F-spin multiplets and alpha-transfer systematics

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It is shown that the Sn-Gd ($N<82$) nuclei form relatively poor F-spin multiplets so that the recently proposed method for predicting the spectroscopic factor ratios in the α-transfer reactions will provide a poor estimate. However, the method may apply to the good F multiplets in the Dy-Pt region ($N<104$).

In the microscopic interacting boson model IBM-2, Arima *et al.*[1] used the subgroup chain $U_p(6)\times U_n(6)\supset U_{p+n}(6)\times SU_F(2)$, where F spin is analogous to the isotopic spin and $F=\frac{1}{2}(N_p+N_n)=\frac{1}{2}N_B$ for F-symmetric states. If H is invariant to the rotation in the p, n boson space, i.e.,

$$[H,F_\pm]=0\ , \tag{1}$$

then the nuclei with the same $N_B=2F$ and with different projections $F_0=\frac{1}{2}(N_p-N_n)$, e.g., those differing by a quartet, will form an F-spin multiplet[2,3] embedded in the U(12) super multiplet group.[3] Harter *et al.*[2] pointed out the similar spectral properties of the neutron-deficient Te-Sm ($N\le 82$) nuclei with $(A,A+4,\dots,)$ and constant $N_B=6$ or 7, supporting (1) for a U(12) multiplet.

Recently, Frank[4] proposed a novel use of the F invariance in a U(12) multiplet for predicting the relative spectroscopic factors of α-transfer reactions among the members of the $(A,A+4,\dots,)$ multiplet of even N, even Z nuclei. In this Comment, we look at the validity of the F-spin invariance in the proposed multiplets, on which his proposal is based.

Since the generators of the F-spin follow[1] the SU(2) algebra, one has

$$\begin{aligned}F_+\,|FF_0-1\sigma\rangle&=[(F-F_0+1)(F+F_0)]^{1/2}\,|FF_0\sigma\rangle\ ,\\ F_-\,|FF_0+1\sigma\rangle&=[(F-F_0)(F+F_0+1)]^{1/2}\,|FF_0\sigma\rangle\ ,\end{aligned} \tag{2}$$

leading to the product nucleus (F,F_0). Here σ denotes all the other quantum numbers corresponding to U(6) and its subgroups. Frank pointed out in that if (1) holds over the U(12) multiplet, one can take the α-particle creation (annihilation) operator equal to $aF_\pm$, where a is a constant. Then, the spectroscopic factors for stripping and pickup reactions are

$$\begin{aligned}S^{\rm str}({\rm g.s.}\to{\rm g.s.})&=|\langle FF_0\sigma\|aF_+\|FF_0-1\sigma\rangle|^2\ ,\\ S^{\rm PU}({\rm g.s.}\to{\rm g.s.})&=|\langle FF_0\sigma\|aF_-\|FF_0+1\sigma\rangle|^2\ ,\end{aligned} \tag{3}$$

and the ratio $r=S^{\rm str}/S^{\rm PU}$ for a given member (F,F_0) of the U(12) group of nuclei will be a simple number obtained from (1), independent of the detailed structure of the target and product nuclei,[4] viz.,

$$r=\frac{(F-F_0+1)(F+F_0)}{(F-F_0)(F+F_0+1)}\ . \tag{4}$$

Thus Frank illustrated the variation of the ratio r for Sn-Gd, $N_B=6,7$ F multiplets with numerical values ranging from 0 to 3 (see Fig. 3 of Ref. 4) and suggested an α-transfer experiment to verify this.

To test the validity of Frank's approach based on (1), i.e., on the existence of similar spectra of the nuclei forming an F multiplet, we illustrate the $R_4=E_4/E_2$ vs Z data, linking the nuclei with same N_B (or F) by the broken lines (Fig. 1). It is apparent that for none of the constant N_B values, the ratio R_4 is approximately constant. Even for the proposed[2,3] F multiplets of $N_B=6,7$, at most three members in the central part, Ba-Nd have nearly the same R_4. In Te, almost the same value is obtained for all N_B. The same is true for Sn (not shown) and partly true for Sm and Gd. Thus at most three nuclei with $N_B=6$ or 7 can be assumed to be embedded in the U(12) super group to which Eq. (1) can apply, so that the assumption of the proportionality of the α-creation (annihilation) operator to the $F_\pm$ operator and the use of (4) will be val-

