

## Monopole strength as a measure of nuclear shape mixing

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A survey of observed  $E0$  transition strength shows that some of the largest strength occurs for transitional nuclei whenever large mixing between almost spherical and largely deformed shapes results. This leads to the conclusion that large  $E0$  strength is not an indication of coexisting shapes but of strong mixing between nuclear states with largely different radii. In addition, we discuss the variation of  $E0$  transition rates in regions where intruder states become the nuclear ground state. We illustrate this by recent results in the  $A \simeq 100$  region concentrating especially on the anomalously large monopoles strength and its variation in the even-even  $N = 60$  isotones as compared with the  $N = 58$  isotones.

### I. INTRODUCTION

The monopole operator is the quantity whose diagonal matrix elements give information about nuclear radii (isotopic and isomeric shifts). The results from nondiagonal matrix elements, such as electromagnetic transitions in nuclei ( $E0$  transitions), are more difficult to interpret in a simple way. Sometimes<sup>1-6</sup> it has been suggested that the observation of large  $E0$  strength is in itself an indication for the occurrence of shape coexistence. Here and in some of the above references too, it is shown that large  $E0$  strength is mainly a result of *strong mixing* of states with largely different shapes and *not* an indication in itself of shape coexistence.

The measurement of monopole transitions in nuclei off the line of stability has only recently been possible for a large number of nuclei through the advances in experimental techniques such as superconducting solenoidal magnets<sup>7</sup> and mini-orange spectrometers.<sup>8</sup> The emergence of these techniques and the ability to measure level lifetimes as low as a fraction of a nanosecond through, for example, the use of the centroid shift technique, have allowed the determination of the absolute rate of  $E0$  transitions, i.e., a determination of the monopole strength. Now, by combining results from several different techniques, it is possible to systematically map the variation of the monopole strength in a series of nuclei which are known to possess coexisting configurations.

In Sec. II, we first review the methods used to calculate  $E0$  transitions (shell model, collective geometric model, interacting boson model). We then show that some of the largest  $E0$  transitions occur in transitional nuclei where large mixing results between an almost spherical and largely deformed shapes. Although a large monopole strength can be expected for the deexcitation of a beta vibration in a nucleus with permanent deformation,  $E0$  transition probabilities between unmixed coexisting shapes with large differences in deformation will probably be small. This is well illustrated by the recent measure-

ment of the  $E0$  strength for the transition between the  $0^+$  fission isomer and the ground state in  $^{238}\text{U}$ . These two states certainly have a large difference in deformation. However, their individual purity (i.e., the fact that they are not mixed) is reflected in the monopole strength which was measured to be one of the smallest known values ( $\rho^2 \simeq 1.7 \times 10^{-9}$ ) by Kantele *et al.*<sup>9</sup> This leads to the conclusion that large  $E0$  strength is an indication of strong mixing between nuclear states with largely different radii. In Sec. III, we point out that the present interpretation is consistent with recent on-line isotope separator (ISOL) and in-beam conversion electron measurements in the  $A \simeq 100$  region. In doing so, we show that there is an anomaly in the  $E0$  strength for the  $N = 60$  nuclei compared with other nuclei in the  $A \simeq 100$  region. We trace the anomalously large jump of a factor of  $\simeq 20$  in the monopole strength in going from  $N = 58$  to  $N = 60$  isotopes to the fact that the states involved in the  $N = 58$  nuclei have a spherical ground state (GS) and a dynamically deformed excited state, while the  $N = 60$  nuclei have a spherical state as the excited state and the GS has permanent deformation.

### II. $E0$ TRANSITIONS AND SYSTEMATIC BEHAVIOR

#### A. $E0$ transition units

Depending on the particular nuclear model, such as the shell model, the collective geometric model (quadrupole vibrator, axial rotor, etc.), or the interacting boson model that is used, the  $E0$  strength can be calculated. The quantity, mainly used to characterize  $E0$  transitions is the  $\rho^2$  value, which has been defined by Bohr and Mottelson<sup>10</sup> as

$$\rho^2 = \left| \frac{\langle \phi_f | \sum e_j r_j^2 | \phi_{ij} \rangle}{eR^2} \right|^2, \quad (2.1)$$

where  $R$  denotes the nuclear radius.

