Quark-gluon plasma versus hadron gas. What one can learn from hadron abundances

Kang S. Lee^{*} and M. J. Rhoades-Brown[†]

Physics Department, State University of New York, Stony Brook, New York 11794

Ulrich Heinz[‡]

Physics Department, Brookhaven National Laboratory, Upton, New York 11973

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We use a phenomenological equation of state to describe the phase transition between a hot and dense hadron resonance gas and a quark-gluon plasma. Our analysis covers the entire temperaturebaryon density plane. The consequences for the phase diagram of strangeness conservation during nuclear collisions are analyzed. The flavor composition of the quark plasma and an equilibrated hadron resonance gas is studied and compared along the phase transition surface. We emphasize the need to compare systems with equal total baryon number and entropy contents in order to be consistent with the dynamics of the hadronization process and to obtain results relevant to nuclear collisions. From our results we conclude that the flavor composition of the hadronization debris from a quark-gluon plasma formed in a nuclear collision is probably hard to distinguish from that of a chemically equilibrated hadron gas, although in both cases the production level of strange and nonstrange antibaryons will be much higher than observed in proton-proton collisions.

I. INTRODUCTION

The possibility of quark-gluon formation in relativistic nuclear collisions (for recent reviews see Ref. 1) is becoming increasingly real as the first experiments with oxygen and silicon projectiles at beam energies of 15 GeV/nucleon and 60 and 200 GeV/nucleon, respectively, are being performed at the Brookhaven Alternating Gradient Synchrotron² and the CERN Super Proton Synchrotron.^{3,4} These exciting developments require reliable theoretical models to aid in the analysis of data. Unfortunately, most of the proposed suggestions to identify quark-gluon plasma formation¹ are based on very simplified assumptions about the collision dynamics, partly reflecting the lack of experimental information even for proton-nucleus collisions. The need for simplification introduces uncertainties in the present theoretical analyses which, in the case of several "plasma signatures", can mean the difference between being useful or obsolete.

It is an unfortunate fact that the majority of particles formed in heavy ion collisions (with or without quark matter formation) are stronger interacting hadrons which are extremely sensitive to the details of the dynamical evolution of the system. In particular in the case of plasma formation, the final hadronization process, by which the plasma decays into the experimentally measured hadronic particles, is very complicated in its theoretical description, but will crucially influence the isotopic composition as well as the hadronic energy spectra recorded in the experiment. At first sight it thus appears that as far as the hadrons are concerned, information on the initial quark-gluon plasma state is well hidden.

A much discussed feature of the quark-gluon plasma is its unique chemical composition:⁵⁻⁸ the plasma contains many gluons which contribute a major fraction of the entropy of the system and are also responsible for a fast rate for flavor equilibration.⁶ In particular, the density of strange quarks and antiquarks is expected to nearly saturate at its thermal equilibrium value during the lifetime of the plasma if its temperature is larger than about 150 MeV,⁶ whereas, in the hadron gas equilibration, times for strangeness production are much longer, due to the higher thresholds.^{8,9} Since it is known from protonproton collisions that the amount of strangeness production by initial hard-scattering processes is small, one expects to see a considerably larger fraction of strange particles emerging from a nuclear collision in which a transient quark-gluon plasma was formed than in a collision which is dominated by the usual hadronic processes also present in proton-proton collisions.⁵

Presented in this way, the argument completely relies on the different strangeness equilibration time scales in a hadron gas and in quark matter, given the initial condition of very little strangeness production by the first hard nucleon-nucleon collisions. A possible danger to this argument could arise from unusual phenomena in high energy nuclear collisions during the initial stopping and particle formation stage through which, even without plasma formation, strangeness would be produced at a higher level than in p-p collisions, thereby facilitating strangeness equilibration even in the hadronic phase. For example, one might think of string formation by multiple collision effects with resulting unusually high color electric fields¹⁰ which would have a larger relative probability to hadronize by $s\overline{s}$ pair formation than the usual strings formed in p-p collisions. In such a scenario, the ensuing hadron gas would be much closer to chemical equilibrium, and a large amount of strange particles would appear from such a collision without intermediate plasma formation.11

In such a case it would be interesting to know whether

there is an additional difference in the production level of strange (and nonstrange) hadrons between a chemically equilibrated hadron gas and a hadronizing quark-gluon plasma which also was initially in equilibrium; this might allow differentation between the two phases even in the limit where the equilibrium time scale in both phases is short compared to the collision time. This was repeated-ly argued to indeed be the case,^{5,7,8,12,13} particularly so for both strange and nonstrange antibaryons. For that purpose one needs a model for the hadronization of light and strange quarks from the plasma into the different types of hadrons and compare the outcome to the abundance levels for those hadrons in an equilibrium hadron gas. Since these comparisons and the conclusions based thereon have been challenged recently,^{14,15} we have decided to reinvestigate this question and try to settle this issue.

To approach the problem systematically, we began by constructing a phenomenological equation of state for hot hadronic matter^{13,16} which reproduces qualitatively the properties of the deconfining phase transition seen in lattice quantum chromodynamics (QCD) calculations at zero baryon density,¹⁷ and extends it in a reasonable way (see Ref. 18) to the baryon rich regime. Thereby we achieve a unified, though phenomenological, description of both the central and the fragmentation regions in nuclear collisions. This is discussed in Sec. II. It was noted that the presence of strangeness as an additional conserved quantum number (on the time scales available in nuclear collisions) has unique and interesting effects on the phase diagram and on the properties of the phase coexistence region.^{16,19-21} The quark-gluon plasma and the hadron gas are separated by a mixed phase region in which, for systems with nonvanishing baryon number, strange and antistrange quarks get, to a considerable degree, separated between the plasma and the hadron gas subvolumes¹⁹⁻²² (see Sec. III). Possible measurable consequences of this were pointed out in Refs. 2 and 23 and will be discussed in the following paper.²⁴

After having identified the phase transition surface in the (T, μ_a, μ_s) parameter space, we proceed to compare the chemical composition of the two phases in order to establish whether anything exceptional is happening in the plasma phase. The crucial point of this paper is that such a comparison has to be made under conditions which are compatible with the expansion dynamics in heavy ion collisions. It is important to realize that the thermodynamics of the hadronization is dominated by the conservation laws for baryon number, net strangeness, and (approximately) for entropy.^{13,25} For a given point (T, μ_q, μ_s) in the phase diagram the entropy per baryon S/A of the plasma phase is considerably higher than in the hadronic phase due to quark deconfinement and the liberation of many gluons. Since during the collision the entropy can never decrease, it is obvious that a plasma state of a given temperature and chemical potential can never dynamically evolve into a hadron gas of the same temperature and chemical potential.

