# Energy spectra of strange particles hadronizing from a quark-gluon plasma

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Using a schematic hydrodynamic description of relativistic heavy-ion collisions in the regime of complete nuclear stopping, we study the freeze-out and strange particle energy spectra from a hadronized quark-gluon plasma. We argue that characteristic changes in the slopes of the $K^+$ and $K^-$ meson energy spectra are a signature for the quark-gluon plasma phase transition.

## I. INTRODUCTION

The collision of nuclei at relativistic energies, and the subsequent phase transition from hadronic matter to quark-gluon matter, continue to be a subject of intense interest.[1]

The possibility of forming a quark-gluon plasma via heavy ion collisions in the laboratory opens up a large number of interesting theoretical problems. To date, a difficult question to answer has been: What experimental signal or combination of signals would unambiguously indicate that a phase transition from hadronic matter to quark matter has taken place? Several signals for the phase transition to quark-gluon matter have been proposed in the literature. These include dilepton[2,3] and photon spectra,[4] suppression in the number of expected $J/\psi$ resonances,[5,6] modification of charmed meson widths,[7] and the relative abundance of strange particles.[8] Although these signals reflect many of the unique properties of the static plasma, our limited understanding of the collision dynamics often makes a critical evaluation of these signals difficult.

In the preceding paper[9] we discussed in some detail the role of strangeness abundance as a signal for the phase transition to the baryon rich plasma. Comparing states connected to each other by a simple dynamical model for the hadronization of a baryon-rich quark-gluon plasma, it was shown that the strangeness of hadronized quark matter and a chemically equilibrated hadron gas are rather similar. A similar result was found by other authors for the baryon free case,[10,11] even when the condition for chemical equilibration of strangeness was relaxed. Therefore, strange particle *abundances,* even if considerably enhanced over the level seen in p-p collisions, by themselves yield ambiguous information as far as the distinction between quark-gluon plasma formation or an equilibrated hadron gas is concerned.

In this paper we show that additional information to resolve this ambiguity can be obtained from the *energy spectra* of strange hadrons, in particular of K mesons. Some of these ideas were briefly discussed in an earlier letter;[12] in this paper we present our ideas in more detail and extend our dynamical calculations to predict changes in the observed energy spectra as a function of the nuclear collision energy.

The signal we propose is, in part, a consequence of the large difference in the entropy density between a hadron gas and a plasma. The larger entropy of the quark-gluon plasma results from the large number of degrees of freedom of the deconfined quarks and gluons. As a consequence, for a given beam energy, the maximum temperature reached in the collision will be much lower if the plasma is formed than if the system always remains in the hadron phase. We will argue that $K^+$ mesons carry information from the early stages of the collision (in particular as a consequence of strangeness separation during the mixed phase[13,14] and can detect such a temperature difference; comparing the slope of their energy spectrum with the slopes of $K^-$, proton, and pion spectra (which probe later stages of the collision) will allow to identify a first order phase transition to quark matter in nuclear collision.

Since the observed energy spectra are strongly influenced by the collective flow in the late phases of the collision, a dynamical model is needed to make definitive predictions. The dynamical model we want to use for the nuclear collision consists of several distinct parts. The initial collision of the relativistic nuclear system is assumed to result in full stopping. This assumption implies that our studies are mostly relevant for the current experimental programs at the Brookhaven Alternating Gradient Synchrotron (AGS). Thus we are particularly interested in heavy ion collisions up to $E_{\text{lab}}/A = 15$ GeV, where a baryon rich plasma is thought to occur.

For our model, the transition for the initial ground state of nuclear matter to quark matter is assumed to occur through shock heating and compression of the stopped nuclei. All the entropy is generated by this process. Following the creation of the quark matter, it is assumed that any further increase in the entropy of the quark-gluon gas is negligible and that the quark-gluon system cools and hadronizes at constant entropy. During the expansion, and at some characteristic temperature, the quark matter starts to hadronize to form bubbles of hadronic matter. During this mixed phase, the effect of the large net baryon density on the hadronization process leads to strangeness separation[15,13,14] between the hadron bubbles and the remaining quark matter. $K^+$ mesons will hadronize predominantly during the early stage of the mixed phase. After completion of the hadronization process, the strongly interacting gas of hot hadrons will further expand until freeze out. At different freeze-out points, particular species of hadrons decouple from the collective hydrodynamical flow of hot matter and subsequently free stream towards the experimental detectors; their observed kinetic energy spectrum is determined by the temperature distribution at this respective freeze-out point. The actual conditions for freeze out depend very strongly on the individual particle interaction cross sections within the hadron gas. The relatively small interaction cross section for the $K^+$ mesons with nucleons implies[12] that in a baryon rich environment these particles will decouple first, and indeed could be carriers of information of the mixed phase dynamics. Our condition for freeze out assumes that the collective flow of the hot hadronic matter can be described by a velocity profile taken from scaling hydrodynamics.[16] We note that the energies and temperatures of interest to us here require a relativistic generalization of these ideas. In this paper we also discuss the sensitivity of our results to the form of the velocity profile.

The final stage of our calculation requires the freeze-out temperature to be related to the laboratory energy spectrum. For this transformation we generalize the ideas of Siemens and Rasmussen,[17] averaging, the thermal energy spectrum over the velocity profile at the freeze-out temperature. We show that the relative slope parameters of the $K^+$ and $K^-$ spectra exhibit a characteristic behavior as a function of beam energy, and that the relative difference of these slopes appears to depend on which phase was reached during the collision. The presented results are in qualitative, although not quantitative agreement with our earlier work,[12] where we neglected the effects from collective flow on the laboratory spectra.

In this paper rather than fully solving the hydrodynamic equations, we will schematically exploit the essential features of hydrodynamics, namely the conservation of total entropy, total energy, strangeness, and the baryon number. We assume thermal and chemical equilibrium in both the quark-gluon plasma and the hadron phases. Using the model equations of state described in our earlier papers,[18,9] constant entropy contours are calculated. Along these constant entropy contours, the radius and the expansion velocity of a spherically expanding system are obtained from the conservation of baryon number and total energy. The price to pay for our simplifications is the need to assume a velocity profile inside the expanding sphere.

In Sec. II after discussion of the initial conditions, we calculate the constant entropy contours from our model equation of state. Comparison is made between the cases with and without plasma formation, and hadronization is studied along the isentropic contours. In Sec. III we calculate freeze-out temperatures for pions, kaons, and protons. In Sec. IV we relate the freeze-out temperature to the final energy distribution of observed particles. In Sec. V we summarize and discuss our results.

