Physical content of pseudopotential interactions

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(Received 30 November 1987)

The partial wave decomposition of the Franey-Love effective nucleon-nucleon amplitude is used to show that it satisfies on-shell unitarity to better than 5% in all partial waves, but its off-shell continuations are strikingly nonunitary.

Since the interaction between nucleons is strong but short ranged compared to the average spacing of nucleons in nuclei, microscopic calculations of nuclear scattering processes are usually carried out in a multiple-scattering framework.[1] This approach has been only moderately successful in nonrelativistic studies of intermediate energy (100–1000 MeV) elastic[2] and inelastic[3] nucleon-nucleus scattering. The failure to reproduce the systematics of measured spin observables has been cited as evidence for relativistic effects and for medium modifications of the effective interaction.[4]

One difficulty in current analyses of these experiments is that they rely on a phenomenological effective nucleon-nucleon interaction known as a "pseudopotential" or "pseudo-T-matrix" fitted to on-shell two-body data. The analytic form of the pseudo-T-matrix, chosen so as to simplify distorted-wave reaction calculations,[5] provides the necessary off-shell continuation. The resulting off-shell behavior has no theoretical foundation; its relation to the behavior predicted by microscopic models and its influence on reaction calculations requires detailed study.

In this paper, we describe the results of a partial wave expansion of the widely used Franey-Love (FL) effective interaction.[5] We then write the off-shell unitarity condition [Eq. (3)] on the partial wave matrix elements. This condition is not satisfied by the off-shell FL amplitudes—a violation that is a direct consequence of the analytic form assumed for the imaginary part of the pseudo-T-matrix [Eq. (1)]. It implies that the FL amplitudes and others similarly constructed cannot provide an accurate representation of the off-shell behavior of the nucleon-nucleon interaction.

In a multiple scattering theory, the interactions between a given pair must be summed to all orders to produce the effective interaction between nucleons. In lowest order, multiple scattering theories require the folding of an off-shell T-matrix element, $\langle \mathbf{k} \,|\, t(E_{\text{eff}}) \,|\, \mathbf{k}' \rangle$, with a nuclear density or transition density. The energy E_{eff} is an effective two-body energy. The operator $t(E_{\text{eff}})$ is usually constructed from a nonrelativistic potential by the solution of a Lippmann-Schwinger (LS) equation in each partial wave at each energy.

To simplify the construction and use of such an effective interaction in distorted-wave reaction calculations, a pseudo-T-matrix is constructed. The simplest and most widely used prescription is

$$t_{\text{eff}}(E)=V_{12}(E)(1-P_{12}) , \tag{1}$$

where $V_{12}(E)$ is a parametrized local, energy-dependent potential, and P_{12} is the exchange operator. The parameters in $V_{12}(E)$ are adjusted at each energy to reproduce on-shell amplitudes which have been fitted to nucleon-nucleon scattering data.[6] The analytic form adopted for the pseudo-T-matrix [Eq. (1)] is characteristic of the lowest-order Born approximation.

An off-shell amplitude contains information about the physics of the model used to build it.[7] For example, the half-shell amplitude $\langle \mathbf{k} \,|\, t(k^2/2\mu) \,|\, \mathbf{k}' \rangle$ determines the nucleon-nucleon relative wave function within the range of the potential. When using an off-shell amplitude, it is therefore important to understand the interior physics implied by the particular off-shell extrapolation employed.

The FL amplitudes are fitted to empirical Wolfenstein coefficients in a spin-isospin decomposition of the on-shell amplitude,[8] not to individual partial-wave amplitudes. This procedure is easy to carry out (compared to building a realistic potential model), and the resulting interaction is nearly local in coordinate space and therefore is simple to use. However, it is difficult to directly extract the nonasymptotic short-range physics implied by the off-shell Wolfenstein amplitudes. We have therefore performed a partial wave analysis of the FL amplitudes both on and off the energy shell. This allows us to address a number of interesting questions.

(1) How well do the fitted amplitudes reproduce the phase shifts? Are they significantly better than the phase shifts predicted by realistic potential models?

(2) How well do the fitted amplitudes satisfy unitarity?

They are constrained to fit on-shell Wolfenstein amplitudes which are unitary since they are built from real phase shifts. But the fitted amplitudes are not directly constrained to be unitary, either on or off the energy shell.