For this reason, a comparison of the two phases at fixed temperature and chemical potential is misleading.¹⁴ If done anyway,^{5,7,8,13} one typically finds considerable

differences (sometimes several orders of magnitude) in abundances of specific particle species obtained from a hadronizing quark-gluon plasma, compared to their hadronic equilibrium level. We will show that, on the other hand, a consistent comparison of systems containing the same amount of entropy and baryon number leads to quite different conclusions: measuring strangeness in units of entropy or baryon number, we will find it to be roughly equally abundant in both the plasma and the hadronic phase if equilibrium systems with equal entropy per baryon S/A close to the phase transition curve are selected. For small values of S/A there is actually more strangeness in an equilibrium hadron gas than in the quark-gluon plasma. Similar conclusions were drawn before in Ref. 36 for the case of a baryon-free system and are here generalized for system of arbitrary baryon number. An analogous comparison of the light antiquark contents shows that they are considerably less abundant in the plasma than in the hadronic phase, which is in contrast to earlier reports.^{12,13}

Our analysis indicates that of all hadronic abundances or ratios only the ratio of strange to light antiquarks (reflected, e.g., in the Λ/\overline{p} ratio) is sufficiently different in the two phases that it may have a chance to survive the hadronization process. This large difference is due to a strange \bar{q} suppression in the quark-gluon plasma (in particular at large baryon density), reflected in a small \bar{q} /entropy ratio. However, we will describe a short back-of-the-envelope calculation showing that gluon fragmentation into quark-antiquark pairs, which is necessary to conserve entropy during hadronization,⁸ tends to largely destroy even this apparently very big gap between the $\overline{s}/\overline{q}$ ratios. Therefore, we conclude that there is no natural large difference in flavor composition between the hadronization products of a chemically equilibrated quark-gluon plasma and an equilibrium hadron gas. Small quantitative differences (less than an order of magnitude) may survive, but will be exceedingly hard to predict reliably due to sensitive model dependence.

Of course, as mentioned above, one has to remember that there is good reason to expect that the hadron gas will not equilibrate fast enough. Therefore, while we claim that there is no large difference between a hadronizing plasma and an *equilibrium* hadron gas, there still will be a considerable difference compared to the nonequilibrium hadronic system which one expects to develop in a collision governed by only the usual hadronic processes. We will not discuss this here, but refer the reader to the extensive work done by Koch *et al.*^{9,8}

Let us now turn to the detailed presentation of our results. In Sec. II we review our model for the nuclear equation of state below and above the deconfinement transition. After constructing and discussing the nuclear phase diagram in Sec. III (with particular emphasis on the effect of strange particles and the conservation of net strangeness) we turn in Sec. IV to a detailed comparison of the equilibrium flavor comparison in both phases near the transition, under conditions which are compatible with the dynamics of the hadronization process in an expanding blob of quark matter. In our concluding Sec. V we interpret our results and comment on what one can expect to learn from experimental data on the flavor composition of the debris from high energy nuclear collisions about the collision process and possible quark-gluon plasma formation.

II. MODEL EQUATIONS OF STATE

As pointed out in the Introduction, we will use a phenomenological approach to the equation of state (EOS) by matching an EOS for a hot gas of hadrons below the deconfining phase transition to a quark-gluon gas with bag pressure above the transition.

A. Plasma phase

We consider the plasma as a free gas of light and strange quarks and gluons, subject to a negative vacuum pressure. Perturbative corrections of this "bag equation of state" due to gluon exchange²⁶ were studied in detail in Ref. 13; as far as the phase transition is concerned it was found that, due to their attractive nature, they can, to a large degree, be absorbed by an appropriate increase of the bag constant which also lowers the pressure in the plasma. Here we will keep the bag pressure *B* as an adjustable parameter to effectively include both effects.

All the thermodynamical quantities can be obtained from the grand canonical potential which in this approximation reads

$$-\Omega_{Q} = P_{Q} = -B + \frac{37}{90}\pi^{2}T^{4} + \mu_{q}^{2}T^{2} + \frac{1}{2\pi^{2}}\mu_{q}^{4} + \frac{1}{\pi^{2}}\int_{m_{s}}^{\infty} de \left[\frac{(e^{2} - m_{s}^{2})^{3/2}}{\exp[\beta(e + \mu_{q} + \tilde{\mu}_{s})] + 1} + \frac{(e^{2} - m_{s}^{2})^{3/2}}{\exp[\beta(e - \mu_{q} - \tilde{\mu}_{s})] + 1}\right],$$
(2.1)

$$q_{\rm res} = -\frac{1}{2} \frac{\partial \Omega_{\varrho}}{\partial r}$$
(2.2)

$$\rho_{s,Q} = -\frac{\partial \Omega_Q}{\partial \tilde{\mu}_s} , \qquad (2.3)$$

$$s_Q = -\frac{\partial \Omega_Q}{\partial T} , \qquad (2.4)$$

$$\varepsilon_Q = (3\mu_q \rho_b + T_s - P)_Q , \qquad (2.5)$$

where $\rho_{b,Q}$ and $\rho_{b,Q}$ are the baryon and strangeness density in the plasma phase, respectively. s_Q and ε_Q are entropy and energy densities, respectively, and $\beta = 1/T$.

