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Basis for relativistic models of nuclear structure in field-theoretic models of the strong interaction

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We make use of the auxiliary (bilocal) field technique in an attempt to understand the relation between modern relativistic models of nuclear structure and quantum chromodynamics. The advantage of the model introduced here is that we are able to discuss and interconnect chiral symmetry breaking, (nontopological) models of hadron structure, and relativistic quantum field theories of nuclear structure. The proposed relation of the auxiliary fields of this model to the fields which play a central role in Dirac phenomenology is an important aspect of our work. It can be seen that the large scalar fields used in Dirac phenomenology, which serve to reduce the mass of the nucleon (or the quarks), are a reflection of a partial restoration of chiral symmetry at finite baryon number, that is, within nuclear matter.

I. INTRODUCTION

In recent years we have seen parallel advances in our understanding of nuclear physics and of quantum chromodynamics (QCD). However, while QCD is considered to be the basic theory of strong interactions, the relation of current models of nuclear structure to QCD is unclear. In particular, the use of the Dirac equation for describing nucleon dynamics in nuclei has become quite popular.[1,2] At the center of the success of the relativistic theories are phenomenological (Lorentz) scalar and vector fields which are coupled to the nucleons. *The nature of these fields is not understood,* and one goal of this work is to put forth a specific interpretation of these fields.

If we include fields with the quantum numbers of the pion and the rho meson, as well as the scalar and vector fields mentioned above, we can create a phenomenological model which is highly successful in describing nuclear properties.[1,2] As we have discussed in detail in earlier works, the various fields $(\sigma,\pi,\rho,\omega,\ldots)$ can be considered to be coupled to the *quarks* in a nucleon.[3–5] From their coupling to the quarks we can infer the coupling of these fields to the *nucleon,* using the techniques developed in Refs. 3–5. *We have stated elsewhere that these fields should not be identified with the physical mesons which have the same quantum numbers.*[1] (For example, an analysis of the structure of the ρ and ω mesons leads to the conclusion that these physical mesons are somewhat larger than the nucleon.[6]) As noted above, it is our goal in this work to provide some further understanding of the nature of the boson fields which play an essential role in modern theories of nuclear structure. At the same time we will provide some basis for our theory of hadron structure, which we have called "covariant soliton dynamics."[3–6]

It is clear that analytic work in strong coupling physics is very limited. Therefore, we are interested in creating a *model* for understanding the relation of QCD to nuclear physics. The construction of this model requires a series of assumptions. We will outline the series of steps which are required in our analysis. We first observe that because of the non-Abelian nature of QCD, that is, the presence of cubic and quartic gluon interactions, the direct application of functional methods in QCD is limited. (These methods are, of course, quite useful for developing Feynman rules after the introduction of the Faddeev-Popov gauge-fixing procedure.)

We wish to use functional techniques in our analysis. Therefore our first major *assumption* is that at an appropriate length scale we can replace QCD with an *effective theory* which is characterized by having a dynamical gluon mass and a "frozen" coupling constant. That is to say, the running coupling constant of the effective theory takes on a large, but more or less constant value, in the momentum range in which the effective theory is applicable. The use of a coupling constant of this type is quite common in studies of chiral symmetry breaking in gauge theories and requires the introduction of a new mass scale which characterizes the behavior of the running coupling constant in the infrared region.[7]

The fact that the gluon has a dynamical mass at low momentum transfer has been demonstrated in lattice gauge simulations of QCD (Ref. 8) (in a specific gauge) and has been discussed by several authors, using various theoretical techniques.[9,10] The notion of a "frozen" coupling constant has also been discussed by Cornwall and collaborators.[11] (In Cornwall's work the use of a frozen coupling constant is related to the generation of a dynamical mass for the gluon.)

The most attractive scheme for introducing a gluon mass term is through the Schwinger mechanism.[12] This mechanism allows for a mass term for the gauge field which does not break the gauge symmetry. For example, in the Landau gauge we can write

$$D^{ab}_{\mu\nu}(q^2)=-(g_{\mu\nu}-q_\mu q_\nu/q^2)\{q^2[1-\Pi(q^2)]\}^{-1}\delta_{ab} , \tag{1.1}$$

for the gluon propagator and assume that as $q^2\to 0$, $\Pi(q^2)\to m_G^2/q^2$. [Here $m_G^2=m_G^2(q^2=0)$ is the value of the dynamical (running) gluon mass at $q^2=0$.]

We believe that the replacement of QCD by an *effective* theory is meaningful at length scales greater than about 0.1 fm. For length scales shorter than this, the various dynamically generated masses, as well as the running coupling constant, will be small.[13] [It may be worth noting at this point, that we will shortly consider still another length-scale restriction for the purposes of discussing the formation of hadrons as nontopological solitons. Hadrons have a characteristic size of about 1 fm or $(200\ \text{MeV}/c)^{-1}$; our discussion of hadron structure will involve the introduction of a second effective Lagrangian appropriate to this larger length scale.]

We analyze our *effective* Abelian theory of quarks and gluons using the method of auxiliary bilocal fields,[14] after performing a Fierz rearrangement of the effective action. The auxiliary fields include some which have the quantum numbers of those boson fields which are central to modern nuclear physics. We include some comments concerning the relation of these fields to the boson-exchange model of nuclear forces[15] and to modern relativistic nuclear physics.[1,2]

We continue our discussion of length scales to clarify some of the ideas introduced above. Consider a bilocal scalar field, $\sigma(x,y)$, and introduce relative and center-of-momentum coordinates,[16]

$$\rho_\mu=x_\mu-y_\mu , \tag{1.2}$$

$$X_\mu=(x_\mu+y_\mu)/2 , \tag{1.3}$$

so that

$$\sigma(x,y)=\int\frac{d^4q}{(2\pi)^4}\int\frac{d^4P}{(2\pi)^4}e^{iq\cdot\rho}e^{iP\cdot X}\sigma_P(q^2) . \tag{1.4}$$

