Relativistic Faddeev calculation of K^+d scattering

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The reaction $K^+d \to K^+d$ has been calculated for incident kaon momenta up to 1.5 GeV/c, using the relativistic Faddeev equations. The effect of multiple scattering corrections and of higher K^+N partial waves has been evaluated, and the sensitivity of the results to the deuteron wave function and to variations of the on- and off-shell behavior of the elementary K^+N amplitude has been studied. Some implications of the results in connection with Glauber's high-energy limit are also discussed.

I. INTRODUCTION

The strangeness $+1$ K^+d system is one of the simplest three-body problems in medium-energy physics. It is simpler than the $\pi^{\pm}d$ system, where one has to always deal with the pion-absorption channel. It is also simpler than the strangeness -1 K^-d system, where the conversion channels $\pi N\Lambda$ and $\pi N\Sigma$ are also open already at zero energy. Thus this reaction offers perhaps the best opportunity to test the relativistic Faddeev method and maybe even to learn something about the elementary K^+N amplitude. Moreover, with the proposed construction of kaon factories in the near future,[1] this reaction will become one of the first systems to investigate experimentally, since its importance lies in the fact that this is not only an exactly soluble three-body system, but it is also the first case of the more general problem of kaon-nucleus scattering.

Historically, the main motivation for studying K^+d scattering[2-10] has been its use as a tool in the process of extracting the isospin zero K^+N amplitude. Since K^+p scattering data provide information only about the isospin one amplitude, then in order to learn about the isospin zero amplitude one has to scatter the K^+ on a neutron, which, however, is only available as a bound particle inside the deuteron. Thus the standard procedure has been to apply the fixed-scatterer impulse approximation[11] to the elastic, breakup, and charge exchange K^+d data, so as to relate these cross sections directly to the elementary K^+N amplitudes. In a few cases, however, some calculations have been performed which go beyond the fixed-scatterer impulse approximation, like, for example, those of Hashimoto,[12] and of Andrade and Ferreira,[13] in which both the single- and double-scattering terms were included; in the first case using Glauber theory and in the second one taking into account the effects of Fermi motion. The first full three-body calculation of K^+d scattering was performed by Hetherington and Schick,[14] who restricted themselves to low energies ($k_{\rm lab} \leq 230$ MeV/c) and in which only S-wave two-body interactions were included. They found that the multiple scattering corrections are of the order of 10–25 %. They also found that using a relativistic phase space could produce changes in the differential cross section of approximately 10% at an incident momentum already as low as 230 MeV/c, which indicates that if one likes to go to energies higher than this, one has to use a formalism that takes into account relativistic effects. More recently, another Faddeev calculation of the K^+d system has been performed by Sañudo,[15,16] who included exactly the NN $^3S_1$ and the K^+N S_{11} channels, and treated the other S and P-wave K^+N channels by means of perturbation theory. He used his results to extract a set of values for the isospin zero K^+N scattering lengths and volumes.

The relativistic Faddeev equations have been used so far mainly to describe the πd system,[17-19] where one has, in addition to the elastic and breakup channels, also the pion-absorption channel. Several prescriptions have been proposed in the literature[20-23] to incorporate the pion-absorption channel into the relativistic Faddeev method, with the result that different formulations lead sometimes to quite different predictions for the observables of the πd system. It has been observed, however, that if the pion-absorption channel is neglected, all the calculations agree with each other.[24] Thus the relativistic Faddeev method appears to be particularly well suited to describe the K^+d system, where there is no absorption channel. Of course, if the energy of the pion is large enough so that the pion-production channels become important, a three-body description may no longer be sufficient (we take into account the production channels only indirectly by using K^+N amplitudes that have inelasticities) and one will have to go at least to a four-body formulation. Thus the validity of our model is probably restricted to momenta of less than 1 GeV/c.

This paper is organized as follows: In Sec. II the method of the relativistic Faddeev equations is briefly reviewed, while in Sec. III the two-body input of the equations is described. In Sec. IV the results of the calculations are presented, and finally the conclusions and a general discussion of the results are given in Sec. V.

II. THE RELATIVISTIC FADDEEV METHOD

The analog of the nonrelativistic Faddeev equations can be obtained also in the relativistic case, by summing all possible Feynmann diagrams of the form where two particles interact with one another while the third parti-

cle acts as spectator. This leads to linear integral equations similar to those of the nonrelativistic three-body problem, but which depend on four-dimensional variables.[25–27] Thus in order to eliminate some of these extra variables without destroying Lorenz invariance and unitarity, two prescriptions have been proposed in the literature. The first one is to carry out a so-called Blankenbecler-Sugar reduction,[28] in which one puts the three particles on the mass shell and performs a dispersion integral in the total energy squared of the system. The second prescription consists simply in putting all the spectator particles on the mass shell and all the exchanged particles off the mass shell.[29,30] We will use for the present calculation the equations obtained[27] with the Blankenbecler-Sugar prescription, although these equations are not completely consistent in the way in which they handle particles off the mass shell, as pointed out in Ref. 30, but they should be adequate for the K^+NN system, where there is no absorption channel. On the other hand, the spectator-on-mass-shell prescription is to be preferred in the cases when particles can be created or absorbed like in the πNN or K^-NN systems. The Blankenbecler-Sugar prescription leads to integral equations which depend only on three-dimensional variables. If, in addition, one introduces the separable or isobar approximation for the two-body amplitudes and performs a partial-wave decomposition, one ends up finally with integral equations that depend only on one continuous variable.

In order to perform the partial-wave decomposition of the three-body equations in the case when the particles have spin, it is convenient to use basis states that are a linear combination of the three-body helicity states constructed by Wick.[31] Thus our basis states are[32]

$$|k_i p_i;JT;\alpha_i\rangle = |k_i p_i;JT;l_i s_i j_i t_i m_i \nu_i\rangle \ , \tag{1}$$

where k_i is the magnitude of the three momentum of particle i as measured in the three-body c.m. frame, and p_i is the magnitude of the relative three momentum of particles j and k as measured in the two-body c.m. frame. J and T are the total angular momentum and total isospin of the three-body system, while l_i, s_i, j_i, and t_i are the orbital angular momentum, spin, total angular momentum, and isospin of the pair jk, and m_i and ν_i the helicities of the pair jk and of particle i.