## II. SHOCK FORMATION, ISENTROPIC EXPANSION, AND HADRONIZATION OF QUARK-GLUON MATTER

### A. Initial conditions

We assume, for a given initial bombarding energy, that the highest compression state which can be reached by the collision is given by the solution of the Rankine-Hugoniot equation[19,20] for one-dimensional shock. All the entropy will be generated during the shock compression. Note that, in taking the results of a one-dimensional shock calculation for our initial condition, we will, for a given collision energy, overestimate the temperature of the shocked matter by neglecting finite mean free path effects and the transverse flow degrees of freedom. However, the qualitative behavior as a function of increasing beam energy will be reasonably well reproduced. For the baryon-rich plasma the initial conditions also depend upon the choice of the equation of state. Nevertheless, the qualitative characteristics of our signal will not be sensitive to the details of the equation of state.

The shock is defined by discontinuities in energy density, pressure, and baryon density. The continuity of energy, momentum, and baryon number fluxes across the shock surface can be summarized by the relativistic shock equation,

$$\left[\frac{(\varepsilon_2+P_2)^2}{\rho_2^2}-\frac{(\varepsilon_1+P_1)^2}{\rho_1^2}\right] =(P_2-P_1)\left[\frac{(\varepsilon_2+P_2)}{\rho_2^2}+\frac{(\varepsilon_1+P_1)}{\rho_1^2}\right], \quad (1)$$

where 1 and 2 indicate the matter on the two sides of the shock front.

When we include the strangeness degree of freedom, we also need to solve an equation for the strangeness density. In this paper we assume that the total strangeness density $\rho_s$ is conserved at its initial zero value throughout the collision. This assumption ignores the influence of weak interactions during the time of the collision and neglects loss of strange mesons due to hard collisions prior to plasma formation, surface losses during expansion[21] and losses due to earlier freeze out of $K^+$ mesons as will be discussed later.

Taking state 1 as nuclear equilibrium ($P=0$, $\rho=0.145$ $\text{fm}^{-3}$, $\varepsilon=130$ $\text{MeV/fm}^3$), these two equations can be solved with the appropriate equations of state for state 2. For the plasma equation for state 2 we find that the shock equation (1) has a solution only for laboratory bombarding energies greater than $\sim 2$ GeV/nucleon.

When comparison with a scenario without a phase transition to quark matter is desired, the hadron gas equation of state (EOS) of Ref. 9 is used for state 2, but without the Hagedorn volume correction factor.

### B. Isentropic expansion of the hot matter

Assuming that the quark-gluon plasma behaves as a perfect fluid (no turbulence, viscosity, or thermal conductivity), a hydrodynamic description of the expansion and cooling of the plasma will conserve entropy. Once the plasma has been formed and all the entropy has been generated, additional sources of entropy will be neglected in our calculation.

Rather than solving the full hydrodynamics, we simulate the properties of the hydrodynamic expansion of our model plasma by calculating contours of constant entropy/baryon ($S/A$) from our equation of state.[22] In Fig. 1, isentropic expansion contours for various values of $S/A$ are shown in the $(T,\rho_b)$ plane, for a phase transition calculated with a bag constant $B=250$ MeV/fm$^3$. The beam energies quoted for each value of $S/A$ are the result of the shock calculation mentioned above. In a full hydrodynamical approach, each point of such an $S/A$ contour would correspond to a certain time. This information on the time evolution is lost by our simplification. We will later recover some of this information using a simple scaling ansatz (see below).

Following an isentropic curve we can see that, as the plasma expands from the highest compression state, the temperature decreases until particles start to hadronize. During the mixed phase the temperature increases; this is due to the large difference in the entropy per baryon between the plasma and the hadronic matter which results in a large latent heat. During the mixed phase, quarks and antiquarks will form hadrons; the gluon will disappear, partly by fragmenting into $q\bar{q}$ pairs, thereby depositing their entropy, partly by building the nonperturbative quantum chromodynamics (QCD) vacuum state. After completion of the hadronization, further expansion of the hadronic matter will decrease the temperature once again until freeze out. The reheating effect in the mixed phase is biggest for low energies (small $S/A$), at higher energies (large $S/A$) the temperature stays nearly constant in the mixed phase.

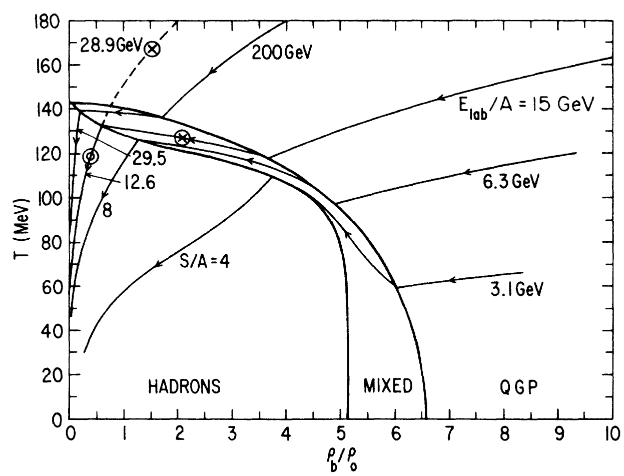


FIG. 1. Isentropic expansion trajectories for a hadronizing quark-gluon plasma, for several values of entropy per baryon ($S/A=4$, 8, 12.6, and 29.5). The phase transition curves are calculated with a bag constant $B=250$ MeV/fm$^3$. The beam energies quoted for each value of $S/A$ are the result of a one-dimensional shock calculation. The freeze-out points for $K^+$ (crosses) and p, $K^-$, $\pi$ (circle) are indicated along the $S/A=12.6$ curve for both scenarios, i.e., with and without quark-gluon plasma formation.

The dotted line shows an isentropic expansion contour corresponding to $S/A=12.6$, but for purely hadronic matter (no phase transition) formed at $E_{lab}/A=28.9$ GeV. This dotted line should be compared with the contour that started in the quark-gluon phase and corresponds to the same entropy value. This line corresponds to an initial bombarding energy of $E_{lab}/A=15$ GeV.

It is important to note that due to the difference in the entropy density, for a given beam energy, the temperature we can reach in the hadron gas is much higher than that with a phase transition to quark-gluon matter. The $K^+$ mesons, which freeze out in an early stage of the evolution because of their small interaction cross section, will preserve this valuable thermal information about the formation of a plasma, as will be discussed in Secs. III and IV.

### C. Hadronization of the quark-gluon plasma

As the temperature of the expanding quark matter becomes lower than the critical temperature, quarks and antiquarks will form hadronic bubbles inside the quark matter which will grow and eventually occupy all the available volume. During this phase conversion the fractional volume $\alpha=V_{had}/V$ occupied by the hadron phase will change from zero to one.

We assume that the microscopic processes leading to hadronization happen fast compared to the mixed phase lifetime.[10,23] Then the mixed phase can be reasonably treated as a system in thermal and chemical equilibrium, and hadronization will occur along a constant entropy contour as described in the previous section, accompanied by reheating.

