

Quenching of magnetic strength in $N = 28$ nuclei

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The influence of ground state correlations on the sum rules for $M1$, $M3$, and $M5$ excitation of $N = 28$ nuclei is investigated. When combined with use of a renormalized single-particle matrix element which takes into account effects of core polarization, isobars, and meson exchange currents, the calculations are able to account for the observed quenching as well as the centroid and width of $M1$ strength below 12 MeV in ^{52}Cr .

Quenching of spin-isospin excitations is commonly observed in (p,n) reactions, and in both electron and proton scattering. Estimates of the expected strengths are often based on very simple models for the ground state wave functions, and it is important to determine corrections due to configuration mixing before drawing conclusions about the magnitude of other effects such as core polarization and mesonic exchange currents. In this Brief Report we examine the role of certain ground state components in the quenching of $M1$, $M3$, and $M5$ strength in $N = 28$ nuclei.

Figure 1 shows the calculated distribution of magnetic strengths in ^{52}Cr for a $f_{7/2}^{-4}$ ground state and excited states which are pure $f_{7/2}^{-5}j$ ($j = p_{1/2}, p_{3/2},$ and $f_{5/2}$). The calculations used the effective interaction of Johnstone and Benson,^{1,2} which is very successful in reproducing energy levels in this mass region when states are assumed to be characterized by a particular number of $f_{7/2}$ holes. Harmonic oscillator radial functions with $b = 1.95$ fm were used to evaluate the required matrix elements of r^2 and r^4 . The summed $M1$, $M3$, and $M5$ strengths are $20.6 \mu_N^2$, $8.15 \times 10^4 \mu_N^2 \text{fm}^4$, and $1.41 \times 10^8 \mu_N^2 \text{fm}^8$. Corresponding centroid energies are 10.0, 7.4, and 6.4 MeV for $T = 2$ states, and 14.3, 12.5, and 11.6 MeV for $T = 3$ states.

Figure 1 also shows the experimental $M1$ strength below 12 MeV, determined from inelastic electron scattering.³ The centroid (9.5 MeV) and width (1.0 MeV) are similar to the calculated values for $E < 12$ MeV (9.5 and 1.2 MeV), but the total observed strength of $5.0 \pm 0.5 \mu_N^2$ is only one-third of that calculated.

Magnetic strengths are dominated by isovector transitions. The contribution of isoscalar transitions to $M1$, $M3$, and $M5$ excitation of the closed-shell state of ^{56}Ni would be only 1%, 4%, and 3% of the total, and in ^{52}Cr (for which isoscalar and isovector amplitudes to $T = 2$ states are out of phase) they reduce the sum rules by just 5%, 7%, and 8% from the pure isovector values. For $N = 28$ nuclei, isovector $B(M_L)\uparrow$ from the $f_{7/2}^{-n}$ state is exhausted by the giant resonance

$$|\psi\rangle = N \sum_j c_L(j) [a_j^\dagger a_{7/2}]_{T_3=0}^{(L,1)} f_{7/2}^{-n} \rangle .$$

Here, N is a normalization factor, and

$$c_L(j) = t_L(j) / \left[\sum_{j'} t_L(j')^2 \right]^{1/2} ,$$

where

$$t_L(j) = \frac{1}{\sqrt{2}} [(jn \| T^{(L)} \| f_{7/2}^{-n}) - (jp \| T^{(L)} \| f_{7/2}^{-p})] ,$$

and the summation is over $j = p_{1/2}, p_{3/2},$ and $f_{5/2}$. The two-particle ($2p$) component in the ground state which exhausts $2p$ transition strength to the giant resonance is

$$|\psi\rangle = N_2 \sum_{jj'} c_L(j') [a_j^\dagger a_{7/2}]^{(L,1)} \times [a_{j'}^\dagger a_{7/2}]^{(L,1)} (0,0) f_{7/2}^{-n} \rangle .$$

If the magnitude of this component is estimated from perturbation theory, the modified sum rule becomes

$$B(M_L)\uparrow = \sum_j \frac{t_L(j)^2}{N^2} \left[\frac{1 - N^2 \langle \psi_2 | V | f_{7/2}^{-n} \rangle}{N_2 \sqrt{3} \Delta E_2} \right]^2 \\ = R_2(L) \sum B(M_L)\uparrow_{\text{sp}} ,$$

where $\sum B(M_L)\uparrow_{\text{sp}}$ is the isovector sum rule value for the pure $f_{7/2}^{-n}$ state (which is reduced by a factor of $1/N^2 = 0.875$ in ^{54}Fe , and 0.75 in ^{52}Cr , relative to its value in ^{56}Ni). In Table I are listed the quenching factors R_2 we calculate for $M1$, $M3$, and $M5$ transitions in ^{52}Cr , ^{54}Fe , and ^{56}Ni using the Kuo-Brown fp -shell interaction for V and ^{57}Ni single-particle energies to give ΔE_2 . (The Johnstone and Benson interaction¹ cannot be used to evaluate the matrix element of

V , since it is diagonal in the number of $f_{7/2}$ holes.)