Using the basis states (1), the relativistic Faddeev equations are written as

$$T_{i1}^{\alpha_i,\alpha_{10}}(k_i,k_{10}) = (1-\delta_{i1})B_{i1}^{\alpha_i,\alpha_{10}}(k_i,k_{10}) + \sum_{j\neq i}\sum_{\alpha_j}\int_0^\infty \frac{k_j^2 dk_j}{2\omega_j(k_j)} B_{ij}^{\alpha_i,\alpha_j}(k_i,k_j) G_j^{\alpha_j}(s_j) T_{j1}^{\alpha_j,\alpha_{10}}(k_j,k_{10}) \ , \tag{2}$$

where $\omega_j(k_j)=(k_j^2+m_j^2)^{1/2}$, and s_j is the invariant mass squared of the pair ki. G_j are the propagators of the interacting pair ki which will be defined in the next section, while the driving terms B_{ij} are given by

$$B_{ij}^{\alpha_i,\alpha_j}(k_i,k_j) = \int_{-1}^{1} d\cos\chi \, A_{ij}^{\alpha_i,\alpha_j}(k_i,k_j,\cos\chi) \frac{g_i^{\alpha_i}(p_i) g_j^{\alpha_j}(p_j)}{2\omega_k(|\mathbf{k}_i+\mathbf{k}_j|)} \frac{\omega_i(k_i)+\omega_j(k_j)+\omega_k(|\mathbf{k}_i+\mathbf{k}_j|)}{S-[\omega_i(k_i)+\omega_j(k_j)+\omega_k(|\mathbf{k}_i+\mathbf{k}_j|)]^2+i\varepsilon} \ , \tag{3}$$

where S is the total invariant mass squared of the three-body system, g_i and g_j are the form factors of the separable two-body amplitudes, and

$$\begin{aligned} A_{ij}^{\alpha_i,\alpha_j}(k_i,k_j,\cos\chi) = {} & (-)^{t_j+\tau_j-T}[(2t_i+1)(2t_j+1)(2j_i+1)(2j_j+1)]^{1/2} \\ & \times W(\tau_j\tau_k T\tau_i;t_i t_j) \sum_{\lambda_j\lambda_k}\sum_{\mu_k\mu_i} b_{\lambda_j\lambda_k}^{l_i s_i j_i} b_{\mu_k\mu_i}^{l_j s_j j_j} (-)^{\sigma_j-\nu_j+\sigma_k+\mu_k} \\ & \times d_{m_j-\nu_j,m_i-\nu_i}^{J}(\chi) d_{m_i,\lambda_j-\lambda_k}^{j_i}(\theta_i) d_{m_j,\mu_k-\mu_i}^{j_j}(\theta_j) d_{\nu_i,\mu_i}^{\sigma_i}(\beta_i) \\ & \times d_{\nu_j,\lambda_j}^{\sigma_j}(\beta_j) d_{\mu_k,\lambda_k}^{\sigma_k}(\rho_k) \ , \end{aligned} \tag{4}$$

with

$$b_{\lambda_j\lambda_k}^{l_i s_i j_i} = [(2l_i+1)/(2j_i+1)]^{1/2} C_{0,\lambda_j-\lambda_k}^{l_i s_i j_i} C_{\lambda_j,-\lambda_k}^{\sigma_j\sigma_k s_i} \ , \tag{5}$$

where τ_i,τ_j,τ_k, and $\sigma_i,\sigma_j,\sigma_k$ are the isospins and spins of the three particles, and the arguments of the rotation matrices χ, θ_i, θ_j, β_i, β_j, and ρ_k are the angles of the Wick triangle.[31,32]

In the integral equations (2) we still have not taken into account the fact that in the kaon-deuteron system two of the particles are identical. Since, as we will see in Sec. IV, our results differ from those of previous calculations,[15,16] it is also important to describe in some detail our antisymmetrization procedure. Thus if we call the kaon particle 1 and the two nucleons particles 2 and 3, it is more convenient to start instead of with the basis states $|1\rangle$, $|2\rangle$, and $|3\rangle$ defined by Eq. (1), with the states $|1\rangle$, $|2'\rangle$, and $|3\rangle$, where the state $|1\rangle$ is obtained by coupling particles 2 and 3 and their resultant to particle

1, the state $|3\rangle$ is obtained by coupling particles 1 and 2 and their resultant to particle 3, and the state $|2'\rangle$ is obtained from the state $|3\rangle$ by interchanging particles 2 and 3. Thus the driving terms with these new states are given in terms of the old ones, as

$$B_{12'}=-B_{13}\ , \tag{6}$$

$$B_{32'}=B_{32}(-)^{\sigma_1+\sigma_3-s_2+\tau_1+\tau_3-t_2+l_2}\ . \tag{7}$$

Equation (6) is a consequence of the fact that state $|1\rangle$ must be antisymmetric with respect to the exchange of particles 2 and 3, while the extra phase in Eq. (7) is due to the different order in which the particles are coupled in states $|2'\rangle$ and $|3\rangle$. Using now Eqs. (6) and (7), it is easy to see that the set of equations (2) which involved originally three coupled amplitudes T_{11}, T_{21}, and T_{31}, reduce to a set with only two coupled amplitudes which symbolically can be written as

$$T_{11}=2B_{13}G_3T_{31}\ , \tag{8}$$

$$T_{31}=B_{31}-B_{32}(-)^{\sigma_1+\sigma_3-s_2+\tau_1+\tau_3-t_2+l_2}G_3T_{31} +B_{31}G_1T_{11}\ , \tag{9}$$

where the driving terms B_{ij} in Eqs. (8) and (9) are the normal ones given by Eq. (3). Finally, as shown in Ref. 32, the set of integral equations (8) and (9) can be decoupled into two independent sets which correspond to the two possibilities of positive and negative parity, by introducing the linear combination of amplitudes

$$T_{i1\pm}^{\alpha_i\alpha_1}=T_{i1}^{\alpha_i\alpha_1}\pm(-)^{J+l_i-s_i-j_i}T_{i1}^{-\alpha_i\alpha_1}\ , \tag{10}$$

where the set of quantum numbers $-\alpha_i$ are those in which the helicities are reversed, that is

$$\{-\alpha_i\}\equiv\{l_is_ij_it_i-m_i-\nu_i\}\ . \tag{11}$$

III. TWO-BODY INPUT

The input of the integral equations (1), are the kaon-nucleon and nucleon-nucleon T matrices which will be taken to be of separable form. Thus in the case of the K^+N subsystem, they are given by

$$\langle k_ip_i;JT;\alpha_i\,|\,t_i\,|\,k_i'p_i';J'T';\alpha_i'\rangle =\delta_{JJ'}\delta_{TT'}\delta_{\alpha_i\alpha_i'}\frac{1}{k_i^2}\delta(k_i-k_i')t_i^{\alpha_i}(p_i,p_i';s_i)\ , \tag{12}$$