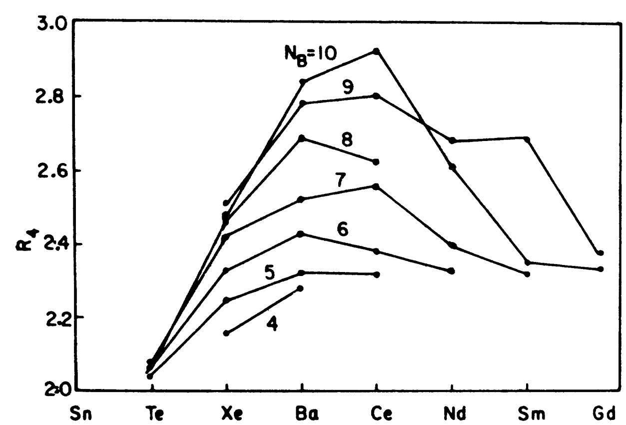

FIG. 1. Variation of the ratio R_4 with atomic number Z. The data points of the same boson number N_B are linked by the broken lines.

TABLE I. The ratio R_4 in the F-spin multiplets of constant N_B.

N_B	Dy	Er	Yb	Hf	W	Os	Pt
12	2.93	3.10	3.12	3.11	3.07	2.92	2.68
13	3.21	3.23	3.23	3.19	3.17	3.02	2.70
14	3.27	3.28	3.27	3.25	3.21	3.09	
15	3.29	3.29	3.29	3.27	3.23		
16	3.30	3.31	3.31	3.28			
17	3.31	3.31	3.31				

id only for these nuclei, and not for the full Sn-Gd $N_B=6$ or 7 F-spin multiplets as suggested by Frank.[4]

Next we look at the numerical values of the ratio r. The calculated ratio r from (4) for $N_B=6$ ^{132}Ba (with ^{128}Xe and ^{136}Ce as targets) is 1.0, and r (^{136}Ce)$=1.2$. Similarly, for $N_B=7$ nuclei ^{130}Ba, ^{134}Ce, and ^{138}Nd, $r=0.94$, 1.07, and 1.25, respectively. (Note the error in Frank's work[4] where squares of these numbers have been taken erroneously, the total variation in r up to Sm being limited to 1.7 and not 3.0.) This already exhausts the possible useful points on the r vs F_0 graph in Fig. 3 of Ref. 4 for $N_B=6,7$, for the reasons explained in the preceding paragraph, so that the predicted[4] large variation in r of up to $r=1.7$ over the full F multiplet will not arise. Thus r is not sensitive to the variation in F_0, lying within $\pm 15\%$ only, for the valid Ba-Nd F multiplet.

A comparison of the calculated value of r for the central member of the triad ($A-4, A, A+4$) with the experimental value could be useful for testing the validity of Eq. (1). But this suffers from another difficulty. Arima *et al.*[1] pointed out that on account of the V_{pn} term in $H_{IBM}=H_p+H_n+V_{pn}$, the term

$$E_0=\mu_p m+\nu_p \tfrac{1}{2}m(m-1)+\mu_n n+\nu_n \tfrac{1}{2}n(n-1)+\nu_{pn}mn \;, \tag{5}$$

(m and n being the number of proton and neutron pairs) could be important for ground state (g.s.) energies, and that if E_0 is removed from H_{IBM}, the differences between proton and neutron bosons are only in their excitations and those are not different, so that the rest of H could be approximated by a scalar in F spin. Thus even if similar collective excitation spectra arise among the members of the F multiplet, the microscopic g.s. properties may still differ. Hence a g.s. to g.s. α-transfer reaction ratio S^{str}/S^{PU} may or may not be equal to the r value derived from (4). Then it will be of interest to compare the experimental value of r with the value derived from (4) for the g.s. relative properties of the triad ($A-4, A, A+4$).

More valid F-spin multiplets do arise in the Dy-Pt region ($N \le 104$) for $N_B=12,13$ (Ref. 5). The ratio R_4 for $N_B=12-17$ for these nuclei vary only slowly with $F_0=\frac{1}{2}(N_p-N_n)$, i.e., with Z (Table I). Also the moment of inertia $\theta=3/E(2_1^+)$ varies slowly with Z in these F multiplets (see Table I of Ref. 6). Hence a test of Eq. (4) should be possible in these F multiplets. However, note that the calculated value of r is again close to one (within 10%) in each of these multiplets, and the suggested[4] large variation in r over the multiplet will not arise. But a test of relative g.s. properties as discussed above should be possible.

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