Neutron-proton scattering observables at 325 MeV, the ϵ_1 parameter, and the tensor force

G. S. Chulick,[a),(b)] Ch. Elster,[(a),(c)] R. Machleidt,[(a),(d)] A. Picklesimer,[(a)] and R. M. Thaler[(a),(b)]
[(a)]Physics Division, Los Alamos National Laboratory, Los Alamos, New Mexico 87545
[(b)]Department of Physics, Case Western Reserve University, Cleveland, Ohio 44106
[(c)]Department of Physics, Kent State University, Kent, Ohio 44242
[(d)]Department of Physics, University of California at Los Angeles, Los Angeles, California 90024
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The sensitivity of neutron-proton elastic scattering observables to variations in the low angular momentum $T=0$ phase shifts is studied at $E_{\rm lab}=325$ MeV. It is found that the $J=1$ coupling parameter ϵ_1 is not well determined by existing data. This uncertainty in ϵ_1 permits models with quite different tensor forces to describe the extant data. Implications and possible experimental resolution of such ambiguities are discussed.

Since the discovery of the quadrupole moment of the deuteron, indicating the existence of a tensor component in the nucleon-nucleon (NN) interaction, the role of the tensor force in nuclear structure physics has been recognized and examined repeatedly.[1] Its effect on the ground state of few-body systems, finite nuclei, and infinite nuclear matter has been extensively studied. The tensor force plays an important role in determining the binding energy of the three-nucleon system[2] and the saturation of nuclear matter and finite nuclei.[3] Calculations of excited states of open shell nuclei indicate that the rate of convergence of core polarization contributions is very sensitive to the strength of the nuclear tensor force.[4]

For these reasons it is of great interest to accurately characterize the NN interaction, particularly the contribution of the tensor force. Beyond any formal theoretical considerations, models of the NN interaction depend crucially on the rigor of the constraints provided by experimental scattering observables. In this paper we analyze the extent to which the ϵ_1 mixing parameter is determined by presently available NN scattering data, and discuss the consequences of this for the tensor force.

Very many neutron-proton (n-p) elastic scattering measurements of the angular distribution and of various spin observables have been performed, most extensively at an energy of 325 MeV in the laboratory system. These data, along with the corresponding complete $T=1$ proton-proton (pp) elastic data, and together with the assumption of charge independence of the nuclear force, have led to what has been more or less accepted as a determination (the phase shift analysis) of the n-p scattering amplitude. However, as is described below, this suggested uniqueness is misleading. Present data still allow for large uncertainties in several partial waves, in particular in $^1P_1$, $^3D_2$, and the ϵ_1 mixing parameter. Important physical conclusions depend upon these phase shifts, one such consequence being the size of the $T=0$ tensor force. Since there is a considerable apparent ambiguity at lower energies, the 325 MeV n-p data analysis has taken on an exaggerated importance in the determination of the $T=0$ tensor force. Thus, we examine the latitude in the ϵ_1 mixing parameter at 325 MeV, which has often served as a measure of the size of the tensor force.

Our interest centers on the $^3S_1$-$^3D_1$ and the $^3D_2$ states, since these are the states most influenced by ambiguities in the $T=0$ tensor force, and on the $^1P_1$ state. We first observe that for angular momenta $J>2$, the phase shifts are reasonably well determined. Both the Arndt phase shift analysis (PSA) (Ref. 5) and the Basque PSA (Ref. 6) are in rough agreement for the higher partial waves. The lower $T=0$ phase shifts at 325 MeV for the two PSA's are shown explicitly in Table I. In addition, the corresponding predicted phase shifts from the Reid,[7] Paris,[8] and Bonn[9] models are presented. The reasonably close agreement between the models and the PSA results for higher partial waves is a reflection of the influence of the unambiguously defined one-pion exchange and the two-pion exchange. For this reason, except as noted, in this work the phase shifts for the high partial waves are taken to be those of the Arndt PSA. We have checked that a substitution of either the Bonn or Paris higher partial wave phase shifts leads to no changes in the present discussion.