For the strange quark mass we take $m_s = 150$ MeV, while the light quarks are assumed to be massless. μ_q is a Lagrange multiplier to control conservation of baryon number [the factor $\frac{1}{3}$ in (2.2) accounts for the fact that each quark carries baryon number $\frac{1}{3}$]; $\tilde{\mu}_s$ controls the strangeness quantum number. Thus, the total chemical potential of the strange quarks which carry both baryon number and strangeness is $\mu_s = \mu_q + \tilde{\mu}_s$. Please note that to satisfy strangeness neutrality in the plasma phase the strange quarks and antiquarks have to have the same total chemical potential; hence, $\mu_s = 0$ in a quark-gluon plasma with vanishing net strangeness.

B. Hadron phase

We know that cold isospin symmetric nuclear matter exists in equilibrium at a baryon density $\rho_b = 0.145$ fm⁻³ with a binding energy of -16 MeV. The compressibility at this point is $\sim 210\pm30$ MeV. Extrapolations from this point to higher densities and finite temperature are only weakly constrained by theoretical considerations, and experimental information is very hard to obtain. There have been attempts to narrow down the behavior of the cold nuclear EOS at high densities from the structure of neutron stars²⁷ and from the phenomenology of supernova explosions,²⁸ and in the hot and dense region using data on pion production and collective flow in heavy-ion collisions at the BEVALAC.²⁹ It is fair to say that these attempts have not yet led to a conclusive picture and a large degree of uncertainty remains.

We, therefore, take for our hadron gas EOS a very simple model, consisting of a mixture of hadron resonances taken from the data booklet; which are being described by Fermi or Bose distribution functions. The only interaction between these hadrons we consider is the hard core repulsion which essentially tells us that these hadrons are not pointlike particles, but have a finite proper volume. An excluded volume correction to the noninteracting gas approximation is implemented in a semi-phenomonological way.³⁰ It is an essential ingredient because in the free-gas approximation there is no phase transition in systems with large baryon density.^{13,31,32}

Recently, more sophisticated methods based on relativistic nuclear mean field theory³³ have been developed,^{32,34} which, in particular, properly account for the binding energy and saturation features of cold equilibrium nuclear matter, but otherwise lead to a qualitatively very similar phase diagram. In order to be applied to our problem, however, a further extension of that work to also include strange particles would be required.

Our ansatz for the pressure in the hadron gas is given by

$$P_{H} = \frac{1}{1 + \varepsilon^{pt} / 4B} \sum_{i} P_{i}^{pt} , \qquad (2.6)$$

where

$$P_i^{p_i} = \frac{d_i}{6\pi^2} \int_{m_i}^{\infty} de \frac{(e^2 - m_i^2)^{3/2}}{\exp[\beta(e + \mu_i)] \pm 1} .$$
 (2.7)

Here d_i is the spin-isospin degeneracy factor, and $\varepsilon^{pt} = \sum_i \varepsilon_i^{pt}$ is the total energy density. The superscript *pt* denotes the thermodynamic expressions for *pointlike* particles. The chemical potential of each particle *i* is written as a combination of μ_q and μ_s :

$$\mu_i = (n_i^q - n_i^{\bar{q}})\mu_q + (n_i^s - n_i^{\bar{s}})\mu_s , \qquad (2.8)$$

where $(n_i^q - n_i^{\bar{q}})$ is the net number of light valence quarks and $(n_i^s - n_i^{\bar{s}})$ is the net number of strange valence quarks contained in hadron species *i*.

The factor $1/(1 + \varepsilon^{pt}/4B)$ is the above mentioned proper volume correction (*B* is the bag constant) and limits the energy density,

$$\varepsilon_H = \frac{1}{1 + \varepsilon^{pt} / 4B} \sum_i \varepsilon_i^{pt} , \qquad (2.9)$$

to 4B, i.e., the value inside a hadron according to the Massachusetts Institute of Technology (MIT) bag model. (A different variant of the excluded volume correction was recently proposed in Ref. 35.) The baryon and net strangeness densities are similarly given by

$$\rho_{b,H} = \frac{1}{1 + \varepsilon^{pt} / 4B} \sum_{i} b_i \rho_i^{pt} , \qquad (2.10)$$

$$\rho_{s,H} = \frac{1}{1 + \epsilon^{pt} / 4B} \sum_{i} s_{i} \rho_{i}^{pt} , \qquad (2.11)$$

where b_i and s_i are the baryon number and strangeness of particle species *i*, respectively. The point-particle quantities ε_i^{pt} and ρ_i^{pt} are given by similar integrals over Fermi and Bose distributions as in Eq. (2.7).

In our calculations the sum over resonances includes the mesons, baryon, and antibaryon ground state octets as well as the Δ and $\overline{\Delta}$ resonances, the ρ and ϕ mesons and the Ω^- and $\overline{\Omega}^-$ and $\overline{\Omega}^+$ baryons.

C. Mixed Phase

By construction (i.e., matching of two different equations of state) the phase transition is of first order, i.e., energy, entropy, and particle densities are discontinuous across the transition¹³ (see Sec. IV). As the thermodynamic parameters T, μ_q , and μ_s cross their critical values, the transition proceeds through a mixed phase in which hadron gas and plasma coexist in thermal and chemical equilibrium, occupying varying and complementary fractions of the total volume of the system. Defining the volume fraction α as the ratio of hadronic subvolume to total volume, $\alpha = V_H/V$, the equation of state in the mixed phase is given by

$$\varepsilon_{M} = \alpha \varepsilon_{H} + (1 - \alpha) \varepsilon_{Q} ,$$

$$\rho_{q,M} = \alpha \rho_{q,H} + (1 - \alpha) \rho_{q,Q} ,$$

$$\rho_{s,M} = \alpha \rho_{s,H} + (1 - \alpha) \rho_{s,Q} .$$
(2.12)

This will be the EOS used by us during the hadronization process. It assumes perfect thermal and chemical equilibrium not only within each subphase but also between them. The reliability of this approximation depends on the time scales for the microscopic processes underlying hadronization compared to the total lifetime of the mixed phase. Our assumption is supported by some hydrodynamic calculations^{36,37} which indicate a rather long lifetime of the mixed phase due to kinematic limits on the speed with which quark-gluon matter of high entropy density can be converted into hadron gas with much lower entropy density.