Within the nuclear shell model in its simplest version, single-particle  $E0$  transitions ( $|l, j\rangle \rightarrow |l, j\rangle$ ) are strictly forbidden (they follow the shell-model selection rules  $\Delta N = \pm 2$ , where  $N$  denotes the major harmonic oscillator quantum number), implying a transition energy of the order of  $2\hbar\omega_0 \approx 20$  MeV. In the more realistic calculations of a nuclear shell-model potential, differences in radii for the orbitals of a given major shell will occur and thus give rise to small  $E0$  transitions, even within a single oscillator shell.

Low-energy monopole transitions can, however, result because of pairing correlations in the nuclear wave function. When orbitals with  $N$  different by one unit are admixed in the pairing wave function, a shell-model estimate can be obtained.<sup>10-12</sup> This results in a value that has been proposed as the  $E0$  single-particle unit (SPU) by Bohr and Mottelson<sup>13</sup> and is given by

$$\rho_{\text{SPU}}^2 = 0.5 A^{-2/3} \equiv (1 \text{ SPU}), \quad (2.2)$$

and whose general trend is shown in Fig. 1(a) (for the Sn nuclei) where it is compared to other types of estimates. The known experimental data for the  $50 < A < 160$  mass region will be discussed in Sec. III.

To the lowest order, the  $E0$  operator for harmonic quadrupole vibrational motion in nuclei can be written as<sup>12-14</sup>

$$T(E0) = \frac{3}{4\pi} ZeR^2 \sum_{\mu} |\alpha_{\mu}|^2, \quad (2.3)$$

where the  $\alpha_{\mu}$  denote the expansion coefficients for the nuclear surface. This operator results in a selection rule:  $\Delta n = 0, \pm 2$ , where  $n$  is the number of quadrupole phonons. The  $E0$  transition from the two-phonon state ( $0_2^+$ )

to the ground state ( $0_1^+$ ; the zero-phonon state) results in a spherical vibrator value

$$\rho_{\text{sph.vibr.}}^2 = \frac{2}{5} \left[ \frac{3}{4\pi} \right]^2 Z^2 \beta_{\text{rms}}^4, \quad (2.4)$$

where  $\beta_{\text{rms}}$  is the root-mean-square value related to the  $B(E2; 2_1^+ \rightarrow 0_1^+)$  transition, in a harmonic limit, via the relation

$$\beta_{\text{rms}} = \frac{[B(E2; 0_1^+ \rightarrow 2_1^+)]^{1/2}}{(3/4\pi)ZeR^2}. \quad (2.5)$$

For the Sn region, as an example, where  $\beta_{\text{rms}} \approx 0.12$  [see Fig. 1(a)], the value of  $\rho_{\text{sph.vibr.}}^2$  is very near to, but slightly smaller than, the single-particle estimate.

For an axial rotor, volume conserving vibrations around the quadrupole equilibrium value  $\beta_0$  give rise to nonvanishing  $E0$  transitions. There, using the same operator as in Eq. (2.3), but transformed into the intrinsic system, one obtains<sup>13-15</sup>

$$\rho_{\gamma \rightarrow g}^2 = 0 \quad (2.6)$$

$$\rho_{\beta \rightarrow g}^2 = 4(3/4\pi)^2 Z^2 \beta_0^2 \langle \bar{\beta} \rangle^2, \quad (2.7)$$

$$\langle \bar{\beta} \rangle = \sqrt{\hbar/2(\sqrt{BC})}. \quad (2.8)$$

More detailed expressions for deformed nuclei (axially nonsymmetric, etc.) have been discussed by Rasmussen<sup>15</sup> and Davydov and Rostovsky.<sup>16</sup>

The expression in Eq. (2.7) can be rewritten as

$$\rho_{\beta \rightarrow g}^2 = \frac{B(E2; 0_2^+ \rightarrow 2_2^+) 4\beta_0^2}{e^2 R^4} \approx 1-2 \text{ SPU}. \quad (2.9)$$