where

$$t_i^{\alpha_i}(p_i,p_i';s_i)=g_i^{\alpha_i}(p_i)G_i^{\alpha_i}(s_i)g_i^{\alpha_i}(p_i')\ , \tag{13}$$

and the propagators of the interacting pair G_i are related to the phase shifts (we use the K^+N phase shifts and inelasticities of Martin[9]) as

$$G_i^{\alpha_i}(s_i)=-e^{i\delta(s_i)}\sin\delta(s_i)\frac{4\sqrt{s_i}}{\pi p_{i0}[g_i^{\alpha_i}(p_{i0})]^2}\ , \tag{14}$$

$$s_i=S+m_i^2-2\sqrt{S}\,(m_i^2+k_i^2)^{1/2}\ , \tag{15}$$

$$p_{i0}^2=[s_i-(m_j+m_k)^2][s_i-(m_j-m_k)^2]/4s_i\ , \tag{16}$$

in the physical region $s_i\geq s_{i0}=(m_j+m_k)^2$, while for the unphysical region $s_i\leq s_{i0}$, we used the extrapolation formula

$$G_i^{\alpha_i}(s_i)=G_i^{\alpha_i}(s_{i0})\frac{1}{(2-s_i/s_{i0})^n}\ , \tag{17}$$

where for the exponent n we have used $n=2$. The form factors g_i were taken as

$$g_i^{\alpha_i}(p_i)=p_i^{l_i}\frac{1}{(\beta^2+p_i^2)^m}\ , \tag{18}$$

with $\beta=1000$ MeV/c. We used for the exponent m the value $m=1$ for the S and P waves ($S_{01},S_{11},P_{01},P_{11},P_{03},P_{13}$) while for the D and F waves ($D_{03},D_{13},D_{05},D_{15},F_{05},F_{15}$) we took $m=2$.

In the case of the nucleon-nucleon subsystem, we constructed a separable T matrix for the $^3S_1-{}^3D_1$ channel, which is a straightforward generalization of Eqs. (12) and (13) to the case of coupled waves. In this case, however, instead of using the phase shifts directly as in Eq. (14), we applied the unitary pole approximation to the Paris potential[33] or so-called PEST1 approximation.[34] Thus the form factors g_i and the propagator of the interacting pair G_i are constructed directly from the S- and D-wave components of the Paris deuteron wave function.[35] This choice, of course, guarantees that the initial and final state deuteron wave function is the Paris wave function[35] which gives a very good description of the deuteron electromagnetic form factor.[36] A similar PEST1 approximation was used for the nucleon-nucleon interaction in the $^1S_0$ channel.[34] Since these T matrices are solutions of the nonrelativistic Lippmann-Schwinger equation, one must then apply the minimal relativity transformation[37] in order to make them solutions of the Blankenbecler-Sugar equation, that is

$$t_i^{\rm BS}(p_i,p_i';s_i)=4M(M^2+p_i^2)^{1/4}(M^2+p_i'^2)^{1/4}t_i^{\rm LS}(p_i,p_i';E_i)\ , \tag{19}$$

where the invariant mass squared of the pair s_i is related to the nonrelativistic energy E_i as

$$s_i=4M(M+E_i)\ . \tag{20}$$

IV. RESULTS

The integral equations (2) were solved using the method of the Padé approximants, where a variable Gauss mesh of 54 points was used to integrate over the logarithmic singularities and discontinuities of the kernel (3) below the breakup threshold of the momentum k_i, while in the region above the breakup threshold of k_i where the kernel is nonsingular, the integrations were performed using a 34-point Gauss mesh. This leads to an estimated accuracy of the numerical solutions of approximately 1%. The integral equations were solved taking as input the six S and P wave kaon-nucleon channels ($S_{01},S_{11},P_{01},P_{11},P_{03},P_{13}$) and the nucleon-nucleon $^3S_1$-

$^3D_1$ and $^1S_0$ channels as described in the previous section. Since multiple scattering effects are important only for the low angular momentum partial waves, the full integral equations were solved for values of the total angular momentum $0 \leq J \leq 5$, while for $6 \leq J \leq 24$ the impulse approximation was used. In addition, the contribution of the kaon-nucleon D and F waves $(D_{03}, D_{13}, D_{05}, D_{15}, F_{05}, F_{15})$ was added in the impulse approximation for $0 \leq J \leq 24$.

As a first application of this theory, we show in Fig. 1 the total and integrated elastic cross sections (the total cross section was obtained via the optical theorem) compared with the full calculation and with the impulse approximation for laboratory momenta up to 1.5 GeV/c. As one can see, at low momenta the impulse approximation leads to gross violations of unitarity, since it predicts $\sigma_{EL} \rangle \sigma_{TOT}$. At momenta larger than $\sim$400 MeV/c, the cross sections are well described by the impulse approximation, since here the multiple scattering corrections increase the elastic cross section by about 10–20%, and have a much smaller effect in the total cross section. The result that the multiple scattering corrections decrease the elastic cross section for $p_{lab} \lesssim 250$ MeV/c, is in agreement with the calculations of Hetherington and Schick.[14] This behavior is also expected on quite general grounds.[11] A somewhat puzzling result is seen as the momentum of the projectile increases, since according to Glauber's high-energy limit,[41–44] the total cross section including single and double scattering terms should be about $\frac{1}{2}$ mb smaller[39] than the impulse approximation $[\sigma_{TOT}(K^+d) = \sigma_{TOT}(K^+p) + \sigma_{TOT}(K^+n) - \Delta\sigma$, where $\Delta\sigma > 0$ if the K^+N amplitudes are purely imaginary], while in Fig. 1 one sees that the result of the full calculation is almost equal to that of the impulse approximation. In order to understand the origin of this behavior, we show in Table I the results for the total cross section at $p_{lab} = 1210$ and 1510 MeV/c, calculated with only the single-scattering term, with single plus double scattering terms, with single plus double plus triple scattering terms and with the full calculation. The double-scattering term is that in which the kaon scatters first on one nucleon and then on the other one. The triple-scattering term is that in which there is first a kaon-nucleon scattering then a nucleon-nucleon scattering, and finally another kaon-nucleon scattering. As one sees in Table I, the double scattering term lowers the cross section by $\sim$0.4–0.5 mb, while the triple and higher order terms raise it again by $\sim$0.3 mb, so that the net effect of the multiple scattering corrections in the total cross section is a decrease of only 0.1–0.2 mb. The triple and higher order terms presented in Table I have been evaluated using a very simple model

