\mathrm{N}')}_{l_1m_1,l_2m_2}\frac{|\langle b\|Q_l\|a\rangle|^2}{(2j_a+1)a_0^{2l-1}}\ , \tag{20}$$

where

$$T^{(\mathrm{N},\mathrm{N}')}_{l_1m_1,l_2m_2}=\frac{1}{2\pi}\int G^{(N)*}_{l_1m_1}G^{(N')}_{l_2m_2}d\chi\sin\theta\,d\theta\ . \tag{21}$$

IV. LASER ASSISTED INTERNAL CONVERSION COEFFICIENT (LA-ICC)

We can make further simplifications in the expressions of $i^{(L,l_sm_s)}_{JM}$ using the formulae (A4) for the spherical harmonics and intergrating over φ. Similarly the integration in $T^{(N,N')}_{l_1m_1,l_2m_2}$ over χ can be carried out. These integrations result: $2\pi\cdot\delta_{L+M+m_1,0}$, $2\pi\delta_{L'+M+m_2,0}$, $2\pi\cdot\delta_{N'-N+m_1-m_2,0}$, respectively, which with the definition of the internal conversion coefficient (ICC) [see Sec. V formulae (28) and (29) in paper I] gives for the LA-ICC

$$\alpha^l_{nj\lambda}=\frac{n_0(j)l[(2l-1)!!]^2}{\pi(l+1)}\left[\frac{\alpha E_0}{E_\gamma}\right]^{2l+1/2}\frac{1}{\sqrt{\alpha}}\left[\frac{Z_{\mathrm{eff}}}{n}\right]^{2l-1}$$
$$\times\left[\frac{2\hbar\omega}{E_\gamma}\right]^{1/2}\cdot\sum_{\substack{N,L,l_1,m_1,l_2,m_2,J\\ |L+m_1|<J}}\begin{bmatrix}\lambda & l & J\\ 0 & 0 & 0\end{bmatrix}^2 i^{(L,l_1m_1)}_{J,-L-m_1}i^{(L+m_1-m_2,l_2m_2)^*}_{J,-L-m_1}\left[\frac{\Delta}{\hbar\omega}+(N-L)\right]^{1/2}T^{(N,N+m_2-m_1)}_{l_1m_1,l_2m_2}\ , \tag{22}$$

where $E_0=mc^2$ is the rest mass of the electron, $\hbar\omega$ is the laser photon energy, α is the fine structure constant, $E_\gamma=E_a-E_b$ is the energy of the gamma photon, $\Delta=E_\gamma-E_B$, $E_B\equiv K$ is the binding energy of the electron in the shell investigated; furthermore,

$$i^{(A,l_sm_s)}_{J,B}=\sqrt{\pi}i^{l_s}K^{B,m_s}_{J,l_s}\int_0^\infty\int_0^\pi P_J^{|B|}(\cos\vartheta)P_{l_s}^{|m_s|}(\cos\vartheta)i^A J_A(b\xi\sin\vartheta)\sin\vartheta\,d\vartheta\,e^{-\xi}f_{n\lambda}(\xi)j_{l_s}(\hat{p}\xi)d\xi\ , \tag{23a}$$

with

$$K^{B,m_s}_{J,l_s}=\sqrt{\hat{l}_s\hat{J}}(-1)^{(B+|B|+m_s+|m_s|)/2}\left[\frac{(J-|B|)!}{(J+|B|)!}\frac{(l_s-|m_s|)!}{(l_s+|m_s|)!}\right]^{1/2}, \tag{23b}$$

and

$$T^{(C,D)}_{l_1 m_1, l_2 m_2} = K^{m_1,m_2}_{l_1,l_2} \int J_c(\beta \sin\theta) J_D(\beta \sin\theta) P_{l_1}^{|m_1|}(\cos\theta) P_{l_2}^{|m_2|}(\cos\theta) d(\cos\theta) \ . \tag{24}$$

The formulae we need to compute $i^{(A,l_s m_s)}_{J,B}$ and $T^{(C,D)}_{l_1 m_1, l_2 m_2}$ are given in Appendix B.

V. SMALL MOMENTUM APPROXIMATION

The SMA results (see Sec. II) that $l_1 = l_2 = 0$, $m_1 = m_2 = 0$ and so we have quantities of types $i^{(L,00)}_{J,-L}$ and $T^{(N,N)}_{00,00}$ appearing in the LA-ICC. Thus (22) giving the LA-ICC ($\alpha^l_{nj\lambda}$) of a nuclear transition of multipolarity l and of an electronic subshell of quantum numbers n, j, λ as

$$\alpha^l_{nj\lambda} = B(j) \sum_{J=|l-\lambda|}^{J=l+\lambda} \begin{bmatrix} l & \lambda & J \\ 0 & 0 & 0 \end{bmatrix}^2 \sum_{\substack{L=-J \\ J-L=\text{even}}}^{J} \sum_{N > -\Delta/\hbar\omega + L} |i_{J|L|}|^2 \tau_{N,L} \ , \tag{25}$$