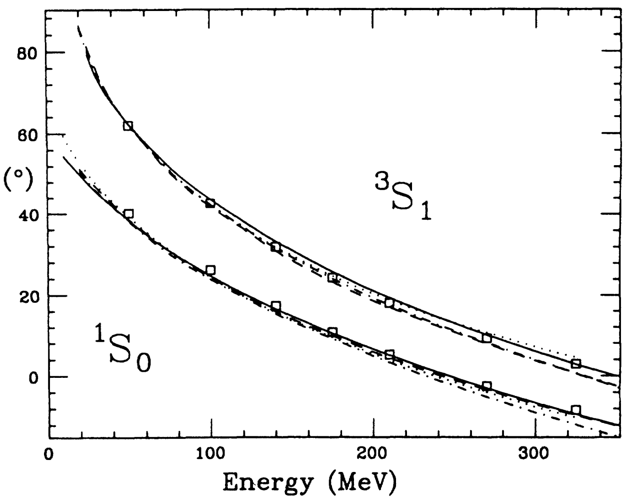

FIG. 1. Nucleon-nucleon S-wave phase shifts from the Franey-Love interaction (boxes), SP86 (solid), Paris-80 (dot-dash), RSC (dash), and full Bonn (dots).

We have investigated the phase shifts implicit in the FL amplitudes for all two-nucleon states with $J \leq 10$. In Fig. 1 we show the S wave phase shifts from a recent analysis,[9] SP86, by the Virginia Tech (VPI) group (solid line), and the phase shifts extracted from the FL Wolfenstein amplitudes (boxes). Also shown are the phase shifts arising from the Reid soft core (RSC), Paris, and (full) Bonn potentials.[10] The coupling parameter ε_1 and the small $^3F_2$ phase shifts (see Fig. 2) show some significant deviations, but the agreement with the phenomenological phase parameters is on the whole excellent—comparable to, but not better than those provided by the Paris and Bonn potentials.

On-shell unitarity is well satisfied by the partial-wave amplitudes. We define on-shell unitarity defects by

$$\Delta U = \|1 - S^{\dagger} S\| , \tag{2}$$

where $\| \cdots \|$ indicates the Euclidean matrix norm (square-root of the sum of the squared moduli of the matrix elements). The defects for the uncoupled states $^1S_0$, $^3D_2$, and the coupled state $J=1$ are shown in Fig. 3. The largest defects are of the order of 5%, with an average defect of 1–2 %.

We next consider the off-shell behavior of the FL amplitudes. If an uncoupled partial-wave amplitude is constructed from a Hermitian potential by solving a two-body Lippmann-Schwinger equation, it will satisfy the unitarity relation

$$\mathrm{Im}\, t_l(k,k';p^2) = -\frac{2\mu p}{\hbar^2} t_l(k,p;p^2) t_l^*(p,k';p^2) . \tag{3}$$

This will hold even if the potential is nonlocal and energy dependent. (We follow the notation and normalizations of Ref. 7.)

The most interesting special case of this relation is on the half-shell, where $k'=p$. If we write the on-shell amplitude