Starting from the  $E0$  operator, as described within the interacting boson model,<sup>17</sup>  $E0$  transitions can easily be described, especially in the dynamical symmetries for even-even nuclei (for modification towards odd-mass nuclei, see Ref. 18). The  $E0$  operator

$$T(E0) = \beta_0 (d + \bar{d})^{(0)} + \gamma_0 (s + \bar{s})^{(0)}, \quad (2.10)$$

can be rewritten as

$$\begin{aligned} T(E0) &= \gamma_0 \hat{N} + \beta'_0 \hat{n}d \\ &= \frac{\beta_0}{\sqrt{5}} \hat{N} - \beta'_0 \hat{n}_s, \end{aligned} \quad (2.11)$$

with  $\beta'_0 = \beta_0/\sqrt{5} - \gamma_0$ ,  $\hat{n}_{s(d)}$  the  $s$  ( $d$ ) boson number operator, and  $\hat{N}$  the total boson number operator.

Within the vibrational limit [ $U(5)$ ],  $E0$  transitions are strictly forbidden. In the  $SU(3)$  limit, the more important “ $\beta$ ” band  $0^+$  to “ground band”  $0^+$   $E0$  transition matrix element becomes

$$\begin{aligned} \langle (2N, 0)K=0, I=0 | T(E0) | (2N-4, 2)K=0, I=0 \rangle \\ = -\beta'_0 \left[ \frac{8}{9} \frac{(N-1)^2}{(2N-1)^2} \frac{N(2N+1)}{(2N-3)} \right]^{1/2}. \end{aligned} \quad (2.12)$$

Finally, within the  $O(6)$  limit, the selection rules are  $\Delta\sigma = 0, \pm 2$ ,  $\Delta\tau = 0$  (no change in the seniority quantum number) and one calculates

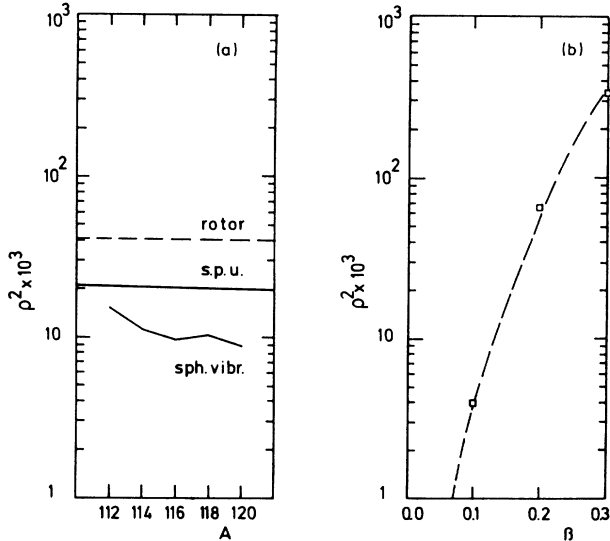


FIG. 1. (a) Schematic representation of the shell-model estimate (SPU), the spherical quadrupole vibrational estimate (sph.vibr.), and the  $\beta \rightarrow g$  axial rotor estimate for  $\rho^2$  and for the  $110 < A < 120$  mass region. (b) Representation of  $\rho^2$  using the mixed wave functions of Eqs. (2.14) as a function of the equilibrium deformation  $\beta$ , characterizing the deformed state.

$$\langle [N], \sigma = N, \tau = 0, I = 0 | T(E0) | [N], \sigma = N - 2, \tau = 0, I = 0 \rangle = -\beta'_0 \frac{1}{2(N+1)} [(N+2)(N-1)(N+3)]^{1/2}. \quad (2.13)$$

Starting from expressions (2.12) and (2.13), a good estimate of  $E0$  transitions in many even-even nuclei can be obtained using only one parameter in the  $E0$  operator, i.e.,  $\beta'_0$ . More detailed calculations require the full IBM Hamiltonian as well as the explicit proton-neutron degrees of freedom in the IBM-2.<sup>19</sup>