The predictions of several models for various np observables together with the PSA results are shown in Fig. 1 along with the existing data. It is evident from these figures that all the phase shift sets under consideration provide acceptable fits to the data. Thus, the differences among them reflect the uncertainty in the uniqueness of the phase shifts and it is this uncertainty we wish to quantify. Although we especially focus on the latitude allowed by present data in the $^3S_1$-$^3D_1$ coupling parameter ϵ_1, it is not our purpose to perform yet another PSA with a somewhat different set of constraints. Rather, we examine the extent to which a reasonable description of the experimental observables will tolerate variations in the phase shift parameters. This suggests experimental studies that would determine ϵ_1 more closely.

Figure 1(a) makes clear that very little further can be learned from a study of the differential cross section unless new precision data become available at forward angles. Similarly, most of the other observables, e.g., the

TABLE I. $T = 0$ phase shifts (in degrees) of neutron-proton scattering at $E_{lab} = 325$ MeV.

State	Ar87[a,f]	Basque[b]	RSC[c]	Paris[d]	Bonn[e]	Variation
$^1P_1$	−28.16	−35.05	−38.23	−26.83	−30.84	8.22
$^3S_1$[g]	2.25	0.93	0.70	0.68	4.54	3.86
$^3D_1$	−24.16	−25.38	−23.84	−25.20	−23.86	2.22
ϵ_1	5.90	6.23	8.17	5.19	3.05	5.12
$^3D_2$	23.06	23.30	25.74	28.32	19.60	8.72
$^1F_3$	−5.79	−5.63		−5.71	−5.50	0.29
$^3D_3$	3.54	2.69		4.74	3.71	2.05
$^3G_3$	−4.60	−4.39		−4.98	−4.86	0.59
ϵ_3	7.41	7.55		7.72	7.19	0.53
$^3G_4$	8.33	7.04		8.65	8.16	1.61

[a]Reference 5.
[b]Reference 6.
[c]Reference 7.
[d]Reference 8.
[e]Reference 9.
[f]The imaginary parts of the $^3S_1$ and the $^3D_1$ phase shifts are not listed here since their effect on the fit to the data is completely negligible.
[g]The $^3S_1$ phase shift vanishes near this energy. Thus, the relative variations in this phase shift appear exaggerated.

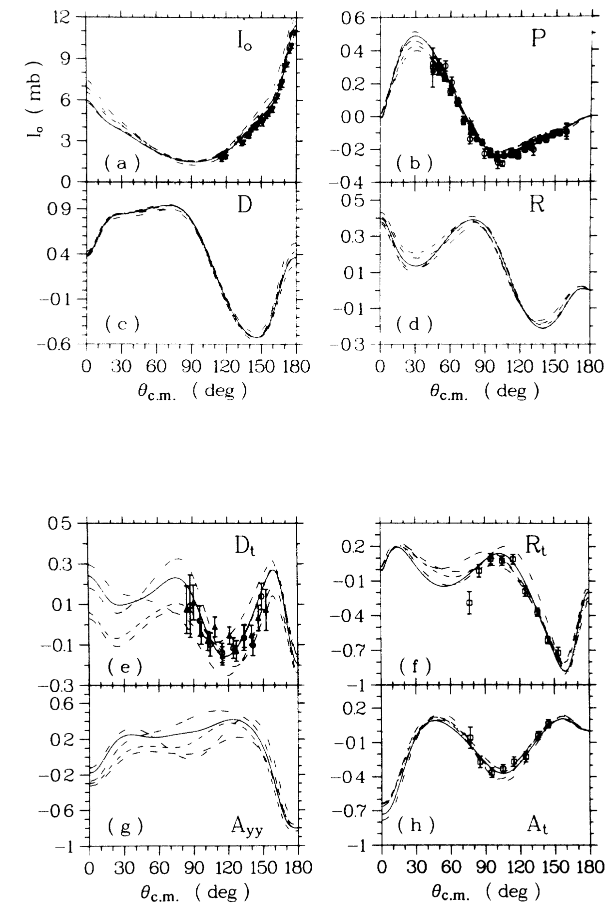

FIG. 1. Predictions for neutron-proton observables at 325 MeV laboratory energy. Full line AR87 (Ref. 5), dashed Basque (Ref. 6), dash-dot RSC (Ref. 7), dash-dot-dot Paris (Ref. 8), and dash-dash-dot-dot Bonn (Ref. 9).