with

$$B(j) = \frac{(2j+1) l [(2l-1)!!]^2}{\pi(l+1)} \left[\frac{Z_{\text{eff}}(j)}{n} \right]^{2l-1} \left[\frac{\alpha E_0}{E_\gamma} \right]^{2l+1/2} \frac{2}{\sqrt{\alpha}} \left[\frac{2\hbar\omega}{E_\gamma} \right]^{1/2} , \tag{25a}$$

and

$$\tau_{N,L} = \frac{1}{2\beta_0} \int_0^{2\beta(N,L)} J_{2|N|}(x) dx \ . \tag{25b}$$

Here $2\tau_{N,L} = T^{(N,N)}_{00,00}$ [see (25) and (26) in paper I], $J_{2|N|}$ denotes the Bessel function of the first kind, $\beta(N,L) = \beta_0 (N - L + \Delta/\hbar\omega)^{1/2}$ with $\beta_0 = ea(2/\hbar\omega E_0)^{1/2}$, $i_{JL} = i^{(L,00)}_{J,-L}$ (see Sec. IV in paper I), and

$$\begin{aligned} |i_{JL}| &= S_{JL}(y)(1+b^2)^{J/2-1} y^{L/2}, \quad J-L = \text{even} \\ |i_{JL}| &= 0, \quad J-L = \text{odd} \end{aligned} \tag{26}$$

where $y = b^2/(1+b^2)$ with b defined in Sec. II,

$$S_{JL}(y) = \sum_{q=0}^{n-1} d_q(n\lambda)(1-y)^{q/2} \sum_{k \geq \frac{J-L}{2}} F_{JLkq}(y) y^k \ , \tag{27a}$$

$$F_{JLkq} = 2\pi N_{JL} f_{JLk} \, {}_2F_1 \left[k+1+\frac{L-J+q}{2}, \frac{L+J-q-1}{2}; L+k+1; y \right] (2k+L-J+q+1)!/[2^{2k+L} k!(L+k)!] \ , \tag{27b}$$

$$N_{JL} = \left[\frac{2J+1}{4\pi} \frac{(J-L)!}{(J+L)!} \right]^{1/2} , \tag{27c}$$

$$f_{JLk} = \frac{\Gamma(k+\frac{1}{2}) k! (J+L)!}{2^L \Gamma\left[\frac{J-L}{2} + k + 1 \right] \Gamma\left[\frac{L+J}{2} + k + 3/2 \right] (J-L)!} \ . \tag{27d}$$

As the L dependence in β can generally be neglected, $\alpha^l_{nj\lambda}$ may be written in a simpler form[10]

$$\alpha^l_{nj\lambda} = B(j) SF_{l\lambda}(n) T \tag{28}$$

with

$$SF_{l\lambda}(n) = \sum_{J=|l-\lambda|}^{J=l+\lambda} \begin{bmatrix} l & \lambda & J \\ 0 & 0 & 0 \end{bmatrix}^2 \sum_{\substack{L=-J \\ J-L=\text{even}}}^{J} |i_{J|L|}|^2 \ , \tag{29}$$

where n in the parentheses indicates that $SF_{l\lambda}$ depends on the constants $d_0(n\lambda), \ldots, d_{n-1}(n\lambda)$,

$$T = \sum_{N > -\Delta/\hbar\omega} \frac{1}{2\beta_0} \int_0^{2\beta(N)} J_{2|N|}(x) dx \tag{30}$$

and $\beta(N)$ is $\beta(N,L)$ with $L = 0$.

VI. RESULTS AND DISCUSSION

At first we discuss the conditions of validity of our results. The laser field should be so intense that the interaction energy of electrons with the laser field would become comparable with the binding energy of electrons in the

atomic potential. This leads to a condition for the applied laser intensity

$$I \gtrsim \left[\frac{Z_{\text{eff}}}{n}\right]^4 [\hbar\omega(\text{eV})]^2 6.31\cdot 10^8 \text{ W/cm}^2 . \quad (31)$$

Another condition is a consequence of the SMA. The SMA is valid and so is our result in the preceding section if $pa_0/\hbar \ll 1$ which yields the condition

$$\Delta \ll E_B , \quad (32a)$$

or more precisely

$$\Delta + (N-L)\hbar\omega \ll E_B . \quad (32b)$$

Finally, our result can be used only at moderately high laser intensities because at extremely high intensities the nonrelativistic formulation loses its validity.