### B. $E0$ transitions for mixed states

The expectation value of the  $E0$  operator [see Eq. (2.3)] within a deformed state gives a measure of the nuclear radius related to this deformed state. Since the wave functions, corresponding to  $0^+$  states with largely different deformation are only poorly overlapping, only that overlap region is relevant when calculating nondiagonal matrix elements of the  $E0$  operator and will normally result in a very small value (see the results of Ref. 9 for an experimental verification of this point). If such intrinsic states, corresponding to largely different equilibrium deformation (quadrupole, octupole, etc.) result in a single nucleus at low energy ( $E_x < 1.5$  MeV) with an important potential energy barrier in the deformation coordinate separating the equilibrium shapes, we speak of shape coexistence. With a decreasing barrier (and thus for experimentally close-lying  $0^+$  levels with typical energy separations of 0.2–0.4 MeV) mixing will increase and modify the  $E0$  transition matrix elements in an important way.

Assuming, in a schematic way, a maximal mixing between the two unperturbed configurations, the spherical ( $| \text{sph.} \rangle$ ) and deformed ( $| \text{def.} \rangle$ ) basis states, one obtains

$$\begin{aligned} |0_f^+\rangle &= 1/\sqrt{2} \{ | \text{sph.} \rangle + | \text{def.} \rangle \}, \\ |0_i^+\rangle &= 1/\sqrt{2} \{ | \text{sph.} \rangle - | \text{def.} \rangle \}, \end{aligned} \quad (2.14)$$

resulting in a  $\rho^2$  value of

$$\rho_{i \rightarrow f}^2 = \frac{1}{4} (\langle \beta^2 \rangle_{\text{def.}})^2 (3/4\pi)^2 Z^2, \quad (2.15)$$

where, in order to obtain the latter expression, we have neglected the small matrix element connecting states with wave functions localized at largely different values of the deformation coordinate.

In Fig. 1(b), we illustrate, for the Sn nuclei, the variation of  $\rho^2$  as a function of the equilibrium deformation for the deformed shape. Here, it becomes clear that for  $\beta \approx 0.22$ ,  $\rho^2 \approx 0.1$  values can result. Only for the *strong* mixing between such nuclear configurations do very fast  $E0$  transitions indeed occur. These are precisely the cases such as <sup>98</sup>Sr, <sup>100</sup>Zr and <sup>102</sup>Mo, as we discuss in more detail in Sec. III.

Besides the nuclei discussed here, large  $E0$  transitions do occur without invoking the mixing arguments such as in light nuclei <sup>12</sup>C, <sup>16</sup>O, <sup>38</sup>Ar, <sup>60,62</sup>Ni, and in several

lanthanides<sup>19</sup> and actinides (the latter two classes since  $E0$  transitions from a  $\beta$  band head to the  $0^+$  ground state can often be strong). We do not discuss such cases in the present article.

In extending the above picture of possible mixing between states with largely different equilibrium values for the nuclear deformation, we can study the following schematic model. Whenever in a series of isotopes (or isotones) one considers a spherical ground state [dashed line in Fig. 2(a)] and an excited intruder configuration [solid line in Fig. 2(a)] which corresponds to a much larger deformed equilibrium shape, compared to the spherical ground state, the  $E0$  transition rate will show a most interesting behavior. We show this for a schematic model where the intruder state has a large but otherwise constant equilibrium deformation (as a function of  $N$  or  $Z$ ), the spherical state a small and constant equilibrium value, and crossing the ground state at some point. A similar picture would apply to odd-mass nuclei as well. If no mixing (or very weak) near the crossing point occurs, almost vanishing  $E0$  transition rates result. On the other hand, if large mixing occurs near the crossing point so as to get wave functions for both the  $0^+$  states as displayed in Eq. (2.14), large  $E0$  transition rates do occur in the crossing nuclei [Fig. 2(b)]. Although Fig. 2 is highly schematic, more realistic situations (see the mass  $A \approx 100$  region in Sec. III) indeed show such a behavior.

