Hamiltonian light-front dynamics of elastic electron-deuteron scattering

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Relativistic calculations of elastic electromagnetic form factors of the deuteron are presented for momentum transfers up to 8 GeV^2. Standard nucleon-nucleon interactions are used to construct a unitary representation of the inhomogeneous Lorentz group on the two-nucleon Hilbert space. Deuteron wave functions represent eigenstates of the four-momentum operator. Existing parametrizations of measured single-nucleon form factors are used to construct a conserved covariant electromagnetic current operator. Deuteron form factors are computed in terms of matrix elements of the current operator and the deuteron wave functions. The results are compared to experiment. The size of relativistic effects, scaling behavior, sensitivity to the nucleon-nucleon interactions, and effects of the uncertainties in measured nucleon form factors are investigated.

I. INTRODUCTION

The electromagnetic properties of the deuteron have received a great deal of attention both experimentally and theoretically. The simple structure of a two-nucleon system should allow definitive conclusions about the relevance of specific nuclear models and the role of the structure of the nucleons. To what extent can the deuteron be successfully described as a two-nucleon system with all the subnucleon degrees of freedom absorbed in the nucleon-nucleon interaction and in the representation of the current operator? At what point does this conventional picture fail and are explicit quark degrees of freedom required? Except at very low values of the momentum transfer, nonrelativistic calculations are inherently unsatisfactory even when their results agree well with experiment. Before formulating specific questions to be addressed in this paper we survey briefly the salient features of relativistic dynamics.

Relativistic models can differ widely, depending on basic assumptions involved in their construction. In quantum particle dynamics, the model Hilbert space of states, $\mathcal{H}_{2\mathrm{N}}$, is the tensor product of one-nucleon Hilbert spaces, $\mathcal{H}_{1\mathrm{N}}$, which are spanned by the physical one-nucleon states $|p,\mu\rangle$, $(\mu=\pm\frac{1}{2})$. Lorentz transformations and space-time translations are represented by unitary transformations of the state vectors. Since the generators of the space and time translations have the physical significance of momentum and energy, the unitary representation of the inhomogeneous Lorentz group (Poincaré group)[1,2] also specifies the dynamics.[3] Covariant-wave-function models,[4-6] on the other hand, are based on assumed relations between various matrix elements of covariant field and current operators between the physical vacuum and physical momentum eigenstates.[7-11] This covariant-wave-function approach and the explicit construction of unitary representations of the Poincaré group lead to quite different relativistic models even when they have the same nonrelativistic limits.

The elastic electron deuteron cross section depends on the matrix elements, $\langle \mu_{\mathrm{d}}',P_{\mathrm{d}}' \,|\, I^{\nu}(0) \,|\, P_{\mathrm{d}},\mu_{\mathrm{d}}\rangle$, of the current-density operator, $I^{\nu}(x)$, where $|P_{\mathrm{d}},\mu_{\mathrm{d}}\rangle \in \mathcal{H}_{2\mathrm{N}}$, $(\mu_{\mathrm{d}}=0,\pm 1)$, are physical deuteron states, represented by wave functions that are eigenfunctions of momentum, energy, and spin.

Conventional bound-state wave functions of a nucleus are eigenfunctions of the energy and spin operators for the nucleus *at rest,* that means they are eigenfunctions of mass and spin operators. Eigenfunctions of the four momentum can be generated by choosing three components of the four momentum to be conserved kinematically, while the remaining component is determined by the mass and the three kinematic components. In the unitary representation of the Poincaré group, which specifies the dynamics, at most the representation of a subgroup (the kinematic subgroup) may be independent of the interactions. The choice of the kinematic components of the four-momentum fixes the choice of the kinematic subgroup and thus determines the "form" of dynamics.[3,12]

If the Euclidean vector $\vec{P}=\{P^1,P^2,P^3\}$ is kinematic, then the Euclidean group (symmetry group of the instant plane $x^0=0$) is the kinematic subgroup, and the dynamics is called "instant form." Let $\vec{n}$ be a fixed unit vector, e.g., $\vec{n}=\{0,0,1\}$, which determines the light front $x^+\equiv x^0+\vec{n}\cdot\vec{x}=0$. The light-front components $\mathbf{P}=\{P^+ \equiv P^0+\vec{n}\cdot\vec{P},\ \vec{P}_T\equiv\vec{P}-\vec{n}\cdot\vec{P}\vec{n}\}$ of the four-momentum transform as a vector under the subgroup which leaves the light front invariant. If the light-front vector $\mathbf{P}$ is kinematic the dynamics is called "front form." The use of conventional nuclear wave functions to describe the deuteron at rest does not prejudice the form of relativistic dynamics. But the wave functions representing the states $|P_{\mathrm{d}},\mu_{\mathrm{d}}\rangle$, which are eigenfunctions of the four-momentum operator, do depend on the choice of the kinematic subgroup. Different choices are related to each other by interaction-dependent unitary transformations.

The current density operator $I^{\nu}(x)$ is a sum of one-body operators plus a short-range two-body operator, which vanishes on states of widely separated nucleons. By definition, a one-body operator is an operator with domain and range in a one-nucleon Hilbert space, $\mathcal{H}_{1N}$; its tensor product with the identity defines the one-body operator on the two-nucleon Hilbert space, $\mathcal{H}_{2N}$. One-body current operators are thus completely determined by the empirical form factors of the nucleons. Historically, two-body current operators have usually been associated with the physical mechanism of meson exchange,[13] and more recently with the quark structure of the nucleons.[14,15] Apart from the effects of specific subnucleon degrees of freedom, current conservation and Lorentz covariance of the current impose essential dynamic consistency conditions on the representations of the current operators and the wave functions. These conditions require nonvanishing two-body contributions to the current operator. The separation into one- and two-body contributions is not invariant under unitary transformations which leave the observable matrix elements invariant. Such transformations in general redistribute correlation effects between the operators and the wave functions. Thus, models with the same mass spectrum and nucleon-nucleon scattering cross sections may require quite different representations of the current operator. In an instant-form dynamics the current matrix elements required for a computation of deuteron form factors are related to each other by the continuity equation,[16,17] and by dynamic Lorentz transformations.[18,19] Hamiltonian light-front dynamics[20] of a fixed number of particles also has been used for the description of elastic electron-deuteron scattering.[14,21]

With front-form dynamics, it is possible to construct consistent relativistic models of elastic electron deuteron scattering using as input conventional deuteron wave functions and one-body currents determined by empirical nucleon form factors, because all required matrix elements of the nucleon current operator are related to the observable nucleon form factors by kinematic transformations.

It is therefore of interest to investigate the sensitivity of such models to different realistic nucleon-nucleon interactions as well as variations in the nucleon form factors that are consistent with present data. While well established isovector exchange currents do not contribute to the elastic deuteron form factors, empirical evidence of isoscalar electroproduction of pions requires exchange currents which must eventually be included in a complete description. These effects are beyond the scope of the present paper. Relativistic properties of the deuteron wave function are discussed in detail in Sec. II. The subjects of Sec. III are the symmetry properties of the matrix representation of the current operators, their relation to deuteron wave functions, and the relations of the form factor to observables. We have computed the deuteron form factors for the Reid soft core (RSC),[22] Argonne v_{14} (AV14),[23] and Paris[24] wave functions, as well as for the deuteron wave functions of three recent Bonn potentials.[25] We use different parametrizations of the nucleon form factors[26–28] to represent the uncertainty in the data. Numerical results are presented in Sec. IV. We find no evidence that the data require the application of perturbative quantum chromodynamics.[29,30] Beyond the scope of this paper is the important theoretical question to what extent the kind of dynamical model considered here can be justified in terms of the quark structure of the nucleons.

II. DEUTERON WAVE FUNCTIONS

Deuteron states, $|\psi\rangle$, in the two-nucleon Hilbert space $\mathcal{H}_{2N}\equiv\mathcal{H}_{1N}\otimes\mathcal{H}_{1N}$ are represented by square integrable functions, $\psi(\mathbf{p}_1,\mu_1,\mathbf{p}_2,\mu_2)$, of the light-front momenta $\mathbf{p}_1,\mathbf{p}_2$, and the spin variables μ_1,μ_2, $(\mu_i=\pm\frac{1}{2})$, of the two nucleons. Boldface letters indicate light-front vectors; the subscript T indicates "transverse" vectors, i.e., vectors perpendicular to $\vec{n}$, $\vec{n}\cdot\vec{p}_T=0$. For noninteracting nucleons the unitary representations of the Poincaré transformations are direct products of unitary operators and the infinitesimal generators are sums of one-body operators. The same is true in the interacting system for the representations of the kinematic subgroup.

It is convenient to introduce the total momentum $\mathbf{P}\equiv\mathbf{p}_1+\mathbf{p}_2$ and appropriate internal variables: the momentum fraction ξ

$$\xi\equiv p_1^+/P^+=1-p_2^+/P^+ , \tag{2.1}$$

and the transverse relative momentum $\vec{k}_T$,

$$\vec{k}_T\equiv\vec{p}_{1T}-\xi\vec{P}_T=-\{\vec{p}_{2T}-(1-\xi)\vec{P}_T\} , \tag{2.2}$$

which transforms as a vector under rotations generated by the longitudinal component of the spin. The momentum fraction ξ and the magnitude of the relative momentum $\vec{k}_T$ are invariant under all kinematic Lorentz transformations.

The state $|P_d,\mu_d\rangle$ is an eigenstate of the four-momentum operator $P=\{P^-,\mathbf{P}\}$, with the normalization condition

$$\langle\mu_d',P_d'\,|\,P_d,\mu_d\rangle=\delta(P_d'-P_d)2\delta(P_d^2+M_d^2)\theta(P_d^0)\delta_{\mu_d',\mu_d} . \tag{2.3}$$

Since $P^2=\vec{P}_T^2-P^+P^-=-M^2$, the "Hamiltonian" $H\equiv P^-$ is related to the mass operator M by

$$H=\frac{M^2+\vec{P}_T^2}{P^+} . \tag{2.4}$$

Two-body interactions can be added to the square of the mass operator to give

$$M^2=\frac{m^2+k_T^2}{\xi(1-\xi)}+4mV_{12}=M_0^2+4mV_{12} , \tag{2.5}$$

where m is the nucleon mass. The Hamiltonian is then

$$H=H_0+4mV_{12}/P^+ . \tag{2.6}$$

Poincaré invariance requires that the interaction operator V_{12} commute with the total light-front momentum **P**, and be independent of **P**. The dynamics so formulated is Poincaré invariant if and only if M^2 commutes with the spin $\vec{j}$, which is defined as a function of the four-momentum P and the Pauli-Lubanski vector[31,32]

$$W_\tau \equiv \tfrac{1}{2} J^{\rho\sigma} P^\nu \epsilon_{\nu\rho\sigma\tau} . \tag{2.7}$$

The generators of infinitesimal Lorentz transformations, $J^{\rho\sigma}$, are related to the angular momentum $\vec{J}$ and the boost generator $\vec{K}$ by

$$J^{kl} = \sum_m \epsilon_{klm} J^m \quad \text{and} \quad K^p = J^{p0} . \tag{2.8}$$

It follows that the Pauli-Lubanski vector W is orthogonal to the four-momentum P, $P\cdot W=0$, and commutes with P.