There are alternative pictures of the hadronization process that allow the system to go out of chemical equilibrium. In several earlier papers[8,18] hadronization was enforced at constant temperature, in which case entropy and baryon number conservation require the system to go out of equilibrium. The actual evolution of the physical system probably lies between these two extremes, but requires a full dynamical study that includes rate equations for all the chemical species.

To focus on the hadronization mechanism, we calculate for a particular hadron resonance the number of particles, $dN/dT$, hadronizing for each temperature interval $dT$. In Figs. 2 and 3. we plot the hadronization rate for several particle species as a function of temperature by following the constant entropy contours of $S/A=8.6$ and $S/A=4$, respectively.

From both figures we can immediately see the difference in the hadronization of $K^+$ meson from other hadrons. The $K^+$ meson hadronizes on average before the $K^-$ meson and other strange particles. This results in strangeness separation between the hadronic and the quark matter subvolumes[15,13,14] and reflects the relative abundance of constituents quarks in the lightest strange mesons $[K^+=(u\bar{s}),\ K^-=(\bar{u}s)]$: In a baryon-rich plasma light quarks are more abundant than antilight quarks, while there are equal numbers of strange and antistrange quarks. Correspondingly, $K^+$ mesons hadronize more easily than $K^-$ mesons. Lambda particles are also suppressed initially relative to the $K^+$ mesons by their large mass (1116 MeV) unless the phase transition happens at very large values of $\mu_q$ (Ref. 24) (in our case this would require a bag constant of order 1000 MeV/fm$^3$). Therefore, initially more $\bar{s}$ quarks hadronize in the form of $K^+$ mesons than $s$ quarks in the form of either $K^-$ mesons or hyperons. Toward the end of the hadronization process the established strangeness imbalance forces an increased hadronization of hyperons and $K^-$ mesons as is seen by the strong peak in their hadronization rates in Figs. 2 and 3. Hyperons do not play a crucial role in this balance because they are suppressed by almost 4 orders of magnitudes, due to both their large mass and the lack of light antiquarks. They also hadronize predominantly at the end of the mixed phase, the higher temperature there reducing the mass suppression.

Due to the large abundance of light quarks, the nonstrange baryons, i.e., nucleons and deltas, hadronize abundantly at the early stage of the hadronization. Up to mass suppression effects, the hadronization rates of $N$ and $\Delta$ (or $\pi$ and $\rho$) behave similarly as the mixed phase is crossed. Comparing the hadronization curves of $N$, $\Delta$, and even $K^+$ with those of the other hadrons, it is seen that the dominant driving force for hadronization is the large baryon density in the plasma which the system strives to reduce. In some cases the baryon number hadronizes so fast that the light quark excess is exhausted towards the end of hadronization, as shown by the decreasing rates in Figs. 3(a), (d), and (e).

The strangeness separation originating in the on average earlier hadronization of the $K^+$ mesons before the $K^-$ mesons and other strange hadrons can become quite appreciable. For the expansion along the $S/A=8.6$ curve, antistrangeness is enriched inside the hadronic subvolume by a factor $\bar{s}/s\sim 6$ near the onset of the hadronization. As $\alpha\rightarrow 1$, this ratio decreases, but then strangeness is increasingly enriched in the plasma subvolume, and at the end of hadronization the $s/\bar{s}$ ratio in the remaining small volume of plasma again reaches $\sim 6$.

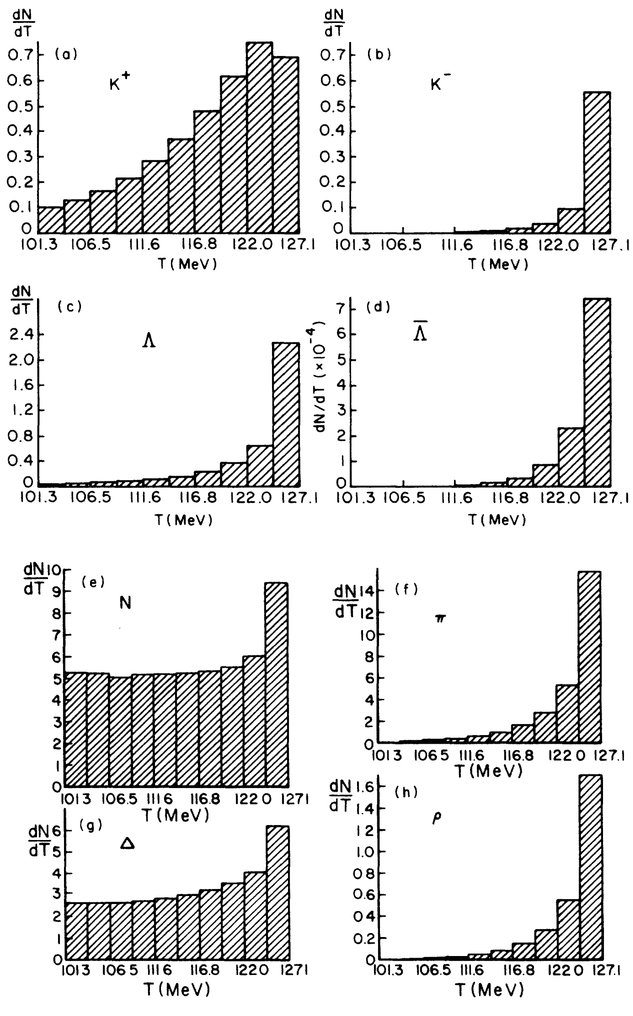


FIG. 2. Hadronization rates along the constant entropy contour with $S/A=8.6$ in Fig. 1, for a system with $A=250$. (In Ref. 12 we mistakenly quoted a baryon number $A=100$ for this figure.) $dN/dT$ is the number of particles hadronized while the system temperature has changed by $dT$ in the mixed phase. Most of the matter hadronizes at the high temperature end of the mixed phase, with the exception of $K^+$ mesons (leading to strangeness separation) and nucleons and $\Delta$'s (whose early hadronization is driven by the large baryon number density).