In vacuum, we can write

$$\sigma_P(q^2)=(2\pi)^4\delta^4(P)\sigma_{\text{vac}}(q^2) . \tag{1.5}$$

In Sec. II, $\sigma_{\text{vac}}(q^2)$ will be related to the dynamical (running) quark mass. The fact that $\sigma_{\text{vac}}(q^2)\neq 0$ is a manifestation of chiral symmetry breaking, which is a phenomenon characterizing the QCD ground state. Note that in vacuum we have $\sigma(x,y)=\sigma(x-y)$, where

$$\sigma(x-y)=\int\frac{d^4q}{(2\pi)^4}e^{iq\cdot(x-y)}\sigma_{\text{vac}}(q^2) . \tag{1.6}$$

A term in an effective action of the form

$$\int d^4x\int d^4y\,\bar{q}(x)\{\delta(x-y)[i\gamma^\mu\partial_\mu-m^{\text{cur}}] -g\sigma(x-y)\}q(y) , \tag{1.7}$$

will then give rise to a quark propagator of the form

$$S_F(q)=[\not{q}-m^{\text{cur}}-\Sigma(q^2)+i\epsilon]^{-1} , \tag{1.8}$$

upon the use of Eq. (1.6). In Eq. (1.8) we have put

$$\Sigma(q^2)=g\sigma_{\text{vac}}(q^2) . \tag{1.9}$$

[Note that for $-q^2$ greater than about 3–4 $(\text{GeV})^2$ we expect that $\sigma(q^2)$ will be quite small.[13]]

It is usually desirable to carry out the analysis in the Landau gauge where in lowest-order calculations one need not worry about terms in $\Sigma(q^2)$ proportional to $\not{q}$. (Such terms are discussed in Ref. 16.) In general, for a system which exhibits translational invariance, we may write the quark self-energy as

$$\Sigma(x-y)=\int\frac{d^4q}{(2\pi)^4}e^{iq\cdot(x-y)}[\not{q}A(q^2)+B(q^2)] \tag{1.10}$$

$$=-i\gamma^\mu\partial_\mu A(x-y)+B(x-y) , \tag{1.11}$$

where

$$A(x-y)=\int\frac{d^4q}{(2\pi)^4}e^{iq\cdot(x-y)}A(q^2) , \tag{1.12}$$

and

$$B(x-y)=\int\frac{d^4q}{(2\pi)^4}e^{iq\cdot(x-y)}B(q^2) . \tag{1.13}$$

As we will see, in the presence of nuclear matter, both the scalar and vector terms in the self-energy will be of major importance since these terms are ultimately related to the scalar and vector potentials used in Dirac phenomenology.

Now for the purpose of studying hadron structure, or nuclear structure, we introduce $\chi(P,q)$, a field which describes the deviation of $\sigma_P(q)$ from its vacuum value due to the presence of hadronic matter

$$\sigma_P(q)=(2\pi)^4\delta^4(P)\sigma_{\text{vac}}(q^2)+\chi(P,q) , \tag{1.14}$$

and work at length scales for which we can neglect the q variation. Then

$$\sigma_P(q)\to(2\pi)^4\delta^4(P)\sigma_{\text{vac}}(0)+\chi(P) , \tag{1.15}$$

and

$$\sigma(x,y)\to\delta^4(x-y)[\sigma_{\text{vac}}(0)+\chi(X)] . \tag{1.16}$$

Using the last approximation, a bilocal coupling term is approximated as follows,

$$g\int d^4x\int d^4y\,\bar{q}(x)\sigma(x,y)q(y)$$

$$\simeq\int\bar{q}(X)[g\sigma_{\text{vac}}+g\chi(X)]q(X)d^4X \tag{1.17}$$

$$=\int\bar{q}(X)[m_q^{\text{dyn}}+g\chi(X)]q(X)d^4X . \tag{1.18}$$

This last form is of particular interest to us in that we have made extensive studies of the effective Lagrangian[4–6,17,18]

$$\mathcal{L}(x)=\tfrac{1}{2}\partial_\mu\chi(x)\partial^\mu\chi(x)-\frac{m_\chi^2}{2}\chi^2(x) +\bar{q}(x)[i\gamma^\mu\partial_\mu-m_q^{\text{dyn}}-q_\chi\chi(x)]q(x) , \tag{1.19}$$

and have shown that a Lagrangian of this form can give a good account of the structure of many hadronic states. (For the study of the charmonium and b-quarkonium systems we have to also introduce large *current* quark masses which have their origin outside of QCD,[17,18] and which do not "run" at the mass scales considered when studying QCD.)

To summarize the above comments we can say that we can identify three length scales in QCD. For momenta greater than about 2 GeV the running coupling constant is small and dynamically generated masses are small.[13] For smaller momenta the coupling constant is larger and dynamical mass generation for quarks and gluons is quite important. If we consider still smaller momenta, we may, in first approximation, consider the dynamical masses to be constant. It is in that regime where simple models of hadron structure can be constructed and where we can use the boson-exchange model of nuclear forces to create a relativistic many-body theory of nuclear structure.[1] (In the study of hadron structure, we can also maintain reference to the momentum dependence of the dynamical quark mass; however, such calculations have not been completed as yet.)

The plan of our work is as follows. In Sec. II we discuss the action for an effective theory. We carry out a Fierz rearrangement and introduce a set of auxiliary fields. We comment upon recent work which indicates that, beyond some critical coupling, models of the type we have introduced will be characterized by chiral symmetry breaking. (In that case the scalar-isoscalar field obtains a vacuum expectation value while the pseudoscalar-isovector field becomes the Goldstone boson.) In Secs. III and IV we indicate how the model developed in Sec. II may be used to develop a theory of hadron and nuclear structure. Section V contains some concluding remarks. Since functional techniques are somewhat unfamiliar in nuclear physics we review some aspects of the formalism in Appendix A.