If $L(P)$ is any Lorentz transformation with the property

$$L(P)\{P^\rho\} = \{M,0,0,0\} , \tag{2.9}$$

then the time component of $L(P)W$ vanishes. The spin vector $\vec{j}$ of the system is defined as a function of the Poincaré generators by

$$\{0,\vec{j}\} = L(P)W/M . \tag{2.10}$$

Obviously $\vec{j}^2 = W^2/M^2$ is Lorentz invariant. Under Lorentz transformations the spin vector undergoes a Wigner rotation,

$$U^\dagger(\Lambda)\vec{j}U(\Lambda) = \mathcal{R}_W(\Lambda,P)\vec{j} , \tag{2.11}$$

where

$$\mathcal{R}_W(\Lambda,P) \equiv L(\Lambda P)\Lambda L^{-1}(P) \tag{2.12}$$

follows from the definition (2.10).

Since $L(P)$ was defined only modulo a rotation, Eq. (2.10) defines an infinite set of spin vectors which differ from each other by rotations, depending on the choice of $L(P)$. The canonical spin $\vec{j}_c$ results if the rotationless Lorentz transformation $L_c(P)$ is chosen.

The generators of the Lorentz transformations Λ_f that leave the light front $x^+ \equiv x^0 + \vec{n}\cdot\vec{x} = 0$ invariant are $K_n \equiv \vec{K}\cdot\vec{n}$, $\vec{E} \equiv \vec{K}_T + \vec{n}\times\vec{J}$ and J_n. The generators K_n and $\vec{E}$ satisfy the commutation relations $[K_n, \vec{E}] = i\vec{E}$.

The Lorentz transformations Λ_f generated by K_n and $\vec{E}$ transform light-front vectors $\mathbf{A} \equiv \{A^+, \vec{A}_T\}$ according to

$$\Lambda_f \mathbf{A} = e^{-i\vec{a}\cdot\vec{E}} e^{-ibK_n} \mathbf{A} e^{ibK_n} e^{i\vec{a}\cdot\vec{E}} = \{e^b A^+, \vec{A}_T + \vec{a} A^+\} . \tag{2.13}$$

Among the Lorentz transformations $L(P)$ there exists one, $L_f(P)$, that leaves the space of light-front vectors invariant,

$$\begin{aligned} \{L_f(P)A\}^+ &= (M/P^+)A^+ , \\ \{L_f(P)A\}_T &= \vec{A}_T - \vec{P}_T(A^+/P^+) . \end{aligned} \tag{2.14}$$

The 2×2 dimensional representations of the Lorentz transformations $L_f(P)$ and $L_c(P)$ are listed in the Appendix. Using $L_f(P)$ in Eq. (2.10) defines the spin operator $\vec{j}$ appropriate for front-form dynamics,

$$j_n = \vec{n}\cdot\vec{j} = W^+/P^+ , \quad \vec{j}_T = (\vec{W}_T - \vec{P}_T j_n)/M . \tag{2.15}$$

This spin operator is invariant under translations and all kinematic Lorentz transformations except rotations about the longitudinal axis. The spin operator $\vec{j}$ so defined must be related to the canonical spin operator $\vec{j}_c$ by the rotation[33,12]

$$\vec{j}_c = L_c(P)L_f^{-1}(P)\vec{j} . \tag{2.16}$$

The rotation

$$\mathcal{R}_M(P) \equiv L_c(P)L_f^{-1}(P) , \tag{2.17}$$

has been called "Melosh transformation."[20,34]

With the choice $L_f(P)$ for $L(P)$ in the definition of the spin, the Wigner rotations associated with the restricted Lorentz transformations Λ_f are equal to the identity,

$$\mathcal{R}_W(\Lambda_f,P) = 1 , \tag{2.18}$$

and the Wigner rotation associated with the pure rotation $\mathcal{R}$ is

$$\mathcal{R}_W(\mathcal{R},P) = \mathcal{R}_M^{-1}(\mathcal{R}P)\mathcal{R}\mathcal{R}_M(P) , \tag{2.19}$$

where $\mathcal{R}_M(P)$ is the Melosh rotation defined in Eq. (2.17).

Note that the longitudinal spin component j_n depends only on the kinematic Poincaré generators. It is invariant under longitudinal boosts. In the "infinite momentum frame" it is equal to the helicity, $\vec{J}\cdot\vec{P}/|\vec{P}|$, of the two-nucleon system, which is, of course, not Lorentz invariant.

In a relativistic system the composition of spins always requires momentum-dependent rotations of the individual spins.[35] Explicitly the spin of the two-nucleon system is given by

$$\vec{j} = i\nabla_k \times \vec{k} + \mathcal{R}_M(\xi,\vec{k}_T,m)\vec{s}_1 + \mathcal{R}_M(1-\xi,-\vec{k}_T,m)\vec{s}_2 , \tag{2.20}$$

where the transverse part of $\vec{k}$ is $\vec{k}_T$, and the longitudinal component k_n is defined as a function of ξ and $\vec{k}_T$,

$$k_n \equiv \frac{1}{2}\left[M_0\xi - \frac{m^2+k_T^2}{M_0\xi}\right] = M_0(\xi - \tfrac{1}{2}) . \tag{2.21}$$

The 2×2 irreducible representation of the Melosh rotation, $\mathcal{R}_M$, in Eq. (2.20) is

$$\langle \mu' | \mathcal{R}_M(\xi,\vec{k}_T,m) | \mu \rangle = \left\{ \frac{m+\xi M_0 - i\vec{\sigma}\cdot(\vec{n}\times\vec{k}_T)}{[(m+\xi M_0)^2 + \vec{k}_T^2]^{1/2}} \right\}_{\mu',\mu} . \tag{2.22}$$

If we use the definition (2.21) to express ξ as a function of $\vec{k}$ then Eq. (2.5) has the form

$$M^2 = 4(\vec{k}^2 + m^2 + mV_{12}) . \tag{2.23}$$

Deuteron wave functions $\chi_{\mu_d}(\xi,\vec{k}_T,\mu_1,\mu_2)$ are defined as eigenfunctions of the mass operator defined in Eq. (2.5) [or equivalently by Eq. (2.23)] and the spin operators, $\vec{j}^2$ and j_n, defined by Eq. (2.20),

$$M^2\chi_{\mu_d}(\xi,\vec{k}_T,\mu_1,\mu_2)=\chi_{\mu_d}(\xi,\vec{k}_T,\mu_1,\mu_2)M_d^2 ,$$
$$\vec{j}^2\chi_{\mu_d}(\xi,\vec{k}_T,\mu_1,\mu_2)=\chi_{\mu_d}(\xi,\vec{k}_T,\mu_1,\mu_2)1(1+1) , \tag{2.24}$$
$$j_n\chi_{\mu_d}(\xi,\vec{k}_T,\mu_1,\mu_2)=\chi_{\mu_d}(\xi,\vec{k}_T,\mu_1,\mu_2)\mu_d .$$

The Jacobian of the variable transformation $\{\xi,\vec{k}_T\}\to\vec{k}$ is

$$\frac{\partial(\vec{k})}{\partial(\xi,\vec{k}_T)}=\frac{\partial k_n}{\partial\xi}=\frac{M_0}{4\xi(1-\xi)} . \tag{2.25}$$

Expressed in terms of the momentum $\vec{k}$ and the composite spin

$$\vec{S}\equiv\mathcal{R}_M(\xi,\vec{k}_T,m)\vec{s}_1+\mathcal{R}_M(1-\xi,-\vec{k}_T,m)\vec{s}_2 , \tag{2.26}$$

the deuteron bound state equation (2.24) has the same form as the nonrelativistic Schrödinger equation. Any conventional nucleon-nucleon potential is a suitable candidate for the interaction operator V_{12}. The deuteron wave function (2.24) is related to the conventional S and D wave functions, $u_L(k)$, by

$$\chi_{\mu_d}(\xi,\vec{k}_T,\mu_1,\mu_2)=\sum_{\mu_1'\mu_2'}\left[\frac{\partial k_n}{\partial\xi}\right]^{1/2}\langle\mu_1\,|\,\mathcal{R}_M^\dagger(\xi,\vec{k}_T,m)\,|\,\mu_1'\rangle\langle\mu_2\,|\,\mathcal{R}_M^\dagger(1-\xi,-\vec{k}_T,m)\,|\,\mu_2'\rangle\chi^c_{\mu_d}(\vec{k},\mu_1',\mu_2') , \tag{2.27}$$

where $\chi^c_{\mu_d}(\vec{k},\mu_1,\mu_2)$ has the familiar forms[36–38]

$$\begin{aligned}\chi^c_{\mu_d}(\vec{k},\mu_1,\mu_2)&\equiv\sum_{L,m_L,\mu}(\tfrac{1}{2},\tfrac{1}{2},\mu_1,\mu_2\,|\,1,\mu)(L,1,m_L,\mu\,|\,1,\mu_d)Y_{L,m_L}(\hat{k})u_L(k)/k\\&=\frac{1}{\sqrt{4\pi}}\left\{\left[\vec{\sigma}\cdot\hat{e}_{\mu_d}u_0(k)/k-\frac{1}{\sqrt{2}}(3\vec{\sigma}\cdot\hat{k}\,\hat{e}_{\mu_d}\cdot\hat{k}-\vec{\sigma}\cdot\hat{e}_{\mu_d})u_2(k)/k\right]\frac{i\sigma_2}{\sqrt{2}}\right\}_{\mu_1,\mu_2} .\end{aligned} \tag{2.28}$$

The standard polarization vectors $\hat{e}_\mu$ have the components $\hat{e}_\pm=\mp(1,\pm i,0)/\sqrt{2}$, $\hat{e}_0=(0,0,1)$, and $\hat{k}$ is the unit vector $\vec{k}/|\vec{k}|$.

The only relativistic effect is in the relation of the eigenvalue of $k^2/m+V$ to the deuteron mass: the eigenvalue is $M_d^2/4m-m$, and M_d-2m in the relativistic and the nonrelativistic case, respectively. The difference

$$\left[\frac{M_d^2}{4m}-m\right]-(M_d-2m)=\frac{(M_d-2m)^2}{4m} \tag{2.29}$$

is negligible for most purposes. The relation between the nucleon-nucleon potentials and the observable scattering cross sections is not affected by the relativistic reinterpretation of the potentials as interaction terms in the square of the invariant mass operator.[39]

The complete deuteron states $|P_d,\mu_d\rangle$ are eigenstates of the four-momentum P. The full deuteron wave functions $\Psi_{P_d,\mu_d}(\mathbf{p}_1,\mathbf{p}_2,\mu_1,\mu_2)$ that represent these states are determined by the eigenfunctions χ_{μ_d} of the Casimir operators M^2 and W^2 and by the Lorentz transformation properties of the states $|P_d,\mu_d\rangle$.