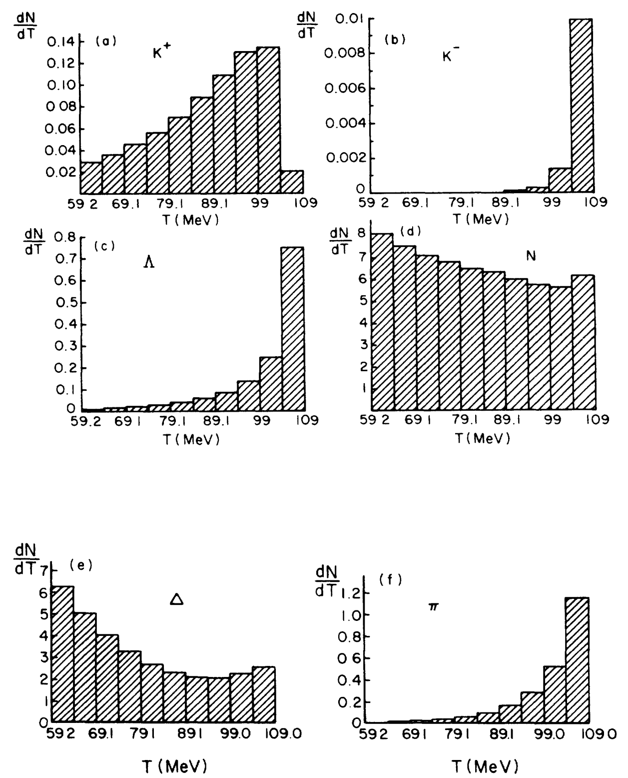


FIG. 3. Hadronization rates along the constant entropy contour with $S/A=4$ in Fig. 1, for a system with $A=500$. In this case nonstrange baryons hadronize initially so fast that towards the end of the mixed phase the light quark excess is essentially exhausted, as shown by the decreasing hadronization rates for $K^+$, N, and $\Delta$.

## III. FREEZE-OUT CONDITION FOR HADRONIC MATTER

After hadronization is complete, the hot and dense hadron gas expands and cools until freeze out. We assume that the system is thermodynamically homogeneous and that the expansion is spherical. Inside the expanding fireball, hadrons move with random thermal motion, collide with each other, and recede from each other by collective radial expansion. The radial expansion will be faster as more thermal energy is converted into collective expansion energy.

Freeze out is defined by us as the condition where the scattering time scale for individual hadron species becomes larger than the rarefaction time constant of the collective or hydrodynamic expansion. (For a review of other variants of the freeze-out condition, please refer to Ref. 25.) After freeze out, collisions between particles are rare and cannot change the thermal distribution of the particles any more. This leads to a breakdown of the hydrodynamic concept and a decoupling of the hadron species from the collective hydrodynamic flow. Thus particles detected by experiment will have information about the thermal distribution at freeze out.

The scattering time for a given hadron species is estimated by $\tau_{\rm scatt}=\lambda/v_{\rm th}=1/\rho_{\rm tot}\sigma_{\rm tot}v_{\rm th}$, where $\rho_{\rm tot}$ is the total density of all particles interacting with the one under consideration, $\sigma_{\rm tot}$ is its appropriately averaged cross section with these particles, and $v_{\rm th}$ is its thermal velocity. (More properly, the mean free path of species $i$ should be written as $1/\lambda_i=\sum_j \rho_j\sigma_{ij}$.) Since we are in a baryon rich environment, we take the mean free paths to be dominated by the interaction cross section with nucleons. For this quantity we adopt the following average values, $\sigma_{\rm tot}(\pi)=\sigma_{\pi\rm N}\simeq 100$ mb, $\sigma_{\rm tot}({\rm p})=\sigma_{\rm NN}\simeq 40$ mb, $\sigma_{\rm tot}({\rm K}^-)=\sigma_{{\rm K}-{\rm N}}\simeq 50$ mb, and $\sigma_{\rm tot}({\rm K}^+)=\sigma_{{\rm K}+{\rm N}}\simeq 10$ mb. This is a rather rough approximation, in particular at higher collision energies when pions start to become an appreciable component and the cross sections with pions should be more properly taken into account.

Without our radial expansion model, the rarefaction time scale can be estimated from the expansion velocity of the surface $\beta_s$, and the velocity profile inside the fireball, $\beta(r)$. For the velocity profile of the expanding fireball, we choose a scaling ansatz of the form $\beta(r)=(r/R)^n\beta_s$, where $R$ is the radius of the expanding fireball. The linear scaling ansatz ($n=1$) arises from the study of so-called self-similar motion in nonrelativistic hydrodynamics.[16] To compare the sensitivity of our freeze-out points and energy spectra to different scaling solutions, $n$ was varied in our calculations between 1 and 2. For our velocity profiles, $R$ is determined from the baryon number $A$, and the baryon density $\rho_b$ at a given point of the expansion along the constant entropy trajectory in Fig. 1, i.e., $R=(3A/4\pi\rho_b)^{1/3}$. The expansion velocity $\beta_s$ of the surface is obtained from energy conservation,

$$E_{\rm c.m.}^{\rm tot}=\int d^3r\,T_{00}(r)=\int_0^R d^3r\left[\frac{\varepsilon+P}{1-\beta^2(r)}-P\right]. \qquad (2)$$

The rarefaction time scale is given by $\tau_{\rm exp}=R/(n+2)\beta_s$. This expression is readily derived by calculating the change in the number density across an expanding spherical surface during an infinitesimal time interval and comparing this result with the equation $d\rho/dt=-\rho/\tau_{\rm exp}$, which defines $\tau_{\rm exp}$. Thus our decoupling or freeze-out criterion is given by the inequality $\tau_{\rm exp}<\tau_{\rm scat}$ or

$$(3A/4\pi\rho_b)^{1/3}/(n+2)\beta_s<1/\rho_{\rm tot}\sigma_{\rm tot}v_{\rm th} \qquad (3)$$

if $\beta(r)=(r/R)^n\beta_s$.

In the top half of Tables I and II, the freeze-out tem-

TABLE I. The freeze-out temperatures $T$ (in MeV), surface expansion velocity $\beta_s$ (in units of $c$), and fireball radius $R_s$ (in fm) at freeze out, and the apparent temperatures $T^{\rm app}$ (in MeV) for kaons, pions, and protons at different beam energies. The values are calculated using a linear velocity profile, $\beta(r)=(r/R)\beta_s$, for a fireball with baryon number $A=100$. Results both without and with a quark-gluon plasma phase transition (PT) are shown. The two values for $\beta_s$ and $R_s$ correspond to the freeze-out points for $\rm K^+$ mesons and the other particles, respectively. Due to the collective expansion flow the energy spectra are nonexponential; we quote two typical values for the apparent temperature, calculated as the local slope parameter at 200 and 600 MeV kinetic energy, respectively, for each particle species.

| | $\beta(r)=(r/R)\beta_s$ | | | | | |
|---|---|---|---|---|---|---|
| | | Without PT | | | with PT | |
| $E_{\rm lab}/A$ (GeV) | 4 | 7 | 15 | 4 | 7 | 15 |
| $T_{\rm K^+}$ | 126.0 | 145.0 | 162.0 | 104.0 | 118.0 | 127.0 |
| $T_{\rm p,K^-,\pi}$ | 93.0 | 106.0 | 118.0 | 88.0 | 110.0 | 121.0 |
| $\beta_s$ | 0.48/0.61 | 0.56/0.68 | 0.66/0.77 | 0.39/0.64 | 0.47/0.66 | 0.60/0.72 |
| $R_s$ | 3.9/5.7 | 4.0/5.9 | 4.2/6.5 | 4.5/5.3 | 5.7/6.2 | 6.8/7.3 |
| $T^{\rm app}_{\rm K^+}$ | 142/158 | 164/187 | 187/220 | 116/126 | 135/150 | 154/161 |
| $T^{\rm app}_{\rm p}$ | 173/171 | 210/211 | 256/266 | 183/177 | 204/208 | 233/242 |
| $T^{\rm app}_{\rm K^-}$ | 129/145 | 149/173 | 173/207 | 130/146 | 149/172 | 165/195 |
| $T^{\rm app}_{\pi}$ | 101/126 | 113/147 | 123/170 | 98/125 | 115/148 | 124/164 |

