PHYSICAL REVIEW C VOLUME 37, NUMBER 4 APRIL 1988

Neutron strengths of the unbound $2h_{11/2}$, $1j_{13/2}$, and $1k_{17/2}$ shell-model states in ^{209}Pb

Ramen Majumdar and Rupayan Bhattacharya
Vivekananda College, Calcutta 700063, India and City College, Calcutta 700009, India
(Received 19 August 1987)

A broad based fragmented pattern of the three $2h_{11/2}$, $1j_{13/2}$, and $1k_{17/2}$ unbound shell-model neutron states in ^{209}Pb has been obtained on the basis of the recent ^{208}Pb(α,^{3}He) reaction data. The present theoretical calculation shows that the shell-model energies of the three neutron states lie at 7.8 MeV excitation energy in ^{209}Pb. The dilution of the neutron strengths of these three high spin states also indicates that the coupling of the neutron particle states with the 11 vibrational states of ^{208}Pb is strong indeed. The theoretical results on the damping of the three neutron states are in sharp contrast with the existing theoretical results on the quasiparticle phonon coupling model calculation.

I. INTRODUCTION

Using the (α,^{3}He) reaction at 183 MeV on the target nucleus ^{208}Pb, the distribution of the three $2h_{11/2}$, $1j_{13/2}$, and $1k_{17/2}$ neutron states in ^{209}Pb has recently been detected within 15 MeV excitation energy by Massolo *et al.*[1] The experiment shows the two discrete very weak fragments of $1k_{17/2}$ states at 3.96 MeV and 4.22 MeV, respectively. Although no fragments of the $2h_{11/2}$ and $1j_{13/2}$ states have been observed in the low excitation energy region, the presence of these three high spin states has been observed within the 2.0 to 15 MeV excitation energy region. Extensive experimental work on the investigation of the shell-model neutron states in ^{209}Pb has been performed by Kovar *et al.*[2] through the ^{208}Pb(d,p)^{209}Pb reaction. The experimental findings[2] reproduce the clean fragmentation of the one high spin $1j_{15/2}$ neutron state that lies below 4 MeV excitation energy in ^{209}Pb but cannot properly identify the presence of the unbound $2h_{11/2}$, $1j_{13/2}$, and $1k_{17/2}$ neutron states in ^{209}Pb. Our previous work on ^{209}Pb (Ref. 3) is centered on the low excitation region, i.e., lying below 4 MeV, and it is based on salvaging the structures of the weak fragments of the $2h_{11/1}$, $1j_{13/2}$, and $1k_{17/2}$ states. In this work we have applied the same core-particle coupling model calculation involving two conspicuous deviations from the earlier one.[3] Firstly, all the collective vibrational states, including the ones detected from the giant resonances in ^{208}Pb, have been taken into consideration in the framework of the core-particle coupling model. Secondly, the shell model energies of the two unbound $2h_{11/2}$ and $1k_{17/2}$ states have been retrieved from the numerical solution of the Schrödinger wave equation with a deep Woods-Saxon potential in ^{208}Pb.

II. RESULTS AND DISCUSSION

The second order differential Schrödinger wave equation has been solved by Numerov's method with a potential well consisting of the Woods-Saxon (WS) and spin-orbit terms[4] to obtain the energies of the $2h_{11/2}$ and $1k_{17/2}$ states. The form of the potential is

$$V=\left\{-V_0\left[1+\exp\left[\frac{r-r_0A^{1/3}}{a_0}\right]\right]^{-1}-\frac{\lambda}{45.2}\frac{1}{r}V_s'\frac{d}{dr}\left[1+\exp\left[\frac{r-r_sA^{1/3}}{a_s}\right]\right]^{-1}\mathbf{L}\cdot\mathbf{S}\right\}.$$

$$=-\left\{V_0\left[1+\exp\left[\frac{r-r_0A^{1/3}}{a_0}\right]\right]^{-1}+V_s\frac{d}{dr}\left[1+\exp\left[\frac{r-r_sA^{1/3}}{a_s}\right]\right]^{-1}\mathbf{L}\cdot\mathbf{S}\right\}. \quad (1)$$

The WS potential parameters for ^{208}Pb (Ref. 4) will not reproduce the negative energy eigenvalues for these neutron states because they are unbound. So we have adjusted the potential parameters for ^{208}Pb to obtain the negative energy eigenvalues for the $2h_{11/2}$ and $1k_{17/2}$ states. For the negative energy of the $1j_{13/2}$ state we need a deeper potential depth V_0 than is required for the $2h_{11/2}$ and $1k_{17/2}$ states. For this reason, we have taken the doublet splitting relation to know the shell model energy of the $1j_{13/2}$ state, and it is given by

$$\Delta E_l=K_l(2l+1)\ , \quad (2)$$

where ΔE_l is the energy splitting between the $1j_{15/2}$ and $1j_{13/2}$ states. For the $1j$ state, K_7 is taken as 0.5 MeV.[5] Consequently the energy of the $1j_{13/2}$ state turns out to be 8.5 MeV in as much as the energy of the $1j_{15/2}$ state

TABLE I. Vibrational of states ^{208}Pb.