polarization P, the depolarization D, and the spin transfer correlation coefficients do not show strong sensitivity to the different models or PSA; for most of them there are also no data available. In particular, if one does not consider correlated initial-state-spin experiments and final state neutron spin detection, then one is left with the additional spin variables D_t, R_t, A_t, A_t', and R_t'. (For a detailed description of the various NN scattering observables we refer to Refs. 10 and 11). The latter three observables show little sensitivity, leaving D_t and R_t. In addition to the extensive np angular distribution and polarization data at 325 MeV, there are a number of measurements of the spin transfer observables D_t and R_t at scattering angles in the backward hemisphere. It is these measurements along with the angular distribution and polarization data which have gone into the determination of the $T = 0$ phase shifts.

We first consider D_t, the transverse polarization transfer normal to the scattering plane [Fig. 1(e)]. This observable shows the largest differences for the theoretical models as well as the PSA, and, in addition, it is the observable most sensitive to the mixing parameter ϵ_1. Since the experimental errors in this observable are relatively large, the question arises as to just how large is the uncertainty in the determination of ϵ_1. Figure 2 shows the sensitivity of D_t to variations in the individual partial waves ϵ_1, $^1P_1$, and $^3D_2$ (all other partial waves are fixed and taken from the Arndt PSA), as well as something of the latitude permitted by the experimental data. Even more latitude is obtained from correlated variations in these partial waves. Because phase shift analyses focus mainly on purely statistical χ^2 criteria, the interplay between χ^2 minimization and the realistic constraints provided by the data are of central interest. The latter depend upon correlated and systematic errors as well as the

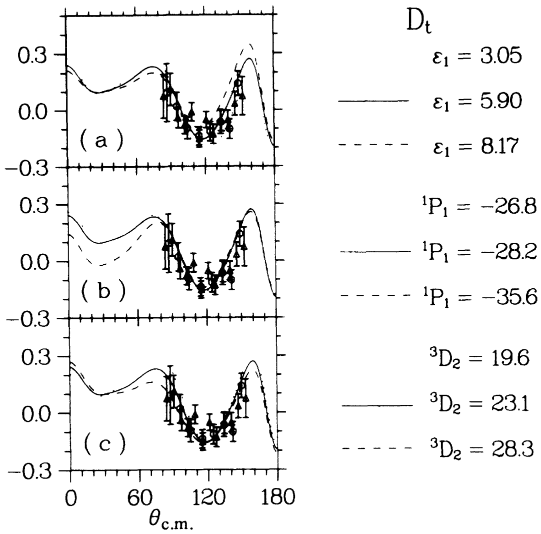

FIG. 2. Effect of changes in the ϵ_1, $^1P_1$, and $^3D_2$ phase shifts on D_t. The solid line represents the Arndt PSA in each of the panels.