TABLE II. Same as in Table I, but for the quadratic velocity profile, $\beta(r)=(r/R)^2\beta_s$. The line for $T(K^+)$ with PT is the same as in Table I because it is calculated from the hadronization rates (the freeze-out condition is already satisfied at hadronization), thus, the velocity profile does not enter. The more rapid expansion near the surface of the fireball gives rise to earlier freeze out than with the linear velocity profile, but the systematic features of the freeze-out temperatures and slope parameters for the different particles with and without the phase transition remain unchanged.

| | $\beta(r)=(r/R)^2\beta_s$ | | | | | |
|---|---|---|---|---|---|---|
| | Without PT | | | With PT | | |
| $E_{lab}/A$ (GeV) | 4 | 7 | 15 | 4 | 7 | 15 |
| $T_{K^+}$ | 139.0 | 161.0 | 180.0 | 104.0 | 118.0 | 127.0 |
| $T_{p,K^-,\pi}$ | 104.0 | 119.0 | 131.0 | 102.0 | 125.0 | 131.0 |
| $\beta_s$ | 0.48/0.65 | 0.57/0.73 | 0.69/0.82 | 0.45/0.68 | 0.55/0.70 | 0.68/0.77 |
| $R_s$ | 3.4/4.9 | 3.5/5.1 | 3.6/5.6 | 3.4/4.6 | 3.7/5.4 | 4.3/6.6 |
| $T^{app}_{K^+}$ | 149/162 | 172/192 | 194/222 | 116/126 | 134/150 | 150/176 |
| $T^{app}_{p}$ | 156/178 | 180/214 | 202/252 | 159/185 | 181/210 | 196/235 |
| $T^{app}_{K^-}$ | 129/151 | 148/178 | 165/204 | 130/154 | 150/178 | 160/194 |
| $T^{app}_{\pi}$ | 108/132 | 122/152 | 132/171 | 107/126 | 126/154 | 132/166 |

peratures for different particle species are shown for both the linear and quadratic velocity profiles, for three values of the initial beam energy. The freeze-out temperatures for the linear velocity profile ($n=1$) were reported in Ref. 12. In addition to the freeze-out temperatures, Tables I and II also show the surface expansion velocities $\beta_s$ and the radius of the fireball in the fireball frame at the freeze-out point. These numbers will be used in the next section when calculating the energy spectra. Two values for $\beta_s$ and $R_s$ are given. The first number corresponds to the $K^+$ mesons which freeze out during the mixed phase; the second corresponds to freeze out of p, $K^-$, $\pi$. Results with and without the phase transition are presented next to each other.

Note that the freeze-out temperatures for the p, $K^-$, $\pi$ particles are the same. Nucleons serve as a heat bath for the other particle species since their cross sections with the nucleons are larger than the N-N cross section itself and their density is small compared to the nuclear density. Once the nucleons have decoupled, their temperature stays constant; any further interaction of the pions and $K^-$ with the decoupled nucleons cannot lead to further cooling of these particles either.

As shown in Tables I and II, both without and with the phase transition, the freeze-out temperatures for the $K^+$ mesons are larger than those for the p, $K^-$, $\pi$ particles. In the absence of a phase transition to the quark-gluon plasma, this freeze-out temperature difference can be as large as 50%, as seen for $E_{lab}/A=15$ GeV. Note however, that when a phase transition has occurred, this temperature difference is considerably reduced.

In the absence of a phase transition, the large difference between the $K^+$ meson and the p, $K^-$, $\pi$ freeze-out temperatures is a consequence of both the large $K^+$ meson mean free path, and the initial high temperature reached in the hadron matter via the shock compression. When a phase transition occurs the initially lower temperature of the plasma and the emission from the mixed phase reduces this difference. In addition, strangeness separation further reduces this difference because the $K^+$ mesons are hadronized during the *early* (i.e., colder) stages of the mixed phase. In Fig. 1 we show the freeze-out points along isentropic trajectory for $S/A=12.6$. These correspond to initial laboratory energies of 28.9 GeV/nucleon for a hadron gas and 15 GeV/nucleon when a phase transition occurs.

Comparing numbers in the top half of Tables I and II indicates that the absolute freeze-out temperatures are sensitive to the form of our scaling ansatz. The quadratic scaling ansatz results in a larger expansion velocity, hence, all particles freeze out earlier at higher temperatures. However the qualitative features discussed above remain the same.

In the next section we want to connect the freeze-out temperature of the various particle species to their kinetic energy spectrum in the nuclear center of mass frame.

## IV. TRANSFORMATION FROM FREEZE-OUT TEMPERATURES TO ENERGY SPECTRA

The emission or freeze-out temperatures should be related to the slope parameter of the measured energy spectra ("apparent temperature") in order to compare with experiments.

In this section, we use the collective flow argument introduced by Siemens and Rasmussen[17] to calculate the energy spectra from the freeze-out temperatures obtained in the previous section. They derived an equation for the intrinsic energy spectra by generalizing the nonrelativistic energy spectra by Bondorf, Garpman, and Zimanyi.[16] One finds

$$\sigma_i = \frac{d^3 n_i}{dp^3} = N_i \int_0^R d^3 r \exp\left[-\frac{\gamma E}{T}\right] \times \left[\left[\gamma + \frac{T}{E}\right]\frac{\sinh\alpha}{\alpha} - \frac{T}{E}\cosh\alpha\right] \times [Z(T)]^{-1} , \quad (4)$$

where $\gamma(r) = [1-\beta(r)^2]^{-1/2}$, $\alpha(r) = \gamma(r)\beta(r)p/T$, and $Z(T)$ is the normalization of a relativistic Boltzmann distribution, calculated at temperature $T$. $N_i$ is a normalization constant for each particle species, here adjusted for optical reasons to make particle cross sections coincide at $E_{\rm kin}^{\rm c.m.} = 10$ MeV. In the above equation we average the cross section over the radius $r$ to take into account the fact that the expansion velocity depends on the radial distance. This averaging, which is absent in Ref. 17, prevents the occurrence of a maximum in the energy spectra at finite values for the particle kinetic energy. (Such a maximum occurs if, as done in Ref. 17, only an expanding shell with constant radial velocity is considered.) From the intrinsic cross section $\sigma_i$, the "apparent temperatures" or slopes of the energy spectra are defined by the relation $T_i^{\rm app} = (-d \ln \sigma_i / dE)^{-1}$.

