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Multiplicity and bombarding energy dependence of the entropy in relativistic heavy-ion reactions

K. G. R. Doss,[a],* H.-Å. Gustafsson,[b] H. H. Gutbrod,[c] D. Hahn,[a] K.-H. Kampert,[d] B. Kolb,[c] H. Löhner,[d] A. M. Poskanzer,[a] H. G. Ritter,[a] H. R. Schmidt,[c] and H. Stöcker[e]

[a]*Lawrence Berkeley Laboratory, Berkeley, California 94720*
[b]*University of Lund, Lund, Sweden*
[c]*Gesellschaft für Schwerionenforschung, Darmstadt, Federal Republic of Germany*
[d]*University of Münster, Münster, Federal Republic of Germany*
[e]*University of Frankfurt, Frankfurt, Federal Republic of Germany*

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The yield ratios of light clusters (d,t,^{3}He,α) to protons were measured for the reactions Nb + Nb and Au+Au for incident beam energies of 150, 250, 400, and 650 MeV/nucleon. The ratios are analyzed in the framework of the quantum statistical model and the specific entropy is extracted as a function of the participant proton multiplicity. The results are compared with predictions of the fireball model and relativistic mean field calculations.

I. INTRODUCTION

There are only a few observables that preserve the signatures of the early high density and high temperature phase of relativistic heavy ion collisions. One of them is considered to be the entropy per nucleon (S/A) produced in the collision. Over the last years a lively debate took place about the mechanisms and significance of entropy and the relationship to nuclear cluster production in high energy heavy ion collisions.[1] On the basis of hydrodynamics[2,3] or Monte Carlo cascade calculations,[4,5] it has been argued that during the expansion phase there is only little change of the entropy which was produced in the initial phase of the heavy ion collision. After the collisions among the constituents of the expanding system have ceased at a certain freeze-out or breakup density, the phase space density stays constant due to Liouville's theorem and the entropy determines the abundances of the produced clusters. Therefore the specific entropy measured via cluster abundances can help to determine the equation of state (EOS) of dense and hot matter. Furthermore, the sudden liberation of new degrees of freedom due to phase transitions (liquid to vapor at low and hadronic to quark matter at high energies) should manifest itself by the extra entropy produced at such a transition.[6–10]

The translation of experimentally measured abundance ratios to values of entropy, however, is model dependent. In the early work of Siemens and Kapusta[11] the yields of composite particles were inferred from inclusive measurements and the deuteron-to-proton ratio $R_{\rm dp}$ was related to the specific entropy via the simple formula $S/A \approx 3.95 - \ln(R_{\rm dp})$. This gives significantly larger entropy values than predicted by dynamic models. This so-called "entropy puzzle" was resolved later by Stöcker *et al.*,[3,7] who showed that the above relation is not appropriate for the (low) bombarding energies considered, and by Gutbrod *et al.*[12] and Doss *et al.*[13] who, in exclusive experiments, showed that the cluster-to-proton ratios increase steadily with the participant proton multiplicity, i.e., with decreasing impact parameter. From these measurements it became evident that the naive use of impact parameter averaged data had been one of the sources of the incorrect entropy determination.

In Sec. II we shall present the results of the measurement of the production of light clusters in the systems Nb + Nb and Au + Au as a function of multiplicity at various bombarding energies. In Sec. III we will employ the quantum statistical model (QSM) to extract entropy information, which will be discussed in Sec. IV. Section V contains a summary.

II. EXPERIMENT AND RESULTS

The experiments to study the production of light fragments ($A<5$) at energies ranging from 150 to 650 MeV/nucleon were performed at the Berkeley Bevalac, using the plastic ball/wall spectrometer.[14] This detector system has full particle identification capability, in the angular range from 9 to 160 deg in the laboratory system, for singly and doubly charged particles as well as positively charged pions. The forward direction (0–9 deg in the laboratory system) is covered by the plastic wall, measuring time of flight, ΔE, and the angle of the particles, thus identifying the nuclear charge and the velocity of the particles. The inner part of the plastic wall is also used to define the trigger of the whole spectrometer. The data discussed here were taken both with a minimum bias trigger and a central collision trigger which enhances the sample of high multiplicity events.