Nevertheless, deviations from equilibrium can be studied by coupling in additional rate equations for light and strange quark creation and annihilation as well as for phase conversion.³⁸ We plan to follow up this approach in a future paper, restricting our attention here to equilibrium properties. Since many of our arguments will be based on entropy considerations and apply equally well in nonequilibrium situations, our conclusions will not be crucially affected by this restriction.

III. THE PHASE DIAGRAM

Given the hadronic and plasma EOS of Sec. II, the region of phase coexistence is obtained from the conditions of thermal, mechanical, and chemical equilibrium between the two phases:

$$T_H = T_Q, \quad P_H = P_Q,$$

 $\mu_{q,H} = \mu_{q,Q}, \quad \mu_{s,H} = \mu_{s,Q}.$ (3.1)

Since the initial hadronic system carries no strangeness and the collision time scale is much smaller than the weak interaction time scale, these conditions have to be supplemented by the one for strangeness conservation:

$$\rho_{s} = \alpha \rho_{s,H} + (1 - \alpha) \rho_{s,Q} = 0 . \qquad (3.2)$$

As discussed before (Sec. II A), in the plasma phase this implies $\mu_s = 0$. In the hadron and mixed phase this condition leads to a nontrivial relation $\mu_s(T,\mu_q)$,^{39,21} and in these phases $\mu_s = 0$ if and only if the net baryon number vanishes, i.e., $\mu_q = 0$.

In Ref. 21 we showed how the phase diagram can be constructed by matching the pressure surfaces $P(\mu_q,\mu_s)|_T$ of the two phases for a series of fixed temperatures T. As one varies the T, the lines of intersection between the two pressure surfaces generate an iglooshaped dome in the three-dimensional (T,μ_q,μ_s) space on whose surface hadron gas and quark-gluon plasma coexist. [This generalizes the usual critical line in the (T,μ_q) plane which is obtained by neglecting the influence of strange particles.] Inside this dome we have hadron gas, and outside there is quark-gluon plasma, neither of which is generally strangeness neutral.

The condition of vanishing net strangeness defines a

two-dimensional surface both inside and outside the igloo. On the plasma side this surface is simply the (T,μ_q) plane, since, there, strangeness neutrality implies $\mu_s = 0$. On the hadron side this surface is more complicated²¹ as shown in Fig. 1. [This slight quantitative difference compared to the similar figure in Ref. 21 is due to Σ (mostly) as well as Ξ and Ω baryons and their antiparticles which were originally omitted. Inclusion of more strange hadrons tends to lower the value of μ_{s} necessary to ensure $\rho_s = 0$ in the hadronic phase.] The two zero-strangeness surfaces inside and outside the dome do not match;¹⁶ rather they cut out a piece from the igloo surface (shown as the nearly vertical strip in Fig. 1) which determines the allowed set of parameters (T,μ_a,μ_s) in the mixed phase compatible with overall strangeness neutrality.

The zero-strangeness surface in the hadronic phase has the following feature: As (for a fixed T) the baryon chemical potential μ_q is increased, μ_s first increases and then decreases again. This behavior can be understood (in the small temperature limit analytically²¹) in the following way: in the baryon poor (small μ_q) region strangeness neutrality is dominantly achieved by balancing $K(\bar{s}q)$ against $\bar{K}(s\bar{q})$ mesons, while hyperons are suppressed by their larger mass. For larger values of μ_q the hyperon mass suppression is eventually overcome by their large baryon chemical potential (due to their light quark content) and a corresponding suppression of antikaons which contain light antiquarks; hence, in this region the strangeness balance is achieved by a competition of kaons and hyperons, with antikaons playing only a minor role.



FIG. 1. The phase diagram for hot and dense nuclear matter with vanishing net strangeness at the transition to a quark-gluon plasma. (Bag constant $B = 250 \text{ MeV/fm}^3$.) Q, M, and H indicate the quark matter, mixed, and hadronic phases, respectively. The solid and dashed lines in the T- μ_q plane show the relationship between temperature and the baryon chemical potential at the beginning and the end of the hadronization phase transition, respectively. The dashed lines in the T- μ_s and μ_s - μ_q planes are projections of the critical line at the hadronic side of the phase transition. The mixed phase is bounded by the solid line in the T- μ_q plane and the solid curve above it. The shape of the isotherm at T = 0 is explained in the text.

μ_s peaks at the point where this role change occurs.

It is important to note that for low temperatures and very large baryon chemical potential (such that $\mu_{\Lambda} > m_{\Lambda}$) the only way to balance the negative strangeness carried by the hyperons is by developing a kaon condensate.²¹ Such a Bose-condensed phase occurs in the hadron phase at large baryon densities if the transition density to quark matter (controlled by the value of the bag constant) is sufficiently high. A similar phenomenon has recently aroused considerable attention in the context of chiral perturbation theory for dense nuclear matter;⁴⁰ there also, the interactions between kaons and strange and nonstrange baryons have been taken into account which further lower the baryon density threshold at which condensation sets in.

In the mixed phase μ_s drops rapidly to zero, accompanied by a finite, but very small change in μ_q ,^{19,21} which was originally overlooked.¹⁶ For nonvanishing μ_q , the values assumed by μ_s while the system passes through the mixed phase do not correspond to strangeness neutrality in either subphase. As noted in Refs. 19, 20, and 23, strangeness separation occurs during the mixed phase in such a way that the strange antiquarks accumulate in the hadronic subvolume (indicated by an overabundance of kaons over hyperons and antikaons), while the strange quarks get enriched in the plasma subvolume. [Rafelski noted in a recent letter²² that this tendency may be reversed under certain conditions: for this to happen (resulting in an enrichment of antistrangeness in the plasma subvolume), the phase transition has to occur at sufficiently high T and/or μ_q . For small $\mu_q \simeq 0$ the direction of the separation switches sign at $T \simeq 320$ MeV.⁴¹ As the volume fractions occupied by the two phases change, the strangeness disbalance shifts in such a way that the system as a whole remains strangeness neutral. The simple and intuitive physical mechanism²³ behind this strangeness separation will be discussed in the following paper²⁴ where we describe the hadronization process in more detail.

