Quenching of g_A in the nuclear medium

R. Parthasarathy
Institute of Mathematical Sciences, Madras, 600113, India

J. Pasupathy
Centre for Theoretical Studies, Indian Institute of Science, Bangalore, 560012, India
(Received 28 August 1987)

The numerical values of g_A are evaluated using quantum-chromodynamic sum rules. The nuclear medium effects are taken into account by modifying the chiral symmetry breaking correlation, $\langle \bar{q}q \rangle$. Our results indicate a quenching of g_A in a nuclear medium. The physical reasons for this fundamental quenching are noted to be the same for the effective mass of the nucleon bound in a nucleus being less than its free space value.

It is widely believed that the properties of nucleon as a free particle are modified when the nucleon is in a nuclear medium. Of particular interest is the axial-vector coupling constant g_A. In a world where chiral symmetry is exact, g_A is expected to be unity. The significant deviation of g_A from this bare value is adequately explained by postulating a partially conserved axial-vector current.[1] In a nuclear medium it has been suggested[2] that the renormalization of the axial vector current is intrinsically modified. As a consequence g_A and other weak couplings are expected to be significantly different from their free nucleon values.

Recently Perez and Buck[3] performed a new and model independent analysis of beta decay and magnetic moments in mirror nuclei $3 \leq A \leq 39$ and concluded that $g_A \simeq 1.00 \pm 0.002$. Rho[4] has analyzed the (p,n) data on giant Gamow-Teller resonances based on the study of Horen *et al.*[5] and Gaarde *et al.*[6] and concluded that $g_A \sim 1.00$ fits the data extremely well. Quite recently Rho[7] himself offered an explanation for the quenching of g_A (in addition to f_π) and for the enhancement of the rms size of the nucleon on the basis of a Skyrmion picture of nucleon, using the axial Ward identities in a nuclear medium.

In this paper we shall consider this problem from the point of view of quantum chromodynamic (QCD) sum rules. The magnetic moments[8] and the axial vector renormalization constants[9,10] of the nucleon have been successfully computed by the method of QCD sum rules, by studying the nucleon current correlation functions in the presence of an external electromagnetic or an axial vector field. In particular the departure of g_A from unity has been shown to arise from the polarization of the QCD vacuum by the external field and the interaction of the external field with the vacuum fields of the quarks and gluons. The nucleon mass according to Ioffe[11] is basically determined by the chiral symmetry breaking correlator $\langle \bar{q}q \rangle$. On the other hand, we know from nuclear physics that the effect of the interaction of a nucleon with the remaining nucleons in a nuclei is to introduce an effective mass m^* for the nucleon which is smaller than its free space value m. From Ioffe's work[11] we know that the nucleon mass is given to a first approximation

$$m \propto | \langle \bar{q}q \rangle |^{1/3} . \tag{1}$$

This suggests in turn that the effect of nuclear medium on other nucleon properties as well can be deduced in an analogous fashion from that of the free nucleon if we replace the nuclear medium effects by an effective chiral symmetry breaking parameter.[12] Given this view point, it is straight forward to account for the quenching of g_A in a nuclear medium due to a reduction in the chiral symmetry breaking parameter, using Eq. (1).

The axial coupling constant g_A is computed by studying the following correlation in the presence of an external axial vector field Z_μ:

$$\prod(p^2)=i \int d^4x \exp(ipx)\langle 0 | T[\eta(x)\bar{\eta}(0)] | 0 \rangle |_{Z_\mu} , \tag{2}$$

where

$$\eta(x)=u^a(x)C\gamma_\mu u^b(x)\gamma_\mu\gamma_5 d^c(x)\epsilon^{abc} , \tag{3}$$

with C as the charge conjugation matrix, $u^a(x)$ and $d^a(x)$ are the up and down quark fields, and a,b,c are color indices. In the sum rule approach $\prod(p^2)$ is calculated as an asymptotic expansion in p^2 using the operator product expansion (OPE) for the product $\eta(x)\bar{\eta}(0)$. The nonperturbative aspect of a QCD vacuum enters by assigning nonzero vacuum expectation values (VEV) for the objects like $\bar{q}q$, $G^a_{\mu\nu}G^{\mu\nu a}$, etc.[13] While in the absence of the external field Z_μ, only Lorentz invariant scalars can have nonzero VEV. In the presence of Z_μ there will be external field induced correlations such as $\bar{q}\gamma_\mu\gamma_5 q$; $\bar{q}\,\tilde{G}_{\mu\nu}\gamma_\nu q$, ($\tilde{G}_{\mu\nu}=\frac{1}{2}\epsilon_{\mu\nu\alpha\beta}G^{\alpha\beta}$), etc., will also be present which should be taken into account. The details of the derivation of these sum rules can be found in Refs. 8–11. To arrive at the value of g_A from an analysis of the sum rule, we shall follow here the procedure of Ref. 10. This is based on the fact that the sum rule in the presence and in the absence of the external field are closely similar and the departure from unity of g_A can be derived from the following ratio function:

$$\frac{M^6/8L^{4/9}+M^2b/32L^{4/9}+a^2L^{4/9}(\frac{1}{6}+\frac{1}{9})+M^2Ka/6L^{68/81}}{M^6/8L^{4/9}+M^2b/32L^{4/9}+a^2L^{4/9}/6} = g_A + AM^2+[\gamma+\delta(W^2-m^2+M^2)]\exp[-(W^2-m^2)/M^2] \ . \quad (4)$$

Here M is the Borel mass parameter,

$$L=\ln(M^2/\Lambda^2)/\ln(\mu^2/\Lambda^2) \ ,$$

Λ is the QCD scale parameter (~ 100 MeV), μ is the renormalization scale (~ 500 MeV), $b=4\pi\langle \alpha_s G^a_{\mu\nu}G^{\mu\nu a}\rangle$ (the gluon condensate), $a=-(2\pi)^2\langle \bar{q}q\rangle$ is the chiral symmetry breaking correlation, $K=-\xi f_\pi 0.2\ \text{GeV}^2/\langle \bar{q}q\rangle$, $f_\pi=133$ MeV, and γ and δ are the coefficients of the continuum state contributions to the sum rules.

In the above, the numerator of the left-hand side is the Borel transform of the correlator [Eq. (2)] in the presence of external field while the denominator is the Borel transform in the absence of the external field M, the Borel mass parameter. It is seen that the asymptotically leading terms of the numerator and denominator in the left hand side are identical. The coefficient of the a^2 term in the numerator of the left hand side of Eq. (4) has an additional factor $\frac{1}{9}$, compared with the coefficient of the a^2 term in the denominator, arising from the interaction of the external field with the soft quark fields, and the last term in the numerator arises from the correlator $\bar{q}\,\tilde{G}_{\mu\nu}\gamma_\nu q$. Its value is not very well known, but has been estimated in Ref. 14. We have defined

$$\langle 0|\bar{q}\,\tilde{G}_{\mu\nu}\gamma_\nu q\,|0\rangle = KZ_\mu\langle 0|\bar{q}q\,|0\rangle$$

with

$$K\langle 0|\bar{q}q\,|0\rangle = -\tfrac{1}{3}\xi f_\pi^2 0.2\ \text{GeV}^2 \ .$$

According to Ref. 14, $\xi=1.0$. The a^2 term is found to be numerically more important than the last term in the numerator on the left hand side of Eq. (4). For the right hand side we have used the ansatz form given in Refs. 10 and 12. Basically, it is the ratio of the correlator [Eq. (2)] evaluated this time in terms of physical intermediate states with and without the external field Z_μ. The nucleon pole contribution in the presence of Z_μ is computed from

$$\langle 0|\eta|N\rangle\langle N|j^5_\mu Z_\mu|N\rangle\langle N|\bar{\eta}|0\rangle \ .$$