The results on composite particle production (Figs. 1 and 2) are given in ratios x/p as a function of $N_{\rm p}$, where x = (d,t,^{3}He,α), and $N_{\rm p}$ is the participant proton multiplicity [$N_{\rm p} = {\rm p} + {\rm d} + {\rm t} + 2(^3{\rm He} + \alpha)$]. The curves are fits to data in the framework of the QSM described below. The experimental ratios are extracted in the region of phase space where the yields of the different species overlap each other. The underlying assumptions are the validity of the basic idea of the coalescence model, namely $D/N \propto N$, and a Boltzmann-type momentum

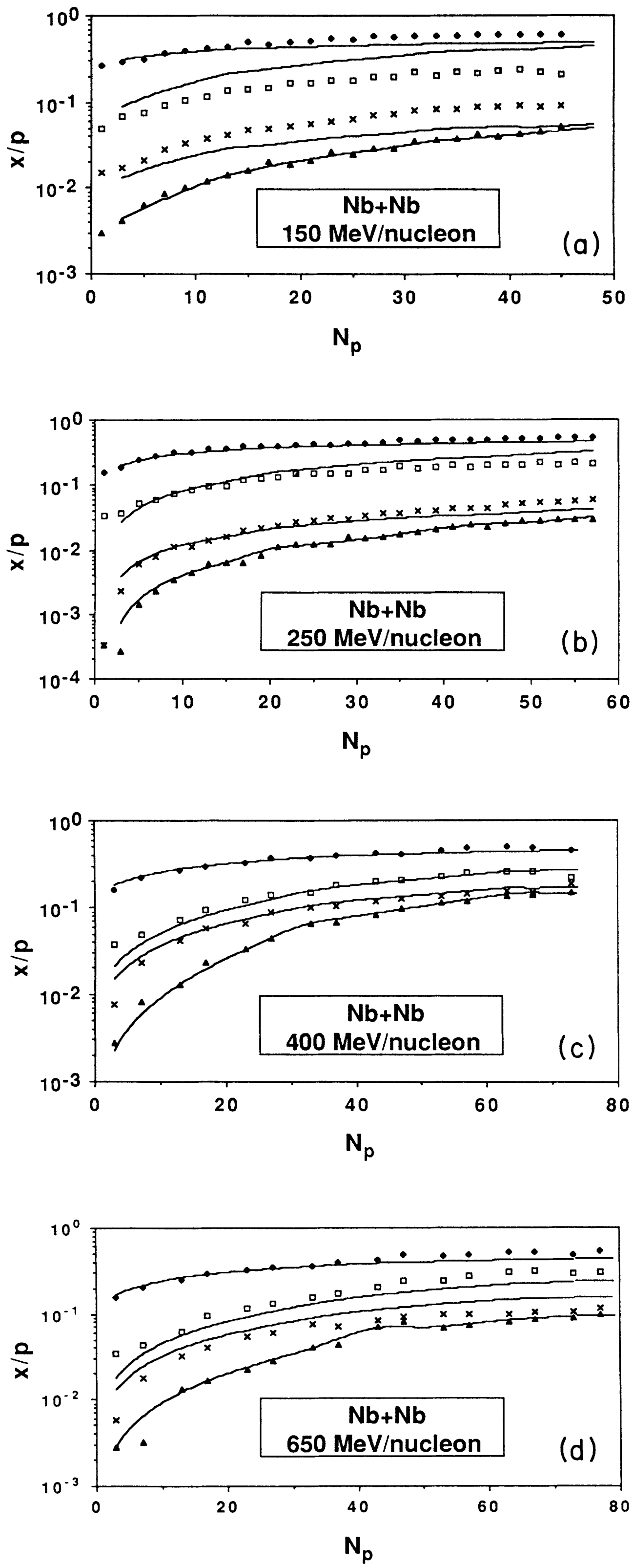

FIG. 1. x/p ratios, where x stands for d (♦), t (□), 0.2 ^{3}He (x), and 0.2 ^{4}He (▲) in (a) and (b) and d (♦), t (□), ^{3}He (x), and ^{4}He (▲) in (c) and (d) as a function of the participant proton multiplicity N_p. The solid curves are fits within the framework of the QSM as described in the text.