For any particle of spin j and mass m the states $|p,\mu\rangle$, $(p^2=-m^2, -j\le\mu\le j)$ transform under Lorentz transformations according to

$$U(\Lambda)\,|\,p,\mu\rangle=\sum_{\mu'}|\,\Lambda p,\mu'\rangle\langle\mu'\,|\,\mathcal{R}_W(\Lambda,p)\,|\,\mu\rangle , \tag{2.30}$$

where the rotation $\mathcal{R}_W$ is the Wigner rotation defined by Eq. (2.12) with $L(p)=L_f(p)$. Since $\mathcal{R}_W(L_f(p),p)=1$, the full deuteron wave function $\Psi_{P_d,\mu_d}(\mathbf{p}_1,\mathbf{p}_2,\mu_1,\mu_2)$ is given by

$$\Psi_{P_d,\mu_d}(\mathbf{p}_1,\mathbf{p}_2,\mu_1,\mu_2)=\left[\frac{\partial(\xi,\vec{k}_T,\mathbf{P})}{\partial(\mathbf{p}_1,\mathbf{p}_2)}\right]^{1/2}\chi_{\mu_d}(\xi,\vec{k}_T,\mu_1,\mu_2)\delta^3(\mathbf{P}-\mathbf{P}_d)(P_d^+)^{1/2} , \tag{2.31}$$

where

$$\frac{\partial(\xi,\vec{k}_T,\mathbf{P})}{\partial(\mathbf{p}_1,\mathbf{p}_2)}=\frac{1}{P^+} \tag{2.32}$$

is the Jacobian of the variable transformation $(\mathbf{p}_1,\mathbf{p}_2)\to(\xi,\vec{k}_T,\mathbf{P})$.

Note that while individual light-front momenta $\mathbf{p}_1$ and $\mathbf{p}_2$ are defined, there are no individual four momenta that transform covariantly under all Lorentz transformations. However, under kinematic Lorentz transformations the four-momenta $\{(m^2+\vec{k}_{iT}^2)/(p_i^+),\mathbf{p}_i\}$ do transform covariantly.

Complete state vectors in Fock-space representations of Lagrangian field theories also satisfy the unitary covariance (2.30), but individual Fock amplitudes are covariant only under the kinematic transformations, that is, respectively, the symmetries of the instant hyperplane or the light front in instant-form or front-form field theories. In perturbative treatments the dynamical symmetries are either realized approximately using expansions[40] in powers of $1/m^2$ or ignored.[29,41] There is reason to doubt the reliability of $1/m^2$ expansions even when the relativistic effects are small.[42]

We emphasize that the deuteron wave functions defined above as eigenfunctions of the four-momentum operator P, and of $\vec{j}^2$ and j_n differ qualitatively and quantitatively from covariant vertex wave functions defined by the matrix elements of a covariant nucleon field operator $\langle \mu',p' \mid \psi(x) \mid P_d,\mu_d \rangle$, between physical deuteron and nucleon states,[4,9] even though both functions may have the same nonrelativistic limit.

III. CURRENT MATRIX ELEMENTS AND FORM FACTORS

A. Covariance of current operators and definitions of form factors

The transformation properties of the current matrix elements $\langle \mu',p' \mid I^\nu(0) \mid p,\mu \rangle$ follow from the covariance of the current,

$$U^\dagger(\Lambda) I^\rho(x) U(\Lambda) = \Lambda^\rho{}_\sigma I^\sigma(\Lambda^{-1}x) , \tag{3.1}$$

and the transformation properties (2.30) of the state vectors $\mid p,\mu \rangle$.[43] The normalization condition for the state vectors $\mid p,\mu \rangle$ is

$$\langle \mu',p' \mid p,\mu \rangle = \delta(p'-p) 2\delta(p^2+m^2)\theta(p^0)\delta_{\mu',\mu} . \tag{3.2}$$

We will also use the notation $\mid \mathbf{p},\mu \rangle$ for state vectors normalized according to

$$\langle \mu',\mathbf{p}' \mid \mathbf{p},\mu \rangle = \delta(\mathbf{p}'-\mathbf{p})\delta_{\mu',\mu} , \tag{3.3}$$

which implies

$$\mid \mathbf{p},\mu \rangle \equiv \mid p,\mu \rangle /(p^+)^{1/2} . \tag{3.4}$$

Theorem: If the matrix elements of the component $I^+(0)$ of a conserved current are known for $p^+ = p'^+ = (m^2+\frac{1}{4}Q^2)^{1/2}$, $\vec{p}'_T = -\vec{p}_T = \frac{1}{2}\vec{Q}$, then the matrix elements of all other components of the current are determined by the transformation properties and current conservation.

Proof: Consider the identity[43]

$$\begin{aligned}\langle \mu',p' \mid I^\nu(0) \mid p,\mu \rangle &= \langle \mu',p' \mid U^\dagger(\Lambda) U^\dagger(\Lambda^{-1}) I^\nu(0) U(\Lambda^{-1}) U(\Lambda) \mid p,\mu \rangle \\ &= (\Lambda^{-1})^\nu{}_\rho \sum_{\bar\mu'} \sum_{\bar\mu} \langle \mu' \mid \mathcal{R}_W^\dagger(\Lambda,p) \mid \bar\mu' \rangle \langle \bar\mu',\Lambda p' \mid I^\rho(0) \mid \Lambda p,\bar\mu \rangle \langle \bar\mu \mid \mathcal{R}_W(\Lambda,p) \mid \mu \rangle .\end{aligned} \tag{3.5}$$

Since the momentum transfer $\mathrm{p}'-\mathrm{p}$ is spacelike we can design the transformation Λ so that the only nonvanishing component of $\Lambda(\mathrm{p}'-\mathrm{p})$ has the direction of the 1 axis: $\Lambda = \mathcal{R} L_f(\mathrm{p}+\mathrm{p}')$, where $\mathcal{R}$ is the rotation that rotates the space part of $L_f(\mathrm{p}+\mathrm{p}')(\mathrm{p}'-\mathrm{p})$ into the direction of the 1 axis. Thus there always exists a Breit frame for which the plus component of the momentum transfer vanishes. Equation (3.5) shows how the general current matrix elements can be obtained from the matrix elements in that frame, i.e., for

$$p^+ = p'^+ = (m^2+\tfrac{1}{4}Q^2)^{1/2}, \text{ and } \vec{p}'_T = -\vec{p}_T = \tfrac{1}{2}\vec{Q} . \tag{3.6}$$

When the momenta are specified by Eq. (3.6) we will use the abbreviated notation,

$$\langle \mu',\mathbf{p}' \mid I^\nu(0) \mid \mathbf{p},\mu \rangle \to \langle \mu' \mid I^\nu(0) \mid \mu \rangle . \tag{3.7}$$

Current conservation requires that the component of the current in the direction of the momentum transfer vanishes,

$$\langle \mu' \mid I_1(0) \mid \mu \rangle = 0 . \tag{3.8}$$

The matrix elements of the components I^- and I_2 can be obtained from I^+ by rotations about the 1 axis by π and $\pi/2$,

$$\langle \mu' \mid I^-(0) \mid \mu \rangle = \langle \mu' \mid U^\dagger\{\mathcal{R}_1(\pi)\} I^+(0) U\{\mathcal{R}_1(\pi)\} \mid \mu \rangle , \tag{3.9}$$

and

$$\begin{aligned}\langle \mu' \mid I_2(0) \mid \mu \rangle = \tfrac{1}{2} \langle \mu' \mid U^\dagger\{\mathcal{R}_1(\pi/2)\} \\ \times [I^+(0) - I^-(0)] U\{\mathcal{R}_1(\pi/2)\} \mid \mu \rangle .\end{aligned} \tag{3.10}$$

This completes the proof of the theorem.

The conditions

$$p^+ = p'^+ \text{ and } \vec{p}'_T - \vec{p}_T = \vec{Q}_T \tag{3.11}$$

remain invariant under all kinematic Lorentz transformations generated by the Lorentz generators K_n and $\vec{E}$. It follows that

$$\langle \mu',\mathbf{p}' \mid I^+(0) \mid \mathbf{p},\mu \rangle = \langle \mu' \mid I^+(0) \mid \mu \rangle \tag{3.12}$$

for all values of the momenta $\mathbf{p}'$ and $\mathbf{p}$ that satisfy the conditions (3.11).

The $(2j+1)^2$ elements of the matrix $\langle \mu' \mid I^+(0) \mid \mu \rangle$ are, of course, not all independent. Since the operator I^+ is Hermitian and invariant under rotations about $\vec{n}$, we have

$$\langle \mu' \mid I^+(0) \mid \mu \rangle = (-1)^{(\mu'-\mu)} \langle \mu \mid I^+(0) \mid \mu' \rangle , \tag{3.13}$$

which yields $j(2j+1)$ independent relations between the matrix elements. Time reversal plus reflection on the plane perpendicular to $\vec{n}$ leaves I^+ invariant.[44] This invariance together with the invariance under rotations about $\vec{n}$ yields the relations

$$\langle \mu' \mid I^+(0) \mid \mu \rangle = (-1)^{(\mu'-\mu)} \langle -\mu' \mid I^+(0) \mid -\mu \rangle \tag{3.14}$$

for the matrix elements. The number of additional independent relations obtained by Eq. (3.14) is $j(j+1)$ for integer spin, and $[(2j+1)/2]^2$ for half-odd integer spin. Thus Eqs. (3.13) and (3.14) leave, respectively, for integer and half-odd integer spin, $(j+1)^2$ and $(2j+3)(2j+1)/4$ independent matrix elements.