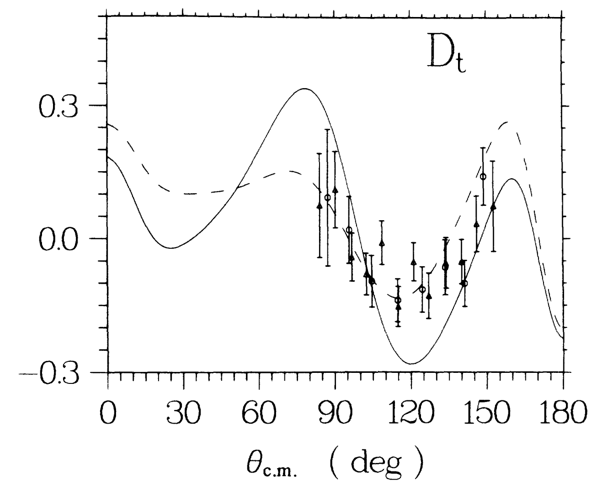

FIG. 3. Variations of different $T=0$ phase shifts, which still provide an adequate description of the data: Starting from the Arndt partial wave analysis (Ref. 5) the solid line is obtained with $\epsilon_1=1°$, $^1P_1=-35°$, and $^3D_2=20°$, the dashed line with $\epsilon_1=7°$, $^1P_1=-28°$, and $^3D_2=28°$.

shallowness of the χ^2 profile. These aspects tend to be more readily apparent in plots such as Fig. 2 and in comparisons of the description of different experimental observables. In general, we provide the χ^2 per data point for comparison with the figures. For Fig. 2, the Arndt PSA (solid lines) yields a value of $\chi^2=1.4$ per data point for the D_t data shown. The remaining six curves listed at the right of Fig. 2 yield, in order, a χ^2 per D_t data point of 1.9, 3.2, 1.4, 1.5, 2.2, and 1.1. Evidently, the data are very forgiving. The χ^2 per data point for the full 325 MeV data set is, of course, also important. The Arndt PSA value is 1.9, whereas the six additional curves of Fig. 2, again in order, yield values of 2.7, 2.1, 2.0, 5.3, 2.4, and 6.7. Here, the two largest values result from a poor description of the exceptionally precise total cross section datum of Ref. 12. Because of the quoted precision of these data, the systematic error is of enhanced consequence. A systematic error of the order indicated in Ref. 12 ($\sim 1\%$, 3 times the random error) reduces the χ^2 per datum of 5.3 and 6.7 to 2.3 and 4.0, respectively. Thus it appears that the full data set is also very forgiving of variations of the type illustrated in Fig. 2.

We note that the sensitivities to the different partial waves occur in different angular regimes, and secondly, that there is a strong interplay between the ϵ_1 and $^3D_2$ partial waves for angles $\theta \geq 60°$. Variations in the $^1P_1$ partial wave show up only in the forward hemisphere. To better address the question about the uncertainty of ϵ_1, that of $^3D_2$ clearly must be taken into account as well. In principle $^1P_1$ must also be considered, but its influence is weaker in the backward hemisphere. The range of typical variations for each of these partial waves is indicated in Table I. In order to find a characterization of the bounds on ϵ_1 we have varied the above mentioned partial waves within these ranges. The result is presented in Fig. 3: $\epsilon_1=1°$ (together with $^1P_1=-35°$, $^3D_2=20°$) as well as $\epsilon_1=7°$ (together with $^1P_1=-28°$, $^3D_2=28°$), all other partial waves being those of Ref. 5, give rough bounds on reasonable fits to the existing D_t data. The χ^2 per D_t data point corresponding to the $\epsilon_1=1°$ and the $\epsilon_1=7°$ curves of Fig. 3 are 3.5 and 1.5, respectively. The corresponding values for the full 325 MeV data set are 3.9 and 6.5. Again, the value for the $\epsilon_1=7°$ case is reduced from 6.5 to 3.8 if one considers the systematic error in the total cross section data of Ref. 12. Modification of the $^1P_1$ and $^3D_2$ phase shifts to $-31°$ and 22°, respectively, then reduces the $\epsilon_1=7°$ χ^2 values to 1.4 and 3.3 for the D_t and the full data sets, respectively. Our conclusion from these considerations is that ϵ_1 at 325 MeV may vary between above about 1° to beyond 7°, if reasonable variations in the $^1P_1$ and $^3D_2$ partial waves are also taken into consideration. Although $\epsilon_1=1°$ represents an extreme lower bound on the value of the $J=1$ mixing parameter, the upper bound we have quoted of $\epsilon_1=7°$ is not so inflexible. For example, for $\epsilon_1=7°$, with all other partial waves taken to be those of the Arndt PSA, we find that χ^2 $(D_t)=2.1$ and that χ^2 (all data)$=1.9$. In fact for $\epsilon_1=8.2°$, and all other partial waves fixed at the Arndt PSA values, we find that χ^2 $(D_t)=3.2$ and that χ^2 (all data)$=2.1$.