The advantage of using the ratio of Eq. (2) with and without Z_μ is that one need not know the value of the coupling λ_n, where $\langle 0|\bar{\eta}|\text{nucleon}\rangle=\lambda_n U(p)$ is the coupling strength of the current to the one nucleon state. The term AM^2 arises from the external field induced transition between the nucleon and excited states. The last term in the first square bracket on the right hand side of Eq. (4) is the contribution of the excited states and is represented by an effective mass W^2 taken to be 2.3 GeV^2.[9-11] To determine g_A from Eq. (4), we first note that if the two sides of the above equation are matched asymptotically, then one finds $g_A+\gamma=1$ (coefficient of the constant term) and $\delta+A=0$ (coefficient of the M^2 term). Then we adopt the procedure of Refs. 10 and 12 which is as follows. Fix δ at an initial value, say $\delta=0$. Start with an arbitrary value for γ. Evaluate (left hand side of Eq. (4) $-\gamma\exp[-(W^2-m^2)/M^2]$) and fit this to $\rho+\sigma M^2$ for M^2 around the nucleon mass. If the output did not satisfy the condition $\rho+\gamma=1$ adopt a new value for γ as (γ input $+1-\rho)/2$ and iterate until $\rho+\gamma=1$. Then the final value of ρ gives g_A. It has been found that the iteration converges quickly and the final value for $\rho(=g_A)$ is independent of the initial choice of γ. The coefficient σ and, consequently, A is small. Changing the initial value of δ produces only small variations in g_A (see Ref. 11). For $W^2=2.3\ \text{GeV}^2$, $\xi=1$, and $a=0.45\ \text{GeV}^3$, it has been found for a free nucleon $g_A=1.38$ and for $\xi=-2.0$, the corresponding value is 1.28. These values are satisfactory in view of the approximations involved in QCD sum rules.

Consider now a nucleon inside a nuclear medium. As remarked earlier, we shall account for the change in g_A for bound nucleon by introducing a modified value for $\langle \bar{q}q\rangle$ in a nuclear medium. Using Eq. (1), we can write approximately, $\langle \bar{q}q\rangle_{\text{nuclear medium}}=\lambda^3\langle \bar{q}q\rangle_{\text{physical vacuum}}$, where $\lambda=m^*/m$, the ratio of the effective mass of the

TABLE I. Axial vector coupling constant g_A using QCD sum rules. Nuclear medium renormalization effects are taken into account by using $a^*=\lambda^3 a$, $f^*_\pi=\lambda f_\pi$, where $\lambda=m^*/m$, $f_\pi=133$ MeV, and $a=0.45\ \text{GeV}^3$.

		g_A		
		$\xi=1.0$		$\xi=-2.0$
λ	a GeV^3	$b=0.5$ (GeV^4)	$b=0.4$ (GeV^4)	$b=0.5$ (GeV^4)
1.0	0.450	1.38	1.39	1.28
0.9	0.328	1.25	1.25	1.15
0.8	0.234	1.14	1.15	1.06
0.7	0.154	1.08	1.08	1.01
0.6	0.097	1.04	1.04	0.99
0.5	0.056	1.02	1.02	0.98

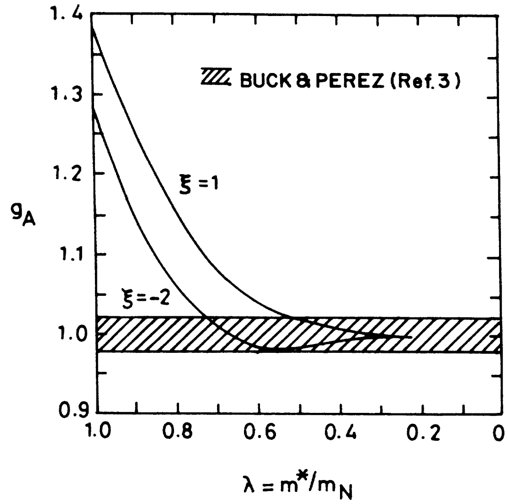

FIG. 1. Variation of g_A with $\lambda(=m^*/m)$ for two values of the parameter ξ. The experimental result of Buck and Perez (Ref. 3) are shown by the shaded region.

nucleon in a nuclear medium to its free space value. As said above, the renormalization of g_A arises both from the change in the coefficient of the a^2 term in the numerator of the left hand side of Eq. (4) and from the nonzero value of the correlator $\langle \bar{q}\, \tilde{G}_{\mu\nu}\gamma_\nu q \rangle$. However, the effect of the latter is numerically less significant than the a^2 term. To consider the nuclear medium effects on $\langle \bar{q}\, \tilde{G}_{\mu\nu}\gamma_u q \rangle$, we take it to be modified in the same way as f_π^2. Following Rho[7] we have

$$f_\pi^*/f_\pi = m^*/m = \lambda \ . \quad (5)$$

In principle, the gluon condensate $\langle G_{\mu\nu}^a G^{\mu\nu} \rangle$ will also be modified.

In Table I we present the results of the numerical calculations using Eq. (4) and with the modified values of the chiral symmetry breaking parameter a. As in Ref. 12, we have considered both the values for ξ as 1.0 and -2.0.