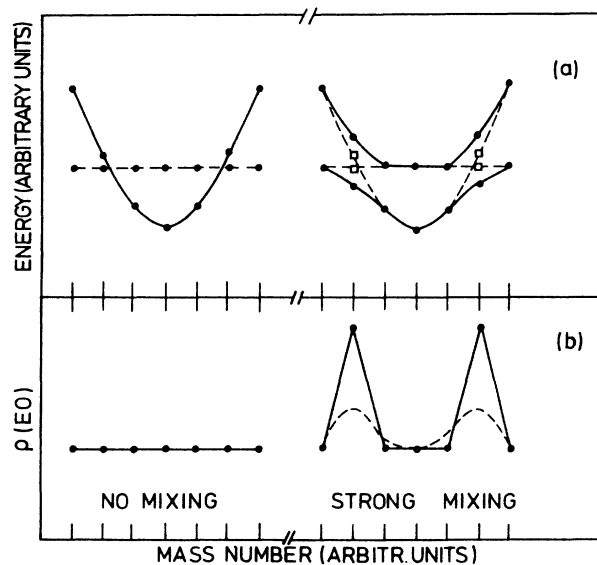


FIG. 2. (a) Schematic model for relating  $E0$  transitions and isotopic shifts where an almost spherical ground state (dashed line) and a deformed intruder state (solid line) cross. In the left part, the figure is drawn without mixing, on the right-hand side, large mixing is considered. (b) Similar figures but now for  $E0$  transitions between the excited and ground state  $0^+$  levels.

### III. MONOPOLE STRENGTH: SYSTEMATICS AND APPLICATION TO THE $A \approx 100$ REGION

In Fig. 3, we indicate the experimentally known  $\rho_{21}^2 \times 10^3$  values for the  $50 < A < 160$  mass region. Here, we exclude, however, most of the  $E0$  transitions in strongly deformed rare-earth nuclei (for such values, see Ref. 20). As can be observed from Fig. 3, most  $E0$  transition strengths are well below the  $\rho_{\text{SPU}}^2$  values (dashed line in Fig. 3). Therefore, the unit  $\rho_{\text{MMU}}^2 = \rho_{\text{SPU}}^2 \times 10^{-3}$  (where MMU denotes milli monopole unit) is often a better unit of the measure of  $E0$  strength. The few exceptions (around the  $A \approx 100$  region) are the nuclei  $^{98}\text{Sr}$ ,  $^{100}\text{Zr}$ ,  $^{102}\text{Mo}$ , where two close-lying  $0^+$  levels are present and where a large shape change for the unperturbed  $0^+$  levels is indeed expected.<sup>2,3,5,6</sup> This phenomenon is also at the origin of the  $E0$  transitions between the  $0_3^+$  and  $0_2^+$  levels in nuclei near  $^{98}\text{Zr}$  (Refs. 1 and 4) as well as for the  $^{112-118}\text{Sn}$  nuclei where  $0_3^+$  and  $0_2^+$  levels (intruder proton two-particle-two-hole configuration and quadrupole two-phonon states as the unperturbed configurations, respectively) are strongly mixed.<sup>21-24</sup>

The specific properties of  $E0$  strength in the  $A \approx 100$  region, as shown in Fig. 4, are now emerging because of a combination of investigation techniques such as ISOL measurements and measurements using in-beam spectroscopy techniques with a superconducting electron spectrometer<sup>7</sup> and the (t,p) reaction on targets of the heaviest stable isotopes of this mass region, i.e., Zr, Mo, Ru, Pd.<sup>25</sup>