At moderate laser intensities the condition $y \ll 1$ is usually fulfilled and in the expression $SF_{l\lambda}(n)$ the only term which contains the lowest power of y is enough to be retained. It can be seen from (26) and (27a) that $|i_{JL}|^2 \sim y^{J_{\min}}$ in this case,[11] thus the $J_{\min} = |l-\lambda|$ power term gives the main contribution to $SF_{l\lambda}$. It means that because of $|L| \leq |l-\lambda|$, L is a small number and the above approximation, i.e., formulae (28), (29), and (30) can be used. [In the computation of y we use $b = 1.07\times 10^{-6}(I^{1/2}/\hbar\omega)\times(n/Z_{\text{eff}})$.]

As numerical examples we discuss the O_4,O_5 shell ICC's of the $E3$ transition of the isomeric state ^{235m}U and the N_1 shell ICC of the $E1$ transition of the isomeric state ^{183m}W. The ICC for these shells is energetically forbidden without the presence of the laser radiation as the gamma energies are less than the binding energies of the electron on the shell in question.

According to Weisskopf's formula[12] for the decay of a $\frac{1}{2}^+$ state of excitation energy 73.5 eV to a $\frac{7}{2}^-$ state by $E3$ transition, the theoretical lifetime for photon emission is 3.1×10^{22} s. Thus the experimental lifetime of 26 min of the ^{235m}U corresponds to a total ICC $\alpha_T = 2\times 10^{19}$ in the laser free case.

We restrict our considerations to lasers of intensity $I = 2.73\times 10^{15}$ W/cm^2 ($\beta_0 = 5$) with photon energy $\hbar\omega = 5$ eV and $I = 1.2\times 10^{14}$ W/cm^2 ($\beta_0 = 9.5$) with $\hbar\omega = 1.16$ eV in the ^{235m}U case. The electronic binding energies E_B of the subshells determine the quantity Z_{eff}/n which can be estimated as $Z_{\text{eff}}/n = (E_B/R_Y)^{1/2}$, where $R_Y = 13.605$ eV is the Rydberg constant. Thus in the cases of O_4,O_5 shells $y \ll 1$. It means that the approximate formulae (28), (29), (30), and the $SF_{l\lambda}(n) \sim y^{J_{\min}}$ approximation can be used to compute ICC. With these approximations one can obtain from (29) with the use of (26) and (27a)–(27d) $SF_{32}(5) = 32\pi y/245$. The number in the parentheses refers to the principal quantum number of the state in question. The comparison of the wave function of the $5d$ state[7] and of its form given by (4c) and (4d) gives for the constants $d_0(52), \ldots, d_4(52)$ necessary for the computation of $SF_{32}(5)$: $d_0 = 0$, $d_1 = 0$, $d_2 = r$, $d_3 = -2r/3$, and $d_4 = 2r/21$ with $r = 4/5!(7!/2!/5)^{1/2}$.

In the ^{183m}W case a $\frac{11}{2}^+$ state decays via $E1$ transition to a $\frac{9}{2}^-$ state with $E_\gamma = 544$ eV and $\tau = 5.3$ s. However, the dipole coupling between low lying states is very weak[12] in the nucleus which is manifested in the fact that a measured half-life against dipole transition can be 10^{-5}–10^{-6} times smaller than the theoretical one computed in the Weisskopf estimation. Thus the total ICC of the $E1$ transition of the ^{183m}W isomer cannot be computed in a similar way as was done in the ^{235m}U case. However, we can give an upper limit for the laser free ICC. For a given transition the largest ICC belongs to the most inner shell; therefore, we estimate the total ICC by the sum of the ICC's of the N_2 and N_3 shells. Their values can be approximated by threshold ICC's which are upper limits. These are not available directly, but the threshold ICC's of M_2, M_3 shells are known,[8] we can compute them for hypothetical transitions of energies 489 and 423 eV corresponding to the binding energies on the N_2, N_3 shells. Thus we obtain $\alpha(N_2) < 490$ and $\alpha(N_3) < 980$, i.e., the total ICC $\alpha_T < 1500$ for the 544 eV transition of ^{183m}W in the laser free case.

We investigate this ICC in two cases: at a laser intensity $I = 9.85\times 10^{15}$ W/cm^2 ($\beta_0 = 9.5$), laser photon energy $\hbar\omega = 5$ eV, and at $I = 2.1\times 10^{14}$ W/cm^2 ($\beta_0 = 12.5$), $\hbar\omega = 1.16$ eV both giving $y \ll 1$, and similarly to the case discussed above we can obtain $SF_{10}(4) = 4\pi y/9$ with $d_0 = 1$, $d_1 = -3$, $d_2 = 2$, and $d_3 = -\frac{1}{3}$.