The matrix elements left independent by the constraints (3.13) and (3.14) are related by the rotational invariance of the charge density, which implies

$$\begin{aligned}\langle \mu', p' \mid I^+(0)+I^-(0) \mid p, \mu \rangle &= \langle \mu', p' \mid U^\dagger\{\mathcal{R}_2(\pi/2)\}[I^+(0)+I^-(0)] \\ &\quad \times U\{\mathcal{R}_2(\pi/2)\} \mid p, \mu \rangle \ .\end{aligned} \tag{3.15}$$

Assume that p and p' are given by Eq. (3.6). When these vectors are rotated by $\pi/2$ they are parallel to the direction of the spin quantization axis. The charge density is then diagonal in the spin quantum numbers,

$$\langle \mu', \mathcal{R}_2(\pi/2)p' \mid I^+(0)+I^-(0) \mid \mathcal{R}_2(\pi/2)p, \mu \rangle (\mu'-\mu) = 0 \ , \tag{3.16}$$

and the matrix elements of $I_1(0)$ vanish for all even values of $\mu'-\mu$,

$$[(-1)^{(\mu'-\mu)}+1] \langle \mu', \mathcal{R}_2(\pi/2)p' \mid I_1(0) \mid \mathcal{R}_2(\pi/2)p, \mu \rangle = 0 \ . \tag{3.17}$$

From Eqs. (2.19) and (2.30) it follows that

$$\begin{aligned}&\langle \mu', \mathcal{R}_2(\pi/2)p' \mid U\{\mathcal{R}_2(\pi/2)\} I^+(0) U^\dagger\{\mathcal{R}_2(\pi/2)\} \mid \mathcal{R}_2(\pi/2)p, \mu \rangle \\ &\quad = \sum_{\bar\mu'} \sum_{\bar\mu} \langle \mu' \mid \mathcal{R}_2(\pi/2)\mathcal{R}_M(p') \mid \bar\mu' \rangle \langle \bar\mu', p' \mid I^+(0) \mid p, \bar\mu \rangle \langle \bar\mu \mid \mathcal{R}_M^\dagger(p)\mathcal{R}_2^\dagger(\pi/2) \mid \mu \rangle \ .\end{aligned} \tag{3.18}$$

The left-hand side of this equation vanishes for even nonzero values of $\mu'-\mu$ according to Eqs. (3.15)–(3.17). Thus the requirement of rotational invariance implies that the right-hand side of Eq. (3.18) vanishes for even $\mu'-\mu \neq 0$,

$$\sum_{\bar\mu'} \sum_{\bar\mu} \langle \mu' \mid \mathcal{R}_2(\pi/2)\mathcal{R}_M(p') \mid \bar\mu' \rangle \langle \bar\mu', p' \mid I^+(0) \mid p, \bar\mu \rangle \langle \bar\mu \mid \mathcal{R}_M^\dagger(p)\mathcal{R}_2^\dagger(\pi/2) \mid \mu \rangle = 0 \ . \tag{3.19}$$

For $j \geq 1$, Eq. (3.19) imposes a nontrivial dynamic constraint on the matrix elements of the operator I^+.

For spin $\frac{1}{2}$ we can define form factors $F_1(Q^2)$ and $F_2(Q^2)$ in terms of the two independent matrix elements

$$\langle -\tfrac{1}{2} \mid I^+(0) \mid -\tfrac{1}{2} \rangle = \langle +\tfrac{1}{2} \mid I^+(0) \mid +\tfrac{1}{2} \rangle \ , \tag{3.20}$$

and

$$\langle -\tfrac{1}{2} \mid I^+(0) \mid +\tfrac{1}{2} \rangle = -\langle +\tfrac{1}{2} \mid I^+(0) \mid -\tfrac{1}{2} \rangle \tag{3.21}$$

by

$$F_1(Q^2) \equiv \langle +\tfrac{1}{2} \mid I^+(0) \mid +\tfrac{1}{2} \rangle \ , \tag{3.22}$$

and

$$F_2(Q^2) \equiv \frac{1}{\sqrt{\tau}} \langle -\tfrac{1}{2} \mid I^+(0) \mid +\tfrac{1}{2} \rangle \ , \tag{3.23}$$

where $\tau \equiv Q^2/4m^2$. It follows from Eqs. (3.12) and (3.20)–(3.22) that

$$\begin{aligned}\langle \mu', \mathbf{p}' \mid I^+(0) \mid \mathbf{p}, \mu \rangle &= F_1(Q^2)\delta_{\mu',\mu} \\ &\quad - i \langle \mu' \mid \sigma_2 \mid \mu \rangle F_2(Q^2)\sqrt{\tau}\end{aligned} \tag{3.24}$$

for all $\mathbf{p}'$ and $\mathbf{p}$ that satisfy Eq. (3.11).

The conventional electric and magnetic form factors $G_E(Q^2)$ and $G_M(Q^2)$ are defined in terms of matrix elements of charge and current densities in the Breit frame ($\vec{p}' = -\vec{p} = \frac{1}{2}\vec{Q}$) with canonical spin components in the direction of $\vec{Q}$.

$$G_E(Q^2) = \sqrt{1+\tau} \langle \tfrac{1}{2}, \tfrac{1}{2}\vec{Q} \mid I^0(0) \mid -\tfrac{1}{2}\vec{Q}, \tfrac{1}{2} \rangle_c \tag{3.25}$$

and

$$G_M(Q^2) = \frac{\sqrt{1+\tau}}{\sqrt{\tau}} \langle \tfrac{1}{2}, \tfrac{1}{2}\vec{Q} \mid I^1(0) \mid -\tfrac{1}{2}\vec{Q}, -\tfrac{1}{2} \rangle_c \ . \tag{3.26}$$

The subscript c indicates the use of the canonical spin with the quantization axis in the direction of the momentum transfer. The matrix elements defining the two sets of form factors are related by a Melosh rotation and a rotation of the spin quantization axis by $\pi/2$. Both rotations are rotations about the 2-axis, and the angle of the Melosh rotation, θ_M, is

$$\begin{aligned}\cos(\tfrac{1}{2}\theta_M) &= \frac{1+\sqrt{1+\tau}}{[(1+\sqrt{1+\tau})^2+\tau]^{1/2}} \ , \\ \sin(\tfrac{1}{2}\theta_M) &= \frac{\sqrt{\tau}}{[(1+\sqrt{1+\tau})^2+\tau]^{1/2}} \ .\end{aligned} \tag{3.27}$$

It follows that the two sets of form factors are related by the equations

$$\begin{aligned}F_1(Q^2) &= \frac{1}{1+\tau}[G_E(Q^2)+\tau G_M(Q^2)] \ , \\ F_2(Q^2) &= \frac{1}{1+\tau}[G_M(Q^2)-G_E(Q^2)] \ .\end{aligned} \tag{3.28}$$

We see from these relations that the form factors defined by Eqs. (3.22) and (3.23) are in fact equal to the standard Dirac and Pauli form factors usually defined in terms of Dirac spinors.[33,45] Note that for quarks the right-hand side of Eq. (3.24) is proportional to $\delta_{\mu',\mu}$ since for quarks F_2 vanishes and $F_1(Q^2)$ is constant.

For the deuteron, which has spin $j=1$, we define form factors F_{0d}, F_{1d}, F_{2d} in terms of the light-front spin matrix elements of the plus component of the current by

$$F_{0d}(Q^2)\equiv\frac{1}{2(1+\eta)}\{\langle +1|I^+(0)|+1\rangle + \langle 0|I^+(0)|0\rangle\} ,$$

$$F_{1d}(Q^2)\equiv\frac{-1}{(1+\eta)}\sqrt{2/\eta}\langle +1|I^+(0)|0\rangle , \quad (3.29)$$

$$F_{2d}(Q^2)\equiv\frac{-1}{(1+\eta)}\langle +1|I^+(0)|-1\rangle ,$$

where $\eta\equiv Q^2/4M_d^2$. The difference, $\langle +1|I^+(0)|+1\rangle - \langle 0|I^+(0)|0\rangle$, between the diagonal matrix elements is determined by the requirements of rotational invariance, Eq. (3.19)

$$\langle +1|I^+(0)|+1\rangle-\langle 0|I^+(0)|0\rangle = F_{2d}-2\eta(F_{0d}+F_{1d}) . \quad (3.30)$$

It vanishes for $Q^2=0$.

Conventional form factors G_0, G_1, G_2 are defined in terms of canonical-spin matrix elements in the standard Breit frame in which the momentum transfer is in the direction of the spin quantization axis.[19]

$$G_0(Q^2)=\tfrac{1}{3}(\langle 0|I^0|0\rangle_c+2\langle +1|I^0|+1\rangle_c) ,$$

$$G_1(Q^2)=-\left[\frac{2}{\eta}\right]^{1/2}\langle +1|I^1(0)|0\rangle_c , \quad (3.31)$$

$$G_2(Q^2)=\tfrac{1}{3}\sqrt{2}(\langle 0|I^0|0\rangle_c-\langle +1|I^0|+1\rangle_c) .$$

The matrix elements in Eqs. (3.29) and (3.31) are related to each other by a rotation of the quantization axis by $\pi/2$ and a Melosh rotation, both about the 2 axis. The Melosh angle θ_M is in this case given by

$$\cos\theta_M=\frac{1}{\sqrt{1+\eta}},\quad \sin\theta_M=\frac{\sqrt{\eta}}{\sqrt{1+\eta}} .$$

For any spin j the matrix relation between the canonical-spin representation and the light-front-spin representation of the current operators is given by the matrix product

$$\langle I^0-I^1\rangle_c=d^j(\pi/2)d^j(\theta_M)\langle I^+\rangle d^j(\theta_M)d^{j\dagger}(\pi/2) . \quad (3.32)$$

For spin $j=\frac{1}{2}$ and 1 the rotation matrices $d^j(\theta)$ are listed in the Appendix.

These transformations establish the relations

$$G_0(Q^2)=F_{0d}+\tfrac{1}{6}F_{2d}-\tfrac{2}{3}\eta\{F_{0d}+F_{2d}+\tfrac{5}{2}F_{1d}\} ,$$

$$G_1(Q^2)=2F_{0d}+F_{2d}+F_{1d}(1-\eta) , \quad (3.33)$$

$$G_2(Q^2)=\frac{\sqrt{8}}{3}\{F_{2d}+\eta(\tfrac{1}{2}F_{2d}-F_{0d}-F_{1d})\} ,$$

between the two sets of form factors defined in Eqs. (3.29) and (3.31), respectively.

B. Relation to observables

The observable electric and magnetic structure functions $A(Q^2)$ and $B(Q^2)$ which appear in the Rosenbluth cross section[4,46]

$$\frac{d\sigma}{d\Omega}=\frac{\alpha^2\cos^2(\theta/2)}{4E^2\sin^4(\theta/2)}\frac{E'}{E}\left[A(Q^2)+B(Q^2)\tan^2\frac{\theta}{2}\right] , \quad (3.34)$$

are related to the form factors G_0,G_1,G_2 defined in Eq. (3.31) by[19]

$$A(Q^2)\equiv G_0^2(Q^2)+G_2^2(Q^2)+\tfrac{2}{3}\eta G_1^2(Q^2) , \quad (3.35)$$

and

$$B(Q^2)\equiv\tfrac{4}{3}\eta(1+\eta)G_1^2(Q^2) . \quad (3.36)$$

The values of the three form factors for $Q^2=0$, are related to the charge, the magnetic moment μ_d, and the quadrupole moment Q_d. We have

$$F_{0d}(0)=G_0(0)=1,\quad F_{2d}(0)=0 ,$$
$$F_{1d}(0)=G_1(0)-2=\frac{M_d}{m}\mu_d-2 , \quad (3.37)$$

and the quadrupole moment, Q_d, is given by

$$Q_d=\lim_{Q^2\to 0}3\sqrt{2}G_2(Q^2)/Q^2 = \lim_{Q^2\to 0}4F_{2d}(Q^2)/Q^2-\frac{1}{M_d^2}[1+F_{1d}(0)] . \quad (3.38)$$

Experimental determination of all three form factors requires that either the target is polarized or the polarization of the recoil deuteron is observed. The tensor polarization T_{20} defined as a ratio of cross sections for the scattering by polarized deuterons and unpolarized deuterons:

$$T_{20}=\sqrt{2}\frac{d\sigma^1-d\sigma^0}{d\sigma} , \quad (3.39)$$

where $d\sigma^1$ and $d\sigma^0$ are the differential cross sections for scattering by deuterons with helicity 1 and 0, respectively. It follows from this definition that T_{20} is given as a function of the form factors by[47]

$$T_{20}(Q^2,\theta) = -\frac{G_2^2+\sqrt{8}G_0G_2+\frac{2}{3}\eta G_1^2[\frac{1}{2}+(1+\eta)\tan^2(\theta/2)]}{\sqrt{2}[A+B\tan^2(\theta/2)]} . \quad (3.40)$$

For sufficiently large values of Q^2 quantum chromodynamics permits a perturbative evaluation of the matrix elements (3.29) which yields simple power laws for these matrix elements.[29,30] However, when the matrix elements are evaluated perturbatively for relatively low values of Q^2, i.e., <10 GeV^2, the η dependent factors in the observables A, B and T_{20} must not be expanded in powers of Q^2.