Eulenberg *et al.*³ note that quenching can be caused by one-particle ($1p$) components in the ground state giving $f_{7/2} \rightarrow f_{7/2}$ amplitudes which interfere destructively with the predominant $f_{7/2} \rightarrow f_{5/2}$ amplitudes in $M1$ transitions to the giant resonance. To determine the magnitude of this effect we have considered the ground state component

$$|\psi_1\rangle = N_1 \sum_j c_L(j) | [a_{7/2}^\dagger a_{7/2}]^{(L,1)} \times [a_{7/2}^\dagger a_{7/2}]^{(L,1)}(0,0) f_{7/2}^- \rangle,$$

which exhausts the $f_{7/2} \rightarrow f_{7/2}$ strength to $|\psi\rangle$. The resulting modification of the sum rule is

$$\begin{aligned} B(M_L)\uparrow &= \sum_j \frac{t_L(j)^2}{N^2} \left[\frac{1 - N^2 \langle \psi_1 | V | f_{7/2}^- \rangle c_L(f_{7/2})}{N_1 \sqrt{3} \Delta E_1} \right]^2 \\ &= R_1(L) \sum B(M_L)\uparrow_{\text{sp}}. \end{aligned}$$

Calculated values of R_1 are given in Table I and lead to an additional 5–6% quenching of $M1$ strength in ^{52}Cr and ^{54}Fe . The effect on $M3$ and $M5$ strengths is smaller, with $M3$ strength actually being slightly enhanced.

Two-particle components other than $|\psi_2\rangle$ in the ground state can affect the summed strengths by (i) reducing the $f_{7/2}^-$ ground state component, and (ii) giving $2p \rightarrow 2p$ and $2p \rightarrow 3p$ contributions to the strength. In the case of ^{56}Ni we find that the summed $M1$ strength from a particular $2p$ 0^+ state to all $2p$ 1^+ states and to all $3p$ 1^+ states averages about 40% and 80% of the $0p \rightarrow 1p$ summed strength. It is therefore to be expected that the effects of (i) and (ii) on the summed strengths will largely cancel each other. A similar argument can be made for the effect of $1p$ components other than $|\psi_1\rangle$. However, since most $2p$ and $3p$ states are expected to lie above 12 MeV in ^{52}Cr , there will be little contribution from (ii) to the $M1$ strength below 12 MeV determined in Ref. 3. It can therefore be argued that the theoretical estimate

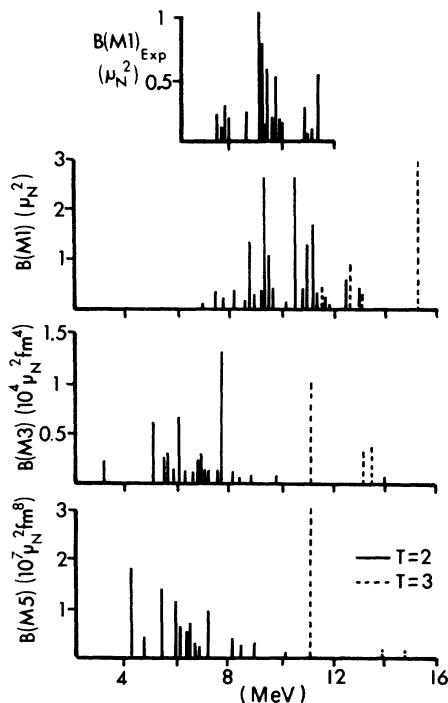


FIG. 1. Calculated magnetic strengths in ^{52}Cr , and the $M1$ strength observed below 12 MeV.

$R_1 R_2 B(M1:0p \rightarrow 1p)$ to be compared with the experimental sum should be reduced by an amount equal to the $2p$ probability in the ground state, which is about 20% with the Kuo-Brown interaction. The theoretical strength below 12 MeV is then about $7 \mu_N^2$.