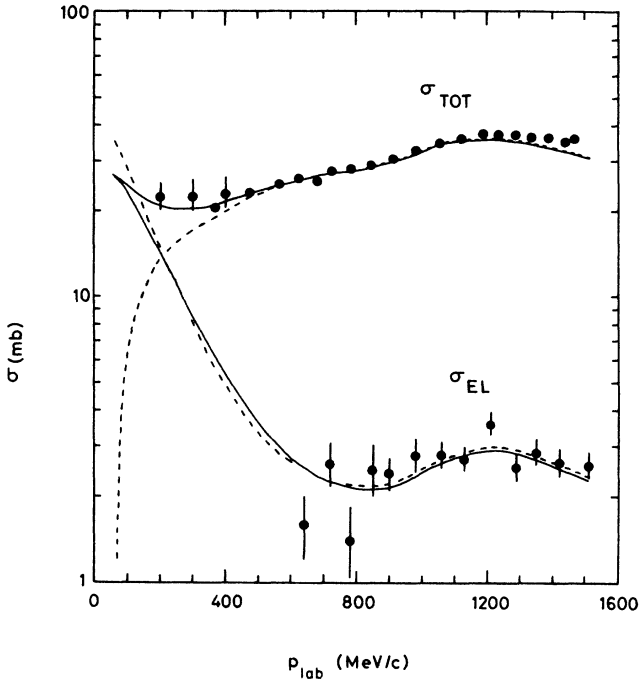

FIG. 1. Energy dependence of the total and integrated elastic K^+d cross sections. The solid lines are the results of the full calculation and the dashed lines are the results of the impulse approximation. The experimental data are from Refs. 7 and 38–40.

for the nucleon-nucleon amplitude and neglecting the pion-production channels, so that our result should be taken only as an indication of the importance of these effects, while a more careful treatment of them must be done in practical applications. Finally, it seems also that the triple-scattering correction should not be neglected when extracting K^+n cross sections from the K^+d data.

The differential cross section at 342 and 470 MeV/c have been calculated before by Sañudo, who found that the multiple scattering corrections decrease the cross section at all angles.[15,16] Our results exhibit the opposite behavior as can be seen in Figs. 2 and 3. We show in Figs. 2–8 the differential cross sections for momenta between 342 and 1510 MeV/c, where the solid lines are the results of the full calculation and the dashed lines the results of the impulse approximation, while the dot-dashed lines in Figs. 3 and 7 show the effect of neglecting the kaon-nucleon D and F waves in the impulse approximation. The data at the lower three momenta are those of Glasser *et al.*,[10] and they cover a considerably large angular region. The data in the last four figures are a representative

TABLE I. Contributions of different terms to the total cross section (in mb) at $p_{lab} = 1210$ and 1510 MeV/c.

p_{lab} (MeV/c)	Single scattering	Single + double scattering	Single + double + triple scattering	Full calculation
1210	35.55	35.04	35.31	35.36
1510	31.02	30.61	30.88	30.90

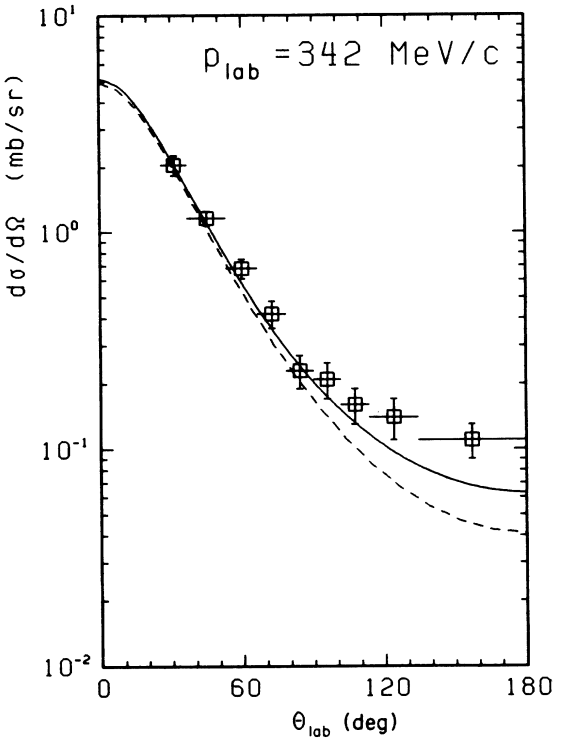

FIG. 2. K^+d differential cross sections at $p_{lab}=342$ MeV/c. The solid line is the result of the full calculation and the dashed line is the result of the impulse approximation. The experimental data are from Ref. 10.

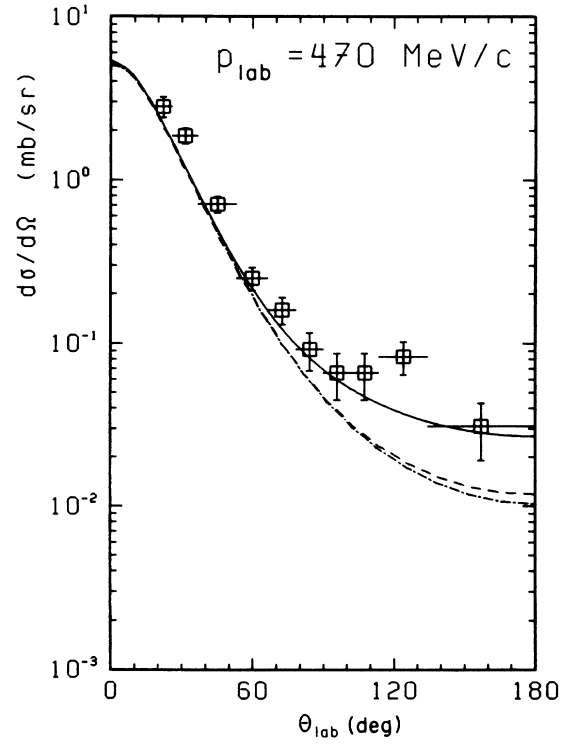

FIG. 3. K^+d differential cross sections at $p_{lab}=470$ MeV/c. The solid line is the result of the full calculation, the dashed line is the result of the impulse approximation, and the dot-dashed line is the result of neglecting the kaon-nucleon D and F waves in the impulse approximation. The experimental data are from Ref. 10.