In order to obtain the LA-ICC's we need the numerical values of the quantity $T(\beta_0, \Delta/\hbar\omega)$ given by (30). It was computed with different $\Delta < 0$ values corresponding to the cases investigated in this article and in paper I (i.e., ^{235m}U, ^{183m}W, and ^{105m}Ag) and the result is depicted in Fig. 1 where $\beta_0 = 1.07\times 10^{-6} I^{1/2}(\hbar\omega)^{-3/2}$. The method of computation is detailed in Appendix C. The numerical data[13] necessary to the calculation and our ICC results are listed in Tables I and II, respectively.

The processes discussed here numerically may be promising to observe as the energy defects are relatively small compared to the laser photon energies.

On the basis of our results one may propose experiments on LA-ICC. The crucial points of such an experiment are the production and collection of nuclei of the desired amount being in the isomeric state and their irradiation by intense and long laser pulses. From an experimental point of view the appearance of multiple charged ions and free electrons in intense ($I > 10^{14}$ W/cm^2) laser

TABLE I. Atomic and nuclear data (Ref. 13) used in the calculation. E_B is the electronic binding energy of the subshell, $\Delta = E_\gamma - E_B$ with $E_\gamma = 73.5$ eV (U) and $E_\gamma = 544$ eV (W). The quantity $SF_{l\lambda}(n)$ is determined by (29), $y = b^2/(1+b^2)$.

Atom	Electronic shell	E_B (eV)	Δ (eV)	$SF_{l\lambda}(n)$
U	$5d\frac{3}{2}(O_4)$	105	−31.5	$SF_{32}(5) = 32\pi y/245$
U	$5d\frac{5}{2}(O_5)$	96	−22.5	
W	$4s\frac{1}{2}(N_1)$	592	−48	$SF_{10}(4) = 4\pi y/9$

TABLE II. The intense field ICC results (α^l_{njq}). $I,\hbar\omega$ are the laser intensity and laser photon energy, respectively, $y=b^2/(1+b^2)$ with $b=1.07\times10^{-6}(n/Z_{\rm eff})(I^{1/2}/\hbar\omega)$ where $Z_{\rm eff}/n=(E_B/R)^{1/2}$, $I=8.73\times10^{11}\,\beta_0^2(\hbar\omega)^3$. The quantity T is defined by (30) and it is given in Fig. 1.

Matter (shell)	$\hbar\omega$ (eV)	β_0	I (W/cm^2)	b	y	T	$\alpha^l_{nj\lambda}$
$U(O_4)$	5.0	5.0	2.7×10^{15}	0.55	0.23	0.27	2.1×10^{15}
$U(O_4)$	1.16	9.5	1.2×10^{14}	0.51	0.20	6.6×10^{-3}	4.6×10^{12}
$U(O_5)$	5.0	5.0	2.7×10^{15}	0.58	0.25	1.2	2.7×10^{15}
$U(O_5)$	1.16	9.5	1.2×10^{14}	0.53	0.22	1.7	1.5×10^{15}
$W(N_1)$	5.0	9.5	9.9×10^{15}	0.44	0.16	3.6	3.3×10^{2}
$W(N_1)$	1.16	12.5	2.1×10^{14}	0.28	7.3×10^{-2}	8.9×10^{-2}	1.80

pulses is disturbing.[14] Thus because of the high electron background, it seems more convenient to detect those soft x rays which are emitted in the recombination processes instead of the detection of slow conversion electrons. The number of emitted soft x-ray photons N_γ can be estimated as

$$N_\gamma=\eta A t\alpha_{\rm las}/\alpha_T\ , \tag{33}$$

where α_T is the total laser free ICC ($\alpha_T\gg1$ in our cases), $\alpha_{\rm las}$ is the LA-ICC of the shell investigated, A is the activity of the sample, t is the integrated laser irradiation time (it is the duration of the laser pulse in an ordinary case), and η is the efficiency of soft x-ray detection including geometrical efficiency.

Taking, e.g., $\alpha_{\rm las}/\alpha_T\sim0.2$ (^{183m}W, $\alpha_{\rm las}=330$, see Table II), $\eta=10^{-2}$ and $t\sim10^{-9}$ s, we obtain that we need an activity $A\sim6\times10^{11}Bq$ (15 Ci) to obtain $N_\gamma\sim1$ in one laser pulse. This high activity may be reached by selecting the isomeric nuclei by a method similar to the one applied by Letokhov *et al.*[15] to sort out ^{189m}Os isomers. This method is based on the fact that the angular momentum is higher in some isomeric states than in the ground state, and this difference gives rise to different molecular spectra because of hyperfine interaction, when the atom with isomeric nucleus is embedded in molecules like OsO_4 or in our case, e.g., WO_3. This method gives the possibility to select molecules containing isomeric nuclei and in this way to produce a sample of low density but high activity.

