Fermi nuclear matrix element of allowed isospin-hindered positron decay of ^{56}Co

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The experimental measurement of the asymmetry parameter of the β^+ decay from the 4^+ ground state of ^{56}Co to the 2.085 MeV 4^+ state of ^{56}Fe not only yields a value for the Fermi nuclear matrix element M_F, but also has significant Fermi-Gamow-Teller mixing and is of interest for a time-reversal invariance test of the weak interaction. To date, nine such measurements have been made and the values of the M_F fall into two groups: $M_F \sim 10^{-5}$ and $M_F \sim (3-5)\times 10^{-4}$. Our theoretical calculation using the Nilsson model and a one-body spheroidal Coulomb potential yields $M_F = 2.3\times 10^{-4}$ for $\beta = 0.1$ and $M_F = 6\times 10^{-4}$ for $\beta = 0.2$, which are in reasonable agreement with the experimental values of $M_F \sim (3-5)\times 10^{-4}$.

INTRODUCTION

Allowed isospin-hindered ($J \neq 0$, $\Delta J = 0$, $\Delta T = \pm 1$, and no parity change) β decays[1] are of great interest in the study of isospin impurity and also in the study of time-reversal[2] invariance. The experimental measurement of the asymmetry parameter from either polarized nuclei or β-γ circular polarization correlations in unpolarized nuclei has been used to yield the Fermi to Gamow-Teller mixing ratio $y = C_v M_F / C_A M_{GT}$, where C_v and C_A denote the usual vector and axial-vector coupling constants. For time-reversal invariance tests in β-decays, the T-violating amplitude is directly proportional to the magnitude of y. Furthermore, the Fermi nuclear matrix element M_F is related[1] to y through the relation

$$| M_F | = \left[\frac{2\ \text{ft (superallowed)}}{\text{ft (decay under study)}} \right]^{1/2} \frac{y}{(1+y^2)^{1/2}} . \quad (1)$$

As Fermi transitions require $\Delta T = 0$, a nonzero value of y therefore implies isospin mixing due to charge-dependent forces.

The positron decay from the 4^+ ground state of ^{56}Co to the 2.085 MeV 4^+ state of ^{56}Fe has significant Fermi-Gamow-Teller mixing and is also of interest for a time-reversal invariance test of the weak interaction. It has been well studied.[3–11] Figure 1 gives nine independent measurements of M_F as a function of time. They fall roughly into two groups: $M_F \sim 10^{-5}$ (Ambler *et al.*[3] and Pingot[9]) and $M_F \sim (3-5)\times 10^{-4}$ (Daniel *et al.*,[4,6] Mann *et al.*,[5] Behrens,[7] Battacherjee *et al.*,[8] and Markey *et al.*[10]). The aim of this paper is to obtain a theoretical value for M_F and to discuss the value so obtained in relation to the above experimental values.

CALCULATION AND RESULTS

Recently,[12–14] we have used the Nilsson model[15] with a one-body spheroidal Coulomb potential to obtain the M_F of a number of transitions. As the results show that the agreement between theory and experiment is within a factor of 1.5, we shall use the same approach.

We assume that the deformed nucleus ^{56}Co has the rotational band $K = 4$ and that the deformed ^{56}Fe has $K = 0$ as shown in Fig. 2, where $| G \rangle$, $| P \rangle$, $| A \rangle$, and $| T_< \rangle$ are the ground state, the parent state, the analog state, and the antianalog state, respectively. By the K-selection rule for β decay of $\Delta K \leq 1$, the β matrix elements with $K = 4$ vanish and thus the experimentally observed decay is due to the admixture of other K bands to the $K = 4$ ground state of ^{56}Co and to the $K = 0$ excited state of ^{56}Fe. Assuming axially symmetric prolate deformation, the initial state is

$$\begin{aligned} | i \rangle = {} & | J=4, M, K=4, T=1, T_z=-1 \rangle \\ & + \bar{\alpha}_1 | J=4, M, K=1, T=1, T_z=-1 \rangle \\ & + \bar{\alpha}_4 | J=4, M, K=4, T=2, T_z=-1 \rangle \\ & + \cdots \end{aligned} \quad (2)$$

and the final state is

$$\begin{aligned} | f \rangle = {} & | J=4, M, K=0, T=2, T_z=-2 \rangle \\ & + a_3 | J=4, M, K=3, T=2, T_z=-2 \rangle \\ & + a_4 | J=4, M, K=4, T=2, T_z=-2 \rangle \\ & + \cdots , \end{aligned} \quad (3)$$

