Speed of sound in asymmetric nuclear matter with Skyrme interactions

R. K. Su*
Center of Theoretical Physics, China Center of Advanced Science and Technology, (World Laboratory), Beijing, People's Republic of China

H. Q. Song
Institute of Nuclear Research, Shanghai, People's Republic of China

T. T. S. Kuo
Department of Physics, State University of New York at Stony Brook, Stony Brook, New York 11794

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The speed of sound v is calculated for asymmetric nuclear matter within the general framework of a finite temperature Green's function method with normal pair cutoff approximation and Skyrme interactions. A rather strong dependence of v on α, the asymmetry coefficient, is observed at high density. Causal violation of v at high density and/or high temperature is studied. This violation may be avoided if the Skyrme interaction parameters at high density may be readjusted to obey the conditions $-1<d<0$, $t_3(3-\alpha^2-2\alpha^2\chi_3)\geq 0$, and $[(3t_1+5t_2)\pm\alpha(t_2-t_1)]\leq 0$. Implications of these conditions are discussed.

Hartree-Fock theories using Skyrme effective interactions have been quite successful in describing the ground state properties of a large number of finite nuclei as well as of nuclear matter.[1-3] Thus it is natural, as has been done by many authors,[4-6] to use these interactions within the framework of a finite temperature mean field theory to derive the nuclear matter equation of state (EOS) at finite temperature. In fact, derivations of the nuclear matter EOS have been a subject of much current interest.[7]

There is, however, one rather serious shortcoming about the EOS derived from Skyrme interactions and within the theoretical framework mentioned above. As pointed out by Osnes and Strottman,[8,9] and by two of the present authors,[10] the speed of sound v calculated in this way violates the causal constraint $v<c$, c being the speed of light, at high density and/or high temperature. This causal violation, usually known as superluminosity, is of course a serious problem and deserves further investigation.

The primary purpose of the present work is to further study the above superluminosity problem. First we want to derive a general expression for calculating v in asymmetric nuclear matter with asymmetric coefficient α. In previous works[8-10] v was calculated only for symmetric nuclear matter. As described later, our derivation is carried out within the general framework of a finite temperature Green's function method with normal pair cutoff approximation.[11,12] We will calculate v in asymmetric nuclear matter for several Skyrme interactions. As discussed later, although v depends rather strongly on α at high density, the superluminosity difficulty persists for any α $(0\leq\alpha\leq 1)$. We next turn to the question whether we can readjust the parameters d, t_0, t_1, t_2, and t_3 of the Skyrme interactions so that superluminosity never occurs. We find that if we choose $-1<d<0$, $t_3(3-\alpha^2-2\alpha^2\chi_3)\geq 0$ and $[(3t_1+5t_2)\pm\alpha(t_2-t_1)]\leq 0$, then the superluminosity difficulty may be entirely eliminated. The feasibility of these conditions and the possibility of modifying the Skyrme interaction parameters at high nuclear matter densities are also discussed later on.

Two basic thermodynamic quantities which are needed in the calculation of the speed of sound in nuclear matter are the internal energy and pressure. In the present work we shall calculate them using a real-time finite temperature Green's function method with normal pair cutoff approximation. Since the details of this method have been given elsewhere,[11,12] in the following we only present our main results.

In our calculation, we employ the Skyrme interaction[1-3]

$$V_{12}=t_0(1+\chi_0P_0)\delta(\mathbf{r}_1-\mathbf{r}_2)+\tfrac{1}{2}t_1[\delta(\mathbf{r}_1-\mathbf{r}_2)k^2+k'^2\delta(\mathbf{r}_1-\mathbf{r}_2)]+t_2\overleftarrow{k}'\cdot\delta(\mathbf{r}_1-\mathbf{r}_2)\mathbf{k}+\tfrac{1}{6}\rho^d t_3(1+\chi_3P_\sigma)\delta(\mathbf{r}_1-\mathbf{r}_2)\ . \tag{1}$$