II. EFFECTIVE ACTION

Here we review the basic idea of the auxiliary field method. We start with the simplest version which only requires the introduction of a *local* auxiliary field. Consider the Lagrangian

$$\mathcal{L}(x)=\bar{q}(x)i\gamma^{\mu}\partial_{\mu}q(x)+\frac{g^2}{2}[\bar{q}(x)q(x)]^2 . \tag{2.1}$$

The vacuum-to-vacuum amplitude is, with $\hbar=1$,

$$e^{iW}=N\int[d\bar{q}][dq]\,e^{i\int d^4x\,\mathcal{L}(x)} , \tag{2.2}$$

where N is a constant and the $q(x)$ are Grassmann variables. We can introduce an integral over a field $\sigma(x)$ without changing the dynamics

$$e^{iW}=N'\int[d\sigma][d\bar{q}][dq]\times\exp\left[i\int d^4x\{\mathcal{L}(x)-\tfrac{1}{2}[\sigma(x)+g\bar{q}(x)q(x)]^2\}\right] \tag{2.3}$$

$$=N'\int[d\sigma][d\bar{q}][dq]e^{i\int d^4x\,\tilde{\mathcal{L}}(x)} , \tag{2.4}$$

where

$$\tilde{\mathcal{L}}(x)=-\tfrac{1}{2}\sigma^2(x)+\bar{q}(x)[i\gamma^{\mu}\partial_{\mu}-g\sigma(x)]q(x) . \tag{2.5}$$

Thus, the four-fermion interaction is replaced by interaction with the field $\sigma(x)$. This field plays the role of the generalized fermion self-energy. The next step is the construction of the effective potential for the theory in terms of the field $\sigma(x)$. In that manner one can study the dynamics of symmetry breaking.

In quantum electrodynamics (QED) one can obtain four-fermion interactions. These interactions arise after "integrating out" the gauge field; however, these four-fermion interactions are not local. The replacement of these interactions by generalized self-energies requires the introduction of *bilocal* fields; however, the basic idea of the method is essentially that described above. The necessity for the use of bilocal fields and their relation to the self-energy of the fermions will become clear as we proceed.

Consider the action in the case of QCD. We have

$$e^{iW}=\mathcal{N}\int[dA_{\mu}][dq][d\bar{q}]\,e^{iS_{\mathrm{QCD}}} , \tag{2.6}$$

where

$$S_{\mathrm{QCD}}=\int d^4x\{-\tfrac{1}{4}G^a_{\mu\nu}(x)G^{\mu\nu}_a(x)+\bar{q}(x)[i\not{D}-m_q^{\mathrm{cur}}]q(x)\} . \tag{2.7}$$

This expression must be supplemented by various gauge fixing terms, and ghost fields, if one chooses a covariant gauge.[19] Here m_q^{cur} is a quark mass matrix, which we will not write explicitly from this point on. As discussed in the Introduction, we wish to introduce an effective theory, appropriate at length scales where dynamical mass generation is important. We define

$$S^{\mathrm{eff}}_{\mathrm{QCD}}=\int d^4x\int d^4y\{\tfrac{1}{2}A^{\mu}_a(x)[(q_{\mu\nu}\Box-\partial_{\mu}\partial_{\nu})\delta^4(x-y)+\Pi_{\mu\nu}(x-y)]A^{\nu}_a(x)+\delta^4(x-y)[\bar{q}(x)i\gamma^{\mu}\partial_{\mu}q(x)+j^{\mu}_a(x)A^a_{\mu}(x)]\} . \tag{2.8}$$

Here $\Pi_{\mu\nu}(x-y)$ is a gluon self-energy term. This term is assumed to arise from the coupling of the gluon to the gluon condensate.[9,10] [We shall assume that the main effects of the cubic and quartic terms in the QCD Lagrangian, in the momentum regime under consideration, is to provide the gluon with a (running) dynamical mass—see Eq. (2.13).] Further, $j^{\mu}_a(x)$ is the quark current,

$$j^{\mu}_a(x)=g\bar{q}(x)\gamma^{\mu}\frac{\lambda^a}{2}q(x)=g\bar{q}_i(x)\left[\gamma^{\mu}\frac{\lambda^a}{2}\delta_{ij}\right]q_j(x) . \tag{2.9}$$

In Eq. (2.9) we have explicitly written the flavor indices, i and j, in anticipation of performing a Fierz rearrangement. We can define (in the Landau gauge)

$$D^{ab}_{\mu\nu}(x-y)^{-1}=[(g_{\mu\nu}\Box-\partial_\mu\partial_\nu)\delta^4(x-y)+\Pi_{\mu\nu}(x-y)]\delta_{ab}$$
$$\equiv\{(g_{\mu\nu}\Box-\partial_\mu\partial_\nu)[\delta(x-y)-\Pi_0(x-y)]\}\delta_{ab}\ , \tag{2.10}$$

so that

$$S^{\rm eff}_{\rm QCD}=\int d^4x\int d^4y[\tfrac{1}{2}A^\mu_a(x)D^{ab}_{\mu\nu}(x-y)^{-1}A^\nu_b(y)]+\int d^4x[\bar q(x)i\gamma^\mu\partial_\mu q(x)+j^\mu_a(x)A^a_\mu(x)]\ . \tag{2.11}$$

We may now integrate out the gluon field, after making a specific gauge choice, to obtain

$$S^{\rm eff}_{\rm QCD}=-\tfrac{1}{2}\int d^4x\int d^4y\, j^\mu_a(x)D^{ab}_{\mu\nu}(x-y)j^\nu_b(y)+\int d^4x\,\bar q(x)i\gamma^\mu\partial_\mu q(x)\ . \tag{2.12}$$

For simplicity, we continue to work in the Landau gauge. (The relations necessary for performing a Fierz rearrangement in a general covariant gauge are given in Appendix B.) Now consider the structure

$$g^2\left[\bar q_i(x)\left[\gamma^\mu\frac{\lambda^a}{2}\delta_{ij}\right]q_j(x)\right]\times D_{\mu\nu}(x-y)\left[\bar q_k(y)\left[\gamma^\nu\frac{\lambda^a}{2}\delta_{kl}\right]q_l(y)\right]\ ,$$

where

$$D_{\mu\nu}(x-y)=-\int\frac{d^4q}{(2\pi)^4}\frac{(g_{\mu\nu}-q_\mu q_\nu/q^2)}{q^2[1-\Pi(q^2)]}e^{iq\cdot(x-y)}\ . \tag{2.13}$$