In Figs. 4–7 we show the results of our cross sections for the p, $\pi$, $K^+$, and $K^-$ particles as a function of their kinetic energy in the nuclear center of mass, both with and without the phase transition. Figures 4–6 are for the linear scaling ansatz at $E_{\rm lab}/A = 4$, 7, and 15 GeV, respectively, and Fig. 7 is for the quadratic scaling form at $E_{\rm lab}/A = 15$ GeV.

For all Figs. 4–7 we see that the slopes for particle species p, $K^-$, $\pi$ are different, despite identical freeze-out points. This reflects the different collective flow energies arising from the different particle masses. Observation of this behavior ($T_\pi^{\rm app} < T_{K^-}^{\rm app} < T_{\rm p}^{\rm app}$) would be a necessary check on the existence of collective flow and on our model for freeze out.

Once this is established, comparison of energy spectra of particles with identical mass, but different cross sections with nuclear matter (e.g., $K^+$ and $K^-$) can provide information on the dynamical history of the system.

From the figures we see that without the phase transition, the apparent temperature for $K^+$ mesons is higher than that of the $K^-$ mesons due to the much higher freeze-out temperature; on the other hand, in the case where a phase transition to quark matter has occurred, $K^+$ mesons have a lower apparent temperature than $K^-$ in spite of their slightly higher freeze-out temperature. The reason is that at the point of $K^+$ freeze out the collective flow has not yet completely developed, while later, when the $K^-$ freeze out, the spherical boost has become much stronger.

Such a behavior would immediately tell us that the $K^+$ meson has undergone an unusual evolution as a consequence of the quark-gluon phase transition. If the quark-gluon plasma is formed, early emission of $K^+$ mesons from the comparatively cool mixed phase result in this characteristic interchange of the slope parameters of energy spectra of $K^+$ and $K^-$ spectra that we propose as a signal for a phase transition to quark-gluon matter.

Comparing Fig. 7 with Fig. 6, we see that the linear or quadratic ansatz for the velocity profile of the expanding hadron gas results in quantitative differences for the observed slope parameters (compare Tables I and II), but that the qualitative features mentioned above remain unchanged. This gives rise to the hope that our qualitative conclusions are rather insensitive to the detailed form of the hydrodynamic flow profile.

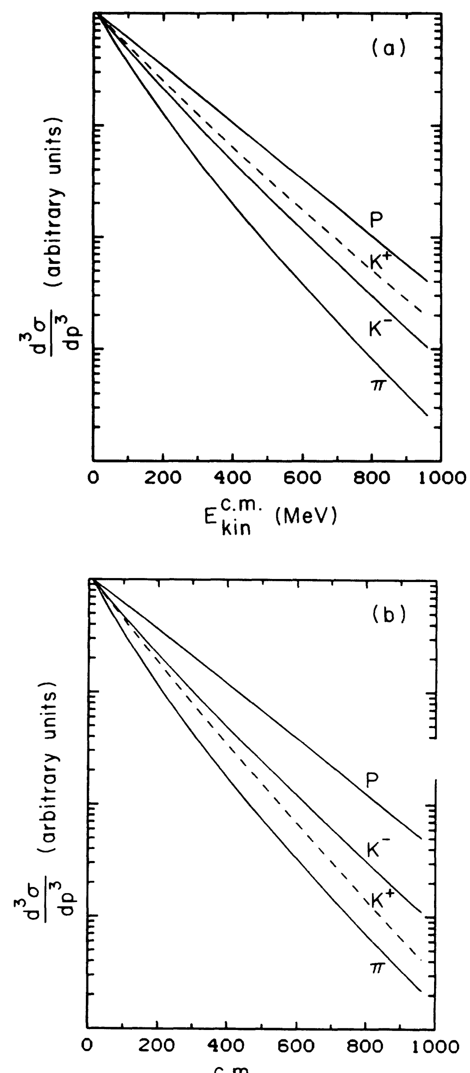


FIG. 4. Energy spectra for several hadrons as a function of their kinetic energy in the nuclear center of mass. Assumed was a spherically expanding fireball with $A = 100$, a linear velocity profile $\beta(r) = (r/R)\beta_s$, and initial conditions corresponding to a collision energy $E_{\rm lab}/A = 4$ GeV. In (a) the fireball was initially taken to be in the hadronic phase, in (b) it started out in the plasma phase. To facilitate comparison of slopes, the spectra were normalized to agree at $E_{\rm kin} = 10$ MeV. Note the relative change in the slopes of $K^+$ and $K^-$ mesons for the cases with and without plasma formation.

In the lower half of Tables I and II we show the "apparent temperatures" for each particle species deduced from the energy spectra. Because of the nonexponential nature of the energy spectra, in Tables I and II we quote values for slope parameters at two typical particle kinetic energies, namely 200 and 600 MeV.

## V. DISCUSSION OF OUR RESULTS

In this paper we have studied, in a schematic model, the hydrodynamic evolution of the baryon-rich plasma as it expands, hadronizes, and freezes out. We assume that the dense matter, after being formed in the initial stage of the collision, is in thermal and chemical equilibrium and evolves adiabatically without entropy production. Depending on the collision energy, the dense matter can be initially in one of three phases: pure hadronic matter, the mixed phase, or the pure plasma.

Instead of solving the hydrodynamic equations, we have used a scaling ansatz for the hydrodynamic flow and the conservation laws to simulate the dynamics. Conservation of baryon number, strangeness, energy, and entropy in the expanding matter were implemented. In assuming strangeness conservation we have ignored the possible loss of strangeness by preferred surface emission of strange particles with a given sign of the strangeness quantum number, a possibility discussed in Ref. 21. For the initial conditions of the hydrodynamic evolution we used the results of one-dimensional shock calculation which assumes full stopping.