For free nucleon ($\lambda=1$) the value $\xi=-2.0$ yields g_A closer to the experiment than with $\xi=1.0$. In either case it is seen that as λ decreases g_A decreases to unity. We have also varied the gluon condensate b which has negligible effect on our numerical estimates of g_A. In nuclear matter $m^* \sim 0.6m$ (Refs. 7 and 15) and then we find $g_A \sim 1.00$.

A few comments are in order. Our results indicate that the physical reasons for the quenching of g_A are the same for the effective mass of the nucleon in a nuclear medium being less than its free space value. For specific nuclei, once m^* is known our procedure gives the corresponding quenched values of g_A. In Fig. 1 we have shown the variation of g_A with λ for $\xi=1.0$ and -2.0 along with the experimental result of g_A by Perez and Buck.[3]

In summary, using the concept that the effective mass of a nucleon bound in a nucleus is smaller than the free nucleon mass and by attributing it to an effective reduction of the chiral symmetry breaking correlator $\langle \bar{q}q \rangle$, our calculations indicate how the quenching of g_A in a nucleus can arise due to nuclear effects.

[1]M. L. Goldberger and S. B. Treiman, Phys. Rev. **110**, 1478 (1958); Y. Nambu, Phys. Rev. Lett. **4**, 380 (1960); W. I. Weisberger, *ibid.* **14**, 1047 (1965); S. Adler, *ibid.* **14**, 1051 (1965).

[2]M. Rho, Phys. Rev. Lett. **18**, 671 (1967); M. Ericson, Ann. Phys. (N.Y.) **63**, 562 (1971); D. H. Wilkinson, in *Nuclear Physics with Heavy Ions and Mesons*, edited by R. Balian, M. Rho, and G. Ripka (North-Holland, Amsterdam, 1978).

[3]B. Buck and S. M. Perez, Phys. Rev. Lett. **50**, 1975 (1983).

[4]M. Rho, Annu. Rev. Nucl. Part. Sci. **34**, 532 (1984).

[5]D. J. Horen *et al.*, Phys. Lett. **95B**, 27 (1980).

[6]C. Gaarde, J. S. Larsen, and J. Rapaport, in *Spin Excitations in Nuclei*, edited by F. Petrorich *et al.* (Plenum, New York, 1983) as quoted in Ref. 4.

[7]M. Rho, Phys. Rev. Lett. **54**, 767 (1985).

[8]B. L. Ioffe and A. V. Smilga, Nucl. Phys. **B232**, 109 (1984); I. I. Balitsky and A. V. Yung, Phys. Lett. **129B**, 328 (1983); C. B. Chiu, J. Pasupathy, and S. L. Wilson, Phys. Rev. D **33**, 1961 (1986); **36**, 1442 (1987); **36**, 1451 (1987); **36**, 1553 (1987).

[9]V. M. Belyaev and Y. I. Kogan, Phys. Lett. **136B**, 273 (1984).

[10]C. B. Chiu, J. Pasupathy, and S. L. Wilson, Phys. Rev. D **32**, 1786 (1985).

[11]B. L. Ioffe, Nucl. Phys. **B188**, 317 (1981).

[12]J. Pasupathy, in *Proceedings of the Symposium on Recent Trends in Nuclear Physics,* edited by K. Srinivasa Rao (in press).

[13]G. K. Savviddy, Phys. Lett. **71B**, 133 (1977); M. A. Shifman, A. I. Vainshtein, and V. I. Zakharov, Nucl. Phys. **B147**, 385 (1979); H. Leutwyler, *ibid.* **B179**, 129 (1981); R. Parthasarathy, M. Singer, and K. S. Viswanathan, Can. J. Phys. **61**, 1442 (1983); D. Kay, R. Parthasarathy, and K. S. Viswanathan, Phys. Rev. D **28**, 3116 (1983).

[14]V. A. Novikov, M. A. Shifman, A. I. VAinshtein, M. B. Voloshin, and V. I. Zakharov, Nucl. Phys. **B237**, 525 (1984).

[15]J. D. Walecka, Ann. Phys. **83**, 491 (1974); B. Serot, in *Hadron Substructure in Nuclear Physics (Bloomington, Indiana),* Proceedings of the 1983 Indiana University Nuclear Physics Workshop on Manifestations of Hadron Substructure in Nuclear Physics, AIP Conf. Proc. No. 110, edited by W. Y. P. Hwang and M. H. Macfarlane (AIP, New York, 1983).