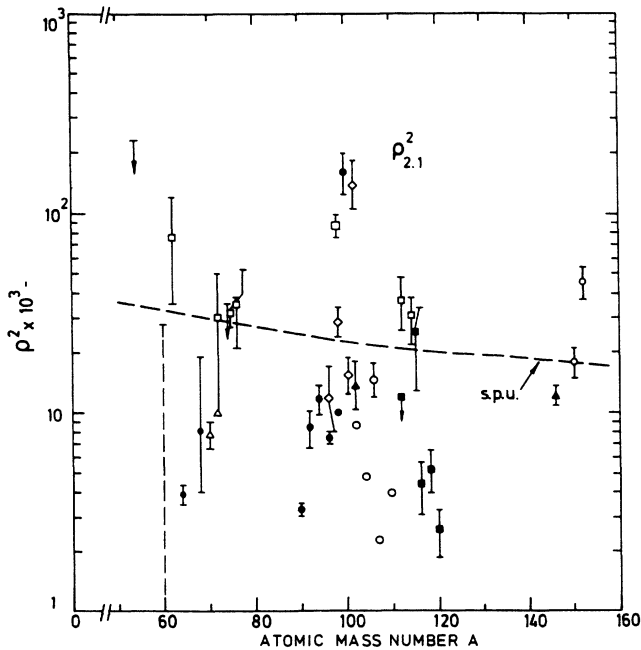


FIG. 3. Comparison for  $\rho_{21}^2(0_2^+ \rightarrow 0_1^+)$   $E0$  transitions) in the  $50 < A < 160$  mass region (the very light nuclei and strongly deformed rare-earth nuclei are excluded). Different symbols are used for different elements where we include Fe, Ni ( $\square$ ), Zn ( $\bullet$ ), Ge ( $\triangle$ ), Se ( $\square$ ), Sr ( $\square$ ), Zr ( $\bullet$ ), Mo ( $\diamond$ ), Ru ( $\blacktriangle$ ), Pd ( $\circ$ ), Cd ( $\square$ ), Sn ( $\blacksquare$ ), Sm ( $\circ$ ), Gd ( $\blacktriangle$ ). The single-particle estimate (SPU) is drawn with a dashed line.

These systematics, shown in Fig. 4 as a function of neutron number ( $N$ ), indicate a striking increase in  $E0$  strength in isotopes when approaching the  $N=60$  neutron number. It is important to recognize that for the even-even Sr, Zr, and Mo nuclei, it is always in the  $N=60$  nucleus that the first excited  $0^+$  level occurs at its lowest excitation energy, e.g., 215.5 keV ( $^{98}\text{Sr}$ ), 331.3 keV ( $^{100}\text{Zr}$ ), 696 keV ( $^{102}\text{Mo}$ ), and thus the largest mixing between the ground state and this low-lying  $0^+$  state is expected.

As is shown in Fig. 4(b), the general trend of the monopole strength for the  $N=58$  and 60 isotones is similar to that suggested in Fig. 2(b) in the preceding section, where here, we observe only half of the suggested trend. The factor of  $\approx 10$  difference in the average monopole strength can be understood within the framework of our recent study where we have shown that coexisting particle-hole excitations alter the structure of the  $A \approx 100$  nuclei.<sup>31</sup> The mechanism involves the promotion of proton pairs into the  $1g_{9/2}$  orbit and a simultaneous polarization of the neutrons to move to the  $1g_{7/2}$  orbit rather than into the  $2d_{5/2}$  orbit. We have also shown in Ref. 27 that these particle-hole excited configurations correspond to increasing deformation when going from near vibrational nuclei at  $N=50$  to gamma-soft [or  $O(6)$ -like] character, in the spectra of  $^{98}\text{Zr}^*$  (i.e., the intruder state structure in  $^{98}\text{Zr}$ ). Once  $N=60$  is reached, however, the  $1g_{7/2}$  neutron orbital becomes populated in the ground state rather than as an excited-state configuration and these nuclei possess ground states with permanent deformation. In fact, as we have shown in Ref. 31, when the intruder states in the  $N \leq 58$  nuclei are juxtaposed with the  $N \geq 60$  GS deformed bands, the series exhibits a smooth onset of deformation rather than a rapid transition. Thus, at first sight, the large jump in  $E0$  strength seems unaccounted for. That is, of the two states that are mixing, one remains more spherical (vibrationlike), while the other is becoming more and more deformed in a smooth way. However, the main difference lies in the deformed state which in the  $N \leq 58$  nuclei is a dynamically deformed intruder deformed configuration. This is in contrast to the  $N=60$  nuclei where the deformed configuration becomes the GS with permanent deformation and hence has a much larger effective deformation in comparison with the excited spherical state.