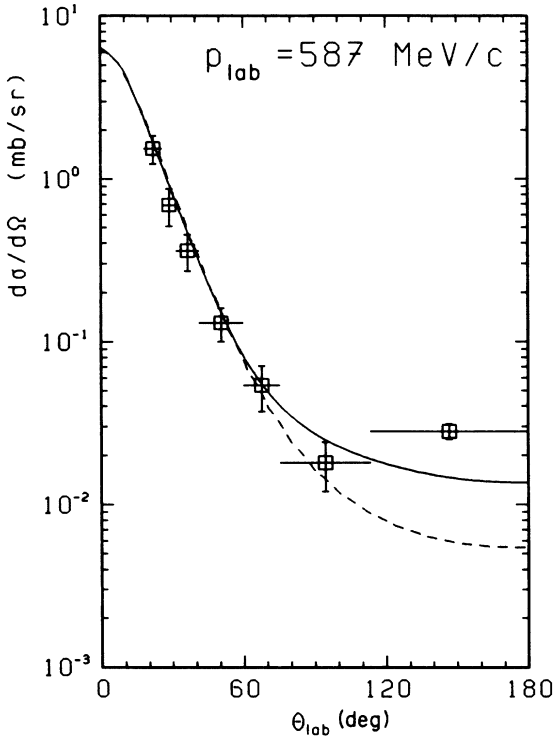

FIG. 4. Same as Fig. 2 for $p_{lab}=587$ MeV/c. The experimental data are from Ref. 10.

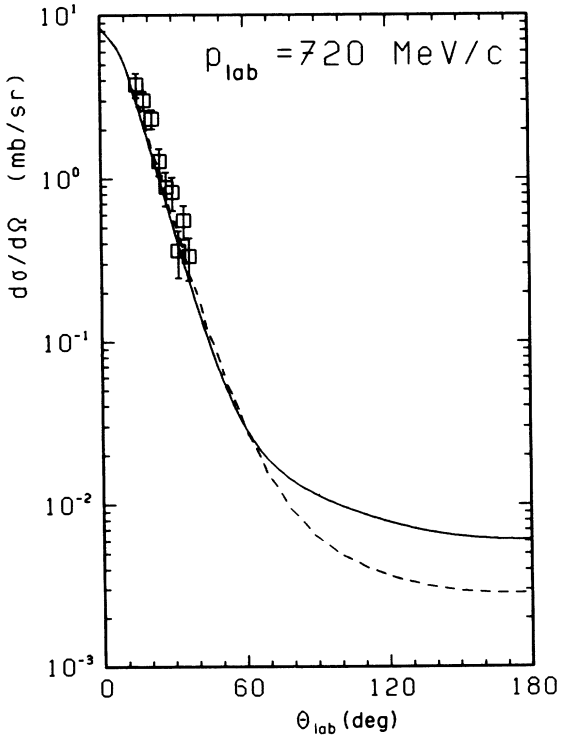

FIG. 5. Same as Fig. 2 for $p_{lab}=720$ MeV/c. The experimental data are from Ref. 7.

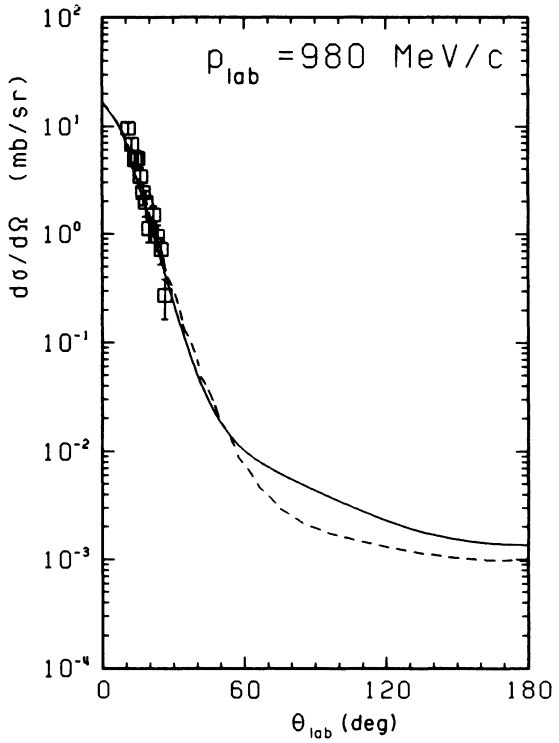

FIG. 6. Same as Fig. 2 for $p_{lab}=980$ MeV/c. The experimental data are from Ref. 7.

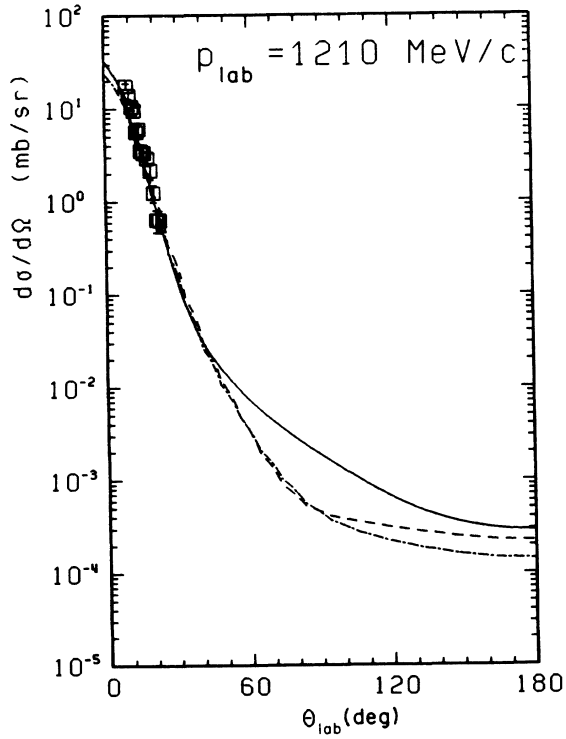

FIG. 7. Same as Fig. 3 for $p_{lab}=1210$ MeV/c. The experimental data are from Ref. 7.

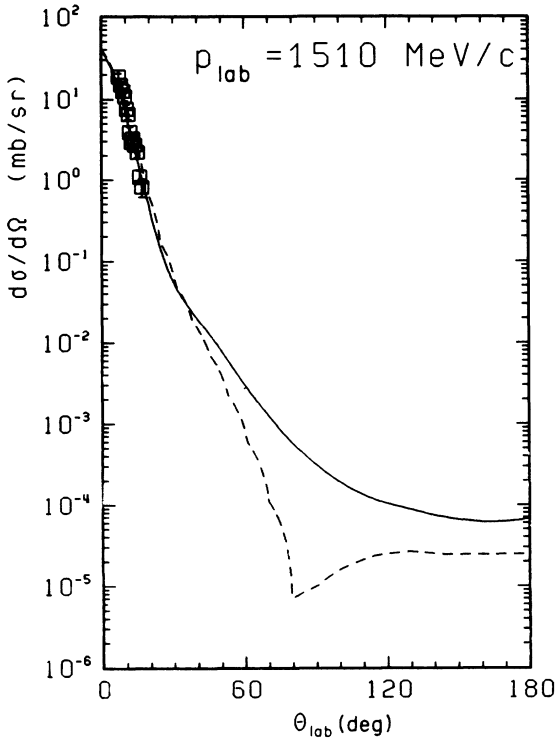

FIG. 8. Same as Fig. 2 for $p_{lab}=1510$ MeV/c. The experimental data are from Ref. 7.