The ^{183m}W isomer can be produced by thermal neutron capture in a neutron beam of high flux. The reaction ^{182}W(n,γ)^{183m}W may have a cross section $\sigma\sim2$ mb estimated by the cross section of the similar ^{184}W(n,γ)^{185m}W reaction[13] where the nuclear angular momenta, which are essential in a thermal neutron capture process, are the same as in the ^{182}W(n,γ)^{183m}W reaction. The abundance of ^{182}W is 26.3%.

Besides the fast isomer separation we need a soft x-ray detection method of high resolution in order to be able to select soft x-rays originated from the recombination of the N_1,N_2, and N_3 shells. A promising method was developed for absolute soft x-ray measurement with a transmission grating spectrometer[16] which may be a candidate for soft x-ray detection in a LA-ICC experiment.

Finally, a regenerative type resonator (amplifier)[17] may be used to confine a laser pulse and to force it to run back and forth through the sample many times, while a laser active material compensates the losses and ensures that its intensity remains approximately constant. In this way the laser irradiation time t may be increased.

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APPENDIX A

The following general mathematical formulae are used in the paper:

$$e^{i\hat{\mathbf{p}}\boldsymbol{\xi}}=4\pi\sum_{l=0}^{\infty}\sum_{m=-l}^{l}i^l j_l(\hat{p}\xi)Y^*_{lm}(\theta,\chi)Y_{lm}(\vartheta,\varphi) \tag{A1}$$

[see Ref. 2, Appendix A, formula (3.5)], here $\hat{p},\theta,\chi$ and ξ,ϑ,φ are the spherical coordinates of vectors $\hat{\mathbf{p}}$ and $\boldsymbol{\xi}$, respectively,

$$\frac{1}{|\mathbf{x}_{\rm p}-\mathbf{R}|}=\sum_{l=0}^{\infty}\sum_{m=-l}^{l}\frac{4\pi}{2l+1}\frac{x_{\rm p}^l}{R^{l+1}}Y_{lm}(\vartheta,\varphi)Y^*_{lm}(\vartheta_{\rm p},\varphi_{\rm p}) \tag{A2}$$

[see Ref. 2, Chap. XII, formula (5.10)], $x_{\rm p}<R$,

$$\sum_{m_1,m_2}\begin{bmatrix}j_1 & j_2 & j\\ m_1 & m_2 & m\end{bmatrix}\begin{bmatrix}j_1 & j_2 & j'\\ m_1 & m_2 & m'\end{bmatrix}=\frac{1}{2j+1}\delta_{mm'}\delta_{jj'} \tag{A3}$$

[see Ref. 7, formula (4.42)],

$$Y_{lm}(\vartheta,\varphi)=(-1)^{(m+|m|)/2}\left[\frac{2l+1}{4\pi}\frac{(l-|m|)!}{(l+|m|)!}\right]^{1/2}$$

$$\times P_l^{|m|}(\cos\vartheta)e^{im\varphi}\ , \tag{A4a}$$

$$Y_{l-m}(\vartheta,\varphi)=(-1)^mY^*_{lm},\quad m\geq0 \tag{A4b}$$

(see Ref. 7, Sec. 1.1.2),

$$P_l^m(x)=(-1)^m(1-x^2)^{(1/2)m}\frac{d^m}{dx^m}P_l(x), \quad m\geq 0 \tag{A5}$$

[see Ref. 18, p. 334, (8.6.6)],

$$\frac{d^m}{dx^m}P_l(x)=(2m-1)!!C_{l-m}^{m+1/2}(x), \quad m\leq l \tag{A6}$$

[see Ref. 18, p. 779, (22.5.37)],

$$C_n^{\alpha}(x)=\frac{1}{\Gamma(\alpha)}\sum_{m=0}^{[n/2]}(-1)^m\frac{\Gamma(\alpha+n-m)(2x)^{n-2m}}{m!(n-2m)!} \tag{A7}$$

[see Ref. 18, p. 775, (22.3.4)], here $[n/2]$ is the largest integer $\leq n/2$,

$$\int_{-1}^{1}(1-x^2)^{\lambda}x^{\rho}dx=\begin{cases}B\left[\frac{\rho+1}{2},\lambda+1\right], & \rho=\text{even}\\ 0, & \rho=\text{odd}\end{cases} \tag{A8}$$

[see Ref. 19, formula (8.380.11)],

$$B(x,y)=\frac{\Gamma(x)\Gamma(y)}{\Gamma(x+y)} \tag{A9}$$

[see Ref. 19, formula (8.384.1)],

$$J_N=(-1)^{(N-|N|)/2}J_{|N|}, \quad N=\text{integer} \tag{A10}$$

(see Ref. 18).

Here j_l denotes the spherical Bessel function, Y_{lm} is the spherical harmonics,

$$\begin{bmatrix} j_1 & j_2 & j \\ m_1 & m_2 & m \end{bmatrix}$$

denotes $3j$ symbol, $P_l(x)$ is the Legendre function, $P_l^m(x)$ is the associated Legendre function of the first kind, $C_n^{\alpha}(x)$ is the ultraspherical (Gegenbauer) polynomial, $\Gamma(x)$ is the gamma function, and J_N stands for the Bessel function of the first kind.