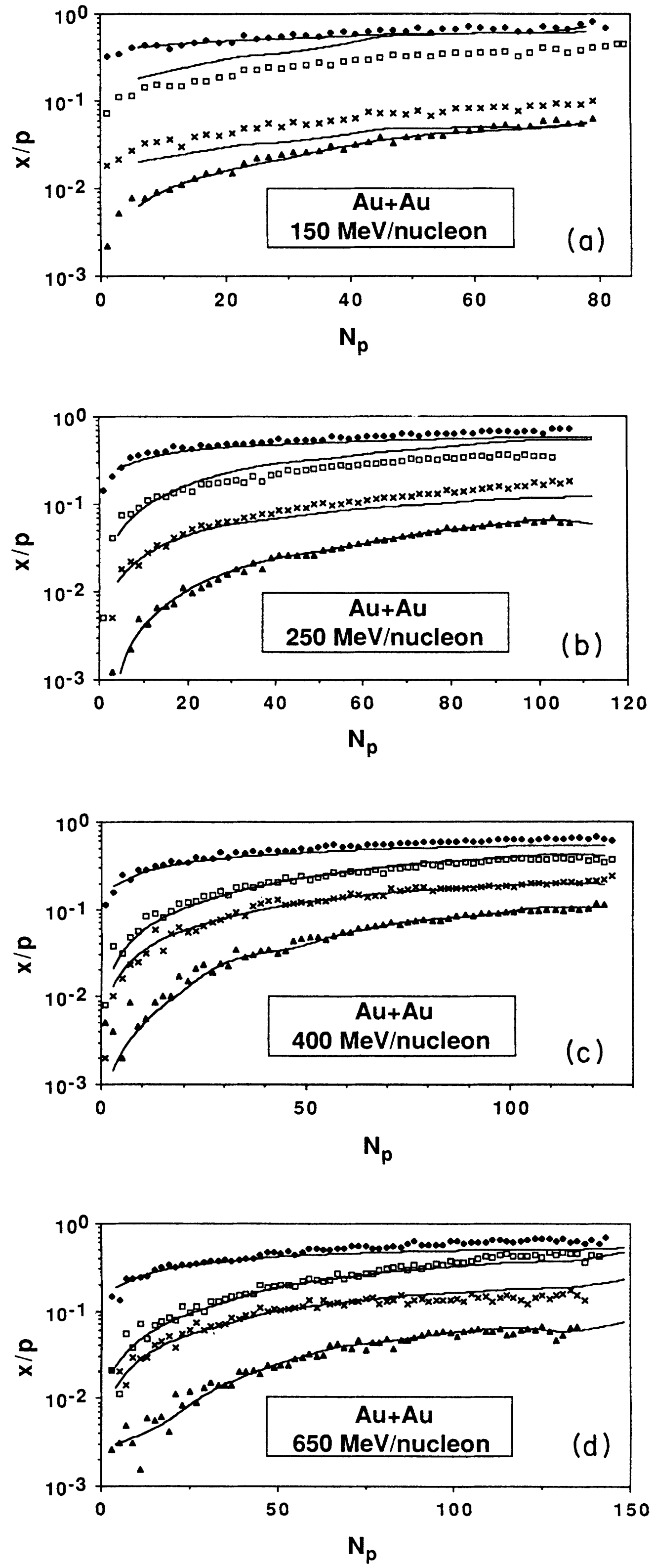

FIG. 2. x/p ratios, where x stands for d (♦), t (□), 0.2 ^{3}He (x), and 0.1 ^{4}He (▲) in (a), d (♦), t (□), 0.5 ^{3}He (x), and 0.2 ^{4}He (▲) in (b), and d (♦), t (□), ^{3}He (x), and 0.5 ^{4}He (▲) in (c) and (d) as a function of the participant proton multiplicity N_p. The solid curves are fits within the framework of the QSM as described in text.