sample of the measurements performed by Giacomelli *et al.*[7] which cover only a very small angular region near the forward direction. The multiple scattering effects, as we see, are largest in the backward direction, where they increase the cross section in some cases by more than a factor of 2. Thus, for example, at 1510 MeV/c, the multiple scattering effects are very large in the backward direction, although in this region the differential cross section has already fallen down by more than 6 orders of magnitude. In the forward direction where the Giacomelli data were taken,[7] the results of the full calculation and of the impulse approximation are essentially identical, which shows that the procedure which was used to extract the K^+N amplitude from the K^+d measurements, as described in the Introduction, is well justified. The low-momentum data of Glasser *et al.*[10] in Figs. 2–4, are more interesting since it covers also the backward direction. As it can be seen in Figs. 3 and 4, the experimental cross section seems to show a raise in the backward direction and this behavior is not followed entirely by the theory. First of all, the multiple scattering effects do tend to rise the cross section at large angles, but not enough as seems to be required, for example, at 342 and 587 MeV/c. Thus it would be very useful if more data at large angles were available, since perhaps this could in turn lead to a better understanding of the elementary K^+N amplitude. We show in Figs. 9–12 the predictions of our model for the polarization observables iT_{11}, T_{20}, T_{21}, and T_{22} at 300 MeV/c (solid lines), 600 MeV/c (dashed lines), and 900 MeV/c (dot-dashed lines). The most outstanding feature of these quantities is their strong energy dependence, which would make them quite

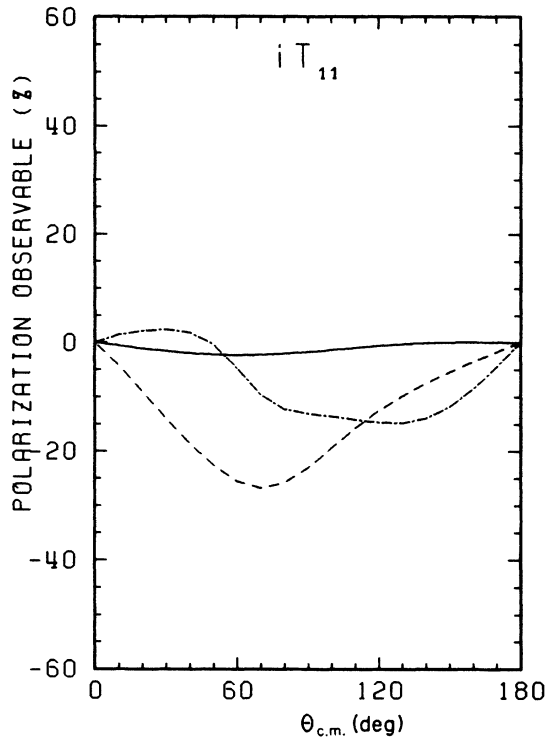

FIG. 9. Vector analyzing power iT_{11} for $p_{\text{lab}}=300$ MeV/c (solid line), $p_{\text{lab}}=600$ MeV/c (dashed line), and $p_{\text{lab}}=900$ MeV/c (dot-dashed line).

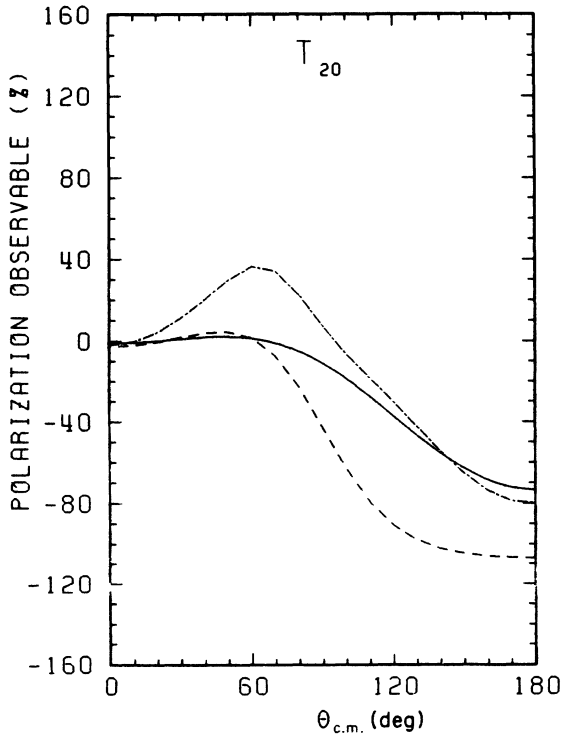

FIG. 10. Same as Fig. 9 for the tensor analyzing power T_{20}.

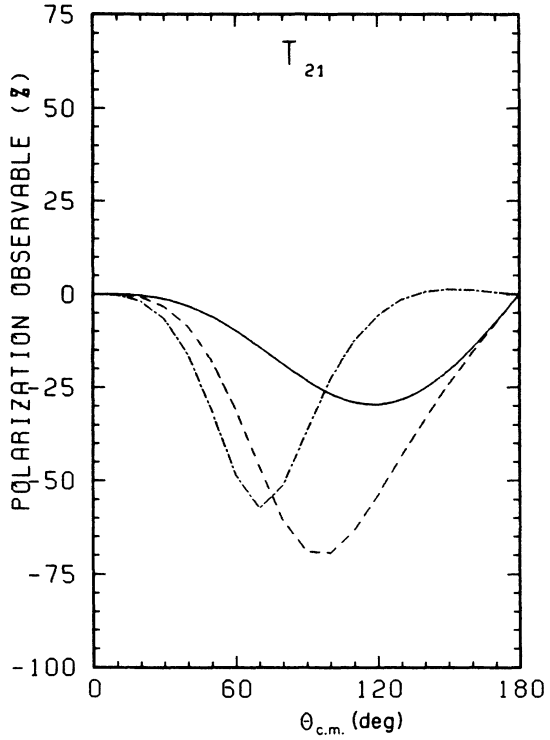

FIG. 11. Same as Fig. 9 for the tensor analyzing power T_{21}.