In addition to quenching arising from $1f2p$ -shell components other than $f_{7/2}^-$ in the ground states, further changes in the magnetic strengths are to be expected from excitation of nucleons into higher shells or out of closed shells, and from the effects of isobars and mesonic exchange currents. To the extent that these additional effects can be represented by a mass-independent renormalization of the single-nucleon operators, the $A=41$ calculation of Towner and Khanna⁴ provides an estimate of the $M1$ renormalization. They find a total reduction of 15.6% in the isovector $f_{7/2} \rightarrow f_{5/2}$ matrix element, leading to an additional quenching factor of 0.71 for the $M1$ strength. The net result of the $|\psi_2\rangle$ and $|\psi_1\rangle$ components in the ground state, the reduction in the $f_{7/2}^-$ component, and the use of the renormalized single-particle matrix element, is then to give a theoretical distribution of $M1$ strength in ^{52}Cr just as in Fig. 1, except that each peak is reduced by a factor of about 3. The calculated strength below 12 MeV, as well as its centroid and width, is then in good agreement with experiment.

McGrory and Woldenthal,⁵ in their calculations of $M1$ strength in Ca isotopes, assume that the deviation of the ^{41}Ca magnetic moment from the Schmidt value is due to a quenching of the spin g factor, and then use this reduced g_s to calculate an effective $f_{7/2} \rightarrow f_{5/2}$ matrix element. This procedure, making use of the ^{41}Sc and ^{41}Ca magnetic moments, gives g_s values for neutrons and protons which are 83% and 87% of the free-nucleon values, and leads to $f_{7/2} \rightarrow j$ single-particle matrix elements which quench the $M1$, $M3$, and $M5$ isovector strengths

TABLE I. Quenching factors for isovector magnetic strengths in ^{52}Cr , ^{54}Fe , and ^{56}Ni due to $2p$ and $1p$ ground state components.

	$M1$	$M3$	$M5$
$R_2(^{52}\text{Cr})$	0.63	0.79	0.81
$R_2(^{54}\text{Fe})$	0.64	0.81	0.83
$R_2(^{56}\text{Ni})$	0.57	0.78	0.80
$R_1(^{52}\text{Cr})$	0.94	1.02	0.98
$R_1(^{54}\text{Fe})$	0.95	1.02	0.98

for $N=28$ nuclei by factors of 0.70, 0.73, and 0.73. The reduction in $M1$ strength is almost exactly that resulting from use of the Towner and Khanna off-diagonal matrix element, but this would appear to be coincidental. Towner and Khanna find that renormalization of the single-particle $M1$ operator results in a sizable $[\sigma \times Y^{(2)}]$ tensor contribution, rather than just a correction to g_s . Moreover, they find that the relative importance of the various quenching processes is rather different for diagonal and off-diagonal matrix elements.

The results of Towner and Khanna apply only to $M1$ (and Gamow-Teller) matrix elements. To investigate how some of the effects they consider depend on multipolarity, we have calculated the corrections to $M3$ and $M5$ off-diagonal single-particle matrix elements arising from isobars and meson exchange currents. The results given in Table II were obtained using methods similar to those used in the ^{17}O calculation of Ref. 6. In general, the isobar and meson exchange corrections for $M3$ and $M5$ are small and out of phase. The renormalization of the $f_{7/2} \rightarrow f_{5/2}$ $M1$ matrix element is somewhat larger than found by Towner and Khanna; this can be traced to our use of a residual interaction based on Landau-Migdal theory, where both direct and exchange particle-hole matrix elements of the random-phase approximation (RPA) series are absorbed in the parameter g' .

In conclusion, we have shown that certain ground state correlations have a large effect on the total magnetic exci-

TABLE II. Isobar (Δ) and meson exchange current (MEC) contributions to f - p shell single-particle matrix elements, given as multiples of free nucleon values.

	Δ	MEC
$\langle f_{7/2} M1 f_{5/2} \rangle$	-0.18	0.05
$\langle f_{7/2} M3 p_{1/2} \rangle$	-0.09	0.02
$\langle f_{7/2} M3 p_{3/2} \rangle$	-0.09	0.03
$\langle f_{7/2} M3 f_{5/2} \rangle$	-0.04	0.01
$\langle f_{7/2} M5 p_{3/2} \rangle$	-0.06	0.02
$\langle f_{7/2} M5 f_{5/2} \rangle$	-0.05	0.02

tation strength for $N=28$ nuclei, and that this quenching can be calculated without knowledge of the complete ground state wave function. In the specific case of ^{52}Cr , our calculations are able to account for the observed $M1$ quenching and strength distribution below 12 MeV, and predict that most of the $T < M3$ and $M5$ strength lies below 8 MeV. It will be interesting to see whether experiment confirms this prediction and can provide a measure of the quenching in these higher multipoles.

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