A further manifestation of the nature of the coexisting structures arises from the general shape of the monopole strength shown in Fig. 4(b). Since it is the n-p interaction that gives rise to the coexisting state, we might expect that the general trend would follow the  $N_p N_n$  principle that has been discussed by Casten.<sup>32</sup> As can be seen in Fig. 4(b), the peak of the values for the  $N=60$  curve is shifted by two mass units with respect to those for the  $N=58$  curve. However, if the two curves are considered as a function of  $N_n N_p$ , the maximum occurs at the same value.

### IV. CONCLUSIONS

In order to establish the origin of the large monopole strength observed in a number of  $A \approx 100$  nuclei, we first

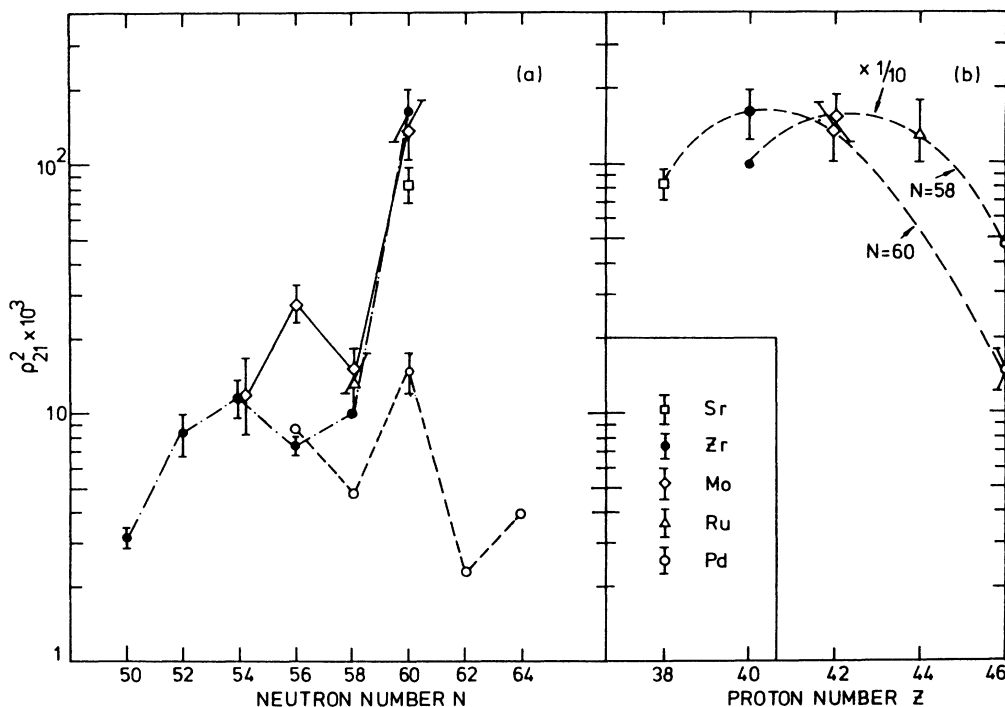


FIG. 4. (a) Detailed results for  $\rho_{21}^2$  in the  $A \approx 100$  mass region. The data, discussed in the present study, are taken from Refs. 1, 4, and 25–30. (b) Monopole strength for the  $N = 58$  and  $60$  isotones. The values for the  $N = 58$  nuclei have been multiplied by a factor of 10 for comparison.

reviewed the correct form of the  $E0$  strength within several different prescriptions. We used this to show that large  $E0$  strengths arise as a result of strong mixing between states with different shapes and that the observation of large  $E0$  strength in itself is not evidence for shape coexistence in a particular nucleus. We went on to show that the difference of a factor of 10 in the monopole strength between  $N = 58$  and  $N = 60$  isotones can be understood within the context of dynamical deformation. That is, the  $N = 60$  nuclei have the deformed state as their ground state and the spherical state as a low-lying excited state. This is in contrast to the  $N = 58$  nuclei where the deformed state not only occurs at a higher energy for the same isotope but exists because of promoted pairs that produce a dynamical deformation. Further, we

have shown that the maximum for both the  $N = 58$  and  $N = 60$  isotones occurs at the same value of  $N_p N_n$  which should be expected for states arising out of the  $n$ - $p$  interaction.

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