C. Deuteron wave functions and nucleon form factors

The current matrix elements that determine the deuteron form factors according to (3.29) are determined by the deuteron wave function (2.31) and the current operators of the interacting proton-neutron system.

$$\langle \mu_d' | I^+(0) | \mu_d \rangle = \sum_{\mu_p',\mu_p} \sum_{\mu_n',\mu_n} \int d^3p_p' \int d^3p_p \int d^3p_n' \int d^3p_n \Psi^*_{P_d',\mu_d'}(\mathbf{p}_p',\mathbf{p}_n',\mu_p',\mu_n')\Psi_{P_d,\mu_d}(\mathbf{p}_p,\mathbf{p}_n,\mu_p,\mu_n)/\mathbf{p}_d^\dagger$$
$$\times[\langle \mu_p',\mathbf{p}_p' | I_p^+(0) | \mathbf{p}_p,\mu_p\rangle \delta_{\mu_n',\mu_n}\delta(\mathbf{p}_n'-\mathbf{p}_n)$$
$$+\langle \mu_n',\mathbf{p}_n' | I_n^+(0) | \mathbf{p}_n,\mu_n\rangle \delta_{\mu_p',\mu_p}\delta(\mathbf{p}_p'-\mathbf{p}_p)$$
$$+\langle \mu_n',\mu_p',\mathbf{p}_n',\mathbf{p}_p' | I_{np}^+(0) | \mathbf{p}_p,\mathbf{p}_n,\mu_p,\mu_n\rangle] \ . \tag{3.41}$$

The single nucleon currents I_n^+ and I_p^+ are determined by the nucleon form factors according to Eq. (3.24). The two-body operator I_{np}^+ vanishes for large separations of neutron and proton. Since all the matrix elements of I_n^+, I_p^+, and I_{np}^+ are related to each other by kinematic Lorentz transformations we may, for any one pair μ_d', μ_d set the two-body current I_{np}^+ in Eq. (3.41) equal to zero without violating Lorentz symmetry or current conservation. We emphasize that we cannot assume that the four-vector current operator, $I^\nu(x)$, of the interacting two-nucleon system is a sum of one-body operators. Such an assumption would be incompatible with covariance requirement (3.1) as well as current conservation:

$$[P_\nu, I^\nu(x)]=0 \ . \tag{3.42}$$

Moreover, it would be inconsistent to assume that all matrix elements of the operator I_{np}^+ vanish. Such an assumption would violate the rotational covariance requirement (3.30). However, we do not need the explicit knowledge of two-body operators which do not contribute to the determination of the form factors by Eqs. (3.29) and (3.41).

For symmetry reasons it is sufficient to calculate the contribution of the proton and then to replace the proton form factors by the isoscalar form nucleon form factors

$$F_{1N}\equiv F_{1p}+F_{1n}, \quad F_{2N}\equiv F_{2p}+F_{2n} \ . \tag{3.43}$$

From Eqs. (3.41), (2.31), (2.32), and (3.24) we get

$$\langle \mu_d' | I^+(0) | \mu_d\rangle = \sum_{\mu_1,\mu_2} \int d^2k_T' \int d^2k_T \int d\xi\, \delta[\vec{k}'_T-\vec{k}_T-(1-\xi)\vec{Q}]$$
$$\times\left[F_{1N}(Q^2)\chi^*_{\mu_d'}(\xi,\vec{k}'_T,\mu_1,\mu_2)\chi_{\mu_d}(\xi,\vec{k}_T,\mu_1,\mu_2) - \sqrt{\tau}F_{2N}(Q^2)\sum_{\mu_1'}\chi^*_{\mu_d'}(\xi,\vec{k}'_T,\mu_1',\mu_2)\langle \mu_1' | i\sigma_2 | \mu_1\rangle \chi_{\mu_d}(\xi,\vec{k}_T,\mu_1,\mu_2)\right] \ . \tag{3.44}$$

Exact relativistic expressions for the magnetic and quadrupole moments can be obtained by expanding the Fourier transform of the delta function in Eq. (3.44) in powers of $q\equiv|\vec{Q}|$:

$$\delta\{\vec{k}'_T-\vec{k}_T-(1-\xi)\vec{Q}\}=\frac{1}{(2\pi)^2}\int d^2x_T e^{-i(1-\xi)\vec{Q}\cdot\vec{x}_T}e^{i(\vec{k}'_T-\vec{k}_T)\cdot\vec{x}_T}$$
$$=\frac{1}{(2\pi)^2}\int d^2x_T e^{i(\vec{k}'_T-\vec{k}_T)\cdot\vec{x}_T}[1-i(1-\xi)qx_1-\tfrac{1}{2}(1-\xi)^2q^2x_1^2+\cdots] \ , \tag{3.45}$$

where $\vec{Q}=\{q,0,0\}$. From Eqs. (3.44) and (3.45) it follows that

$$F_{1d}(0)=\sqrt{8}M_d\sum_{\mu_1,\mu_2}\int d^2x_T\int d\xi\, i\left[F_{1N}(0)\chi^*_+(\vec{x}_T,\xi,\mu_1,\mu_2)(1-\xi)x_1\chi_0(\vec{x}_T,\xi,\mu_1,\mu_2) + \frac{F_{2N}(0)}{2m}\sum_{\mu_1'}\chi^*_+(\vec{x}_T,\xi,\mu_1',\mu_2)\langle\mu_1'|\sigma_2|\mu_1\rangle\chi_0(\vec{x}_T,\xi,\mu_1,\mu_2)\right] \ , \tag{3.46}$$

and

$$\lim_{Q^2\to 0}F_{2d}(Q^2)/Q^2=\sum_{\mu_1,\mu_2}\int d^2x_T\int d\xi\left[F_{1N}(0)\chi^*_+(\vec{x}_T,\xi,\mu_1,\mu_2)\tfrac{1}{2}(1-\xi)^2x_1^2\chi_-(\vec{x}_T,\xi,\mu_1,\mu_2) + \frac{F_{2N}(0)}{2m}\sum_{\mu_1'}\chi^*_+(\vec{x}_T,\xi,\mu_1',\mu_2)\langle\mu_1'|(1-\xi)x_1\sigma_2|\mu_1\rangle\chi_-(\vec{x}_T,\xi,\mu_1,\mu_2)\right] \ , \tag{3.47}$$

where $\chi_{\mu_d}(\vec{x}_T,\xi,\mu_1,\mu_2)$ is the Fourier transform of $\chi_{\mu_d}(\vec{k}_T,\xi,\mu_1,\mu_2)$.

D. Nonrelativistic approximation

The numerical computations of the integrals (3.44) are greatly simplified if we evaluate the form factors (3.29) for arbitrary Q^2 in the limit $m \to \infty$. For the evaluation of $F_{1\mathrm{d}}$ we need to expand the delta function $\delta\{\vec{k}'_T-\vec{k}_T-(1-\xi)\vec{Q}\}$ and the Melosh rotations to first order in powers of $1/m$. In that approximation

$$\frac{m+\xi M_0-i\vec{\sigma}\cdot(\vec{n}\times\vec{k}_T)}{[(m+\xi M_0)^2+\vec{k}_T^{\,2}]^{1/2}}\approx 1-\frac{i\vec{\sigma}\cdot(\vec{n}\times\vec{k}_T)}{2m}\ , \tag{3.48}$$

and

$$\delta\{\vec{k}'-\vec{k}-(1-\xi)\vec{Q}\}=\frac{1}{(2\pi)^3}\int d^3x e^{-i(1-\xi)\vec{Q}\cdot\vec{x}}e^{i(\vec{k}'-\vec{k})\cdot\vec{x}}$$
$$\approx\frac{1}{(2\pi)^3}\int d^3x e^{-i/2\vec{Q}\cdot\vec{x}}e^{i(\vec{k}'\cdot\vec{x})}\left[1-\frac{1}{2m}\vec{Q}\cdot\vec{x}\frac{\partial}{\partial x_3}\right]e^{-i(\vec{k}\cdot\vec{x})}\ . \tag{3.49}$$

Thus in the limit $m \to \infty$ Eq. (3.44) reduces to

$$\lim_{m\to\infty}\langle\mu'_{\mathrm{d}}\,|\,I^+(0)\,|\,\mu_{\mathrm{d}}\rangle=F_{1\mathrm{N}}(Q^2)\sum_{\mu_1,\mu_2}\int d^3x\, e^{-i/2\vec{Q}\cdot\vec{x}}\chi^{c*}_{\mu'_{\mathrm{d}}}(\vec{x},\mu_1,\mu_2)\chi^{c}_{\mu_{\mathrm{d}}}(\vec{x},\mu_1,\mu_2)\ , \tag{3.50}$$

and

$$\lim_{m\to\infty}-\left[\frac{2}{\eta}\right]^{1/2}\langle+1\,|\,I^+(0)\,|\,0\rangle=\left[\frac{2\tau}{\eta}\right]^{1/2}\sum_{\mu_1,\mu_2}\int d^3x e^{-i/2\vec{Q}\cdot\vec{x}}\left[F_{1\mathrm{N}}(Q^2)\chi^{c*}_{+}(\vec{x},\mu_1,\mu_2)x_1\frac{\partial\chi^c_0(\vec{x},\mu_1,\mu_2)}{\partial x_3}\right.$$
$$\left.+F_{2\mathrm{N}}(Q^2)\sum_{\mu'_1}\chi^{c*}_{+}(\vec{x},\mu'_1,\mu_2)\langle\mu'_1\,|\,i\sigma_2\,|\,\mu_1\rangle\chi^c_0(\vec{x},\mu_1,\mu_2)\right]\ . \tag{3.51}$$