APPENDIX B

The quantities $T_{l_1m_1,l_2m_2}^{(C,D)}$ and $i_{J,B}^{(A,l_sm_s)}$ given by (23a), (23b), and (24) are discussed here. First we are dealing with $T_{l_1m_1,l_2m_2}^{(C,D)}$. We have to evaluate integrals of the type

$$\tau_{l_1m_1,l_2m_2}^{C,D}=\int J_C(\beta\sin\theta)J_D(\beta\sin\theta)P_{l_1}^{|m_1|}(\cos\theta)\times P_{l_2}^{|m_2|}(\cos\theta)d(\cos\theta), \tag{B1a}$$

and so

$$T_{l_1m_1,l_2m_2}^{(C,D)}=K_{l_1,l_2}^{m_1,m_2}\tau_{l_1m_1,l_2m_2}^{C,D} \tag{B1b}$$

[see (23b) and (24)]. The product J_CJ_D is written in another form using (A10) and the new variable $t=\cos\theta$ as

$$J_{|C|}[\beta(1-t^2)^{1/2}]J_{|D|}[\beta(1-t^2)^{1/2}]=\sum_{r,s=0}^{\infty}\frac{(-1)^{r+s}(1-t^2)^{(|C|+|D|+2r+2s)/2}}{r!s!}\times\frac{\beta^{|C|+|D|+2r+2s}}{\Gamma(|C|+r+1)\Gamma(|D|+s+1)2^{r+s+[|C|+|D|]/2}} \tag{B2}$$

(see Ref. 20). Using (A5) and (A6) we can obtain

$$\tau_{l_1m_1,l_2,m_2}^{C,D}=(-1)^{(C+D-|C|-|D|)/2+|m_1|+|m_2|}(2|m_1|-1)!!(2|m_2|-1)!!\times\int_{-1}^{1}\sum_{r,s=0}^{\infty}(-1)^{r+s}(1-t^2)^{(|C|+|D|+|m_1|+|m_2|)/2+r+s}C_{l_1-|m_1|}^{|m_1|+(1/2)}(t)C_{l_2-|m_2|}^{|m_2|+(1/2)}(t)dt\times\beta^{|C|+|D|+2r+2s}/[r!s!\Gamma(|C|+r+1)\Gamma(|D|+s+1)2^{r+s+(|C|+|D|)/2}]. \tag{B3}$$

Taking into account (A7) we can conclude that (B3) contains only integrals of the type of (A8) and so (B3) can be integrated term by term using (A8).

The other quantity can be written as

$$i_{J,B}^{(A,l_sm_s)}=\sqrt{\pi}i^{l_s}K_{J,l_s}^{B,m_s}i^{A}\int_0^{\infty}\varepsilon_{J,B}^{A,l_sm_s}(\xi)e^{-\xi}f_{n\lambda}(\xi)j_{l_s}(\hat{p}\xi)\xi^{1-l}d\xi \tag{B4}$$

with the notation

$$\varepsilon_{J,B}^{A,l_sm_s}=\int P_J^{B}(\cos\vartheta)P_{l_s}^{|m_s|}(\cos\vartheta)J_A(b\xi\sin\vartheta)\sin\vartheta\,d\vartheta. \tag{B5}$$

At first we deal with $\varepsilon_{J,B}^{A,l_sm_s}$. We use (A5) and (A6) again and with the new variable $t=\sin\vartheta$

$$\varepsilon_{J,B}^{A,l_s m_s}=(-1)^{|B|+m_s}(2|m_s|-1)!!(2|B|-1)!!2\int_0^1 t^{|B|+m_s}C_{J-|B|}^{|B|+(1/2)}[(1-t^2)^{1/2}]C_{l_s-|m_s|}^{|m_s|+(1/2)}[(1-t^2)^{1/2}]$$
$$\times J_A(b\xi t)\frac{tdt}{(1-t^2)^{1/2}} \quad \text{if } J+l_s-|B|-|m_s|=\text{even}\ ,$$
$$\varepsilon_{J,B}^{A,l_s m_s}=0 \quad \text{if } J+l_s-|B|-|m_s|=\text{odd}\ . \tag{B6}$$

Using (A7) and the integral formula

$$\int_0^1 t^{\mu-(1/2)}(1-t^2)^{\lambda}J_\nu(b\xi t)(b\xi t)^{1/2}dt$$
$$=\frac{(b\xi)^{\nu+1/2}}{2^{\nu+1}\Gamma(\nu+1)}B(\lambda+1,\tfrac{1}{2}[\mu+\nu+1])\,{}_1F_2\left[\frac{\mu+\nu+1}{2};\nu+1,\frac{\mu+\nu+3}{2}+\lambda;-\frac{b^2\xi^2}{2}\right] \tag{B7}$$

(see Ref. 21), the integration over t can be performed. The remaining integration with respect to ξ can be carried out with the use of (A4) of paper I, as with the use of the definition of the ${}_1F_2(a;b,c;y)$ function, it can be written in power series form of $-(b^2\xi^2/2)$. The resulting formulae in both cases (T and i) are so cumbersome that we omit their publication. The formulae given above are suitable for constructing computer program to calculate LA-ICC in the $\hat{p}\gtrsim 1$ case.