We also define, for later use,

$$D_0(x-y)=\int\frac{d^4q}{(2\pi)^4}\frac{e^{iq\cdot(x-y)}}{q^2[1-\Pi(q^2)]}\ . \tag{2.14}$$

We need, therefore,

$$[(\gamma^\mu)_{ab}(\gamma_\mu)_{cd}-(\not q)_{ab}(\not q)_{cd}/q^2]$$
$$=\tfrac{3}{4}[(\mathbb{1})_{ad}(\mathbb{1})_{cb}+(i\gamma_5)_{ad}(i\gamma_5)_{cb}]-\tfrac{1}{4}[(\gamma^\mu)_{ad}(\gamma^\nu)_{cb}+(\gamma_5\gamma^\mu)_{ad}(\gamma_5\gamma^\nu)_{cb}]\left[g_{\mu\nu}+\frac{2q_\mu q_\nu}{q^2}\right]$$
$$-\tfrac{1}{8}[(\sigma^{\mu\nu})_{ad}(\sigma^\rho_\mu)_{cd}](g_{\nu\rho}-4q_\nu q_\rho/q^2)\ . \tag{2.15}$$

We restrict ourselves to up and down quarks and write

$$\delta_{ij}\delta_{kl}=a_0\delta_{il}\delta_{kj}+a_1(\tau)_{il}\cdot(\tau)_{kj}\ , \tag{2.16}$$

with $a_0=\frac{1}{2}$ and $a_1=\frac{1}{2}$.

Finally, we note that

$$\left[\frac{\lambda^a}{2}\right]_{\alpha\beta}\left[\frac{\lambda^a}{2}\right]_{\gamma\delta}=\tfrac{4}{9}\delta_{\alpha\delta}\delta_{\gamma\beta}-\frac{1}{3}\left[\frac{\lambda^a}{2}\right]_{\alpha\delta}\left[\frac{\lambda^a}{2}\right]_{\gamma\beta}\ , \tag{2.17}$$

where we will keep only the color-singlet terms. (The motivation for that step lies in the fact that in nuclear physics, the quarks are clustered into color-singlet nucleons and exchange of *colored* order parameters will not be important for a mean-field theory.) Once we have made the approximation of keeping only the first term on the right-hand side of Eq. (2.17), we see that the color matrices, λ^a, play no role in the analysis. [The factor of $(\frac{4}{9})$ can be absorbed into the coupling constant.] If we drop the color matrices we see that we have an Abelian theory with eight (massive) gluons which are uncoupled from one another. (In this approximation the effective theory has a local gauge symmetry.)

Let us now concentrate on the scalar-isoscalar term in the effective action. We have, with $a_{00}=(\frac{4}{9})(\frac{3}{4})a_0=\frac{1}{6}$,

$$S_{\rm eff}=\int d^4x\int d^4y[\bar q(x)i\gamma^\mu\partial_\mu q(x)\delta^4(x-y)-\tfrac{1}{2}g^2a_{00}\bar q(x)q(y)\times D_0(x-y)\bar q(y)q(x)]\ . \tag{2.18}$$

Now, without changing the dynamics we can include a constant of the form

$$\int[d\sigma]\exp\left[i\int d^4x\int d^4y\{\tfrac{1}{2}[\sigma(x,y)-g(a_{00})^{1/2}D_0(x,y)\bar q(x)q(y)]D_0^{-1}(x,y)[\sigma(x,y)-g(a_{00})^{1/2}D_0(x,y)\bar q(y)q(x)]\}\right]\ , \tag{2.19}$$

in the vacuum-to-vacuum amplitude, $\exp(iW)$. (Other auxiliary fields may be included for each of the terms which arise upon Fierz rearrangement of the action.) At this point, we have for the vacuum to vacuum amplitude,

$$e^{iW}=\mathcal{N}'\int[d\sigma][dq][d\bar q]\exp\left[i\int d^4x\int d^4y\{\tfrac{1}{2}\sigma(x,y)D_0^{-1}(x,y)\sigma(x,y)+\bar q(x)[\delta^4(x-y)i\gamma^\mu\partial_\mu-g_\sigma\sigma(x,y)]q(y)\}\right]\ , \tag{2.20}$$

where $g_\sigma=g(a_{00})^{1/2}$. Inclusion of a pionlike order parameter would yield the more general result,

$$e^{iW}=\mathcal{N}'\int[d\sigma][d\pi][dq][d\bar q]\exp\left[i\int d^4x\int d^4y(\tfrac{1}{2}\sigma(x,y)D_0^{-1}(x,y)\sigma(x,y)+\tfrac{1}{2}\pi(x,y)D_0^{-1}(x,y)\cdot\pi(x,y)+\bar q(x)\{\delta^4(x-y)i\gamma^\mu\partial_\mu-g_\sigma[\sigma(x,y)+i\pi(x,y)\cdot\tau\gamma_5]\}q(y))\right]\ . \tag{2.21}$$

Equation (2.21) exhibits a manifest chiral symmetry. This is essentially the action studied recently by Morozumi and So in their analysis of strong-coupling QED.[16] (A correspondence to the work of Ref. 16 may be made if we drop the isospin label for the π field and the dynamical gluon mass.) In the Landau gauge, these authors find a critical value, λ_c, for the coupling constant, $\lambda=3e^2/4$:

$$\lambda_c=3e_c^2/4=\pi^2 \text{ or } e_c^2/(4\pi)=\pi/3 .$$

[We can compare our action to that of Ref. 16 if we were to drop the gluon mass term. We would have $\lambda_c^{\rm QCD}=g_c^2/6=\pi^2$, or $g_c^2/(4\pi)=3\pi/2$ for the critical value of the coupling constant of the effective theory. One should be cautious, however, in that chiral symmetry breaking is seen in lattice simulations at significantly smaller values of $e^2/(4\pi)$ in the case of strong-coupling QED.[20]]

The analysis of Morozumi and So also requires that the critical coupling constant be an ultraviolet fixed point.[21] The assumption avoids the untenable result that the dynamically generated mass is proportional to the cutoff used to analyze an approximate Schwinger-Dyson equation. [That equation is used to determine the self-energy, $\Sigma(q^2)$.[22]] We will not discuss this matter here and refer the reader to the work of Morozumi and So[16] for an analysis of the effective potential for the QED Lagrangian expressed in terms of bilocal fields.