The spin summations in Eqs. (3.50) and (3.51) can be done algebraically using Eq. (2.28) and the angle integrations can be performed analytically. In this manner we obtain from Eq. (3.50) approximations for the deuteron from factors $F_{0\mathrm{d}}$ and $F_{2\mathrm{d}}$ defined in Eq. (3.29):

$$F_{0\mathrm{d}}\approx F^{\mathrm{NR}}_{0\mathrm{d}}\equiv F_{1\mathrm{N}}[C_{u0}+C_{w0}-\tfrac{1}{4}C_{uw}+\tfrac{1}{8}C_{w2}]\ ,$$
$$F_{2\mathrm{d}}\approx F^{\mathrm{NR}}_{2\mathrm{d}}\equiv F_{1\mathrm{N}}\tfrac{3}{2}[C_{uw}-\tfrac{1}{2}C_{w2}]\ . \tag{3.52}$$

Here C_{u0}, C_{w0}, C_{w2}, and C_{uw} are Fourier-Bessel transforms of products of wave functions familiar in nonrelativistic form factor calculations:

$$C_{u0}(Q^2)\equiv\int dr\,u^2(r)j_0(\tfrac{1}{2}qr)\ ,$$
$$C_{w0}(Q^2)\equiv\int dr\,w^2(r)j_0(\tfrac{1}{2}qr)\ ,$$
$$C_{uw}(Q^2)\equiv\int dr\sqrt{2}u(r)w(r)j_2(\tfrac{1}{2}qr)\ ,$$
$$C_{w2}(Q^2)\equiv\int dr\,w^2(r)j_2(\tfrac{1}{2}qr)\ . \tag{3.53}$$

The corresponding approximation for the form factor $F_{1\mathrm{d}}$ is obtained from Eq. (3.51):

$$F_{1\mathrm{d}}\approx F^{\mathrm{NR}}_{1\mathrm{d}}\equiv\left[\frac{\tau}{\eta}\right]^{1/2}(F_{2\mathrm{N}}C_s+F_{1\mathrm{N}}\tfrac{3}{2}[C_{w2}-\tfrac{1}{2}C_{w0}])\ , \tag{3.54}$$

where

$$C_s(Q^2)\equiv C_{u0}-\tfrac{1}{2}C_{w0}+\tfrac{1}{2}C_{uw}+\tfrac{1}{2}C_{w2}\ . \tag{3.55}$$

This approximation is useful in gaining qualitative insight into the effects of feature of the wave function on the form factor. We will test its reliability by comparisons with exact calculations in the next section.

Eqs. (3.37) and (3.54) yield an approximation for the magnetic moment μ_{d} of the deuteron,

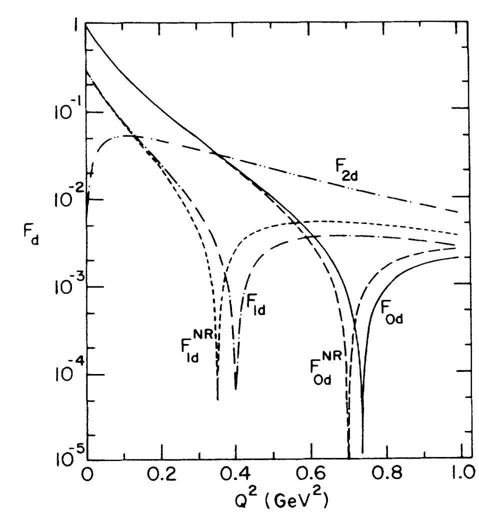

FIG. 1. Deuteron form factors $F_{0\mathrm{d}}$, $F_{1\mathrm{d}}$, and $F_{2\mathrm{d}}$, calculated with Paris wave functions and dipole nucleon form factors, at low Q^2. The dashed lines are the nonrelativistic approximations obtained from Eqs. (3.52) and (3.54). For $F_{2\mathrm{d}}$ the approximation (not shown) is very close to the exact result.

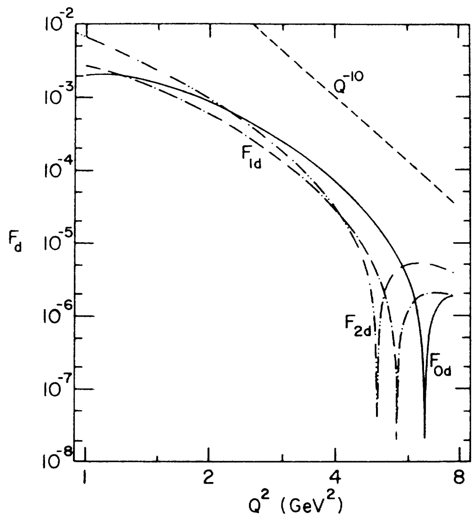

FIG. 2. The same deuteron form factors, F_{0d}, F_{1d}, and F_{2d}, as in Fig. 1 at high Q^2.

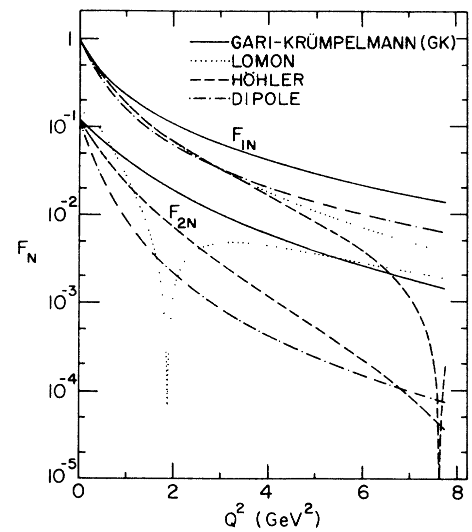

FIG. 4. Comparison of different parameterizations of the nucleon form factors F_{1N} and F_{2N}.

$$\mu_d \approx \frac{m}{M_d}[F_{1d}^{NR}(0)+2] = F_{2N}(0)(1-\tfrac{3}{2}P_D)-\tfrac{3}{4}P_D+\frac{2m}{M_d}$$
$$=(\mu_p+\mu_n)(1-\tfrac{3}{2}P_D)$$
$$+\tfrac{3}{4}P_D+\frac{2m-M_d}{M_d}\,, \qquad (3.56)$$

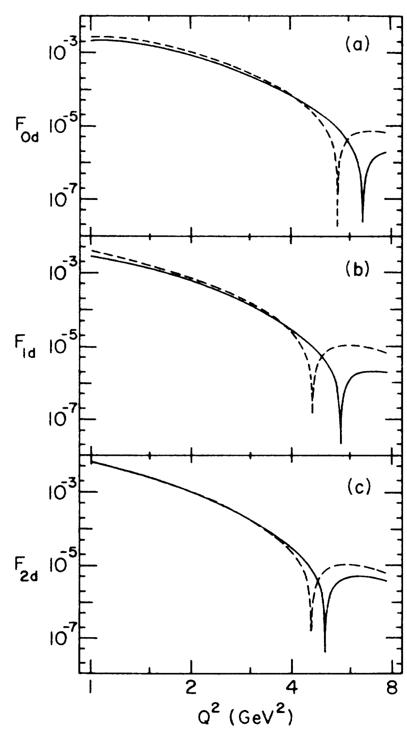

FIG. 3. Deuteron form factors F_{0d}, F_{1d}, and F_{2d} at high Q^2. The dashed lines show the nonrelativistic approximations, Eqs. (3.52) and (3.54). Paris wave functions and dipole nucleon form factors were used.

which differs from the standard nonrelativistic result by the small last term. This term appears among the relativistic corrections to the magnetic moment obtained by Kondratyuk and Strikman.[48]

Nonrelativistic approximations to the observables are ambiguous in that they depend on where η and/or τ are neglected or not neglected. The standard nonrelativistic expressions[49] result from using $\eta=0$ in Eq. (3.33) as well as $\tau=0$ in Eq. (3.28).

IV. NUMERICAL RESULTS

Figure 1 shows typical low-momentum-transfer results for the form factors F_{0d}, F_{1d}, and F_{2d} together with their nonrelativistic approximations, using the Paris deuteron wave functions and dipole nucleon form factors, $G_{ED}\equiv 1/(1+Q^2/0.71)^2=G_{Ep}$, and $G_{MD}=(\mu_p+\mu_n)G_{ED}$. The qualitative features of these curves are the same for

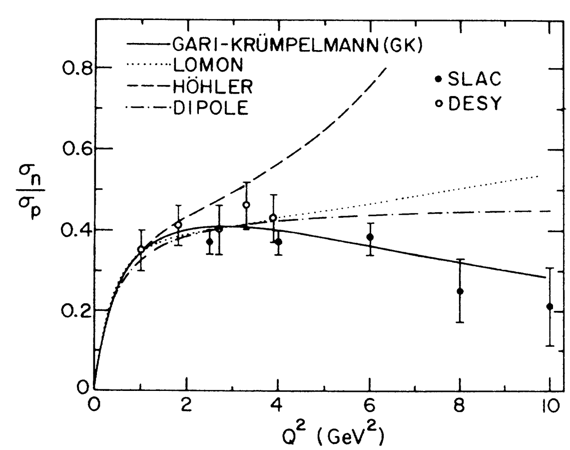

FIG. 5. Ratios of neutron and proton cross sections calculated with different nucleon form factors compared to data (Ref. 50).

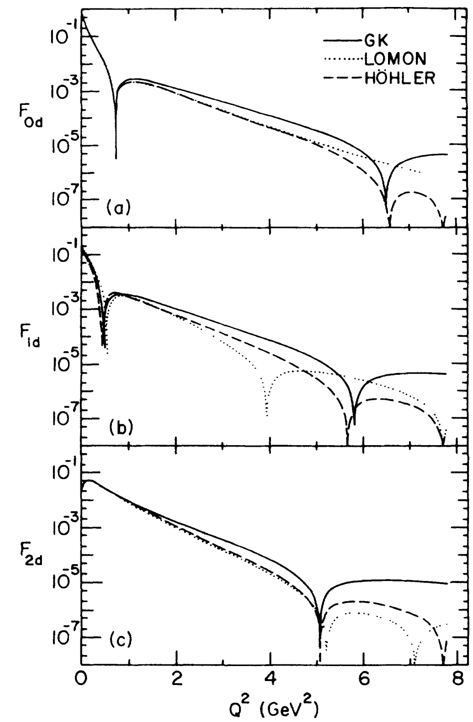

FIG. 6. Sensitivity of the deuteron form factors, F_{0d}, F_{1d}, and F_{2d}, to variations in the nucleon form factors.

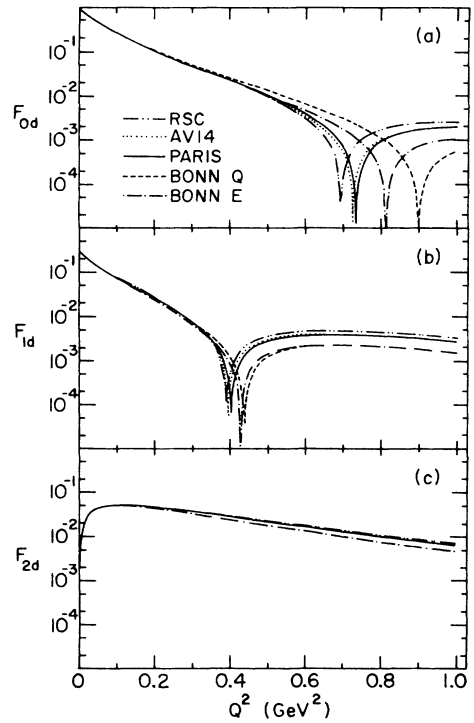

FIG. 8. Sensitivity of the deuteron form factors, F_{0d}, F_{1d}, and F_{2d}, to features of the deuteron wave function at low Q^2. The form factor $F_{2d}(Q^2)$ for AV14 is indistinguishable from Paris and for Bonn Q it is very close to Bonn E. The curves shown are calculated with dipole nucleon form factors.