distribution of the particles. Here D and N stand for the number of deuterons and nucleons, respectively. The choice of an overlap area in phase space, where the particle momenta have been scaled by $(1/m)^{1/2}$, ensures that the above relation is also fulfilled locally. This procedure is necessary since the plastic ball does not have full particle identification in the full phase space. N_p is, on the other hand, determined from all well-identified particles except for the spectators. These were excluded by applying software cuts to the data. A detailed description of the procedure for extracting the ratios and the N_p numbers is given in Ref. 13.

The abundance ratios of Figs. 1 and 2 exhibit the following features:

(i) They all show the same behavior of increasing production of composite particles with increasing N_p.

(ii) The curves tend towards an asymptotic value at high N_p values. The asymptotic limit is reached faster for the curves corresponding to the higher projectile energies.

This can qualitatively be explained by simple phase space considerations[12] taking both the size of the light fragment and the participant volume into account. The saturation at high N_p values is, however, a very reassuring result, because it indicates that we are close to an infinite volume, thermodynamic limit, where finite source size effects[15] should be negligible.

III. ENTROPY IN THE FRAMEWORK OF THE QUANTUM STATISTICAL MODEL

The QSM (Refs. 3, 7, and 16) takes into account, simultaneously, particle unstable nuclides up to mass 20 and ground state nuclei up to mass 130, as well as Bose condensation of the integer spin nuclides and excluded volume effects. In particular, the model describes the dependence of the ratios of deuteronlike to protonlike particles (d_{like}/p_{like}), as defined in Ref. 13, and x/p as a function of the specific entropy of the system. The dependence of this relation on the breakup temperature T_b and the breakup density ρ_b is extensively discussed in Ref. 16. Here we only want to point out that the curve $x/p(S/A)$, which is calculated for a grandcanonical ensemble, i.e., for infinite nuclear matter, can also be employed at finite multiplicities. It has been shown[16] that only at very low particle numbers the deviations from a classical microcanonical treatment are worth mentioning; for $A=10$ they are of the order of 20%. This demonstration of the applicability of the QSM at finite multiplicities constitutes a decisive improvement over previous methods to extract specific entropies from experimental data:

(i) In a previous publication[13] the ratio d_{like}/p_{like} was extrapolated to infinity employing a coalescence-model inspired formula. Due to the lever arm from finite to infinite multiplicities the extracted values S/A^{∞} were insensitive to fine differences in the experimental ratios at different bombarding energies or colliding systems.

(ii) At lower bombarding energies the fraction of clusters heavier than $A=4$ contributing to the participant proton multiplicity N_p becomes increasingly important. These clusters, however, were not measured in the plastic ball spectrometer in the experiments under consideration. Therefore the coalescence formula, which relies on the measurement of the "true" N_p values, must fail at lower bombarding energies.

(iii) Fitting all cluster ratios *simultaneously* puts a very stringent condition on the finally extracted S/A values.

The solid curves in Figs. 1 and 2 are fits to the data with the QSM. The least square fits were done simultaneously for all four ratios at ten selected multiplicities. The lines are interpolated between the fitted values. Since the QSM treats clusters explicitly, the multiplicity N_p used in the model calculations could be defined in the same way the participant proton number is defined experimentally, not imposing any restrictions due to nonmeasured heavy clusters at lower bombarding energies. The breakup temperature T_b and the breakup density ρ_b were the only free parameters.

IV. DISCUSSION

Generally the fits to the data are quite satisfactory considering the fact that only two parameters were adjusted to fit, simultaneously, the relative yields of the different fragments. However, at the lower bombarding energies the model does not fit the t/p and ^{3}He/p ratios. The theory predicts t/^{3}He>1 at all energies for the neutron-rich systems Au + Au and Nb + Nb, in agreement with the data except for 150 MeV/nucleon incident energy. As the bombarding energy is increased the agreement gets better. At the highest energies the model yields too little tritium and too much ^{3}He. Also in a coalescence picture it is not conceivable that in neutron-rich systems like Au + Au and Nb + Nb the neutron-poor ^{3}He is produced more abundantly than tritium. Therefore we look for possible experimental causes for this discrepancy:

(i) The phase space acceptance of the plastic ball is best at the lower bombarding energies and therefore the overlap regions in phase space, where the ratios were taken, are largest at these energies. Therefore the discrepancy between experiment and the QSM at the lower bombarding energies for the t/p and ^{3}He/p ratios is unlikely to be due to simple experimental cuts.

