Comparison of Vlasov-Uehling-Uhlenbeck model with 4π heavy ion data

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Streamer chamber data for collisions of Ar + KCl and Ar + BaI_2 at 1.2 GeV/nucleon are compared with microscopic model predictions based on the Vlasov-Uehling-Uhlenbeck equation, for various density-dependent nuclear equations of state. Multiplicity distributions and inclusive rapidity and transverse momentum spectra are in good agreement. Rapidity spectra show evidence of being useful in determining whether the model uses the correct cross sections for binary collisions in the nuclear medium, and whether momentum-dependent interactions are correctly incorporated. Sideward flow results do not favor the same nuclear stiffness parameter at all multiplicities.

Theoretical estimates of the peak density attained during the compressional phase of relativistic nucleus-nucleus collisions are typically in the range two to four times normal nuclear matter density. Model simulations indicate that certain observables stabilize at about the same time that the nuclear density reaches its maximum, and remain essentially unchanged during the subsequent stages of the collision process.[1,2] Collective sideward flow is one such observable, and shows promise of providing valuable information about the equation of state (EOS) of compressed nuclear matter. Fluid dynamic models[3] were the first to predict collective nuclear flow, but lack the detailed predictive power of a microscopic approach. The intranuclear cascade,[4] which neglects compressional potential energy, was the first microscopic model to successfully reproduce a wide range of experimental results; however, the current consensus is that the cascade model yields a collective flow signature that is finite,[5-8] but consistently smaller than experimentally observed.[9,10,5-8] There have been previous comparisons[11-13,8] between experimental flow data and microscopic models with realistic EOS implementation over the full range of nuclear densities. Due to statistical errors, or uncertainties associated with filtering the predictions to simulate experimental sample selection criteria and detector inefficiencies, these comparisons yielded only preliminary estimates of EOS properties. In addition, more basic questions have yet to be resolved–uncertainties in the nucleon-nucleon cross section in the nuclear medium,[14,15] and the neglect of momentum dependence[16-18] in models with EOS implementation through a local density-dependent mean field potential.

The model[12,2] used in this study is a microscopic simulation which can be considered a solution of the Vlasov-Uehling-Uhlenbeck[19] (VUU) equation. It proceeds in terms of a cascade of binary collisions between nucleons, Δ resonances, and pions according to the experimental scattering cross sections for free particles, corrected by a Pauli blocking factor. The isospin of each particle is explicitly incorporated. The dependence on the equation of state enters via the acceleration of nucleons in the nuclear mean field. It is assumed that the local potential, U, is determined by the density of nucleons within a radius of 2 fm, with a functional form $U(\rho)=a\rho+b\rho^{\gamma}$. The parameter γ fixes the incompressibility, K, and the remaining two parameters are constrained by nuclear equilibrium conditions. $\gamma=2$ corresponds to $K=380$ MeV, and implies a "stiff" EOS, while $\gamma=\frac{7}{6}$ corresponds to $K=200$ MeV, usually characterized as either a "medium" or "soft" EOS. A special "supersoft" case, in which $\partial U/\partial\rho=0$ above $\rho=\rho_0$ (equilibrium nuclear density), conforms to the assumptions of the intranuclear cascade model. Since K is defined in terms of the second derivative of the binding energy at ρ_0, both the K value *and* the functional form $U(\rho)$ must be specified in order to fix the EOS at higher densities.

This paper presents both inclusive and exclusive parameters in a more detailed comparison between previously reported[8] experimental samples from the Bevalac streamer chamber and a relatively large set of VUU model events. In order to minimize the difficulty of correctly filtering model predictions to simulate the experimental sample selection criteria and detector distortions, cuts have been imposed to remove the projectile and target spectator regions. These cuts (see below) remove $Z\geq 2$ spectator fragments which are not correctly identified in the streamer chamber, and for which a production mechanism is not incorporated in most models. The experimental samples contain a total of 1357 1.2 GeV/nucleon ^{40}Ar beam events with observed charged multiplicity $M\geq 30$. Of these, 571 were collisions on a KCl target, the remaining 786 on a BaI_2 target. The condition $M\geq 30$ selects just over 20% of the inelastic cross section in the case of the KCl target, and just under 40% in the case of the BaI_2 target. The streamer chamber, trigger, particle identification criteria, and additional experimental particulars are described elsewhere.[8,9,20] For each of the three values of EOS stiffness mentioned above, we have generated model statistics amounting to typically five times the experimental samples, using a total of about 50 h of Cray X-MP CPU time.