It is also worth commenting upon the limiting case, where we take the mass of the exchanged gluon to be quite large. Let us write

$$D_0(x-y)\cong-\delta^4(x-y)/m_G^2 . \tag{2.22}$$

Then Eq. (2.18) becomes

$$S^{\rm eff}=\int d^4x \left[\bar{q}(x)i\gamma^\mu\partial_\mu q(x) + \frac{1}{2}\frac{g^2a_{00}}{m_G^2}\{[\bar{q}(x)q(x)]^2+[\bar{q}i\gamma_5\tau q(x)]^2\}\right] , \tag{2.23}$$

if we also include the pionlike order parameter, as in Eq. (2.21). Note that *in the large-mass limit we obtain the Lagrangian of the Nambu-Jona-Lasinio model*.[23] This model is known to give a quite good account of the pattern of chiral symmetry breaking and has been extensively investigated. It may be of interest to study this model using Eq. (2.13) for $D_0(x-y)$ rather than the approximation of Eq. (2.22).

Finally, we note that Weber and collaborators[24] have used the Fierz rearrangement technique to identify various fields which play an important role in the nucleon-nucleon interaction. These authors use a quark-interchange model which has some basis in QCD.

III. STRUCTURE OF HADRONS AND OF NUCLEI

An essential point of our analysis is the identification of several length scales for our discussion of QCD. The first length scale, $(2-3 \text{ GeV})^{-1}$, is that boundary which separates perturbative QCD from the region where the physics is dominated by dynamical mass generation for quarks (chiral symmetry breaking) and for gluons. To a first approximation, one can begin to see Bjorken scaling for momenta greater than about 2–3 GeV. Therefore the up and down quarks can be considered massless at that momentum scale. We have suggested that there is an effective theory relevant to the mass scale below 2–3 GeV. In addition, we can consider the physics of hadron formation. The relevant mass scale here is smaller. Thus we can introduce order parameters which describe the local condition of the vacuum with respect to the degree of symmetry breaking. More precisely, we assume, as in the Introduction, that we can write

$$\sigma_P(q)=\delta^4(P)(2\pi)^4\sigma_{\rm vac}(q^2)+\chi(P,q) , \tag{3.1}$$

where $\chi(P,q)$ is a deviation field excited by the presence of quarks. A quite useful assumption is that the scale of variation with respect to P can be separated from the variation with respect to q. To the extent that we do not investigate small length scales, we can write

$$\sigma_P(q)\simeq\delta^4(P)(2\pi)^4\sigma_{\rm vac}(0)+\chi(P) , \tag{3.2}$$

which will lead to a model of the kind we have used to study hadron structure.[4,5] Similar approximations can be made for the bilocal fields with the quantum numbers of the ρ, ω, and π mesons.

We note that it is necessary to calculate the effective potential for the auxiliary fields. [Otherwise, each auxiliary field would have the same mass—see Eqs. (2.21) and (2.22), for example.] In particular, a study of chiral symmetry breaking[7] is essential to obtain the correct dynamics for the sigma and pionlike fields. After a calculation of the effective potential, one can, in principle, obtain a Lagrangian of the form

$$\mathcal{L}(x)=\bar{q}(x)\{i\gamma^\mu\partial_\mu-g_\sigma[\sigma(x)+i\boldsymbol{\pi}(x)\cdot\boldsymbol{\tau}\gamma_5]-g_\omega\gamma^\mu\omega_\mu(x)-g_\rho\gamma^\mu\boldsymbol{\rho}_\mu(x)\cdot\boldsymbol{\tau}\}q(x)+\tfrac{1}{2}\partial_\mu\sigma(x)\partial^\mu\sigma(x)+\tfrac{1}{2}\partial_\mu\boldsymbol{\pi}(x)\cdot\partial^\mu\boldsymbol{\pi}(x) -\lambda[\sigma^2(x)+\boldsymbol{\pi}^2(x)-\sigma_{\rm vac}^2]^2-\tfrac{1}{4}\boldsymbol{\rho}^{\mu\nu}(x)\cdot\boldsymbol{\rho}_{\mu\nu}(x)+m_\rho^2\boldsymbol{\rho}_\mu^2(x)-\tfrac{1}{4}\omega^{\mu\nu}(x)\omega_{\mu\nu}(x)+m_\omega^2\omega_\mu^2(x) . \tag{3.3}$$

This Lagrangian contains the various effective fields needed to understand the phenomena of low-energy nuclear physics. (For many applications it is preferable to use the Weinberg transformation to replace the pseudoscalar coupling of the pion by pseudovector coupling.) It is also useful to introduce the field $\sigma'(x)=\sigma(x)-\sigma_{\rm vac}$, as well as a mass term for the pion. Once the coupling of the various bosonic fields ($\sigma,\pi,\rho,\omega,\ldots$) to the quarks is specified, we can calculate the coupling of these fields to the nucleon. For example, we had[3,4]

$$G_{\sigma NN}=g_\sigma F_S(0) \tag{3.4}$$

where $G_{\sigma NN}$ is the coupling constant describing the coupling of the scalar-isoscalar field to the *nucleon* and $F_S(0)$ is the scalar form factor of the *nucleon*, evaluated at $q^2=0$. Similar relations may be obtained which relate boson-*quark* coupling constants to boson-*nucleon* coupling constants in the case of the ρ, ω, and π fields.[3,4]

IV. RELATIVISTIC NUCLEAR PHYSICS AND DIRAC PHENOMENOLOGY

If one uses the Dirac equation to describe the interaction of a nucleon with a nucleus one finds that the optical potential gives a good description of the data if it contains strong scalar and vector fields.[1,2,25] The significance of these fields is best understood at the quark level. For a large nucleus the *ambient* mean fields seen by a quark can be taken to be constants and for an even-even nucleus ($N=Z$), we need only consider scalar-isoscalar and vector-isoscalar fields, if we use the Hartree approximation. We now need to refer explicitly to the bilocal vector field

$$\omega^\mu(x,y)=\int \frac{d^4q}{(2\pi)^4}\int \frac{d^4P}{(2\pi)^4}e^{iq\cdot(x-y)}e^{iP\cdot[(x+y)/2]}\omega_P^\mu(q^2)\ , \tag{4.1}$$

and consider nuclear matter, for simplicity.