λ^{π}	$\langle H_{\text{vib}} \rangle$ (MeV)	$\langle \alpha_\lambda \rangle$
3_1^-	2.61	0.046
5_1^-	3.20	0.017
5_2^-	3.71	0.010
7^-	4.04	0.100
2_1^-	4.08	0.025
4^+	4.32	0.024
6^+	4.42	0.015
8^+	4.61	0.010
$1^{-'}$	13.60	0.010
$2^{-'}$	10.50	0.037
$3^{-'}$	17.50	0.011

can be taken to be 1.4 MeV.[6] The unperturbed energies of the $4s_{1/2}$, $3d_{3/2}$, $3d_{5/2}$, $2g_{7/2}$, $2g_{9/2}$, and $1i_{11/2}$ states have been taken from the experimental estimates that reproduce the main fragment of the respective states.[2] These ten neutron states are coupled with the 3_1^-, 5_1^-, 5_2^-, 7^-, 2_1^+, 4^+, 6^+, 8^+, $1^{-'}$, $2^{+'}$, and $3^{-'}$ states of ^{208}Pb (Refs. 7 and 8) to frame the Hamiltonian matrices for the $\frac{11}{2}^-$, $\frac{13}{2}^-$, and $\frac{17}{2}^+$ states of ^{209}Pb. $1^{-'}$, $2^{+'}$, and $3^{-'}$ are the high lying vibrational states arising from the giant resonances in ^{208}Pb.[7] The model for the calculation has been discussed in our recent work on ^{207}Tl.[8] The eigen-

TABLE II. Woods-Saxon potential parameters and the shell-model energies of the $2h_{11/2}$, $1j_{13/2}$, and $1k_{17/2}$ states with respect to the $2g_{9/2}$ ground state energy of ^{209}Pb.

$V_0=48.479$ MeV; $r_0=1.31$ fm; $a_0=0.718$ fm;
$V_s=27.74$ MeV; $r_s=1.246$ fm, and $a_s=0.391$ fm

nlj_2	$\rightarrow$	$1j_{13/2}$	$2h_{11/2}$	$1k_{17/2}$
ϵ_{j_2} (MeV) (Calculated)	$\rightarrow$	8.500	6.892	8.563
ϵ_{j_2} (MeV) (Optimized)	$\rightarrow$	7.800	7.800	7.800

values (E) and the wave functions of the three spin states are obtained from the diagonalization of the Hamiltonian matrices H where the wave function for the state with spin $J^{\pi}=j_1$ is written as

$$\psi_J=\sum_{\lambda j_2} a_{\lambda j_2} \mid n_\lambda;\lambda j_2,j_1\rangle \ . \tag{3}$$

Here j_2 and λ are the angular momenta of the shell model states and the collective vibrational states of ^{208}Pb. n_λ is the number of quanta. Also vectorially $\boldsymbol{\lambda}+\mathbf{j}_2=\mathbf{j}_1$. The diagonal terms of the Hamiltonian matrix H are the sum of H_n and H_{vib}. The expectation values of H_n are the shell model neutron energies and that of H_{vib} are the energies of the vibrational states of the ^{208}Pb core nucleus. H_{int} stands for the interaction Hamiltonian whose matrix elements in this basis [Eq. (3)] is

$$\langle n_\lambda=1;\lambda j_2,j_1 \mid H_{\text{int}} \mid n_\lambda=0;0j_1,j_1\rangle=\langle j_2 \mid K(r) \mid j_1\rangle\langle j_2\|Y_\lambda\|\,j_1\rangle(2j_1+1)^{-1/2}\langle\alpha_\lambda\rangle \ . \tag{4}$$

TABLE III. Spectroscopic factors for the fragments of the $1j_{13/2}$, $2h_{11/2}$, and $1k_{17/2}$ states of ^{209}Pb. E is the energy of the fragments in MeV and $a_{0j_2}^2$ is the squared amplitude of the zero phonon coupled states. The values in parentheses indicate experimental values (Ref. 1).