(ii) The overlap region was chosen in the space where the particle momenta have been scaled by $(1/m)^{1/2}$, where m is the mass of the different species (p,d,t,^{3}He,α). This scaling is introduced by the assumption that the particle momenta are Boltzmann distributed in momentum space coming from a common source with temperature parameter T (see Ref. 13 for a detailed description). Recently, it has been observed[17] that ^{3}He is emitted with almost twice the mean transverse energy per particle than tritium, which has the same transverse energy per particle as the p, d, and α particles. If this translates into a higher "temperature" for the ^{3}He source, this would have the following consequence: At low bombarding energies, where the excluded region cuts into the low side of the momentum distribution, relatively more

^{3}He than tritium would be inside the overlap region due to the "boost" of the higher apparent temperature. At the highest bombarding energy, the excluded region cuts into the high energy tail of the momentum distribution and we would have relatively more tritium than ^{3}He in the overlap zone. This is exactly what is observed experimentally. If this explanation holds, it constitutes a very interesting observation in itself.

At the low energy the extracted entropies are based on the d/p and α/p ratios only. The resulting specific entropies as a function of the reduced multiplicity N_p/N_p^{max} are shown in Figs. 3(a) and (b) for the systems Nb + Nb and Au + Au, respectively. (For a definition of N_p^{max} see Ref. 18.) The reduced multiplicity has been chosen in order to compare the S/A value for a given system at about the same impact parameter. Observe that there is an increase of S/A with bombarding energy at all multiplicities. In this plot no indication of significant extra entropy production due to a phase transition at the lowest bombarding energy can be seen. From these curves we extract the entropy per nucleon at a finite multiplicity, i.e., $N_p/N_p^{max}=1$ for the various bombarding energies. The result is shown in Fig. 4 together with two model calculations. The experimentally

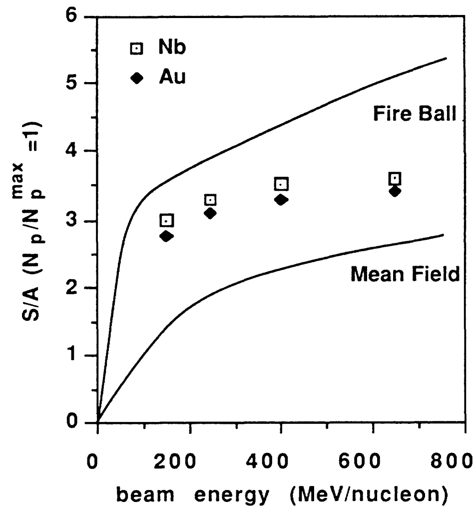

FIG. 4. Comparison of the bombarding energy dependence of S/A with the fireball and a hydrodynamic mean field model. The experimental points *cannot* be compared directly with the theoretical models, because the data are for finite multiplicities, i.e., $N_p/N_p^{max}=1$.

extracted entropy is smaller than the one predicted by the fireball model, where *all available kinetic energy* is converted into random thermal motion. The fact that this model yields too high entropy values indicates that compressional effects play an important role for the entropy production in heavy ion reactions. The curve, labeled as mean field, is a hydrodynamical calculation using an equation of state based on the relativistic mean field theory of Ref. 19. It lies below the data, reflecting mainly the lower entropy per nucleon for infinite nuclear matter, for which the calculation was done. An extrapolation of S/A to infinite multiplicities,[16] subject to the aforementioned uncertainties, would yield, e.g., for Nb + Nb at 400 MeV/nucleon a value of 1.6±0.4 in agreement with the hydrodynamical prediction. The error of 0.4 units of entropy is estimated from the variation of S/A with the breakup density ρ_b.