We can take $\omega_{\rm vac}^\mu(q^2)=0$ and write

$$\omega_P^\mu(q^2)=(2\pi)^4\delta^4(P)\omega_{\rm NM}\delta_{\mu,0}\ , \tag{4.2}$$

where $\omega_{\rm NM}$ is the ambient field in *nuclear matter*. Similarly we can put

$$\sigma_P(q^2)=(2\pi)^4\delta^4(P)(\sigma_{\rm vac}+\sigma_{\rm NM})\ . \tag{4.3}$$

Thus

$$\sigma(x,y)\simeq\delta^4(x-y)(\sigma_{\rm vac}+\sigma_{\rm NM})\ , \tag{4.4}$$

and

$$\omega^\mu(x,y)\simeq\delta^4(x-y)\delta_{\mu,0}(\omega_{\rm NM})\ . \tag{4.5}$$

We have, for nuclear matter,

$$\int d^4x\int d^4y\,\bar q(x)[\delta^4(x-y)i\gamma^\mu\partial_\mu-g_\sigma\sigma(x,y)-g_\omega\gamma^\mu\omega_\mu(x-y)]q(y)=\int d^4x\,\bar q(x)(i\gamma^\mu\partial_\mu-\tilde m_q-g_\omega\gamma^0\omega_{\rm NM})q(x)\ , \tag{4.6}$$

where

$$\tilde m_q=g_\sigma(\sigma_{\rm vac}+\sigma_{\rm NM})\ . \tag{4.7}$$

The fact that $\tilde m_q$ is less than m_q reflects a partial restoration of chiral symmetry in nuclear matter.

We can estimate $\sigma_{\rm NM}$ and $\omega_{\rm NM}$ in terms of the scalar and baryon densities of nuclear matter, $\rho_S^{\rm NM}$ and $\rho_V^{\rm NM}$. One has[5]

$$\sigma_{\rm NM}=-G_{\sigma\rm NN}\rho_S^{\rm NM}/m_\sigma^2\ , \tag{4.8}$$

and

$$\omega_{\rm NM}=G_{\omega\rm NN}\rho_V^{\rm NM}/m_\omega^2\ , \tag{4.9}$$

where $G_{\sigma\rm NN}$ and $G_{\omega\rm NN}$ are the coupling constants of the σ and ω fields to the nucleon. These can be taken from some typical boson-exchange models of the nucleon nucleon interaction.[15] One has $G_{\sigma\rm NN}\simeq7.6$ and $G_{\omega\rm NN}\simeq13$ so that $\sigma_{\rm NM}\simeq-37$ MeV and $\omega_{\rm NM}\simeq32$ MeV. If we now estimate the scalar and vector fields seen by the nucleon, we would find in the Hartree approximation, $U_S\simeq G_{\sigma\rm NN}\cdot\sigma_{\rm NN}\simeq-281$ MeV and $U_V\simeq G_{\omega\rm NN}\cdot\omega_{\rm NM}\simeq422$ MeV. There is a significant correction due to correlations to be made for U_V which brings U_V down to about 300 MeV.[1] Further, exchange (Fock) terms give a contribution to U_S of the order of -100 MeV.[1] Thus, after corrections are made for short-range correlation effects and Fock terms, one can estimate $U_S\simeq-400$ MeV and $U_V\simeq300$ MeV, which are values typical of those used in Dirac phenomenology. (Similar numbers are obtained in relativistic Brueckner-Hartree-Fock theory.[1])

V. DISCUSSION

We have presented a simple model which we suggest describes QCD at relatively low momentum transfer. As we have seen, the model is closely related to the model of Nambu-Jona-Lasinio.[23] (The latter model may be obtained from ours if we make a large-mass approximation for the exchanged gluon.) It is well known that the Nambu-Jona-Lasinio model gives a good account of the dynamics of chiral-symmetry breaking.

Our model also leads to order parameters with the quantum numbers of ρ and ω mesons. These order parameters are required in any boson-exchange model of the nuclear force or in relativistic models of nuclear structure.[1,2]

For example, in one particular boson-exchange model of the nuclear force[26] one finds the following fields:

J	$T=0$	$T=1$
0^+	σ (500 MeV)	δ (960)
0^-	η (548.5)	π (138.5)
1^-	ω (782.8)	ρ (763)
	ϕ (1020)	

The number in the parentheses represents the masses assigned to these fields in the study of nucleon-nucleon scattering. From the point of view adopted here, the assignment of the masses of the *physical* mesons to the field with the same quantum numbers can only be a rough approximation. (In principle, these masses should be obtained from a study of the effective potential of the model.) It is interesting to note that we find bilocal fields describing axial-vector mesons with $T=0$ and $T=1$. These appear to be rather unimportant in phenomenological studies of the nucleon-nucleon force. Indeed, in the boson-exchange model one can make satisfactory fits using only the fields with the smallest masses: σ, ρ, π, and ω.

We believe that we can use the model introduced in this work to understand the significance of the fields which play an important role in Dirac phenomenology. (Thus far, there has been only minimal understanding of the nature of these fields.) From our work, we see that the large scalar fields of the relativistic theories are intimately connected to the modification of the quark self-energy in the nuclear medium. (Essentially one is seeing a partial restoration of chiral symmetry in nuclear matter.) The quark self-energy is, in general, a gauge-dependent quantity and for detailed calculations one needs to make a choice of gauge. While the vector-isoscalar order parameter can be taken to be zero in vacuum, it has as its source the quark baryon density. Therefore, the mean-field value of that field is nonzero in nuclear matter.

It is well known that it is the interplay of the scalar and vector fields which leads to the success of the relativistic approach to nuclear structure physics.[1,2] This work represents an attempt to relate these fields to QCD, the fundamental theory of strong interaction.