APPENDIX C

The computation of the numerical values of T given by (30) is based on the following formulae:[18]

$$\int_0^z J_{2N}(t)dt=\int_0^z J_0(t)dt-2\sum_{k=0}^{N-1}J_{2k+1}(z)\ , \tag{C1}$$

$$\int_0^z J_0(t)dt=1-\int_z^\infty J_0(t)dt\ . \tag{C2}$$

Fortunately there is available an approximate solution[22] for

$$E=\int_z^\infty J_0(t)dt\ , \tag{C3}$$

which gives E for $z\geq 8$ ($\beta_0\geq 4$) with an error less than $2\cdot 10^{-9}$. So we can obtain $\int_0^z J_0(t)dt$ with the same error if we use (C2). In this way a program was constructed where a subroutine producing $J_N(x)$ with an error less than 10^{-10} was used. Thus with the help of (C1) we can obtain one term of the sum T with an error less than $2N\cdot 10^{-9}$ (if $\beta_0\geq 4$, $z\geq 8$).

In the $z<8$ ($\beta_0<4$) range we used the formula[23]

$$\int_0^z t^\mu J_\nu(t)dt=\frac{z^{\mu+\nu+1}\,{}_1F_2\left[\frac{1}{2}[\mu+\nu+1];\frac{1}{2}[\mu+\nu+3],\nu+1;-\frac{z^2}{4}\right]}{2^\nu(\mu+\nu+1)\Gamma(\nu+1)}\ , \tag{C4}$$

with $\mu=0$ and $\nu=2|N|$ in order to compute $I_2=\int_0^z J_{2|N|}(t)dt$. From the asymptotic form[18]

$$J_N(z)\sim\frac{1}{\sqrt{2\pi N}}\left[\frac{ez}{2N}\right]^N,\quad N\to\infty \tag{C5}$$

it is clear that above a critical value of N the integral I_2 rapidly goes to zero. This rapid decrease of I_2 versus increasing N was observed in each case calculated. For each point of our T curve (Fig. 1) we also obtained a number N_{MAX} characterizing the ICP. Those N photonic processes played a dominant role in the LA-ICP which had $N<N_{\mathrm{MAX}}$. Thus to an arbitrarily small number ε belongs an N_{MAX} so that the contribution of N photonic processes $N>N_{\mathrm{MAX}}$ to the total LA-ICC is less than ε. The N_{MAX} values are important as with their help we can check the validity of condition (32b) at a given laser intensity.

At first sight it seems that besides the O_4,O_5 shell ICC's, the P_1,P_2 ICC's of the ^{235m}U can be computed in the SMA as they fulfill (32a). However, the detailed calculation (i.e., the N_{MAX} values) show that for these shells condition (32b) is not fulfilled and the SMA cannot be used at moderate laser intensities.

APPENDIX D

There have been contradictory claims in the literature on the methods, about how can handle problems in intense radiation fields. On the one hand it is stated that the momentum translation method (MTA) is a very compact useful one to describe intense field phenomena.[5,24]

On the other hand,[25,26] it is claimed that the MTA wave function represents a noninteracting wave function.[26] As our initial and final states given in Sec. II are in the radiation gauge, and the initial state has the form which is called MTA wave function,[24] we want to make clear the situation here. The MTA is a gauge-specific technique appropriate to Coulomb-gauge;[24] therefore, we formulate our problem at first in the Göppert-Mayer (xE) gauge in order to avoid the above problem and then we transform the result into the radiation (pA) gauge.

The electric field strength of a circularly polarized wave can be written in the dipole approximation as

$$\mathbf{E}=E_0[\hat{\mathbf{e}}_1\sin(\omega t)+\hat{\mathbf{e}}_2\cos(\omega t)] , \tag{D1}$$

which can be deduced from a vector potential of the form (3) with $a=cE_0/\omega$.

The Hamiltonian corresponding to (2) in the electric field (xE) gauge is

$$H_0=p^2/2m+V(R)-e\mathbf{E}\mathbf{R} . \tag{D2}$$

Here $V(R)$ denotes the Coulomb potential. As was mentioned in the Introduction, there has been no exact solution of this problem, yet.