IV. HADRON ABUNDANCES

A. General remarks on comparing the flavor composition of the two phases

Comparing the thermodynamic quantities below and above the phase transition, one immediately notes its strong first order nature: energy and entropy density as well as the density of baryon number and strangeness increase by considerable fractions as matter goes from the hadron gas to the quark matter phase. For the specific EOS studied here, this has been discussed in detail in Refs. 13 and 16, to which we refer the interested reader.

The larger density of both light and strange quarks and antiquarks in the plasma, compared to the density of valence (anti)quarks in a hadron gas of the same temperature, is mostly due to the restoration of chiral symmetry and the resulting small quark masses in the plasma phase. This increase in quark densities has been the underlying motivation⁵ to study clustering probabilities of strange and nonstrange quarks into strange hadrons^{42,8} and also of light antiquarks into antibaryons and light antinuclei;^{12,13} the idea was to see whether these particles through their final abundances and ratios can remember the abnormally high initial quark and antiquark densities in the plasma, thereby serving as unique plasma signatures. Positive results were claimed for both strange hadrons⁸ (in particular antihyperons and multistrange antibaryons) and antibaryon clusters,¹³ in some cases with gain factors of several orders of magnitude relative to hadronic equilibrium abundances. We will now critically reassess these claims.

The crucial issue is, of course, that particle densities, however reliably calculated by the theory, have little informational value if the volume of the system under consideration is not specified.¹⁴ Entropy and baryon number conservation force the volume to expand considerably during the hadronization process, to correct for the large drop in the entropy and baryon densities between the initial plasma and the final hadron phase. Therefore, not the ratio of particle number to volume for a specific kind of particles (i.e., their density), but the ratio of particle number to entropy or baryon number should be compared in both phases, if an answer is desired which is independent of the dynamical evolution of the system as dictated by the conservation laws.³⁶ The importance of entropy conservation was first noted^{14,43} in the context of the charged K/π ratio, which originally was computed by simply studying the ratio of strange to light (anti)quarks in the plasma,⁴² until it was realized that many additional pions are produced from fragmenting gluons in order to absorb the large amount of gluonic entropy, thereby drastically diluting this particular ratio.^{7,43}

Comparing systems with equal baryon number or entropy content thus becomes an essential step in properly normalizing the theoretical results before making predictions for the outcome of heavy-ion experiments. We will now show that following this basic rule leads to drastic changes in the interpretation of almost all existing calculations of hadronic particle abundances from a hadronizing quark-gluon plasma.

In Fig. 2 we show the entropy per baryon S/A for the two phases along the critical surface (which is parameter-



FIG. 2. Entropy per baryon S/A for a hadronic matter (H) and a plasma (Q) along the critical surface. At fixed temperature and chemical potential, the quark-gluon plasma has a higher entropy per baryon than hadronic matter.

ized by T and μ_q). Clearly S/A is always higher in the plasma phase than in a hadron gas of the same temperature and chemical potential, due to the liberation of quarks and gluons. Although for a given T the change in S/A is only a factor 2 or so, the S/A curves are very flat at large μ_q (corresponding to the region where T is small, but rapidly varying with μ_q). Therefore, a system which attempts to hadronize while conserving baryon number and entropy will end up at considerably smaller baryon chemical potential and larger temperature.²⁵

In Fig. 3 we identify trajectories of constant S/A in the temperature-density plane, which show quite clearly the reheating and dilution of hot nuclear matter undergoing isentropic hadronization. If additional entropy were produced during the mixed phase, the change in T and μ_q between beginning and end of the hadronization process would be even bigger. The reheating effect is most strongly present in quark matter of low entropy per baryon which begins to hadronize at large values of μ_q and low temperature; on the other hand, systems with very large S/A ratios remain hot and expand to very small values of μ_q before they begin to hadronize, and there the effect of reheating is only minor (although the relative change in μ_q is still considerable).

B. Strangeness

We now wish to compare the strangeness and quark contents in a quark-gluon plasma just above the phase transition (more accurately, in what remains of it after hadronization, given a specific scenario for the hadronization process) with the corresponding values in an equilibrium hadron gas near the deconfinement transition, requiring both systems to contain the same total number of baryons and entropy. In light of the above discussion, we have to take into account that this "critical" equilibrium hadron gas has a higher temperature and smaller chemi-



FIG. 3. Lines of constant entropy per baryon in the temperature-density plane. A system expanding and hadronizing isentropically along these lines will experience considerable reheating and dilution during its passage through the mixed phase, particularly for low values of S/A.

cal potential than the plasma started out with when it began to hadronize. Even in cases where this temperature difference is small, due to the exponential dependence of particle densities on the temperature and baryon chemical potential it has important consequences; it implies that we will find a quite different chemical composition in this hadron gas than if we had studied instead an equilibrium hadron gas at the same temperature and chemical potential at which the plasma began to hadronize. The reason, of course, is the different S/A contents in the latter case.

To appreciate the importance of this effect let us first remind the reader of the findings in previous studies, in which the decay products of a quark-gluon plasma [beginning to hadronize at a certain point $(T_c, \mu_{q,c})$] are compared to an equilibrium hadron gas which is taken to have the same temperature T_c and chemical potential $\mu_{q,c}$.^{13,8} Such a hadron gas is then usually found to have a much smaller density of the hadronic species of interest than the plasma decay product, for two reasons: (1) the hadron phase is more dilute, i.e., its baryon density is lower; (2) it contains less entropy.

The first point is easily corrected for by plotting not the particle densities, but their number in units of entropy or baryon number; this takes care of the dilution effect caused by entropy and baryon number conservation. In Fig. 4 we show the strangeness per entropy (this ratio is more convenient than strangeness/baryon because it avoids the artificial singularity of the latter in baryon-free systems) in both the plasma and equilibrium hadron phase along the phase transition which is parameterized by $T(\mu_q)$. Although still higher in the plasma phase, in particular at low temperatures, there is never more than about a factor of 2 difference for this ratio when compared at constant values of T or μ_q .¹⁴

However, our view of the relationship between the two phases changes even more drastically once we compare instead systems with the same entropy per baryon. In Fig. 5 we plot the strangeness content (again measured in units of the entropy) in a critical quark-gluon plasma (Q)against S/A and compare it to the strangeness/entropy



FIG. 4. Strangeness per unit entropy in a hadron gas (H) and a quark-gluon plasma (Q) as a function of temperature and chemical potential along the critical surface. The difference between the two phases is never more than a factor of 2.