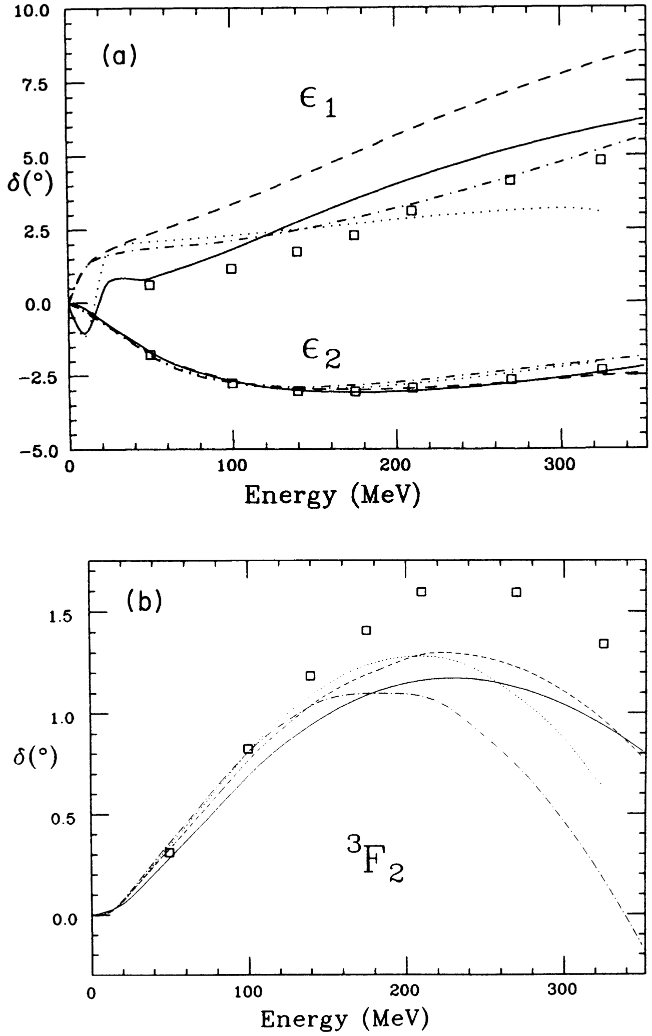

FIG. 2. (a) Nucleon-nucleon phase coupling parameters ε_1 and ε_2; (b) nucleon-nucleon phase shift in the $^3F_2$ state. Codes as in Fig. 1.

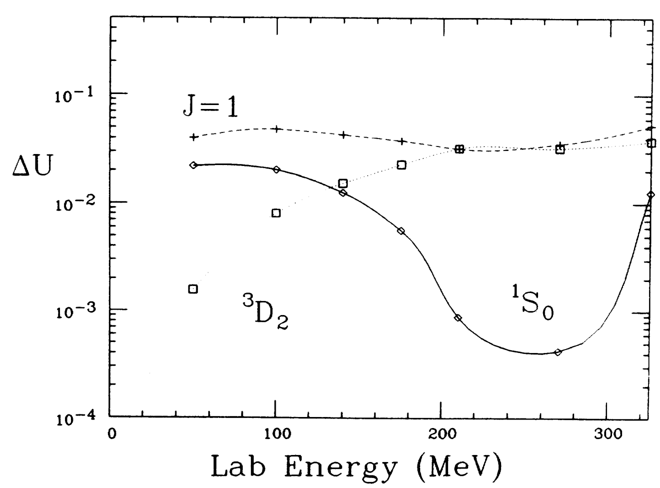

FIG. 3. Unitarity defects for the Franey-Love S matrices in the states $^1S_0$, $^3D_2$, and $J=1$.

$$t_l(p,p;p^2) = -\frac{\hbar^2}{2\mu p} e^{i\delta_l(p)} \sin\delta_l(p) \tag{4}$$

and set $k'=p$ in Eq. (3), we get

$$\mathrm{Im}t_l(k,p;p^2) = e^{-i\delta_l(p)} \sin\delta_l(p) t_l(k,p;p^2) \ . \tag{5}$$

Since $\mathrm{Im}t_l$ is a real number, this implies that the half-shell T matrix has the form

$$t_l(k,p,p^2) = e^{i\delta_l(p)} \tau_l(k,p;p^2) \ , \tag{6}$$

where τ_l is a real function.

We use two different criteria to express the violations of off-shell unitarity. First, define the "off-shell phase shift" by

$$\tan\delta_l(k;p) = \mathrm{Im}t_l(k,p;p^2)/\mathrm{Re}t_l(k,p;p^2) \ . \tag{7}$$

If the amplitude satisfies half-shell unitarity, $\delta_l(k;p)$ should be independent of k and should agree with the on-shell phase shift. Second, define the off-shell K matrix by the Heitler equation evaluated on the half-shell. The result is

$$k_l(k,p;p^2) = [1 - i\tan\delta_l(p)] t_l(k,p;p^2) \ . \tag{8}$$

If the amplitude satisfies half-shell unitarity, k_l should be real.

Both measures of the violation of off-shell unitarity are displayed in Table I for the half-shell amplitudes at $E_{\mathrm{lab}} = 140$ MeV ($p = 1.3$ fm^{-1}) at $k = 3.0$ fm^{-1} for the first three uncoupled singlet states. We plot in Fig. 4 the dependence of $\delta_l(k;p)$ extracted from the FL amplitudes as a function of k. If the amplitude satisfied half-shell unitarity, it would be equal to the constant shown by the solid line. The on-shell point is indicated. According to either criterion, off-shell unitarity is badly violated by the pseudo-T-matrix.