With this interaction, the internal energy per nucleon $(\overline{U})$ for nuclear matter can be readily derived. For symmetric nuclear matter, this has been done by Su *et al.*[11,12] In a similar way, we have derived $\overline{U}$ for asymmetric nuclear matter as

$$U=C_0(\rho,\alpha)+I_p(\rho,\alpha,T)+I_n(\rho,\alpha,T)\ , \tag{2}$$

where

$$C_0(\rho,\alpha)=\frac{t_0}{8}\rho(3-\alpha^2-2\alpha^2\chi_0)+\frac{t_3}{48}\rho^{d+1}(3-\alpha^2-2\alpha^2\chi_3)\ , \tag{3}$$

$$I_p(\rho,\alpha,T)=\left[\frac{8}{m\rho}+3t_1+5t_2+\alpha(t_1-t_2)\right]\frac{1}{16\pi^2}\times\int q^4 n_p(q)dq\ , \quad (4)$$

$$I_n(\rho,\alpha,T)=\left[\frac{8}{m\rho}+3t_1+5t_2-\alpha(t_1-t_2)\right]\frac{1}{16\pi^2}\times\int q^4 n_n(q)dq\ . \quad (5)$$

The nuclear matter density is denoted by ρ, and the asymmetric coefficient α is defined by

$$\rho_n=(1+\alpha)\rho/2 \text{ and } \rho_p=(1-\alpha)\rho/2\ , \quad (6)$$

where ρ_n and ρ_p are, respectively, the neutron and proton nuclear matter density. The neutron and proton Fermi-Dirac distribution functions are denoted, respectively, by n_n and n_p and are calculated from a self-consistent Green's function method.[11,12]

It may be noted that C_0 is independent of the temperature T where the I's are temperature dependent. Therefore, they may be referred to as the compressional and thermal internal energy, respectively. As T becomes zero, $\bar{U}$ becomes the ground state energy per nucleon E_0. Clearly E_0 is different from C_0.

The speed of sound v in a medium is given, in units of c, by the well-known relation $v^2=(\partial p/\partial\bar{\varepsilon})_S$ where $\bar{\varepsilon}$ is the internal energy density $\rho(m+\bar{U})$, m being the nucleon mass. S denotes the entropy. p is the total pressure which is equal to the sum of the proton and neutron partial pressures. Since $p=\rho^2(\partial\bar{U}/\partial\rho)_S$, v^2 can be expressed directly in terms of $(\partial\bar{U}/\partial\rho)_S$ and $(\partial^2\bar{U}/\partial\rho^2)_S$. We have derived $\bar{U}$ as given by Eq. (2). The calculation of the density derivatives of C_0 is straightforward. To calculate the density derivatives of I_p and I_n, we have employed the relation $p_{id}(i)=\frac{2}{3}\rho_i I_i$, $i=$p,n where p_{id} is the ideal gas partial pressures and the I's are the thermal internal energies defined in Eqs. (4) and (5). In this way the speed of sound in asymmetric nuclear matter is derived as

$$v^2=\frac{2\rho C_0'+\rho^2 C_0''+J_pI_p+J_nI_n}{m+C_0+C_0'+[1+b(\mathrm{p})]I_p+[1+b(\mathrm{n})]I_n}\ , \quad (7)$$

where C_0, I_p, and I_n have been given by Eqs. (3)–(5). The other quantities in the above equation are, for $i=$p,n,

$$J_i=b(i)+b(i)^2+\rho b'(i)\ ,$$
$$b(i)=\frac{2}{3}-\frac{\rho}{m_i^*}\left[\frac{\partial m_i^*}{\partial\rho}\right]_\alpha\ , \quad (8)$$

with the effective mass given by

$$m_i^*=m\left\{1+\frac{m}{4}[2(t_1+t_2)\rho+(t_2-t_1)\rho_i]\right\}^{-1}\ . \quad (9)$$

In the above, the primes denote $\partial/\partial\rho$. And in calculating v^2, we first perform a self-consistent finite temperature Green's function calculation[11,12] to determine the Fermi-Dirac distribution functions.