We take two different approximate solutions of (D2) as in and out states. The initial state can be an unperturbed one of hydrogenic-type[7] [see formulae (4c) and (4d) in Sec. II], as we are searching for internal conversion of inner shells, which are "near" to the nucleus, where the shielding of the external radiation field is strong.[27] The final state is described by the exact time dependent solution of (D2) without the Coulomb term. We hope this is a good approximation, as in the final state the atomic electrons shield the Coulomb field of the nucleus and thus for the outgoing electron the laser-electron interaction can be considered dominant. Thus the exact solution of the time dependent Schrödinger equation

$$ih\frac{\partial}{\partial t}v_f'=(p^2/2m-e\mathbf{E}\mathbf{R})v_f' \tag{D3}$$

has the form

$$v_f'=e^{i(eE_0/\hbar\omega)\mathbf{R}[-\hat{\mathbf{e}}_1\cos(\omega t)+\hat{\mathbf{e}}_2\sin(\omega t)]}u(\mathbf{R},t) \tag{D4}$$

with

$$u(\mathbf{R},t)=V^{-1/2}e^{i(\mathbf{p}\mathbf{R}-\hat{E}t)/\hbar}e^{i(eE_0/m\hbar\omega^2)\mathbf{p}[\hat{\mathbf{e}}_1\sin(\omega t)+\hat{\mathbf{e}}_2\cos(\omega t)]} , \tag{D5}$$

and $\hat{E}=E+e^2E_0^2/2m\omega^2$, which will be approximated as $\hat{E}=E$ further on. The prime over v_f denotes that the state is given in the xE gauge. One can recognize that formally (D4) can be written as

$$v_f'=e^{-ie\mathbf{A}\mathbf{R}/\hbar c}u(R,t) . \tag{D6}$$

where $\mathbf{A}$ is the vector potential in the Coulomb gauge.

Now we transform the initial and final wave functions into the radiation gauge.[26] The results are

$$v_i=e^{ie\mathbf{A}\mathbf{R}/\hbar c}\phi_{n\lambda\mu}(\mathbf{R})e^{i(Kt/\hbar)} \tag{D7}$$

and

$$v_f=u(\mathbf{R},t) . \tag{D8}$$

Thus (D7) is the noninteracting state in the radiation gauge[26] though it looks like an MTA wave function.[24] Using (3)′ (D5), the definition of the frame of reference $(\hat{\mathbf{e}}_1,\hat{\mathbf{e}}_2,\hat{\mathbf{e}}_3)$; furthermore, the Jacobi-Anger formulae[28]

$$e^{iz\sin\alpha}=\sum_{N=-\infty}^{\infty}e^{iN\alpha}J_N(z) , \tag{D9}$$

$$e^{iz\cos\alpha}=\sum_{L=-\infty}^{\infty}i^Le^{iL\alpha}J_L(z) , \tag{D10}$$

the identities

$$e\mathbf{A}\mathbf{R}/\hbar c=(eaR/\hbar c)\sin\vartheta\cos(\omega t+\varphi)$$
$$=b\xi\sin\vartheta\cos(\omega t+\varphi) , \tag{D11}$$

$$(eE_0/m\hbar\omega^2)\mathbf{p}[\hat{\mathbf{e}}_1\sin(\omega t)+\hat{\mathbf{e}}_2\cos(\omega t)]$$
$$=\beta\sin\theta\sin(\omega t+\chi) , \tag{D12}$$

with the notation $b=eaa_0/\hbar c$, $R=\xi a_0$, $a=cE_0/\omega$, $\beta=eap/mc\hbar\omega$, we obtain the in and out states in the radiation gauge in the form given in Sec. II.

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[10]The relations between quantities B, SF_l of paper I and $B(j)$, $SF_{l\lambda}$ of this paper are the following:

$$B(j)=\frac{(2j+1)l[(2l-1)!!]^2 2}{\pi(l+1)}B\ ,$$

$$SF_{l0}=\begin{pmatrix} l & 0 & l \\ 0 & 0 & 0 \end{pmatrix}^2 SF_l \Big/ \left[\frac{2l[(2l-1)!!]^2}{\pi(l+1)(2l+1)}\right]$$

$$\text{and}\quad \begin{pmatrix} l & 0 & l \\ 0 & 0 & 0 \end{pmatrix}^2 = \frac{1}{2l+1}\ .$$

Unfortunately, some mistakes occurred in paper I. These are the following: a factor of π is missing in (24) and a factor of 2 ought to stand in formulae (30), (31), (34a), and (36) instead of 4 and 8, respectively, causing that all the results in Tables II and III in paper I have to be divided by 2.

[11]For $y \ll 1$ the hypergeometric function ${}_2F_1(a,b;c;y)$ is approximated by unity. Furthermore, the $1+b^2 \sim 1$ and $1-y \sim 1$ approximations are used.

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