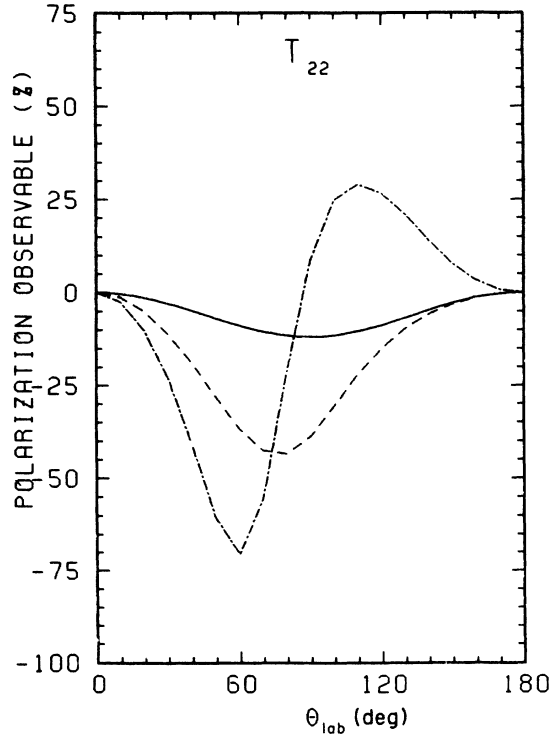

FIG. 12. Same as Fig. 9 for the tensor analyzing power T_{22}.

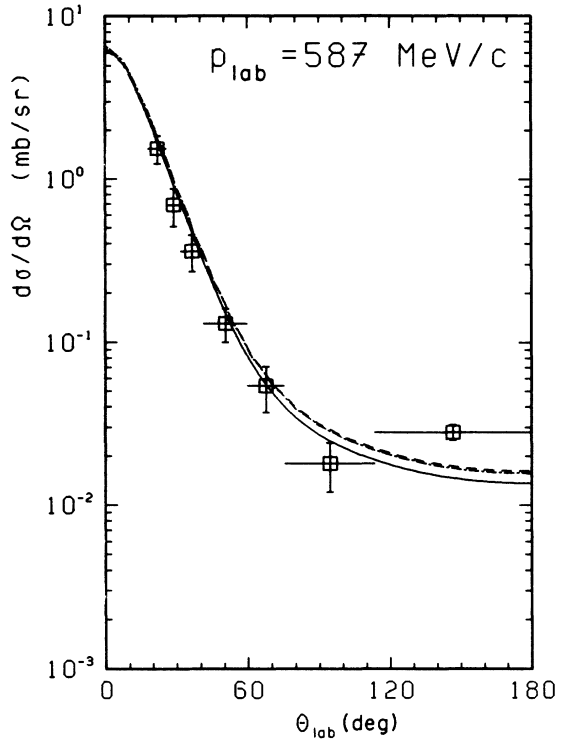

FIG. 13. Sensitivity of the $K^+d \to K^+d$ reaction to the deuteron wave function. The solid line is the result of the Paris wave function, the dashed line is the result of the McGee wave function, and the dot-dashed line is the result of the Moravcsik wave function. The experimental data are from Ref. 10.

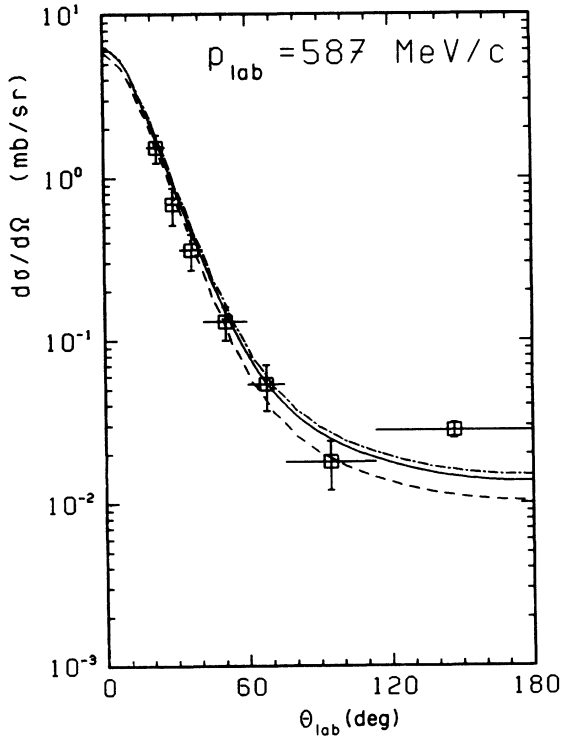

FIG. 14. Off-shell dependence of the $K^+d \to K^+d$ reaction. The different results correspond to using for the off-shell parameter β in Eq. (18), the values $\beta = 500$ MeV/c (dashed line), $\beta = 1000$ MeV/c (solid line), and $\beta = 1500$ MeV/c (dot-dashed line). The experimental data are from Ref. 10.

valuable as a tool to study the energy dependence of the two-body input. These observables could be very easily measured nowadays using the newly developed vector and tensor polarized deuteron targets.[45,46]

The results that have been presented in Figs. 1–12 are based on a particular model of the two-body input representing the K^+N and NN vertex functions and two-body amplitudes. Thus one can study the model dependence of the results by varying the two-body inputs within their allowed forms. For example, in Fig. 13, we have calculated the differential cross section at 587 MeV/c, using for the deuteron wave function the wave functions of Paris[35] (solid line), McGee[47] (dashed line), and Moravcsik[48] (dot-dashed line). As one can see, the use of the McGee and Moravcsik wave functions leads to a raise of the cross section in the backward direction of about 15%. In Fig. 14 we have investigated the off-shell dependence of this process, by performing the calculation using for the off-shell parameter β in Eq. (18) the values $\beta = 500$ MeV/c (dashed line), $\beta = 1000$ MeV/c (solid line), and $\beta = 1500$ MeV/c (dot-dashed line). As one can see, the differential cross section increases monotonically with β, so that for $\theta \leq 100°$ the curve with $\beta = 500$ MeV/c is the one that fits the data better, although for the last point it is lower by approximately a factor of 3. In Fig. 15 we have studied the sensitivity of the results to the behavior of the elementary K^+N amplitude in the unphysical region $s_i \leq (m_j + m_k)^2$, by performing the extrapolation indicated in Eq. (17) with $n = 1$ (dashed line), $n = 2$