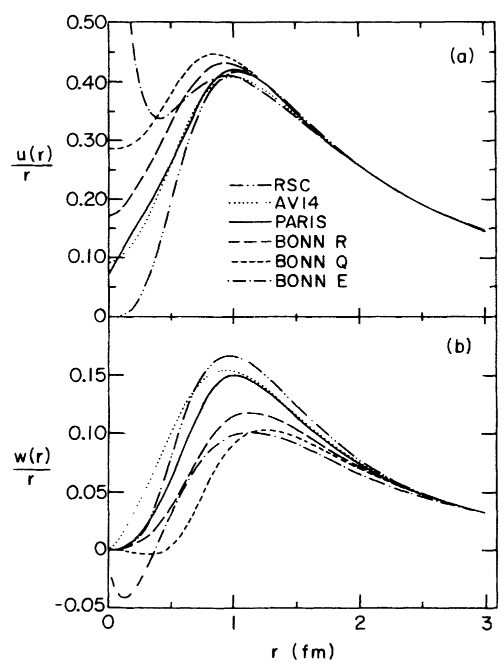

FIG. 7. Deuteron S and D wave functions for the Reid soft core, Paris, Argonne v_{14}, and Bonn potentials (Ref. 25). Bonn E is the energy dependent "full model."

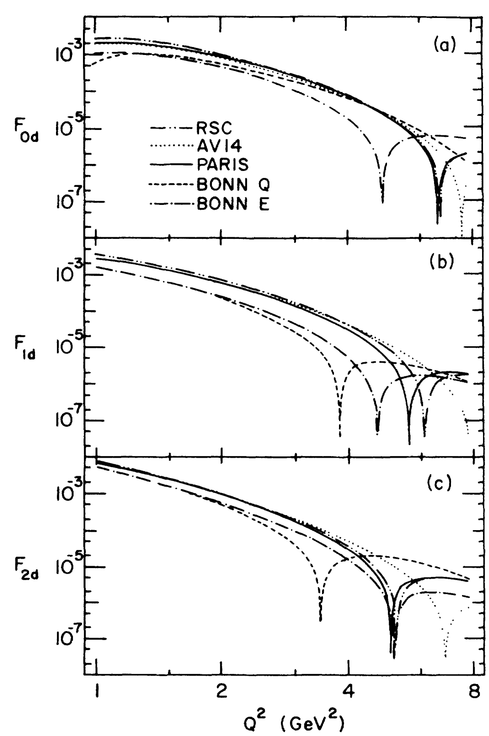

FIG. 9. Sensitivity of the deuteron form factors, F_{0d}, F_{1d}, and F_{2d}, to features of the deuteron wave function at high Q^2. The nucleon form factors are dipole as in Fig. 8.

TABLE I. Table of the zeros of the deuteron form factors $F_{0d}(Q^2)$ for different combinations of deuteron wave functions and nucleon form factors.

Potentials	P_D (%)	GK		Höhler			Lomon		Dipole	
RSC	6.47	0.691	6.60	0.693	6.67	7.73	0.690		0.695	6.71
AV14	6.08	0.724	7.45	0.727	7.43		0.723		0.728	7.56
Paris	5.77	0.732	6.50	0.735	6.58	7.72	0.731		0.737	6.61
Bonn R	4.81	0.804		0.807	7.62		0.803		0.808	
Bonn Q	4.38	0.894		0.899	7.61		0.895		0.901	
Bonn E	4.25	0.811	4.84	0.813	4.86	7.67	0.810	4.97	0.815	4.86

all potentials and nucleon form factors we considered. Both the nonrelativistic and the exact calculations show similar patterns. The main difference is in the position of the zeros of F_{0d} and F_{1d}. The zero of F_{0d} is primarily determined by the diffraction pattern of the density u^2+w^2, i.e., the sign change of $C_{u0}+C_{w0}$, but its location is also influenced by the subtraction of the interference term C_{uw}. In F_{1d}, on the other hand, all the individual integrals are positive and the zero is the result of a cancellation which depends on features of the D-wave function. We show in Fig. 2 the same form factors for 1 $\text{GeV}^2<Q^2<8$ GeV^2 on a double log plot to exhibit the power decrease of the form factors. Perturbative QCD predicts that for very large Q^2 the leading contribution to the matrix elements in Eq. (3.29) decreases as Q^{-10}. To guide the eye we also show the function Q^{-10}. A comparison of the exact and the nonrelativistic calculations is shown in Fig. 3. The nonrelativistic calculations appear to give a remarkably good approximation for $Q^2<4$ GeV^2. The quality of the approximation may depend significantly on possible exotic short-range features of the deuteron wave functions and also on the nucleon form factors.

A comparison of different parametrizations of the nucleon form factors[26–28] is shown in Fig. 4. The experimental uncertainty is primarily in the neutron form factors. In Fig. 5 we compare calculated ratios of neutron to proton cross sections with data.[50] We use these form factors in order to exhibit in Fig. 6 the sensitivity of the deuteron form factors to variations in the nucleon form factors.

The sensitivity of the form factors to the different deuteron wave functions, shown in Fig. 7, is exhibited in Figs. 8 and 9 for the ranges $Q^2<1$ GeV^2 and 1 $\text{GeV}^2<Q^2<8$ GeV^2, respectively. Differences in the short-range properties of the wave functions clearly affect the form factors. The three recent Bonn models[25] differ substantially from the other models as well as from each other. (We use the label Bonn E to designate their energy-dependent "full model.") The zeros of the deuteron form factors as functions of Q^2 conveniently characterize the differences. They are listed in Tables I–III for all combinations of potentials and nucleon form factors, together with the D-state probability of each model.

Calculated elastic structure functions A and B are compared in Figs. 10 and 11 with data[51,52] for A and data[53–55] for B. Assuming the nucleon form factors of Gari and Krümpelmann[28] (GK) both Paris and AV14 wave functions give good agreement with the data for A, but only AV14 agrees well with the data for B. The difference in the results for B is associated with the larger D-wave function at short distances as shown in Fig. 7(b). In Figs. 10(b) and 11(b) we see that with Höhler nucleon form factors[27] the data require two-body-current effects in the structure functions $A(Q^2)$ and $B(Q^2)$. The same conclusion is obtained if Lomon nucleon form factors[26] are used.

Recent data[52] for A at small Q^2 are far more accurate than the plot in Fig. 10. In Fig. 12 we see that the GK form factors with the Paris wave function are in disagreement with the data. A 50% reduction of the neutron electric form factor G_{E_n} brings agreement with the data. Figure 13 shows that even at these small values of the momentum transfer there are significant differences between Paris and Bonn and between the different Bonn models. Höhler nucleon form factors were used in Fig. 13. Comparison with Fig. 12 also illustrates the form factor dependence on this scale. On the scale of Figs. 12 and 13, AV14 is indistinguishable from Paris.

In Fig. 14 we compare the exact results for A with the standard nonrelativistic results for four models, Paris, AV14 and the two energy independent Bonn models:

TABLE II. Table of the zeros of the deuteron form factors $F_{1d}(Q^2)$ for different combinations of deuteron wave functions and nucleon form factors.

Potentials	P_D (%)	GK		Höher			Lomon			Dipole	
RSC	6.47	0.464	6.26	0.414	6.16	7.70	0.522	4.36	7.73	0.391	6.14
AV14	6.08	0.478	7.72	0.422	7.57		0.539			0.396	
Paris	5.77	0.492	5.81	0.431	5.66	7.70	0.554	3.95	7.72	0.404	5.64
Bonn R	4.81	0.546	5.82	0.460	5.14	7.60	0.616	3.08		0.424	5.05
Bonn Q	4.38	0.597	5.13	0.486	3.98	7.52	0.663	2.61		0.439	3.83
Bonn E	4.25	0.565	4.63	0.467	4.71		0.635			0.428	4.72

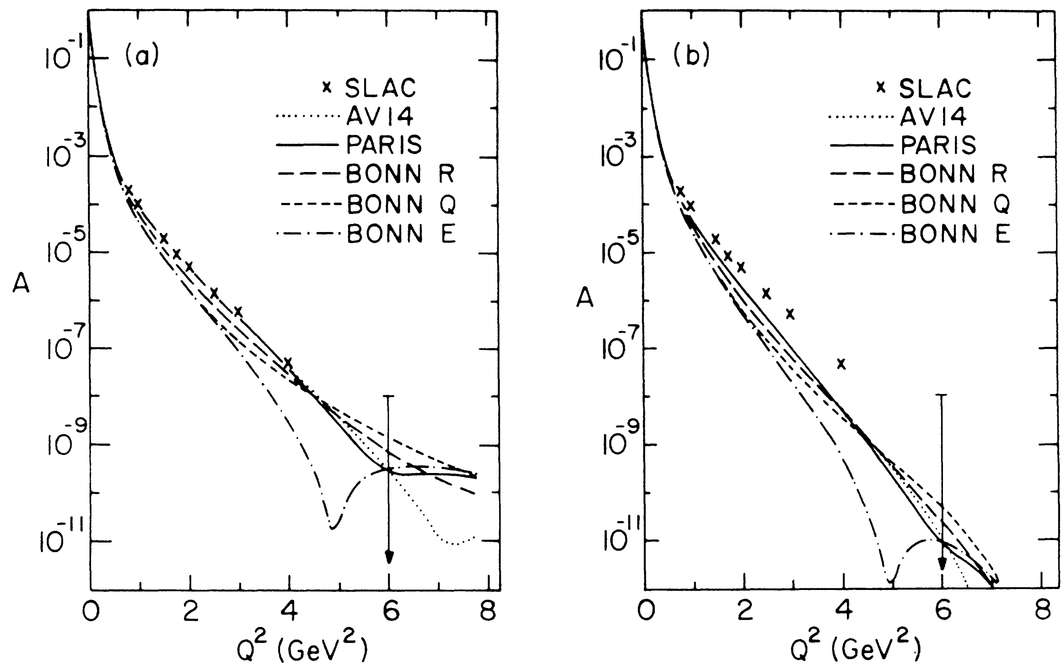

FIG. 10. Deuteron structure function $A(Q^2)$ for different deuteron wave functions compared to data. (a) GK nucleon form factors. (b) Höhler nucleon form factors. The low-Q^2 Saclay data (Ref. 52) shown in Figs. 12 and 13 are indistinguishable from the solid line.