where $\bar{a}_1$ is the admixture amplitude of $K = 1$ in the initial state, a_3 and a_4 are those of $K = 3$ and $K = 4$ in the final state, respectively, and $\bar{\alpha}_4$ is the isospin impurity amplitude given by

$$\bar{\alpha}_4 = - \frac{\langle K=4, T=1, T_z=-1 | V_c | K=4, T=2, T_z=-1 \rangle}{\Delta E} , \quad (4)$$

where ΔE is the separation energy and V_c the Coulomb potential. The Fermi matrix element is

$$M_F = \langle f \mid T_- \mid i \rangle = 2\bar{a}_4 a_4 \tag{5}$$

and the Gamow-Teller (GT) matrix element is calculated from the relation

$$M_{\mathrm{GT}}^2 = \frac{1}{2J+1} \sum_{\mu, M_i, M_f} \left| \langle J, M_f, K_f, T_f, T_{zf} \mid D_{\mathrm{GT}}(\mu) \mid J, M_i, K_i, T_i, T_{zi} \rangle \right|^2 . \tag{6}$$

When the operator $D_{\mathrm{GT}}(\mu)$ is transformed into the body-fixed coordinate system, we obtain

$$M_{\mathrm{GT}}^2 = \left| \frac{\bar{a}_1}{\sqrt{2}} \langle \chi_0 \chi_{T_z=-2}^{T=2} \mid D'_{\mathrm{GT}}(-1) \mid \chi_1 \chi_{T_z=-1}^{T=1} \rangle + \frac{a_3}{\sqrt{5}} \langle \chi_3 \chi_{T_z=-2}^{T=2} \mid D'_{\mathrm{GT}}(-1) \mid \chi_4 \chi_{T=-1}^{T=1} \rangle + \frac{2a_4}{\sqrt{5}} \langle \chi_4 \chi_{T_z=-2}^{T=2} \mid D'_{\mathrm{GT}}(0) \mid \chi_4 \chi_{T_z=-1}^{T=1} \rangle \right|^2 , \tag{7}$$

where $|\chi_{K_i} \chi_{T_{zi}}^{T_i}\rangle$ and $|\chi_{K_f} \chi_{T_{zf}}^{T_f}\rangle$ are the intrinsic states, which depend on the deformation parameter β. A recent theoretical calculation[16] gives $\beta \sim 0.1$ which is consistent with the value obtained by Gallagher and Morzkowski.[17] Using this value of β, it was found that the value of

$$\frac{1}{\sqrt{2}} \langle \chi_0 \chi_{T_z=-2}^{T=2} \mid D'_{\mathrm{GT}}(-1) \mid \chi_1 \chi_{T_z=-1}^{T=1} \rangle = -0.0002 ,$$

$$\frac{1}{\sqrt{5}} \langle \chi_3 \chi_{T_z=-2}^{T=2} \mid D'_{\mathrm{GT}}(-1) \mid \chi_4 \chi_{T_z=-1}^{T=1} \rangle = 0.0684 , \tag{7a}$$

and

$$\frac{2}{\sqrt{5}} \langle \chi_4 \chi_{T_z=-2}^{T=2} \mid D'_{\mathrm{GT}}(0) \mid \chi_4 \chi_{T_z=-1}^{T=1} \rangle = -0.9384 .$$

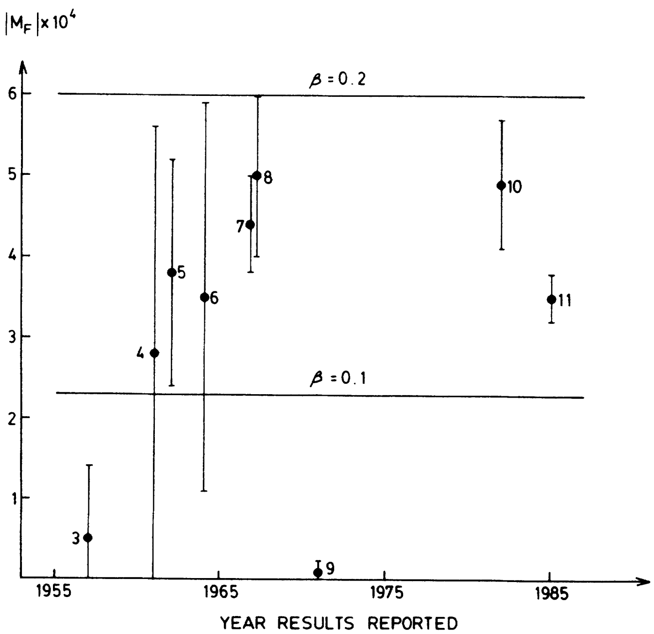

FIG. 1. Plot of all experimental values of M_F that have been reported. The numbers that label the data points refer to references. The two horizontal lines are theoretical values of M_F for $\beta=0.1$ and $\beta=0.2$.