[*Note added in proof.* Since completion of this work we have gained further understanding of dynamical mass generation for quarks and gluons in the gluon condensate (see Ref. 27).]

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APPENDIX A

In this paper we have carried out our analysis in real (Minkowski) space, with the understanding that our calculations can be justified in Euclidean space. However, the path-integral formalism has a well-defined meaning only in Euclidean (or imaginary-time) space. Therefore, we review the functional method, in this appendix, using Euclidean space. (To obtain physical quantities in real space, we have to perform an analytic continuation.)

In Euclidean space, we use the variable $\bar{x}_\mu$ to replace Minkowski variables: $\bar{x}_\mu=(\bar{\mathbf{x}}=\mathbf{x};\ \bar{x}_4=ix_0=it)$. Further, $\bar{k}_\mu=(\bar{\mathbf{k}}=\mathbf{k};\ \bar{k}_4=ik_0)$. Here $\bar{x}_4$ and $\bar{k}_4$ are real, since t and k_0 have been taken to be imaginary.

We consider a (real) scalar field theory for simplicity. The generating functional $Z[J]$, which is the vacuum-to-vacuum amplitude in the presence of an external source $J(x)$, is given by

$$Z[J]=N\int[d\phi]\exp\left[i\int d^4x\{\mathcal{L}[\phi(x)]+J(x)\phi(x)\}\right], \tag{A1}$$

where

$$\mathcal{L}[\phi(x)]=\mathcal{L}_0[\phi(x)]-V[\phi(x)], \tag{A2}$$

with $\mathcal{L}_0$ given by

$$\mathcal{L}_0(\phi)=\frac{1}{2}\partial_\mu\phi(x)\partial^\mu\phi(x)-\frac{\mu^2}{2}\phi^2(x). \tag{A3}$$

The corresponding quantity in Euclidean space is

$$Z_E[J]=N\int[d\phi]\exp\left[\int d^4\bar{x}\{\mathcal{L}_0[\phi(\bar{x})]-V[\phi(\bar{x})]+J(\bar{x})\phi(\bar{x})\}\right], \tag{A4}$$

where

$$\mathcal{L}_0[\phi(\bar{x})]=-\frac{1}{2}\left[\left(\frac{\partial\phi}{\partial\tau}\right)^2+(\nabla\phi)^2\right]-\frac{\mu^2}{2}\phi^2. \tag{A5}$$

The Euclidean generating functional

$$Z_E[J]=N\int[d\phi]\exp\left\{-\int d^4\bar{x}\left[\frac{1}{2}\left(\frac{\partial\phi}{\partial\tau}\right)^2+\tfrac{1}{2}(\nabla\phi)^2+\frac{\mu^2}{2}\phi^2+V(\phi)-J(\bar{x})\phi(\bar{x})\right]\right\}, \tag{A6}$$

may be written as

$$Z_E[J]=\exp\left[-\int d^4\bar{x}\,V\left(\frac{\delta}{\delta J}\right)\right]Z_E^0[J], \tag{A7}$$

where

$$Z_E^0[J]=N\int[d\phi]\exp\left[\int d^4\bar{x}(\mathcal{L}_0+J\phi)\right] \tag{A8}$$

is the free-field generating functional.

The quantity $-(\partial\phi/\partial\tau)^2-(\nabla\phi)^2$ in Eq. (A6) can be replaced by $\phi(\partial^2/\partial\tau^2+\nabla^2)\phi$ because the difference is a total four-divergence. Then, $Z_E^0[J]$ becomes

$$\begin{aligned}Z_E^0[J]&=N\int[d\phi]\exp\left\{-\tfrac{1}{2}\int d^4\bar{x}\left[\phi(\bar{x})\left[-\frac{\partial^2}{\partial\tau^2}-\nabla^2+\mu^2\right]\phi(\bar{x})-J(\bar{x})\phi(\bar{x})\right]\right\}\\&=N\int[d\phi]\exp\left[-\tfrac{1}{2}\int d^4\bar{x}\int d^4\bar{y}\,\phi(\bar{x})\Delta^{-1}(\bar{x},\bar{y})\phi(\bar{y})+\int d^4\bar{z}\,J(\bar{z})\phi(\bar{z})\right].\end{aligned} \tag{A9}$$

Here,

$$\Delta^{-1}(\bar{x},\bar{y})=\delta^4(\bar{x}-\bar{y})\left[-\frac{\partial^2}{\partial\tau^2}-\nabla^2+\mu^2\right] \tag{A10}$$

is the inverse propagator of the scalar field in Euclidean space. Note that

$$\Delta(\bar{x},\bar{y})=\int\frac{d^4\bar{k}}{(2\pi)^4}\frac{e^{i\bar{k}\cdot(\bar{x}-\bar{y})}}{\bar{k}^2+\mu^2}. \tag{A11}$$

As $\bar{x}$ and $\bar{y}$ are continuous indices, $Z_E^0[J]$ of Eq. (A9) can be considered an infinite-dimensional $(M \to \infty)$ Gaussian integral of form

$$\int d\phi_1 \cdots d\phi_M \exp\left[-\tfrac{1}{2}\sum_{ij}\phi_i(\Delta^{-1})_{ij}\phi_j + \sum_k J_k\phi_k\right] = [\det\Delta^{-1}]^{-1/2}\exp\left[\tfrac{1}{2}\sum_{ij} J_i\Delta_{ij}J_j\right] . \quad \text{(A12)}$$

In this way the functional integral in Eq. (A9) can be performed and we obtain, up to an inessential multiplicative factor,

$$Z_E^0[J] = N'\exp\left[\tfrac{1}{2}\int d^4\bar{x}\int d^4\bar{y}\, J(\bar{x})\Delta(\bar{x},\bar{y})J(\bar{y})\right] . \quad \text{(A13)}$$

The corresponding formula in Minkowski space is given by

$$Z_0[J] = N'\exp\left[\frac{-i}{2}\int d^4x\int d^4y\, J(x)\Delta_F(x,y)J(y)\right] , \quad \text{(A14)}$$

where

$$\Delta_F(x,y) = \int \frac{d^4k}{(2\pi)^4}\frac{e^{-ik\cdot(x-y)}}{k^2-\mu^2+i\eta} . \quad \text{(A15)}$$

In the main text, the corresponding result is given by Eq. (2.12).