Next, we consider R_t, the transverse polarization transfer in the scattering plane [Fig. 1(f)]. Here a serious problem arises. None of the phase shift combinations under study reproduces the most forward data point. Small departures from present models cannot account for so negative a value of R_t as it is shown in the forward point of Fig. 1(f). We are therefore forced to treat this datum as spurious. If R_t turns out to be strongly negative in the 60°–90° angular range, we may be forced to reevaluate our entire picture of the NN interaction. We have tested the sensitivity of this observable to the different $T=0$ low partial waves, and it turns out that it is basically sensitive to the same partial waves as D_t. The effect of varying the mixing parameter ϵ_1 as well as the partial waves $^1P_1$ and $^3D_2$ on the different angular regimes is shown in Fig. 4. There are only small variations in the backward hemi-

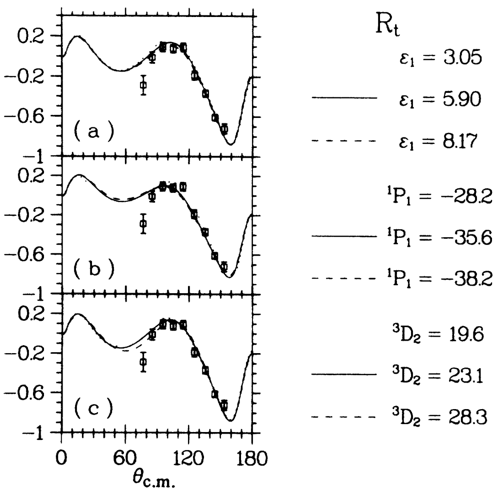

FIG. 4. Effect of changes in the ϵ_1, $^1P_1$, and $^3D_2$ phase shifts on R_t. The Arndt PSA is represented by the solid line in panels (a) and (c) and by the dotted line in (b).

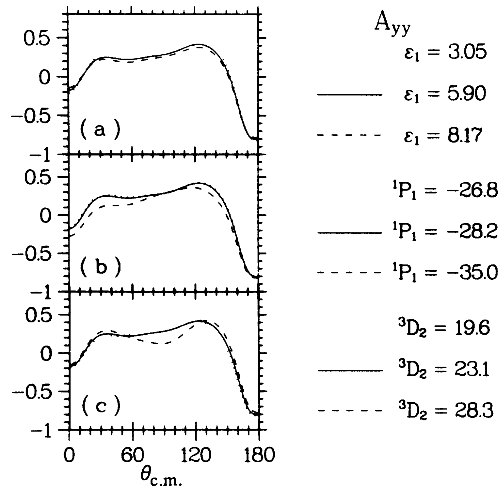

FIG. 5. Effect of changes in the ϵ_1, $^1P_1$, and $^3D_2$ phase shifts on A_{yy}. The solid line represents the Arndt PSA in each of the panels.

sphere and a sensitivity to $^3D_2$ around $\theta=90°$. The $^1P_1$ partial wave determines the forward dip structure. In general these sensitivities are small compared to those discussed in connection with D_t and discrimination requires quite accurate data. The χ^2 per R_t data point for the Arndt PSA is 1.9, which is to be compared to that for the other six curves in Fig. 4 which are, in order, 3.2, 1.4, 2.3, 2.6, 2.2, and 1.1. The corresponding values of χ^2 per data point for the full 325 MeV data set are 2.7, 2.1, 5.3, 8.0, 2.4, and 6.7. As discussed in connection with D_t, taking the systematic error in the datum of Ref. 12 into account reduces the χ^2 values of 5.3, 8.0, and 6.7 to 2.3, 2.9, and 4.0, respectively. We conclude that the behavior of R_t does not significantly affect our conclusions regarding the uncertainty in the ϵ_1 mixing parameter.