We assume that the K admixture amplitudes are of the same order of magnitude, so that, neglecting the first two terms of Eq. (7a),

$$M_{\mathrm{GT}}^2 = \tfrac{4}{5} a_4^2 \left| \langle \chi_4 \chi_{T_x=-2}^{T=2} \mid D'_{\mathrm{GT}}(0) \mid \chi_4 \chi_{T_z=-1}^{T=1} \rangle \right|^2$$

$$= \tfrac{4}{5} a_4^2 \left| \frac{1}{\sqrt{2}} \left\langle \frac{5^-}{2}[303]\mathrm{p} \mid D'_{\mathrm{GT}}(0) \mid \frac{5^-}{2}[303]\mathrm{n} \right\rangle - \frac{1}{\sqrt{2}} \left\langle \frac{3^-}{2}[312]\mathrm{p} \mid D'_{\mathrm{GT}}(0) \mid \frac{3^-}{2}[312]\mathrm{n} \right\rangle \right|^2 , \tag{8}$$

from which we obtain $|a_4| = 1.066 |M_{\mathrm{GT}}|$.

The value of M_{GT} can be obtained from the following relation:[1]

$$|M_{GT}| = \frac{C_v}{C_A} \left[\frac{2 \text{ ft (superallowed)}}{\text{ft (decay under study)}} \right]^{1/2} \frac{1}{(1+y^2)^{1/2}} . \tag{9}$$

Owing to the smallness of the experimental value of y, we shall obtain essentially the same value of M_{GT} irrespective of whichever experimental value[3–11] of y we use.

For the calculation of the isospin impurity as given by Eq. (4), we take V_c to be the one-body spheroidal Coulomb potential given by[18]

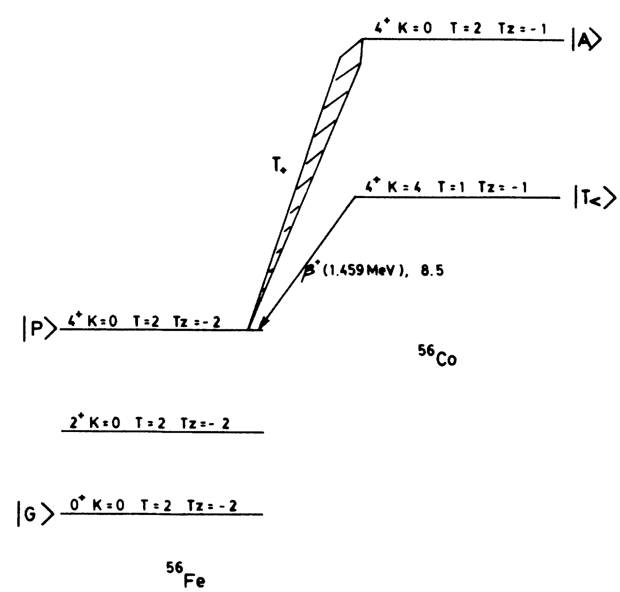

FIG. 2. Partial decay scheme of ^{56}Co.

$$V_c = \frac{(Z-1)e^2}{R}[\tfrac{3}{2} - \tfrac{1}{2}(r/R)^2] + a(r/R)^2 Y_{20} \quad \text{for } r < R ,$$
$$= \frac{(Z-1)e^2}{r} + a(R/r)^3 Y_{20} \quad \text{for } r > R , \qquad (10)$$

where R is the nuclear radius and a is related to the Bohr deformation parameter β by

$$a = \tfrac{3}{5}\beta(Z-1)e^2/R . \qquad (11)$$

The calculations were carried out for both $\beta=0.1$ and $\beta=0.2$ with the following results:

$$|M_F|_{\text{theor}} = 2.3\times10^{-4} \quad \text{for } \beta=0.1 ,$$
$$= 6.0\times10^{-4} \quad \text{for } \beta=0.2 .$$

In Fig. 1 we have drawn the lines corresponding to M_F for $\beta=0.1$ and $\beta=0.2$. Except for the results of Ambler *et al.*[3] and Pingot,[9] all experimental values of M_F lie between these two lines. Although the experimental value of β is not available, the rather well-developed rotational band of ^{56}Fe indicates reasonable deformation and this implies that our theoretical values are in disagreement with those of Ambler *et al.* and Pingot but are in good agreement with the experimental values of all other workers.

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