$1j_{13/2}$ state		$2h_{11/2}$ state		$1k_{17/2}$ state	
E	$a_{0j_2}^2$	E	$a_{0j_2}^2$	E	$a_{0j_2}^2$
2.566	0.050	2.315	0.123	4.068(3.96)	0.140(0.038)
2.814	0.185	3.045	0.090	4.392(4.22)	0.010(0.023)
3.567	0.040	3.249	0.029	4.466	0.017
4.295	0.032	3.413	0.010	5.081	0.026
5.015	0.005	4.537	0.032	5.130	0.032
5.334	0.020	5.194	0.014	5.332	0.073
5.833	0.012	5.310	0.019	5.401	0.020
6.346	0.010	5.885	0.025	8.058	0.490
9.778	0.490	9.965	0.533	10.576	0.032
10.431	0.017	11.916	0.010	12.614	0.137
12.029	0.020	12.714	0.063		Total=0.977
12.185	0.010		Total=0.948		
12.311	0.050				
12.441	0.023				
12.510	0.010				
	Total=0.957				

The $\langle \alpha_\lambda \rangle$ are the amplitudes of the λ-mode vibrational states, and these are shown in Table I. The matrix elements of $K(r)$ have been kept fixed at 50 MeV.[8] The energies of (ϵ_{j_2}) of the three $1k_{17/2}$, $1j_{13/2}$, and $2h_{11/2}$ states are adjusted around their estimated values to reproduce the main $2g_{9/2}$, $1i_{11/2}$, and $2g_{7/2}$ fragments and the four distinct fragments of the $1j_{15/2}$ state lying at 1.424 MeV, 3.052 MeV, 3.556 MeV, and 3.716 MeV excitation energies, respectively. The optimized ϵ_{j_2} along with their estimated values from the numerical solution are displayed in Table II. The energies of the fragmented states and the corresponding single particle strengths $a^2_{0j_2}$ (the squared amplitude of the zero-phonon coupled state) are depicted in Table III. From Table III we observe the damping of the single particle strengths of the three high spin states where almost 50% of the entire shell-model sum-rule strengths are distributed over a large number of weak states. This inherently suggests that the coupling of the shell-model states with the collective vibrational states is rather strong to strip off a sizable amount of the neutron strength from the respective main shell-model states that lie at 9.778 MeV, 9.965 MeV, and 8.058 MeV for the $1j_{13/2}$, $2h_{11/2}$, and $1k_{17/2}$ states, respectively (Table III). Furthermore, the experimental results indicate (Fig. 4, Massolo *et al.*[1]) that the majority of the particle strengths for the three states must center around 10 MeV and the presence of the states will lie within the 4.5–14.5 MeV excitation energy region. This picture is quite obvious from our Table III. The only theoretical calculation[1] based on the quasiparticle phonon nuclear model shows that the bulk of the single particle strengths of the $2h_{11/2}$, $1k_{17/2}$, and $1j_{13/2}$ states are centered around 6.5 MeV, 8 MeV, and 9.5 MeV, respectively, and to some extent the theoretical distribution pattern (Sp factor vs excitation energy) does not corroborate with the experimental one (Fig. 4, Massolo *et al.*[1]). But in the present calculation, we observe the almost complete overlapping of the spectrum of these three states with the experimental distribution (Table III). As a result of this, the bulk of the single particle strength of each of the three states practically lies around 9 MeV (Table III). This supports the experimental observation, as the peak value of the experimental cross section of the three neutron states centers around 9.5 MeV (Fig. 4 of Ref. 1). The major contribution for the high lying fragments exclusively arises from the coupling of the vibrational states (arising from the giant resonances) with the shell-model states.

III. CONCLUSION

From the present research it can be concluded that the detailed fragments of the unbound states of ^{209}Pb can only be explained within the particle vibrational coupling scheme, taking care of the effect of high lying vibrational states from the giant resonances in ^{208}Pb. To our knowledge, this is the first attempt to understand the structures of the high spin states of ^{208}Pb that lie in the high excitation region. We further advocate that the shell model energies of the $1j_{13/2}$, $2h_{11/2}$, and $1k_{17/2}$ states must lie at 7.8 MeV excitation energy of ^{209}Pb.

[1]C. P. Massolo, F. Azaiez, S. Gales, S. Fortier, E. Gerlic, J. Guillot, E. Hourani, and J. Maison, Phys. Rev. C **34**, 1256 (1986).

[2]D. G. Kovar, N. Stein, and C. K. Boekelmen, Nucl. Phys. **A231**, 266 (1974).

[3]P. Mukherjee, I. Mukherjee, and R. Majumdar, Nucl. Phys. **A294**, 73 (1978).

[4]J. Blomqvist and S. Wahlborn, Ark. Fys. **16**, 545 (1960).

[5]B. L. Cohen, P. Mukherjee, R. H. Fulmer, and A. L. McCarthy, Phys. Rev. **127**, 1678 (1962).

[6]R. Majumdar, Ph.D. thesis, Vivekanamda College, Calcutta, 1979 (unpublished).

[7]V. G. Soloviev, Ch. Stoyanov, and V. V. Voronov, Nucl. Phys. **A394**, 141 (1983).

[8]R. Majumdar, J. Phys. G **12**, 641 (1986).