A third observable which shows significant differences among the existing models, the spin correlation parameter A_{yy}, has been discussed in Ref. 13. A_{yy} again shows sensitivity to the partial waves $^3D_2$ and $^1P_1$, and to the mixing parameter ϵ_1 (Fig. 5). A slight sensitivity to the other $J=1$ triplet partial waves, which is not shown here, is also observed. Since A_{yy} is sensitive to $^3D_2$ and $^1P_1$ in nearly orthogonal angular regimes, it could be used to pin down at least $^3D_2$, which shows its largest variation around $\theta=90°$. The sensitivity to $^1P_1$ is largest in the forward hemisphere, whereas the variation due to ϵ_1 is small. Therefore A_{yy} seems less suited for drawing conclusions about the magnitude of ϵ_1 but is better suited for constraining the $^3D_2$ phase shift in order to eliminate uncertainties concerning this partial wave in the other observables under discussion.

A somewhat similar study to ours at ~50 MeV was performed by Binstock and Bryan[10] in 1974. They found that the spin observables A_{zz}, C_{pp}, A_t', C_{kk}, A_t, D_t, C_{nn}, and A_{xx} to be the most sensitive to variations in ϵ_1, listed in the order of decreasing sensitivity. At that time there were no data at this energy on spin observables except for polarization data. Since then some data on A_{yy} (C_{nn}) have been added, but the ϵ_1 parameter is still not well determined as may be seen from Fig. 6, in which the results of the most recent PSA (Ref. 5) are shown. At 325 MeV the sensitivities are different from those at 50 MeV and the polarization transfer parameter D_t is the most sensitive to variations in ϵ_1.

The present status of the ϵ_1 parameter from 0 to 325 MeV is summarized in Fig. 6. On the basis of the PSA alone and given the apparent ambiguity at lower energies, it is evident why PSA values of ϵ_1 around 300 MeV assume a central role in static potential models. In such models the magnitude of the tensor force is largely determined by the value of ϵ_1 at ~325 MeV. From Fig. 6 one sees that this is not a reliable procedure: As emphasized by our "uncertainty bar" in Fig. 6, the apparent lack of ambiguity in ϵ_1 from the PSA is misleading. Because the situation at lower energies is even more uncertain, at present there is evidently little in the way of convincing