FIG. 5. Strangeness contents in a hadron gas (H) and a quark-gluon plasma (Q) at the critical surface as a function of entropy per baryon. Small values of S/A correspond to low temperature in Fig. 4, while the high temperature, baryon-free end of that diagram corresponds to $S/A \rightarrow \infty$. At fixed S/A, the strangeness content of both phases is very similar.

content of a "critical" equilibrium hadron gas (H) with the same S/A. Small values of S/A correspond to low temperatures in Fig. 4, whereas the high-temperature, baryon-free end of that diagram corresponds to $S/A \rightarrow \infty$. It is clearly seen that the plasma is never "stranger" than the hadron gas by more than about 15%, and in the baryon-rich region (small S/A) contains even considerably less strangeness than an equivalent hadron gas. If we assume a larger value for the bag constant and thereby shift the transition temperature at $\mu_q = 0$ to a higher value, say 200 MeV the equilibrium plasma even at $\mu_q = 0 (S/A = \infty)$ turns out to contain less strangeness than the equilibrium hadron phase.¹⁵

This comparison, although correctly referring to systems with equal entropy per baryon, still neglects the possibility of producing additional $s\overline{s}$ pairs during hadronization (say, by fragmentation of gluons⁸), or of $s\overline{s}$ annihilation. Depending on one's microscopic model for the hadronization process, such mechanisms can become important. Clearly, the entropy contained initially in the gluons has to go somewhere during hadronization, and it is necessary to assume that it reappears through the production of additional quark-antiquark pairs on top of those initially present in the plasma, by gluon fragmentation.⁸ The actual influence of these processes on the strangeness/entropy ratio depends on model assumptions as to what fraction of these gluon decays produce light rather than strange quark-antiquark pairs.

Our point is that any possible anomaly in strangeness abundances has to originate in such details of the hadronization mechanism, and that it is not easy to obtain order of magnitude effects this way. For example, in a $\mu_q = 0$ plasma hadronizing at T = 200 MeV, there are about 2 gluons per light quark and about 3.5 gluons per strange quark. If 15% of these gluons produce additional $s\bar{s}$ pairs (a canonical assumption⁸ based on the fragmentation of gluon jets in high energy e^+e^- collisions, although presumably subject to modification in systems with nonzero baryon density) the strangeness/entropy ratio will be raised by about 50% above its initial value in the plasma. In any case, it is not correct to state that even in equilibrium (i.e., before the onset of hadronization) the plasma is much stranger than the hadron gas; even after including gluon fragmentation, the total amount of strangeness contained in the hadronization debris of a quark-gluon plasma is still comparable to the strangeness content of a critical equilibrium hadron gas.

C. Light antiquarks and antibaryon clusters

In Fig. 6 we compare the light antiquark contents (measured again in entropy units) of a critical equilibrium quark-gluon plasma (Q) and an equilibrium hadron gas (H) with the same S/A. Contrary to the picture obtained when simply studying the antiquark densities at equal temperatures in the two phases,¹³ it is now seen that the relative antiquark content is considerably lower (by a factor 5 or more) in the plasma than in the hadron gas one can think of all the entropy being carried by the valence quarks and antiquarks inside the hadrons, whereas in the plasma a large entropy fraction is contributed by the gluons, with the quarks and antiquarks playing a minor role in comparison.

From Fig. 6 we have to conclude that the earlier speculation,^{12,13} that the larger antiquark density in the plasma may lead to an increased clustering probability of antiquarks and to an anomalously high formation rate of antibaryons and antibaryon clusters during hadronization, was to a large degree based on a misleading way of comparing the two phases.

One might be tempted to deduce from this figure that the hadronization debris from a quark-gluon plasma will actually be depleted of nonstrange antibaryons. However, this is not necessarily true: additional $q\bar{q}$ production by fragmenting gluons trying to deposit their entropy before vanishing from the particle spectrum tends again to replenish the antiquark to entropy ratio. Indeed, if these processes happened fast enough to ensure instantaneous



FIG. 6. The number of light antiquarks per unit entropy in a hadron gas (H) and a quark-gluon plasma (Q) at the critical surface, as a function of entropy per baryon. Note the relative suppression of \overline{q} in the plasma phase, due to the large gluonic contribution to the entropy.

chemical equilibrium they would be guaranteed to finally yield the ratio shown as the hadronic curve. Thus any deviation from hadronic equilibrium will presumably be small and strongly depend on the details *how* chemical equilibrium is violated during the hadronization process. We will further elaborate on this statement in the following subsection.

D. The $\overline{s}/\overline{q}$ ratio and its consequences for the antihyperon/antinucleon ratio

In quark-gluon plasma regions where μ_q is large, light antiquarks are suppressed relative to strange antiquarks (whose chemical potential vanishes), and the $\overline{s}/\overline{q}$ ratio becomes large. A similar rise in the $\overline{s}/\overline{q}$ ratio for large μ_q is also seen in the hadronic phase; there, however, the strange valence quark chemical potential is a nonzero fraction of the light valence quark chemical potential (see Sec. III), and the $\overline{s}/\overline{q}$ ratio, therefore, is smaller than in the plasma phase. This is shown in Fig. 7 where the $\overline{s}/\overline{q}$ ratio is plotted in both phases along the phase transition, comparing systems with equal S/A. The gap between the equilibrium values of this ratio widens from a factor of 5 near $\mu_q = 0$ ($S/A = \infty$) to more than 2 orders of magnitude in the small T, high μ_q ($S/A \rightarrow 0$) limit.