While the S/A value as a function of the impact parameter (or the reduced multiplicity) is appropriate for comparisons within one system, it is instructive to choose the participant proton multiplicity for the comparison of different systems at the same bombarding energy. The similarity of the extracted entropy values for the two systems (Fig. 5) at a given bombarding energy shows that the specific entropy is mainly dependent on the number of particles in the reaction volume. Since the reaction volumes are also proportional to first order to the number of particles, the relevant variable for S/A is the mean particle density. The slight excess of S/A for Au + Au over Nb + Nb at the highest multiplicities might indicate an effect of contributions of less central collisions: At a given (high) multiplicity the two nuclei nearly fully overlap for Nb + Nb, while at the same multiplicity the reaction Au + Au is still more peripheral, resulting in a less dense participant region and hence a higher entropy.

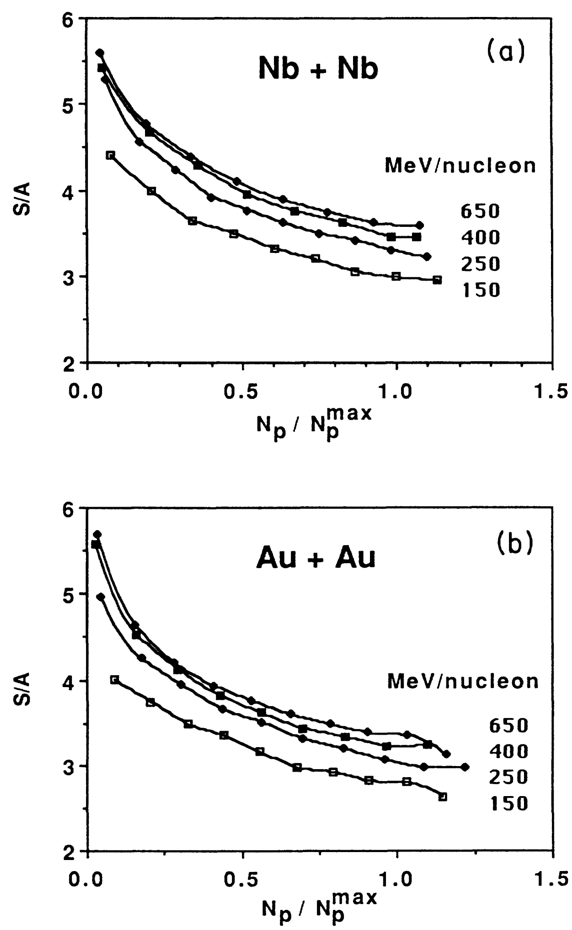

FIG.3. Entropy values (S/A) extracted from the x/p ratios as a function of the reduced multiplicity N_p/N_p^{max}.