The kinematic cuts remove particles with momentum (momentum per nucleon in the case of composites) <0.27 GeV/c in the rest frames of the target and projectile. Figure 1 shows distributions of M', the multiplicity of charged particles after imposition of these cuts. In correcting for observational losses and remaining $Z \geq 2$ composites, the detector filtering process reduces M' for each VUU event by about 12%; otherwise, the plotted VUU spectra are unaffected by filtering. Below $M' \sim 25$, the sample selection criterion $M \geq 30$ causes the rolloff in the M' spectra, and events in this lower tail of M' are discarded in the subsequent analysis. The consistently good agreement between experiment and VUU in Fig. 1 is an indication that matching M' distributions is an effective way to establish correct impact parameter averaging for a model.

Before making detailed comparisons of charged particle exclusive parameters, it is appropriate to verify that inclusive spectra are adequately reproduced by the model. Figure 2 shows rapidity distributions, after applying the above spectator cuts and the condition $M' \geq 24$. The dotted curves (labeled $0.7\sigma_{2\text{-body}}$) correspond to a version of the VUU model in which all binary collision cross sections have been reduced by 30%. The total number of two-body collisions decreases by about the same factor. Likewise, the dot-dash curve demonstrates the effect of an increase in collision cross sections. These curves demonstrate that rapidity spectra are useful both for determining whether the model uses the correct two-body collision cross sections,[14,15] and for addressing questions about momentum-dependent interactions[16–18] (MDI), which influence the number of collisions. Thus, these spectra can fulfill the need[18] for collective flow signatures (sensitive to both the EOS and MDI) to be supplemented by another parameter sensitive to just one of these. The factors 0.7 and 1.4 were chosen in light of the study by Bertsch et al.[15] of the effect of varying the cross sections over a two to one range, and the finding of Aichelin et al.[18] that MDI reduce the number of nucleon-nucleon collisions by 30% in the case of La + La at 0.8 GeV/nucleon. The current agreement between VUU (which does not incorporate MDI) and the experimental rapidity spectra suggests that any reduction in collisions due to MDI may need to be counteracted by an increase in the collision cross sections, possibly attributable to in-medium effects.

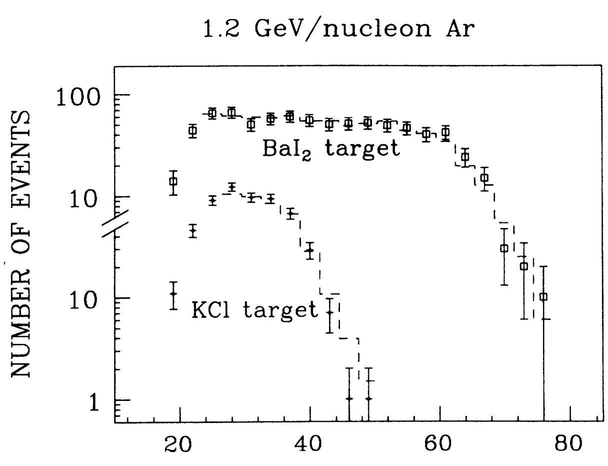

FIG. 1. Distributions of M', the total multiplicity of charged particles after cuts (see text). The dashed lines are the predictions of the VUU model, normalized to the same total number of events. Since the three VUU equations of state give essentially the same spectra, the three predictions have been averaged together in this plot. (The same is true for Figs. 2 and 3.)

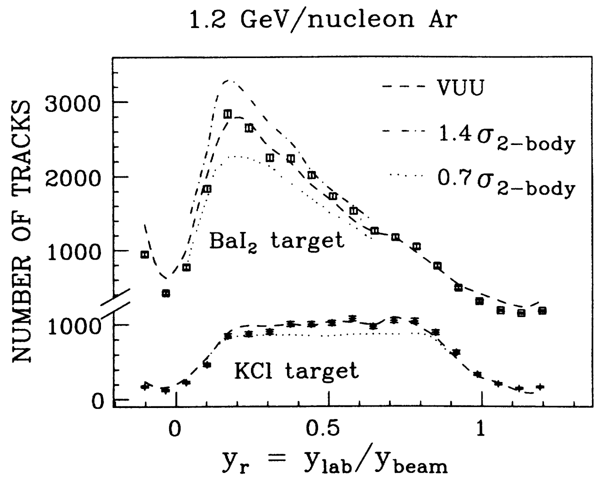

FIG. 2. Nucleon rapidity distributions for $M' \geq 24$, with spectator cuts. The results for the modified binary collision cross sections are shown only at rapidities where there is a significant difference between this calculation and the unmodified VUU model.

Figure 3 presents distributions of transverse momentum per nucleon in three rapidity intervals. The good overall agreement between predictions and experiment again confirms that the VUU model accurately reproduces parameters which are not sensitive to the nuclear EOS.