For symmetric nuclear matter, $\alpha=0$ and Eq. (7) readily reduces to the equation for the speed of sound given by Ref. 10. We would like to point out, however, that Eq. (7) is different from the corresponding equation given by Refs. 8 and 9. Here C_0 is the compressional internal energy given by Eq. (3). It is the temperature independent part of the internal energy, and it is not equal to the ground state energy E_0. In addition, our thermal internal energy I is defined as the temperature dependent part of the internal energy. In Refs. 8 and 9, C_0 was treated as the ground state energy E_0 and, in addition, I was treated as constants. In the present work C_0 and I of Eq. (7) are those given by Eqs. (3)–(5) and have been calculated as a function of ρ, α, and T.

Using several Skyrme interactions, we have calculated $\bar{v}^2$ of Eq. (7). Some representative results, using SkM* and SkIII, are presented in Figs. 1–3 and may be summarized as follows.

(i) In earlier calculations of the speed of sound in symmetric nuclear matter, a problem of much concern has been the occurrence of superluminosity at high nuclear matter density.[8–10] A main purpose of the present work is to investigate this problem for asymmetric nuclear matter. As shown by the figures, the superluminosity problem still arises for asymmetric nuclear matter at high density. However, as illustrated by Fig. 1, the neutron excess appears to delay its occurrence. For example, for $k_BT=10$ MeV and SkM* superluminosity takes place at $\rho=9.82\rho_0$ for $\alpha=0.4$ but at $\rho=20.51\rho_0$ for $\alpha=1.0$ where ρ_0 is the normal nuclear matter density.

(ii) The shapes of the v^2 vs ρ/ρ_0 curves for various α

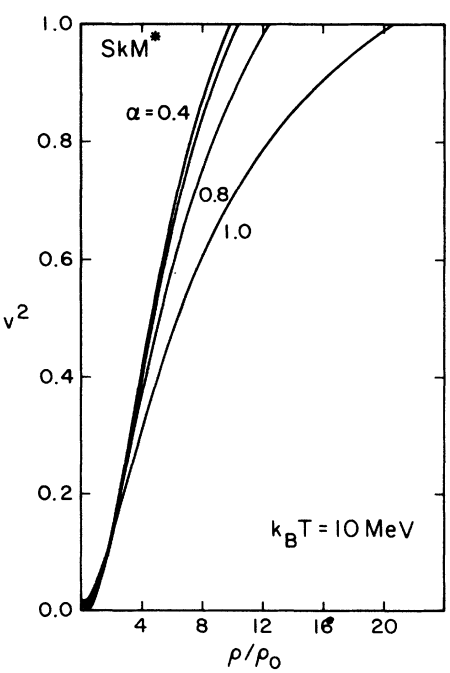

FIG. 1. Dependence of the speed of sound in nuclear matter on the asymmetric coefficient α, for interaction SkM* and temperature $k_BT=10$ MeV. ρ_0 is the normal nuclear matter density (0.16 fm^{-3}).

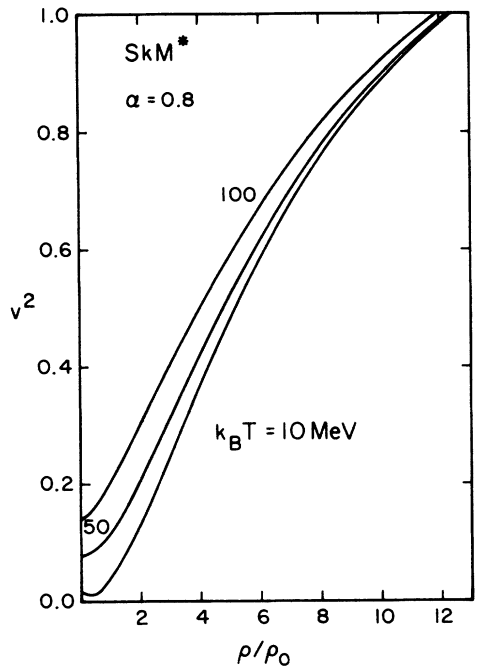

FIG. 2. Dependence of the speed of sound in nuclear matter on temperature, for $\alpha=0.8$ and SkM*.

values are generally similar to each other. And for most cases, v^2 decreases as the neutron excess increases as seen from Fig. 1. The dependence of v^2 on α is in fact strong at high densities.