Were this ratio frozen in at the plasma level throughout the hadronization process, a large effect on the antihyperon/antiproton $(\overline{Y}/\overline{N})$ ratio could be expected.⁸ Except for the mass suppression of antihyperons compared to antinucleons (at temperatures $T < m_Y - m_N \sim 200$ MeV there will be an increased driving force for the \overline{s} quarks to hadronize through the



FIG. 7. The ratio of strange to light antiquarks in the hadronic (H) and quark-gluon plasma phase (Q) along the phase transition, as a function of entropy per baryon. See text for comments on the relevance of the apparently large difference between the two curves as a possible plasma signature.

lighter K^+ , K^0 mesons rather than as antihyperons—this effect strengthens exponentially with dropping temperature), this ratio is directly proportional to the $\overline{s}/\overline{q}$ ratio in the plasma.

However, the same process required to ensure entropy conservation can be seen to also have the potential of very effectively diluting the \bar{s}/\bar{q} ratio: light antiquark production from fragmentating gluons contributes more to the denominator than to the numerator. Using the expressions for the \bar{s}, \bar{q} and gluon densities in the plasma $(m_s = 150 \text{ MeV})$,

$$\rho_{\overline{s}} = \frac{6}{2\pi^2} \int_{m_s}^{\infty} de \frac{e(e^2 - m_s^2)^{1/2}}{e^{\beta e} + 1}$$

$$\simeq 0.178 \left[\frac{T}{200 \text{ MeV}} \right] K_2 \left[\frac{150 \text{ MeV}}{T} \right] \text{fm}^{-3} ,$$

$$\rho_{\overline{q}} = \frac{12}{2\pi^2} \int_0^{\infty} \frac{p^2 dp}{e^{\beta(p + \mu_q)} + 1}$$

$$\simeq 1.266 \left[\frac{T}{200 \text{ MeV}} \right]^3 e^{-\mu_q/T} \text{fm}^{-3} ,$$

$$\rho_g = \frac{16}{2\pi^2} \int_0^{\infty} \frac{p^2 dp}{e^{\beta p} - 1} = 2.029 \left[\frac{T}{200 \text{ MeV}} \right]^3 \text{fm}^{-3}$$

the following instructive back-of-the-envelope analysis can be performed.

(1) For a plasma with S/A = 2.55, beginning to hadronize at $(T, \mu_q) = (40 \text{ MeV}, 460 \text{ MeV})$, $\overline{s}/\overline{q} \simeq 8200$ and thus about a factor 4000 larger than in an equilibrium hadron gas with the same S/A on the other side of the transition. However, for every \overline{q} in this plasma there are 1.6×10^5 gluons. Assuming (as before) a 6:1 margin for fragmentation into $q\overline{q}$ over $s\overline{s}$ pairs (although at such low T and large μ_q this splitting ratio should probably be smaller because of Pauli suppression of the light quark final states), already a fraction of 3% of all gluons would be sufficient to adjust the $\overline{s}/\overline{q}$ ratio down to its hadronic equilibrium value.

(2) A somewhat more realistic set of parameters for the projectile and target fragmentation regions in nuclear collisions may be a plasma with S/A = 5.1, beginning to hadronize at $(T,\mu_q) = (72.5 \text{ MeV}, 419 \text{ MeV})$. For such a system $\overline{s}/\overline{q}$ is a factor 120 higher in the plasma than in the hadron gas on the other side of the transition. Under the same assumptions as above just 37% of the gluons $(g/\overline{q} \simeq 500)$ need to fragment in order to adjust the $\overline{s}/\overline{q}$ ratio to its hadronic equilibrium value.

(3) To see what happens closer to the baryon-free limit, we finally consider a plasma with S/A = 11.7, hadronizing at $(T,\mu_q) = (115 \text{ MeV}, 300 \text{ MeV})$. Here a gap in the $\overline{s}/\overline{q}$ ratios of a factor 20 has to be overcome by gluon fragmentation. [This gap further reduces to about a factor of < 5 as $\mu_q \rightarrow 0$ $(T \rightarrow 143 \text{ MeV})$.] Now, however, even fragmenting all thermal gluons into $q\overline{q}$ and $s\overline{s}$ pairs (with a probability ratio of 6:1 as above) is not enough: after that the $\overline{s}/\overline{q}$ ratio will still lie 80% above the hadronic equilibrium value. Additional bremsstrahlung gluons⁸ are necessary to complete the chemical equilibration. But even without them, the factor < 2 difference between a hadronized quark-gluon plasma and an equilibrium hadron gas is too marginal to offer hope for experimental distinction between the two cases from, say, the $\overline{Y}/\overline{N}$ ratio.

In summary, these simple numerical estimates show that in the regions where $(\overline{s}/\overline{q})_H \ll (\overline{s}/\overline{q})_Q$, the gap in this ratio between the two phases is most susceptible to annihilation by gluon fragmentation. On the other hand (and perhaps surprising at first thought), this process is least effective in the central region where, however, the difference is not very large to begin with. In neither case do we expect order of magnitude effects on the $\overline{Y}/\overline{N}$ ratio relative to the hadronic equilibrium value.

We would like to close this section with a short comment about the large $\overline{Y}/\overline{N}$ ratios from a hadronizing quark-gluon plasma quoted in Sec. 4.4 of Ref. 8, which seem to contradict our conclusions. These results were obtained using the combinatoric recombination model by Biró and Zimányi,⁴² modified for gluon fragmentation. Its essential ingredient is the assumption that, after correcting for the different combinatoric probabilities for 3 (anti)quarks or a quark and an antiquark of various flavor combinations to find together, the actual formation rate is the same for all mesons and for all baryons independent of their mass. So, for example, in terms of the number of available light antiquarks $\tilde{N}_{\overline{q}}$ and strange antiquarks $\tilde{N}_{\overline{s}}$, the number of antinucleons formed is assumed as

$$N_{\bar{N}} = \frac{1}{3!} \beta \tilde{N}_{q}^{3} ,$$

whereas the number of antihyperons is given by

$$N_{\overline{Y}} = \frac{1}{2!} \beta \tilde{N}_{q}^{2} \tilde{N}_{\overline{s}} ,$$

with the same recombination constant β . No allowance is made for the larger mass of \overline{Y} compared to \overline{N} , which intuitively (given an approximately thermal distribution of quarks from which these antibaryons arise) might be expected to further suppress \overline{Y} relative to \overline{N} .⁴⁴ Indeed, a study by Kämpfer et al.,45 who incorporated Boltzmann factors ~ $\exp(-m_i/T)$ into the recombination constants to take care of this mass effect, shows that then the $\overline{Y}/\overline{N}$ ratio tends to come out much closer to its hadronic equilibrium value. Here, introduced through the exponential mass and temperature dependence, one sees indeed order-of-magnitude uncertainties occurring in different microscopic models: Although one deals with the same amount of $\overline{s}/\overline{q}$ on average, the combinatoric break-up model strongly favors the \overline{s} to end up in an antihyperon instead of in K mesons. We would like to caution that since these models are not under sufficient theoretical control, large effects based on such differences should not be considered theoretically safe.