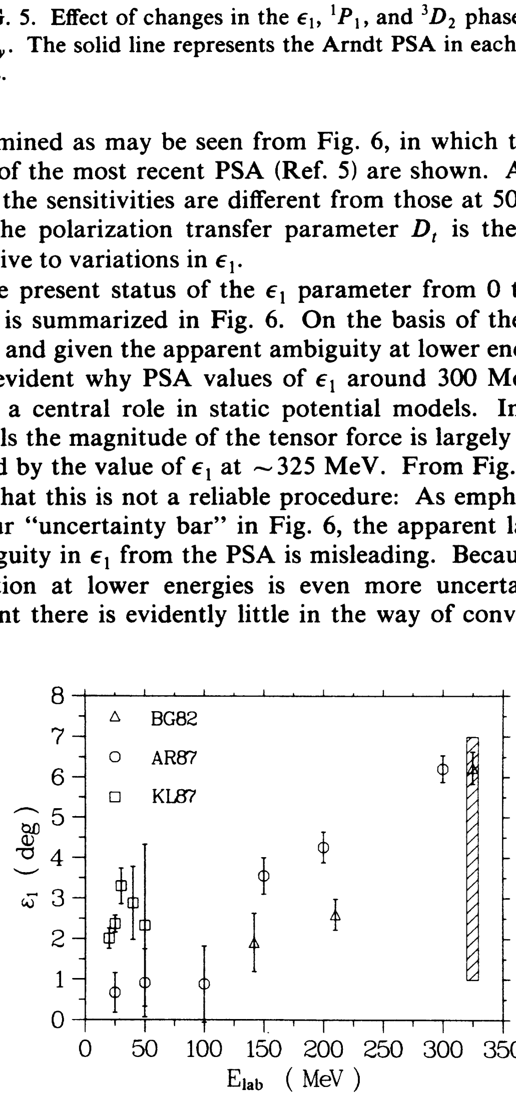

FIG. 6. Experimental data points for the ϵ_1 mixing parameter in the energy range from 0 to 325 MeV laboratory energy. The shaded bar represents the uncertainty which should be allowed given the variations of the NN observables discussed above. The data points are taken from the following references: AR87 (Ref. 5), BG82 (Ref. 6), and KL87 (Ref. 14).

information regarding the ϵ_1 parameter. For static potential models a precise determination of ϵ_1 at a single energy could indeed serve to constrain the magnitude of the tensor force. However, there is now considerable interest in nonstatic NN interactions constructed from meson theory.[9] Given the possibility of a nonstatic interaction, even a precise value of ϵ_1 at a single energy provides an inadequate constraint, unless supplemented by the correlated spin measurements needed to distinguish between explicit energy-dependent and tensor force effects.

In summary, we have verified that the 325 MeV n-p elastic scattering data can be analyzed with the $T=1$ phase shifts taken from p-p scattering and the higher partial wave phases taken from a meson theoretic model such as Bonn or Paris. The higher partial wave phase shifts predicted by both models are very close to those of the Arndt phase shift analysis and are completely consistent with pionic exchanges. In this framework we have examined and discussed the sensitivity of the observables to variations in the low partial wave phase shifts. We have argued that purely statistical χ^2 analyses of the data tend to yield an apparent constraint on the ϵ_1 mixing parameter which is more stringent than is actually warranted by the data. In addition to the fact that the χ^2 minima are not sharp, this apparent constraint neglects correlated and systematic errors. Because the gradients with respect to variations in the phase shifts (especially ϵ_1 and $^3D_2$) in the vicinity of the Arndt PSA χ^2 minimum are not large, relatively small changes in the data can easily shift the minimum. The presence of correlated and systematic errors, which are not fully taken into account in a purely statistical χ^2 approach, is also evident in the data. Examples include the total cross section datum discussed herein in regard to Figs. 2 and 4, the presence of two incompatible D_t data sets ($\chi^2 \sim 1$ for one data set yields χ^2 well outside the statistical error limit for the other) and the apparent incompatibility with the cross section data ($\chi^2 \sim 1.3$, due to a consistent overprediction of the experimental values), and the polarization data ($\chi^2 \sim 2.9$) in the Arndt PSA. For these reasons, it is not surprising that Figs. 2–4, and more generally the full 325 MeV data set, show a tolerance to variations in ϵ_1 which is not fully characterized by a χ^2 minimization criterion alone.

Our principal conclusion is that the $J=1$ coupling parameter ϵ_1 is only loosely constrained[15] by the existing data to be ϵ_1 (325 MeV) $\simeq 1°-7°$. Since the strength of the tensor interaction in static potential models is closely related to this value of ϵ_1, it follows that the tensor force cannot be precisely determined by the existing data at this energy. The situation at lower energies is even more ambiguous and the deuteron quadrupole moment does not provide a definitive constraint either. From Figs. 1 and 3, and from experimental realities involving the difficulty of correlated spin and final state neutron spin experiments, it appears that high precision measurements of D_t are most promising in removing this ambiguity. Because of uncertainties concerning energy dependences in realistic potential models, definitive results at lower energies, in addition to the 300 MeV regime, are needed to discriminate between tensor effects and explicit energy-dependent effects. Precise results for ϵ_1 are required to verify the reliability of current potential models. Unambiguous results for ϵ_1 strongly constrain the tensor force, and this is an essential ingredient for nuclear structure and nuclear reactions.

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