(iii) For a given α, v^2 depends rather strongly on the temperature at low densities but rather weakly at high densities, as shown in Fig. 2. This is because at high densities the terms in the numerator and denominator of Eq. (7) which are temperature dependent are each overwhelmed by the respective temperature independent terms. As a result, the causal boundaries shown in Fig. 3 are nearly straight lines parallel to the temperature axis. Here we see that for $\alpha=0.8$, v^2 does not become superluminous until $\rho\cong 5\rho_0$ for SkIII while the corresponding boundary is at $\rho\cong 10\rho_0$ for SkM*. That the causal

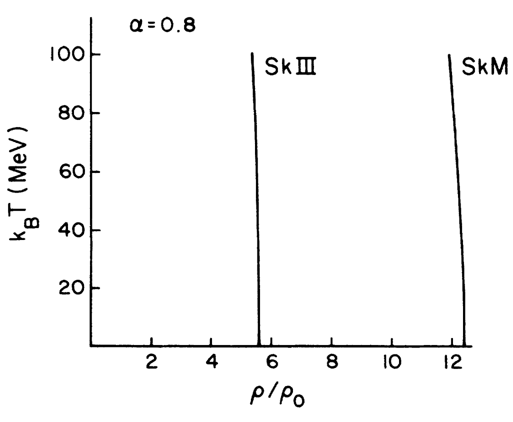

FIG. 3. Causal boundaries for $\alpha=0.8$ calculated with SkM* and SkIII.

boundary for SkM* is considerably further out than SkIII is mainly because the latter interaction has a much stronger density dependent component. (Both interactions have a density dependent term of the form ρ^α. The value of α is $\frac{1}{6}$ for SkM*, while for SkIII it is 1.)

Let us now study why the above superluminosity takes place and under what conditions it may be avoided. It is convenient to study the symmetric nuclear matter first. For this case we have[10]

$$v^2=\frac{2\rho C_0'+\rho^2 C_0''+(b+b^2+\rho b')I}{m+C_0+\rho C_0'+(1+b)I}\ , \tag{10}$$

where

$$C_0=\tfrac{3}{8}\rho\left[t_0+\frac{t_3}{6}\rho^d\right]\ , \tag{11}$$

$$b=\frac{2}{3}-\frac{d\ \ln m^*}{d\ \ln\rho}=\frac{2}{3}-\frac{\rho}{m^*}\frac{\partial m^*}{\partial\rho}\ . \tag{12}$$

Consider first the situation that I is small so that it can be neglected. Then Eq. (10) may be rewritten as

$$v^2=\frac{2\tilde{C}\frac{\rho}{\rho_0}+\tilde{C}D(d+1)(d+2)\left[\frac{\rho}{\rho_0}\right]^{d+1}}{m+2\tilde{C}\frac{\rho}{\rho_0}+\tilde{C}D(d+2)\left[\frac{\rho}{\rho_0}\right]^{d+1}}\ , \tag{13}$$

where

$$\tilde{C}\equiv\tfrac{3}{8}t_0\rho_0 \tag{14a}$$

and

$$D\equiv\frac{1}{6}\frac{t_3}{t_0}\rho_0^d\ . \tag{14b}$$

From Eq. (13) we see that if $\tilde{C}D>0$ and $-1<d<0$, we will always have $v^2<1$. Since $\tilde{C}D=\frac{1}{16}t_3\rho_0^{d+1}$, the condition $C\tilde{D}>0$ is satisfied by the Skyrme interactions SkI→SkVI and SkM* which all have $t_3>0$. But the condition $-1<d<0$ is *not* satisfied by any of these interactions. (SkI→VI have $d=1$, and SkM* has $d=\frac{1}{6}$.)