We calculated the hadronization rate along isentropic expansion contours, and the freeze-out temperatures by comparing the collision time scale (determined by the particle densities and cross sections) and the expansion

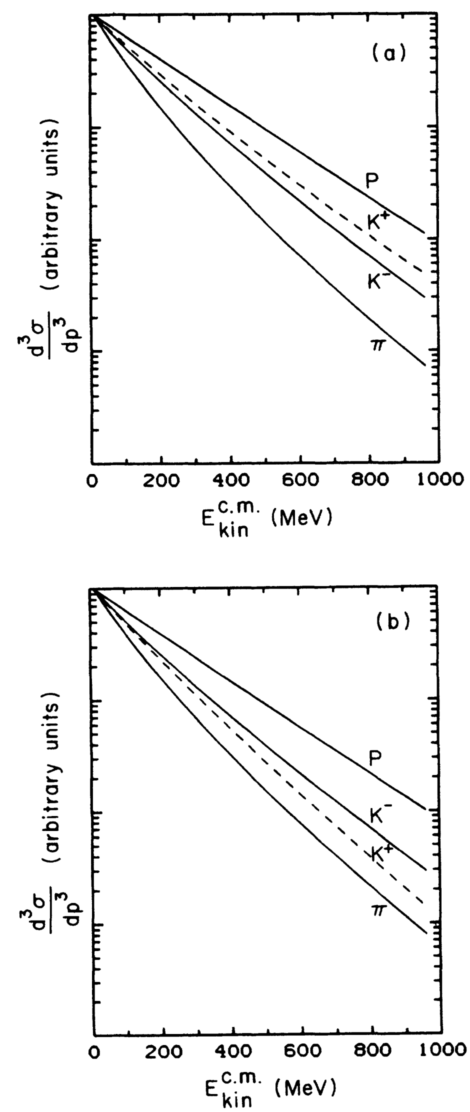


FIG. 5. Same as Fig. 4, but for a different collision energy, $E_{\rm lab}/A = 7$ GeV.

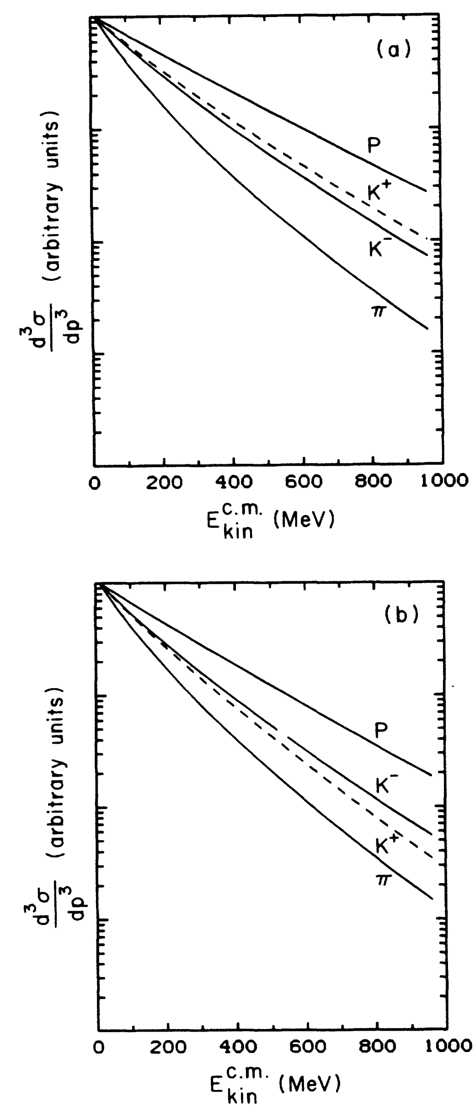


FIG. 6. Same as Fig. 4, but for a collision energy $E_{\rm lab}/A = 15$ GeV. As the collision energy increases, the difference in the kaon slopes decreases somewhat

time scale (determined from energy conservation using a scaling ansatz for the flow profile). By boosting the thermal distribution at the freeze-out point with the assumed flow profile we calculated the final energy spectra for pions, kaons, and protons.

Our calculated hadronization rates showed that, compared to all other mesons, $K^+$ mesons hadronize at a rather early stage of the mixed phase; the same is true for nucleons and $\Delta$ resonances which demonstrates a strong tendency of the plasma to get rid of its large light quark excess by hadronization. Since in the $K^+$ mesons the light quarks come with a strange antiquark, this leads to strangeness separation between the hadron and plasma subvolumes during the mixed phase.

Due to the larger entropy per baryon in the plasma phase, the mixed phase is initially much cooler than the hadronic matter after completion of hadronization. The $K^+$ mesons carry this information in the form of their energy spectra, and as a consequence of their small interaction cross section they are able to preserve this information until they are detected. Most of other hadrons, like $K^-$, pions, and protons, interact more strongly, and through their energy spectra, remember a much later phase of the expansion. This leads into the suggestion that, while their *abundance*, i.e., the $K^+/\pi^+$ ratio, is not an unambiguous signal for plasma formation,[10,11] their energy spectra should contain rather unique information about the evolution of the expanding fireball and the possible existence of a first order phase transition corresponding to quark deconfinement.

The particular signal we suggest is that without a transition to quark matter the slope parameter for $K^+$ mesons should always exceed that for $K^-$ mesons, reflecting the hotter temperature at earlier stages; if a phase transition has occurred, the $K^+$ should appear with a smaller slope parameter than the $K^-$ mesons, due to the smaller temperature in the mixed phase and the less strongly exhibited collective flow.

We would like to close with a few remarks about possible future improvements of our analysis. In our calculation we have assumed both thermal and chemical equilibrium throughout the collision. There are several layers of sophistication at which this approximation can be relaxed. Simply relaxing the chemical equilibrium condition in the hadron gas would require coupling chemical rate equations for each hadron species to source terms in the hydrodynamic equations. This has not been done to date for the hadronization from a baryon rich plasma with the strangeness degree of freedom, and would seem to be a worthwhile task for the future. In such a treatment also, losses of entropy and strangeness by surface emission of pions and kaons could be included, possibly leading to accumulation of net strangeness in the fireball and corresponding modification of the expansion trajectory through the phase diagram. We expect the increase of entropy by nonequilibrium processes during hadronization to be partly off set by these surfaces losses, thereby supporting our simple assumption of adiabatic expansion in this paper.

Relativistic kinetic theories, that would allow complete relaxation of thermal and chemical equilibrium in the quark-gluon gas, have been developed in the mean field limit,[26] but we are still a long way from utilizing these formalisms for the hadronization process.

For the initial conditions of the plasma we assume the validity of one-dimensional shock and, hence, full stopping of the nuclear matter. If stopping is less immediate (due to the finite nuclear mean free path, which increases beam energy) and also transverse degrees of freedom are available, the temperature in the plasma will be lower. In this sense, the shock adiabats indicated in Fig. 1 reflect too high a temperature and bias the formation of the plasma at too low a bombarding energy. More realistic initial conditions would imply mostly scaling up the beam energy needed to reach a given value of $S/A$ or of the initial temperature; the gross features of the signal discussed in this manuscript would not qualitatively be