The existence of a violation of off-shell unitarity is not unexpected, but the magnitude of the violation is striking. It is a direct consequence of the analytic form assumed for the pseudo-T-matrix. Equation (1) gives the T matrix the structure of a first Born amplitude. This is reasonable for the real part of T, which is often well described by the Born form, albeit with substantially modified strengths. The imaginary part of T, however, vanishes in first Born approximation and the first Born form (1) is inappropriate. Indeed, it is clear from the form of the unitarity relation (3) that $\mathrm{Im}T$ is a separable function of its momentum arguments.

TABLE I. Half-shell T matrix, K matrix, and off-shell phase shifts in the Franey-Love model. $E_{\mathrm{lab}} = 140$ MeV, $p = 1.3$ fm^{-1}, and $k = 3.0$ fm^{-1}.

State	$^1S_0$	$^1P_1$	$^1D_2$
$t(k,p;p^2)$	15.213	2.146	-1.435
(MeV fm^3)	$+0.512i$	$-1.336i$	$-0.201i$
$k(k,p;p^2)$	15.373	2.560	-1.452
	$+4.242i$	$-0.661i$	$-0.075i$
$\delta(k;p)$ (deg)	1.928	-31.904	7.981
$\delta(p)$ (deg)	17.353	-17.458	5.047
$\lvert \Delta U(\mathrm{on}) \rvert$	1.2%	1.4%	1.6%

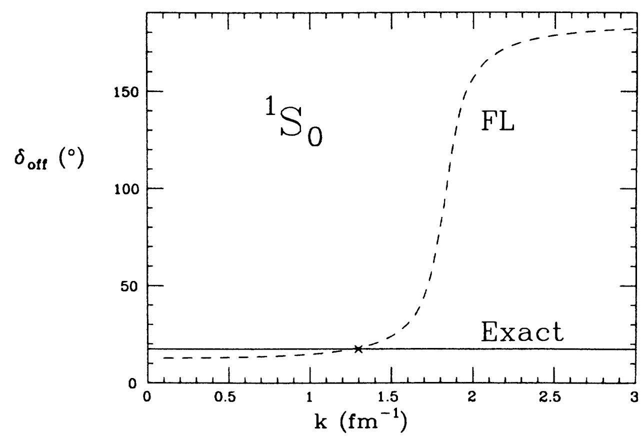

FIG. 4. Half-shell phase shift $\delta_l(k,p)$ for the FL $^1S_0$ state at 140 MeV (dashed line). If the half-shell amplitude satisfied unitarity it would be constant (solid line). The on-shell point is marked by an $\times$.

This is more than a formal problem. Every serious model of nuclear phenomena, whether it is a quark model, a model with mesons and deltas, a relativistic, or a nonrelativistic model, leads to an effective nucleon-nucleon potential when the extra degrees of freedom are formally eliminated. This potential may be strongly nonlocal and energy dependent, but it cannot be non-Hermitian below the pion production threshold. It will therefore satisfy the unitarity relation (3). Half-shell unitarity requires that we have a partial-wave scattering wave function of the form

$$\psi_l^{(+)}(r;p) = e^{i\delta_l(p)} \phi(r;p) \ , \tag{9}$$

where ϕ is real. A violation of half-shell unitarity implies that the wave function ϕ becomes complex inside the range of the potential.

The violation of off-shell unitarity by pseudo-T-matrices of the form (1) comes from the inappropriate analytic form assumed for $\mathrm{Im}T$ and is potentially a serious problem as it takes pseudo-T-matrix amplitudes outside the realm of models usually considered as descriptions of nuclear phenomena. Since existing inelastic scattering programs require effective interactions local in coordinate space, the quantitative implications for intermediate-energy nucleon-nucleus scattering calculations cannot yet be easily assessed. From an analysis of the plane-wave impulse approximation in momentum space, we expect the largest effects to be in the excitation of high-spin states, where the form factors peak at large momentum transfers ($q \simeq 2-3$ fm^{-1}). Rough estimates indicate that at the peaks of the inelastic cross sections T matrix elements enter whose initial and final relative momenta can differ by $0.5-1.0$ fm^{-1} where very large violations of unitarity occur (see Fig. 4).

The important issues being addressed by inelastic nucleon-nucleus scattering are the importance of medium modifications of the effective interaction, relativistic effects, and other nonlocalities. Spin observables, which are sensitive to relative phases, play a crucial role in such studies. Under these circumstances, the use of an effective interaction which seriously violates off-shell unitarity runs the risk of obscuring the physical effects of primary interest.

This work was supported in part by the U.S. Department of Energy.

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