For the general case, the thermal energy is nonzero and depends on temperature and density. Then to have $v^2<1$ we need not only $t_3>0$ and $-1<d<0$ but also

$$b^2+b'\rho\le 1\ . \tag{15}$$

For symmetric nuclear matter, we have[10]

$$m^*=\frac{m}{1+A\rho} \tag{16a}$$

with

$$A=\frac{m}{8\hbar^2}(3t_1+5t_2)\ . \tag{16b}$$

Substituting Eqs. (12), (16a), and (16b) into Eq. (15) leads to the condition

$$A\rho\le\tfrac{15}{48}\ . \tag{17}$$

Since we would like this inequality to hold for any ρ, the above means $(3t_1+5t_2)\leq 0$. And this is *not* satisfied by any of the Skyrme interactions SkI$\rightarrow$VI and SkM*.[11]

The above analysis can be carried out in a very similar way for asymmetric nuclear matter, starting from Eq. (7). Since the derivation is rather straightforward, let us just state our results. First we consider the case that the thermal energies I_p and I_n are both small and can both be neglected. Then to have $v^2<1$ we need $-1<d<0$ and $t_3(3-\alpha^2-2\alpha^2X_3)>0$. For the general case of $I_p\neq 0$ and $I_n\neq 0$, the additional conditions for $v^2<1$ are

$$b^2(\mathrm{p})+\rho b'(\mathrm{p})\leq 1 \qquad (18a)$$

and

$$b^2(\mathrm{n})+\rho b'(\mathrm{n})\leq 1\ , \qquad (18b)$$

where $b(\mathrm{p})$ and $b(\mathrm{n})$ are given by Eq. (8). We rewrite Eq. (9) as

$$m_{\mathrm{p}}^*=m[1+A(\mathrm{p})\rho]^{-1}\ ,$$
$$m_{\mathrm{n}}^*=m[1+A(\mathrm{n})\rho]^{-1}\ , \qquad (19a)$$

where

$$A(\mathrm{p})=\frac{3t_1+5t_2-\alpha(t_2-t_1)}{8\hbar^2}m\ ,$$
$$A(\mathrm{n})=\frac{3t_1+5t_2+\alpha(t_2-t_1)}{8\hbar^2}m\ . \qquad (19b)$$

Then expressions (18a) and (18b) lead to the condition $(3t_1+5t_2)\pm\alpha(t_2-t_1)\leq 0$. Clearly this condition is not satisfied by the Skyrme interactions mentioned earlier.

The above discussion indicates clearly that the speed of sound in asymmetric nuclear matter calculated from the usual Skyrme interactions and with the pair cutoff Green's function method is bound to violate the causal constraint at high density and/or high temperature, as proved above and as also demonstrated by our numerical results (see, for example, Fig. 3). This is, of course, unphysical. The following conclusions are in order.

We may proceed in two directions to remedy the above superluminosity problem. We have shown that if the Skyrme force parameters satisfy the conditions $-1\leq d\leq 0$, $t_3(3-\alpha^2-2\alpha X_3)\geq 0$, and $[(3t_1+5t_2)\pm\alpha(t_2-t_1)]\leq 0$, the above superluminosity will never happen. These conditions, however, are not all satisfied by the existing Skyrme interactions. In other words, force parameters satisfying these constraints are not compatible with the empirical nuclear matter properties near normal nuclear matter density. Thus a possible way to avoid the superluminosity is to let the Skyrme force parameters be density dependent. For example, at low density we have $d>0$ while after some high density we use $d<0$. This kind of spline approach is in fact closely related to the spirit of the equation of state proposed by Sierk and Nix[13] where the density dependences of the equation of state at low and high density are taken to be different.

Another direction for avoiding the superluminosity is towards the method used in calculating v^2. All calculations in this paper are based on the first order normal pair cutoff approximation of the real time finite temperature Green's function.[11] Improved methods, such as summing up the ring diagrams to all orders,[14] may significantly change the value of v^2 at high density and/or high temperature. Such studies are being planned.

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*Permanent address: Department of Physics, Fudan University, Shanghai, People's Republic of China.

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