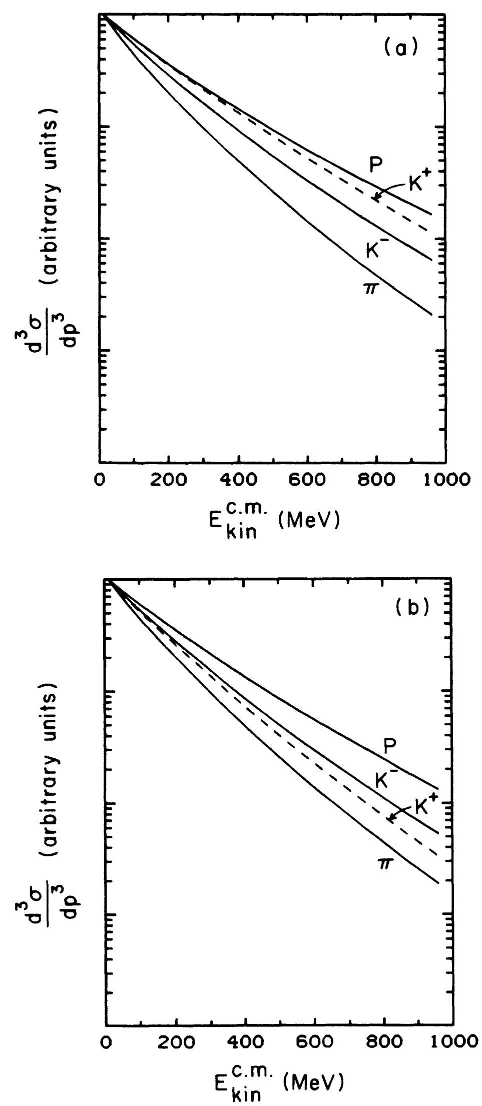


FIG. 7. Same as Fig. 6, but for a quadratic velocity profile, $\beta(r)=(r/R)^2\beta_s$. The slope parameters of all particles are not as clearly separated than for the linear velocity profile, but the qualitative features remain unchanged.

affected. Although the absolute values of the freeze-out temperatures and slope parameters would undoubtedly change, the relative slopes of the K mesons would still have to reflect the large entropy per baryon (and resulting low temperature) in the quark phase and the small $K^+$-baryon interaction cross section.

If, in addition the interaction cross section of the $K^+$ meson with the quark-gluon plasma in the mixed phase turns out to be large, the difference between the $K^+$ and $K^-$ meson energy spectra shown in this paper will be less distinct.

The energy spectra, calculated within our model, resulted from the Lorentz boost of the local thermal Boltzmann distribution of particles in the co-moving frame to the nuclear center of mass frame. As is summarized in Ref. 27, there remains a theoretical ambiguity in explaining the existing data on energy spectra from the heavy-ion collisions at BEVALAC energy. The low energy part of the pion spectra is subject to modifications by $\Delta$-resonance decay, leading also to a nonexponential shape of the spectrum which has nothing to do with collective flow. The spectra for kaons will similarly be affected by decay of excited hyperon resonances. Therefore it will be necessary to observe energy spectra for as many different particle species as possible to resolve ambiguities of this kind, to establish the collective flow picture and draw conclusion about possible plasma formation.

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[1] *Quark Matter '83,* Proceedings of the Third International Conference on Ultra-relativistic Nucleus-Nucleus Collisions, edited by T. W. Ludlam and H. E. Wegner [Nucl. Phys. **A418** (1984)]; *Quark Matter '84,* Proceedings of the Fourth International Conference on Ultra-relativistic Nucleus-Nucleus Collisions, edited by K. Kajantie (Springer, Berlin, 1985); *Quark Matter '86,* Proceedings of the Fifth International Conference on Ultra-relativistic Nucleus-Nucleus Collisions, edited by L. S. Schroeder and M. Gyulassy [Nucl. Phys. **A461** (1987)].

[2] L. McLerran and T. Toimela, Phys. Rev. D **31**, 545 (1985).

[3] K. Kajantie, J. Kapusta, L. McLerran, and A. Mekjian, Phys. Rev. D **34**, 2746 (1986).

[4] J. D. Bjorken and L. McLerran, Phys. Rev. D **30**, 63 (1984).

[5] T. Matsui and H. Satz, Phys. Lett. B **178**, 416 (1986).

[6] F. Karsch, M. T. Mehr, and H. Satz, Brookhaven National Laboratory Report BNL-40122, 1987.

[7] T. H. Hansson, Su H. Lee, and I. Zahed, State University of New York, Stony Brook, Report, 1987.

[8] P. Koch, B. Müller, and J. Rafelski, Phys. Rep. **142**, 167 (1986).

[9] K. S. Lee, M. J. Rhoades-Brown, and U. Heinz, Phys. Rev. C **37**, 1452 (1988).

[10] J. Kapusta and A. Mekjian, Phys. Rev. D **33**, 1304 (1986).

[11] T. Matsui, B. Svetitsky, and L. D. McLerran, Phys. Rev. D **34**, 2047 (1986).

[12] U. Heinz, K. S. Lee, and M. J. Rhoades-Brown, Phys. Rev. Lett. **58**, 2292 (1987).

[13] U. Heinz, K. S. Lee, and M. J. Rhoades-Brown, Mod. Phys. Lett. **A2**, 153 (1987).

[14] C. Greiner, P. Koch, and H. Stöcker, Phys. Rev. Lett. **58**, 1825 (1987).

[15] B. Lukács, J. Zimányi, and N. L. Balazs, Phys. Lett. B **183**, 27 (1987).

[16] J. P. Bondorf, S. I. A. Garpman, and J. Zimányi, Nucl. Phys. **A296**, 320 (1978).

[17] P. J. Siemens and J. O. Rasmussen, Phys. Rev. Lett. **42**, 880 (1979).

[18] U. Heinz, P. R. Subramanian, H. Stöcker, and W. Greiner, J. Phys. G **12**, 1237 (1986); U. Heinz, *Physics of Strong Fields* (Plenum, New York) (in press).

[19] Y. B. Zel'dovich and Y. P. Raizer, *Physics of Shock Waves and High-Temperature Hydrodynamic Phenomena,* (Academic, New York, 1967), Vols. I and II.

[20] L. D. Landau and E. M. Lifshitz, *Fluid Mechanics* (Pergamon, Oxford, 1959).

[21] H. Liu and G. L. Shaw, Phys. Rev. D **30**, 1137 (1984).

[22] P. R. Subramanian, H. Stöcker, and W. Greiner, Phys. Lett. B **173**, 468 (1986).

[23] M. Kataja, P. V. Ruuskanen, L. McLerran, and H. von Gersdorff, Phys. Rev. D **34**, 2755 (1986).

[24] J. Rafelski, Pys. Lett. B **190**, 167 (1987).

[25] R. B. Clare and D. Strottman, Phys. Rep. **141**, 177 (1986).

[26] U. Heinz, Phys. Rev. Lett. **51**, 351 (1983); H. Th. Elze, M. Gyulassy, and D. Vasak, Phys. Lett. B **177**, 402 (1986); Nucl. Phys. **B276**, 706 (1986); M. Carrington, M. J. Rhoades-Brown, and M. Ploszajczak, Phys. Rev. D **35**, 3981 (1987).

[27] R. Stock, Phys. Rep. **135**, 259 (1986).