V. CONCLUSIONS

Within a phenomenological model for the phase transition between a hot and dense hadron gas and a quarkgluon plasma, we comparatively analyzed the flavor contents of an equilibrium quark phase just above the phase transition and of an equilibrium hadron gas just below the transition. We pointed out the importance of performing the comparison under thermodynamic conditions which are compatible with the dynamical evolution of the hadronization process in nuclear collisions, and of properly normalizing the results such that baryon number is conserved and entropy does not decrease while crossing the phase transition. This forbids comparing plasma and hadron gas at equal temperatures and chemical potentials, a widely employed procedure.

Our analysis led to the following conclusions.

(1) A quark-gluon plasma just above the transition contains a very similar amount of strangeness per unit entropy as an equilibrium hadron gas just below the phase transition which has the same total entropy and baryon number content. This is true both for the nuclear fragmentation regions and for the baryon-poor central rapidity region of ultra-high energy collisions. If there is a difference between the two phases, its tendency is that the equilibrium hadron gas is "stranger."

(2) The plasma is depleted in light antiquarks relative to an equilibrium hadron gas: the \bar{q} / entropy as well as the \bar{q}/\bar{s} ratios are lower in an equilibrium plasma by factors ranging from ~5 near $\mu_q = 0$ to orders of magnitude near T = 0 (μ_q large). This singles out the \bar{s}/\bar{q} ratio as a candidate for a flavor-based quark matter signature.

(3) However, the mechanisms most easily appealed to for explaining entropy conservation despite the vanishing of the gluons during hadronization, namely gluon bremsstrahlung followed by gluon fragmentation into quarkantiquark pairs (additional to those already thermally present in the plasma), have a strong tendency to reduce the gap between the equilibrium $\overline{s}/\overline{q}$ ratios in the two phases. It is hard to construct models which balance the entropy without diluting the $\overline{s}/\overline{q}$ ratio to a level close to the hadronic equilibrium value.

(4) Given this level for the overall $\overline{s}/\overline{q}$ ratio, different hadronization scenarios may predict different relative tendencies of the \overline{s} quarks to end up as (multiply) strange antibaryons or as K and ϕ mesons. This was shown to lead to possibly large effects, say, on the $\overline{Y}/\overline{N}$ ratio, but these results are strongly model dependent and should be taken with appropriate caution.

What does this imply for the planned quark-gluon plasma searches using relativistic nuclear collisions? Throughout this paper we have assumed perfect thermal and chemical equilibrium in the plasma at the point of hadronization, and in our comparisons we dealt with a gas of hadron resonances which was also taken to be completely equilibrated. Thereby we have analyzed the question to which degree hadronic abundance ratios will enable us to distinguish between a hadronizing quarkgluon plasma and an equilibrium hadron gas formed in a nuclear collision. Our results are rather discouraging.

However, as mentioned in the Introduction, this does not negate the well-investigated fact that, on the basis of our present understanding of hadronic dynamics in the absence of a quark plasma, we do not expect strangeness and in particular strange antibaryons to become even nearly saturated at their equilibrium level in collisions without a phase transition.^{9,8} Similarly, nonstrange antibaryons and light antinuclei will be far below equilibrium levels if the plasma is never formed. Therefore, we think that observing in nuclear collisions strangeness and antibaryons, even at the hadronic equilibrium value, would be highly exciting and require for its explanation new dynamical processes which are absent in usual hadron collisions. This prospect in itself is, in our opinion, more than sufficient motivation to measure hadronic, in particular, strange hadron and antibaryon abundances in nuclear collisions. Since we expect a large amount of entropy to be generated in these collisions, large numbers of strange particles per event can be expected if there is sufficient time to (chemically) equilibrate the system; this does not depend on the nature of the processes leading to equilibration, nor does it, intrinsically, require plasma formation. In this sense the observed level of strangeness is predominantly an indicator for the relevant time scales for flavor equilibration.

However, it will be hard to prove plasma formation from such an analysis. Since, for a given amount of entropy produced in the collision, plasma hadronization leads to a final state which, in its flavor composition, is rather similar to an equilibrium hadron gas to the same entropy and baryon content, how would one know it came from a quark-gluon plasma? Clearly, more detailed information than just the relative hadronic abundances is needed to make progress on this question. In the following paper²⁴ we suggest that also measuring the energy spectrum of certain hadronic species will be very helpful in sorting out some of the dynamics of the expansion and hadronization phase. Of course, other, nonhadronic data like direct photon and dilepton spectra may serve this purpose even better. On the other hand, without such additional information essentially only two possible routes offer themselves to resolve the question "plasma or not?:" (i) one excludes the possibility that chemical equilibrium could have been achieved by hadronic processes alone, or (ii) one has a sufficiently accurate model for the dynamics of the collision (in particular the initial hardscattering and particle creation stage) to reliably predict the total amount and distribution of entropy produced for a given collision system and energy with and without phase transition, and thus rejects, say, the latter possibility by showing that it could not have yielded the entropy per baryon level extracted from the measured hadronic spectra. Before essential progress on plasma formation is made in these directions, information gained from hadronic abundances will be ambiguous.

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- *On leave of absence from Chonnam National University, Yongbong-dong, Kwangju 500, Korea.
- Present address: Brookhaven National Lab, Upton, NY 11973.
- Present address: Institut Für Theoretische Physik, Universitat Regensburg, Postfach 397, D-8400 Regensburg, West Germany.
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