For convenience in passing between the Euclidean and Minkowski-space formalism we present the correspondence between various quantities in the two spaces in Table I.

APPENDIX B

In this appendix we provide some relations which are useful if we are to introduce auxiliary fields in the Landau gauge. The Landau gauge is usually used when studying chiral symmetry breaking, since if one writes the quark self-energy in the form $\Sigma(p) = A(p^2)\not{p} + B(p^2)$, one can put $A(p^2)=0$ in lowest order calculations in that gauge. [This choice is made by Morozumi and So.[16] Further, Larsson discusses the general form for $\Sigma(p)$ in the deep Euclidean region, $-p^2 \to \infty$. His analysis also yields $A(p^2)=0$ in the Landau gauge in the presence of quark and gluon condensate parameters.[12]]

Consider $D^{AB}(q^2)$ of Eq. (1.1). Then,

$$(\gamma^\mu)_{ab}D^{AB}_{\mu\nu}(q^2)(\gamma^\nu)_{cd} = -\left[(\gamma^\mu)_{ab}(\gamma_\mu)_{cd} - \frac{(\not{q})_{ab}(\not{q})_{cd}}{q^2}\right]\{q^2[1-\Pi(q^2)]\}^{-1}\delta_{AB} . \quad \text{(B1)}$$

We have,

$$\frac{(\not{q})_{ab}(\not{q})_{cd}}{q^2} = A_S(\mathbb{1})_{ad}(\mathbb{1})_{cb} + A_P(i\gamma_5)_{ad}(i\gamma_5)_{cb} + A_V(\gamma^\mu)_{ad}(\gamma_\mu)_{cb} + A_{PV}(\gamma_5\gamma^\mu)_{ad}(\gamma_5\gamma_\mu)_{cb}$$
$$+ A_T(\sigma^{\mu\nu})_{ad}(\sigma_{\mu\nu})_{cb} + A'_V(\not{q})_{ad}(\not{q})_{cb}/q^2 + A'_{PV}(\gamma_5\not{q})_{ad}(\gamma_5\not{q})_{cb}/q^2 + A'_T(\sigma^{\mu\nu}q_\nu)_{ad}(\sigma_{\mu\rho}q^\rho)_{cb}/q^2 , \quad \text{(B2)}$$

with

$$A_S=\tfrac{1}{4},\quad A_P=\tfrac{1}{4},\quad A_V=-\tfrac{1}{4},\quad A_{PV}=-\tfrac{1}{4},\quad A'_V=\tfrac{1}{2},\quad A'_{PV}=\tfrac{1}{2},\quad A_T=\tfrac{1}{8},\quad A'_T=-\tfrac{1}{2} .$$

Using Eq. (B2) and the result of a Fierz transformation of the first term of Eq. (B1), we have the following result in

TABLE I. Correspondence between Minkowski and Euclidean-space quantities.

Euclidean space	Minkowski space
$f(\bar{x})$	$f(x)$
$\int d^4\bar{x}$	$i\int d^4x$
$\delta^4(\bar{x}-\bar{y})$	$-i\delta^4(x-y)$
$\int d^4\bar{x}\, f(\bar{x})\delta^4(\bar{x}-\bar{y}) = f(\bar{y})$	$\int d^4x\, f(x)\delta^4(x-y) = f(y)$
$\bar{\partial}^2$	$-\partial^2$
$\Delta^{-1}(\bar{x}-\bar{y})$	$i\Delta_F^{-1}(x-y)$
$\Delta^{-1}(\bar{x}-\bar{y}) = (-\bar{\partial}^2+\mu^2)\delta^4(\bar{x}-\bar{y})$	$i\Delta_F^{-1}(x-y) = -i(\partial^2+\mu^2)\delta^4(x-y)$
$\int d^4\bar{k}$	$-i\int d^4k$
$\Delta(\bar{x}-\bar{y})$	$i\Delta_F(x-y)$
$\Delta(\bar{x}-\bar{y}) = \int \frac{d^4\bar{k}}{(2\pi)^4}\frac{e^{i\bar{k}\cdot(\bar{x}-\bar{y})}}{\bar{k}^2+\mu^2}$	$i\Delta_F(x-y) = i\int \frac{d^4k}{(2\pi)^4}\frac{e^{-ik\cdot(x-y)}}{k^2-\mu^2+i\eta}$
$\int d^4\bar{z}\,\Delta^{-1}(\bar{x}-\bar{z})\Delta(\bar{z}-\bar{y}) = \delta^4(\bar{x}-\bar{y})$	$\int d^4z\,\Delta_F^{-1}(x-z)\Delta_F(z-y) = \delta^4(x-y)$

a general covariant gauge:

$$[(\gamma^\mu)_{ab}(\gamma_\mu)_{cd}-(1-\alpha)(\not q)_{ab}(\not q)_{cd}/q^2]$$
$$=[(\mathbb{1})_{ad}(\mathbb{1})_{cb}+(i\gamma_5)_{ad}(i\gamma_5)_{cb}][1-\tfrac{1}{4}(1-\alpha)]+[(\gamma^\mu)_{ad}(\gamma_\mu)_{cb}+(\gamma_5\gamma^\mu)_{ad}(\gamma_5\gamma_\mu)_{cb}][-\tfrac{1}{2}+\tfrac{1}{4}(1-\alpha)]$$
$$-[(\not q)_{ad}(\not q)_{cb}+(\gamma^5\not q)_{ad}(\gamma^5\not q)_{cb}]\left[\frac{1-\alpha}{2}\right]-(\sigma^{\mu\nu})_{ad}(\sigma_{\mu\nu})_{cb}\left[\frac{1-\alpha}{8}\right]+\frac{(\sigma^{\mu\nu}q_\nu)_{ad}(\sigma_{\mu\rho}q^\rho)_{cb}}{q^2}\left[\frac{1-\alpha}{2}\right]. \quad \text{(B3)}$$

The general result for a Fierz transformation in a covariant gauge has previously been given by Shrauner[16] in a particularly transparent notation.

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