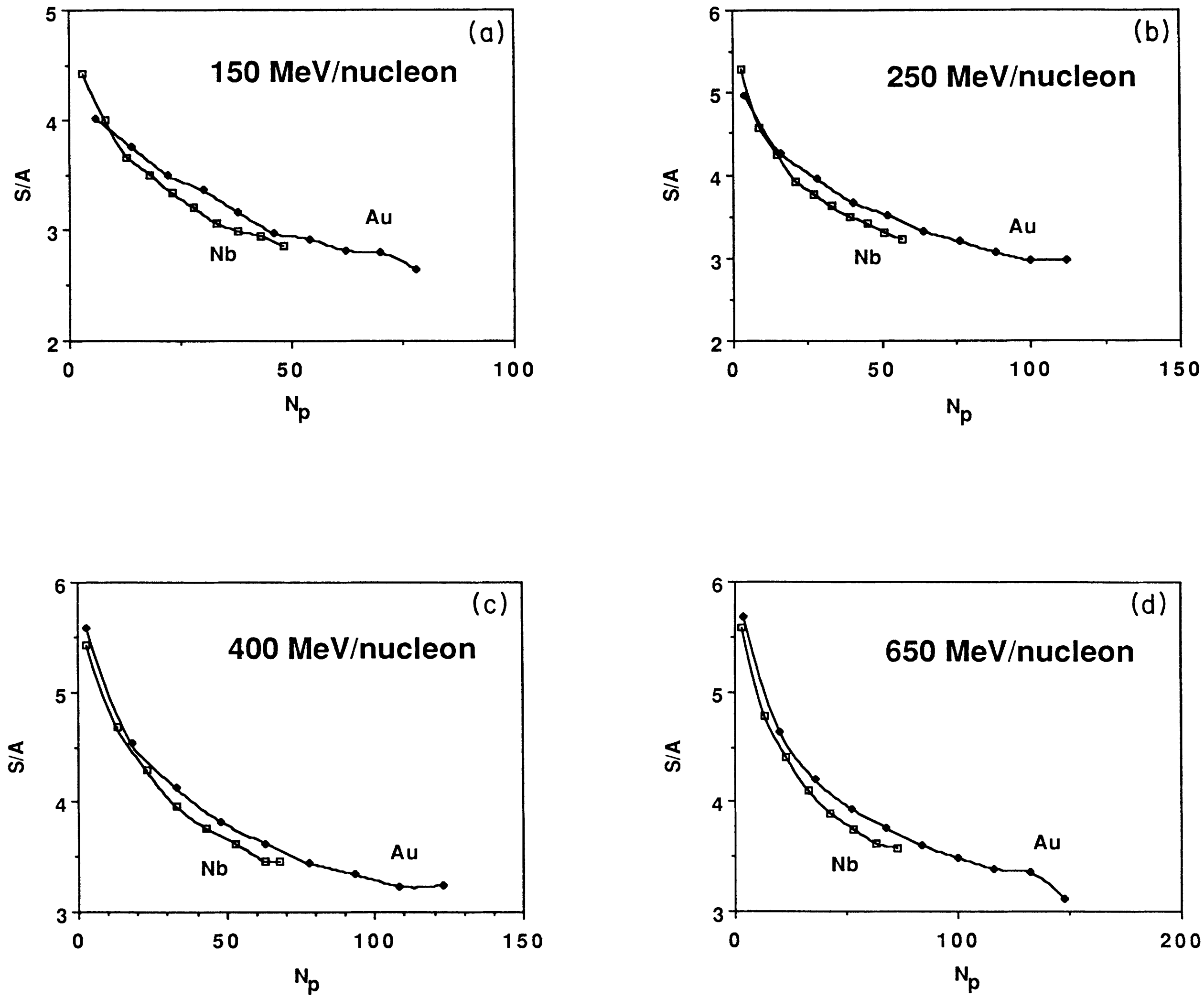

FIG. 5. Comparison of the mass dependence of S/A as a function of the participant proton multiplicity N_p.

In a recent paper, studying the effects of momentum-dependent interactions,[20] it was claimed that the deuteron-to-proton ratio, at least for heavy systems, is sensitive to the nuclear equation of state. The theoretical value of $(d/p)_{max} \approx 0.62$, assuming a rather hard equation of state without momentum-dependent forces, is in perfect agreement with our data at all projectile energies.

V. SUMMARY

We have measured the multiplicity dependence of light cluster production for the medium heavy and heavy systems Nb + Nb and Au + Au at bombarding energies ranging from 150 to 650 MeV/nucleon. The specific entropy production has been extracted using the quantum statistical model by Hahn and Stöcker.[16] A comparison of S/A at different bombarding energies gave no evidence for a significant amount of extra entropy at the lowest bombarding energy (150 MeV/nucleon) as would be expected from a phase transition, as far as this can be judged from the monotonically decreasing experimental entropies. A comparison of S/A for the two colliding systems shows at all energies only little dependence on the system. A comparison with models gives further evidence for significant compression effects in relativistic heavy ion reactions and for a rather stiff equation of state.

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*Present address: Linear Accelerator Laboratory, University of Saskatchewan, Saskatoon, Saskatchewan, Canada S7N 0W0.

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