Bonn Q and Bonn R. For $Q^2 < 2$ GeV^2 the relativistic effect in A is quite small and remarkably model independent. Nevertheless, it is not negligible when precise measurements of A are used to determine the electric neutron form factor.[52,56] The relativistic analysis implies a larger neutron form factor than the nonrelativistic analysis. For larger values of Q^2 both the sign and the size of the effect are strongly model dependent.

Results for the tensor polarization T_{20} are shown in Fig. 15. The curves were calculated with GK nucleon form factors. For $Q^2 < 2$ GeV^2 there are no significant differences in T_{20} obtained with different nucleon form

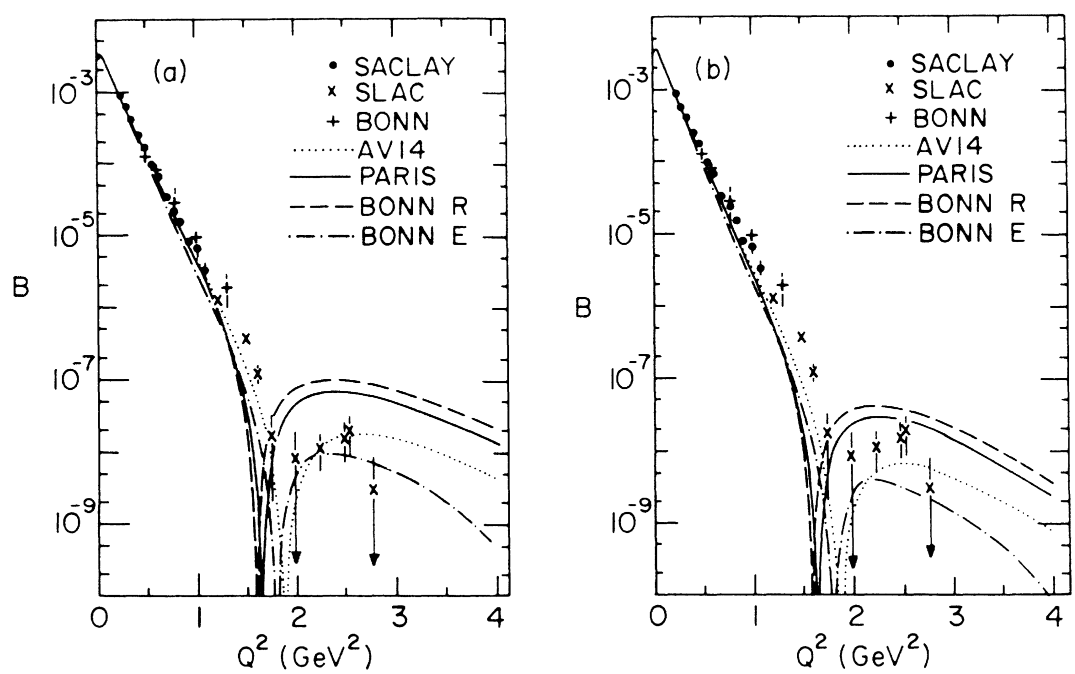

FIG. 11. Deuteron structure function $B(Q^2)$ for different deuteron wave functions compared to data. (a) GK nucleon form factors. (b) Höhler nucleon form factors. The curve for Bonn Q (not shown) is quite close to that for Bonn R.

TABLE III. Table of the zeros of the deuteron form factors $F_{2d}(Q^2)$ for different combinations of deuteron wave functions and nucleon form factors.

Potentials	P_D (%)	GK	Höhler		Lomon		Dipole
RSC	6.47	5.71	5.22	7.72			5.23
AV14	6.08	6.65	6.79				6.88
Paris	5.77	5.04	5.06	7.71	5.21	7.11	5.06
Bonn R	4.81	4.60	4.51	7.69	4.10		4.51
Bonn Q	4.38	3.52	3.46	7.67	3.21		3.44
Bonn E	4.25	5.09	5.16	7.75			5.15

factors. Possible effects of different deuteron wave functions are illustrated by the difference between the Paris and Bonn models. The data shown[57] are at too low a momentum transfer to discriminate between different models. Future measurements[58] at higher Q^2 could play a decisive role in clarifying the role of subnucleon degrees of freedom.

V. CONCLUSIONS

We have demonstrated that Hamiltonian light-front dynamics allows the construction of internally consistent Poincaré invariant models of elastic electron-deuteron scattering that use conventional deuteron wave functions and empirical nucleon form factors. These deuteron wave functions are based on models which give the best fits to nucleon-nucleon scattering data. The calculated observables are quite close to those of the "nonrelativistic" limit described in the text, though the size of the difference is model dependent. We find that AV14 wave functions and Gari-Krümpelmann form factors give satisfactory agreement with all existing data except that the electric neutron form factor must be reduced for Q^2 less than 1 GeV^2 by about 50%. There is no intrinsic breakdown of these models or massive disagreement with existing data that would call for a perturbative-QCD cure. The models considered here bypass the important question how the short-range charge-current structure and the short-range nuclear force are to be understood in terms of the fundamental quark degrees of freedom. While these models are mathematically consistent for arbitrarily high momentum transfer it remains to be seen where they become physically irrelevent. A more precise determination of the nucleon form factors, both experimentally and theoretically, would be an important step.

There is clearly a need for better experimental determination of the neutron form factors, which could greatly reduce the spread in predicted electron-deuteron observables. Measurements of the tensor polarization[58] at higher Q^2 could provide definite evidence for an explicit role of subnucleon degrees of freedom.

ACKNOWLEDGMENTS

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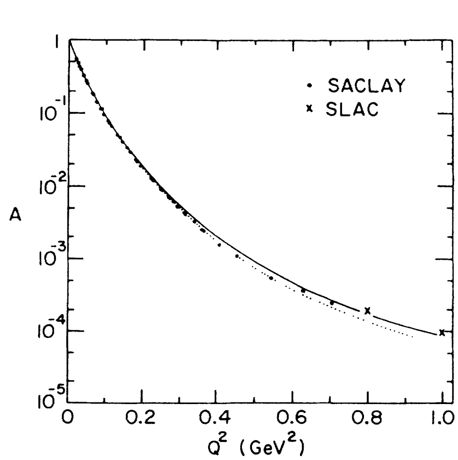

FIG. 12. Sensitivity of the structure function $A(Q^2)$ to changes in the neutron electric form factor. (GK nucleon form factors.)

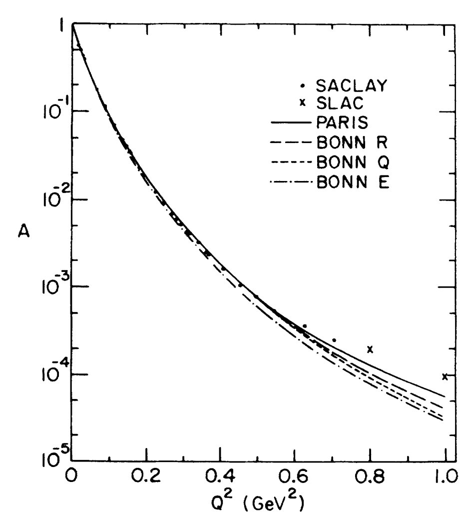

FIG. 13. The deuteron structure function $A(Q^2)$ at low Q^2 for different deuteron wave functions and Höhler nucleon form factors compared to data.

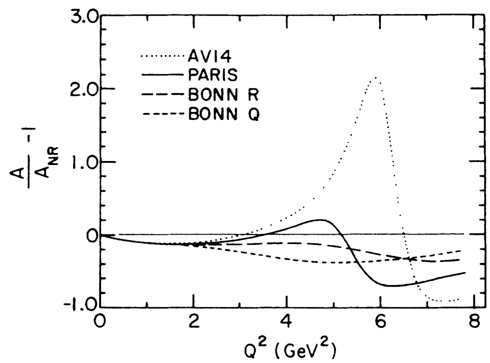

FIG. 14. Wave-function dependence of relativistic effects in the structure function A.

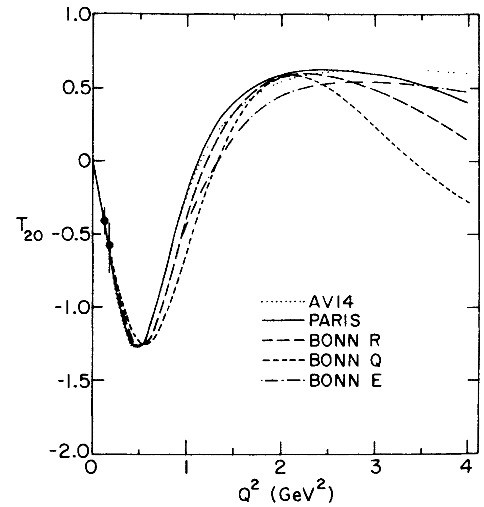

FIG. 15. Deuteron tensor polarization as a function of Q^2 for $\theta=70°$. The curves are calculated with GK nucleon form factors.

APPENDIX

The Minkowski representation $\Lambda^\rho{}_\nu$ of the Lorentz transformation Λ and the 2×2 unimodular representation $u(\Lambda)$ are related by[2]

$$\Lambda^\rho{}_\nu=\tfrac{1}{2}\mathrm{Tr}(\sigma_\rho u\sigma_\nu u^\dagger)\ , \tag{A1}$$

where σ_0 is the unit matrix and $\sigma_1,\sigma_2,\sigma_3$ are the standard Pauli matrices.

The unimodular representations $u_c(P)$ and $u_f(P)$ of the Lorentz transformations $L_c(P)$ and $L_f(P)$ are given by

$$u_c(P)=\frac{M+P^0-\vec{\mathrm{P}}\cdot\vec{\sigma}}{[2M(M+P^0)]^{1/2}}\ , \tag{A2}$$

and

$$u_f(P)=\frac{M+P^+-(P^+-M)\vec{\mathrm{n}}\cdot\vec{\sigma}-\vec{\mathrm{P}}_T\cdot\vec{\sigma}+i\vec{\sigma}\cdot(\vec{\mathrm{n}}\times\vec{\mathrm{P}})}{2\sqrt{MP^+}}\ . \tag{A3}$$

The irreducible representations $d^j(\theta)$ of rotations about the 2 axis are for $j=\frac{1}{2}$

$$d^{1/2}(\theta)=\cos(\theta/2)+i\sigma_2\sin(\theta/2)\ , \tag{A4}$$

and for $j=1$

$$d^1(\theta)=\begin{bmatrix}\frac{1}{2}(1+\cos\theta) & \frac{1}{\sqrt{2}}\sin\theta & \frac{1}{2}(1-\cos\theta)\\ -\frac{1}{\sqrt{2}}\sin\theta & \cos\theta & \frac{1}{\sqrt{2}}\sin\theta\\ \frac{1}{2}(1-\cos\theta) & -\frac{1}{\sqrt{2}}\sin\theta & \frac{1}{2}(1+\cos\theta)\end{bmatrix}\ , \tag{A5}$$

where the rows and columns are labelled in the order $+1,0,-1$.

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