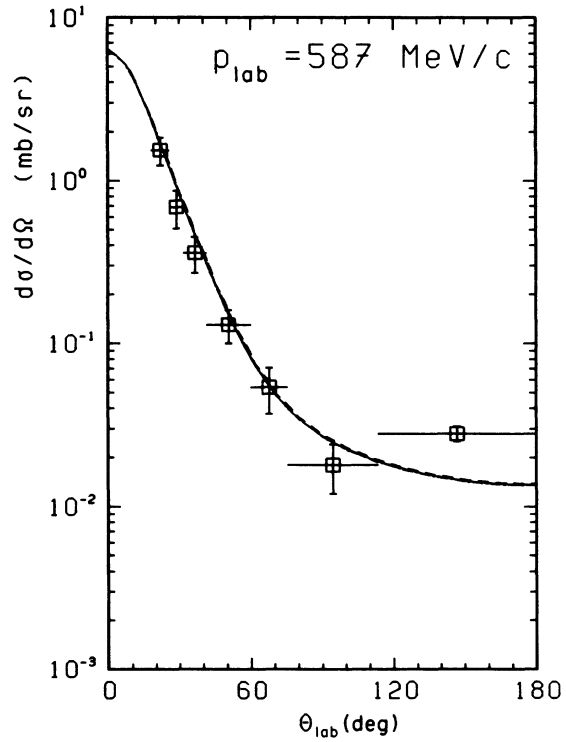

FIG. 15. Sensitivity of the $K^+d \to K^+d$ reaction to the behavior of the K^+N amplitude in the unphysical region $s_i \leq (m_j + m_k)^2$. The different results correspond to using for the exponent n in Eq. (17), the values $n = 1$ (dashed line), $n = 2$ (solid line), and $n = 3$ (dot-dashed line). The experimental data are from Ref. 10.

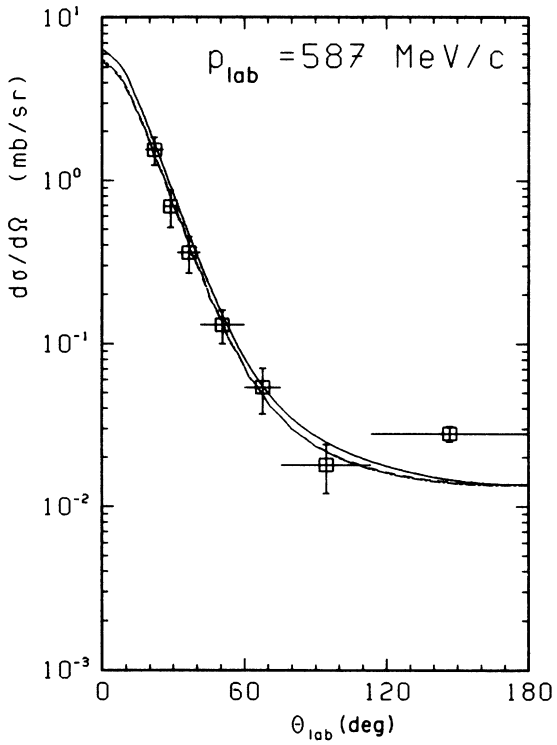

FIG. 16. Sensitivity of the $K^+d \to K^+d$ reaction to different parametrizations of the isospin one K^+N amplitude. The solid line is the result of the amplitudes of Martin (Ref. 9), the dashed line is the result of model 1 of Ref. 49, and the dot-dashed line is the result of model 4 of Ref. 49. The experimental data are from Ref. 10.

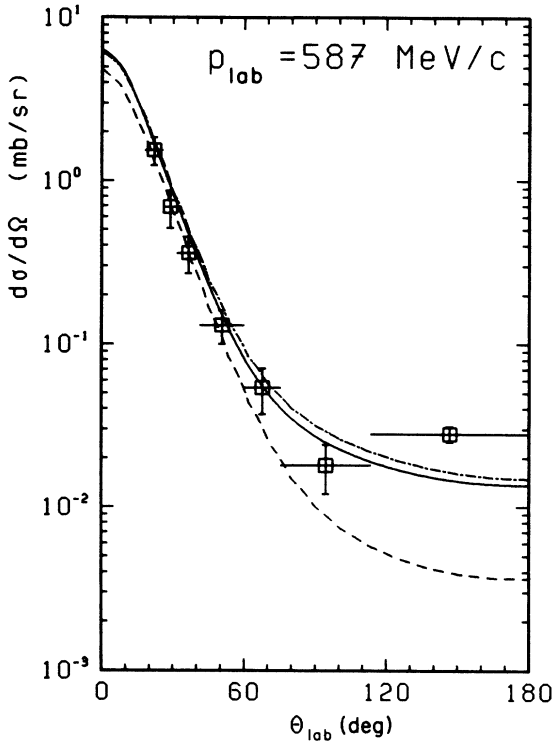

FIG. 17. Sensitivity of the $K^+d \to K^+d$ reaction to different parametrizations of the isospin zero K^+N amplitude. The solid line is the result of the amplitudes of Martin (Ref. 9), the dashed line is the result of model *A* of Ref. 8, and the dot-dashed line is the result of model *D* of Ref. 8. The experimental data are from Ref. 10.

(solid line), and $n=3$ (dot-dashed line). As one can see, there is essentially no sensitivity to the assumed form of the K^+N amplitude in the unphysical region (at lower energies, particularly very close to threshold, the results are more sensitive to the behavior in the unphysical region). In Fig. 16 we have studied the sensitivity of the results to different existing parametrizations of the isospin one K^+N amplitude, by comparing the results obtained with the amplitudes of Martin[9] (solid line), and those obtained using for the isospin one amplitude the models 1 (dashed line) and 4 (dot-dashed line) of Ref. 49. In Fig. 17 we show the corresponding sensitivity to the isospin zero K^+N amplitude, where we compare the results obtained using the amplitudes of Martin[9] (solid line), with those obtained using for the isospin zero amplitude the models *A* (dashed line) and *C* (dot-dashed line) of Ref. 8. As one can see, there are considerable variations in the theoretical results, particularly in the case of the isospin zero amplitudes. Thus the taking of new data in this region could be of great help in order to decide among the various isospin zero solutions.

V. CONCLUSIONS

We have performed a relativistic Faddeev calculation of K^+d scattering up to 1.5 GeV/*c*. Our results show that multiple scattering effects are important mainly in the backward direction where they increase the cross section by about a factor of 2. The main corrections to the impulse approximation in the high-energy region are the double and triple-scattering terms where the last one contains an intermediate nucleon-nucleon scattering. The differential cross sections exhibit great sensitivity to the use of different existing isospin zero phase shift solutions of the K^+N subsystem, which indicates that an accurate measurement of the K^+d differential cross section would be of great help in deciding among the various solutions. Our predictions for the polarization observables show quite a strong energy dependence, which shows that if measured, these data could contribute greatly towards a complete understanding of this system.

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