The transverse momentum method[6] is now widely accepted[12,8,13,21–23] as the most useful parametrization of sideward flow; for experimental data, this approach involves estimating the reaction plane for each event using the vector

$$\mathbf{Q} = \sum w_\nu \mathbf{p}_\nu^\perp ;$$

$$w_\nu = \pm 1 \text{ for baryons with rapidity } y_{\text{c.m.}} \gtrless \pm\delta$$

$$= 0 \text{ otherwise ,}$$

where $\mathbf{p}_\nu^\perp$ is the transverse momentum per nucleon for the νth track. The quantity $\langle p^{x'}(y) \rangle$ is the mean component of transverse momentum per nucleon in the estimated reaction plane:

$$p_\nu^{x'} = \frac{\mathbf{p}_\nu \cdot \mathbf{Q}_\nu}{Q_\nu} , \quad \mathbf{Q}_\nu = \sum_{\mu \neq \nu} w_\mu \mathbf{p}_\mu^\perp .$$

The component in the true reaction plane, p^x, is systematically larger than the component in the estimated plane, $p^{x'}$:

$$\langle p^x(y)\rangle = \frac{\langle p^{x'}(y)\rangle}{\langle \cos\Delta\phi\rangle} ,$$

$$\langle \cos\Delta\phi\rangle \simeq \langle \omega p^{x'}\rangle \left[\frac{\langle W^2 - W\rangle}{\langle Q^2 - \sum (wp^{\perp})^2\rangle}\right]^{1/2} ,$$

where $W=\sum |w|$. In the context of an event-generating model, the true reaction plane can immediately be obtained from the initial orientation of the nuclei, and hence a more direct calculation of $\langle p^x\rangle$ is possible.

Figure 4 shows the observed $\langle p^x(y)\rangle$, along with VUU predictions for the three equations of state. While the multiplicity M' is still defined as in Fig. 1, with target and projectile spectator cuts, the projectile spectator cut has been omitted when calculating p^x. This has been done because the best sensitivity to the EOS coincides with rapidities above $y_r \sim 0.7$ in the upper half of the available multiplicity range as plotted in Fig. 1, and this region is excessively depopulated when the projectile spectator cut is applied. Ionization measurements on comparable samples confirm that the level of $Z \geq 2$ spectatorlike fragments in this region is not large enough to distort the p^x comparisons.

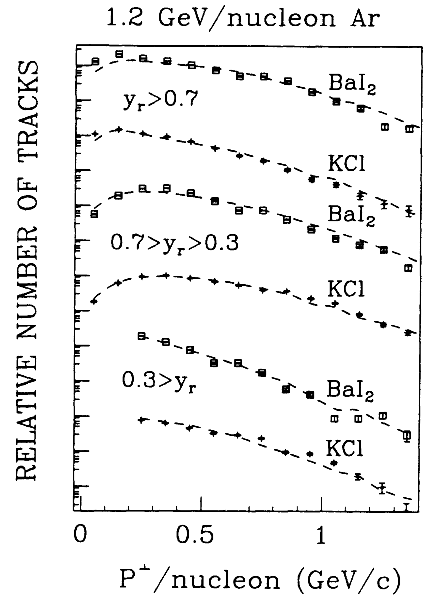

FIG. 3. Transverse momentum spectra for experiment and VUU in three rapidity intervals, where $y_r = y_{lab}/y_{beam}$. The vertical scale is in arbitrary logarithmic units.

Over the relatively narrow multiplicity interval available for Ar + KCl, no significant dependence of $\langle p^x\rangle$ on M' can be detected. We have confined the VUU comparisons to the rapidity region where the overall detector efficiency is high, and there is useful sensitivity to K. The Ar + KCl results in Fig. 4 favor incompressibilities in the medium to stiff range; a similar conclusion[13] is indicated by data for 1.8 GeV/nucleon Ar + KCl.[6] Between 1.2 and 1.8 GeV/nucleon, the transverse flow signature for Ar + KCl increases $\sim 40\%$,[24] and the VUU model predicts a comparable increase at constant K. Streamer chamber data for samples of several thousand Ne + NaF events also indicate an increase in transverse flow (averaged over forward rapidities) with beam energy between 0.4 and 1.2 GeV/nucleon, and between 1.2 and 2.1 GeV/nucleon.[20,25] Doss *et al.*[21] have reported a plateau or a decrease in the transverse flow with beam energy above 0.65 GeV/nucleon, but point out that it is well possible that this effect is influenced by the plastic ball response. Moreover, Doss *et al.* parametrized the flow in terms of the slope of $\langle p^x(y)\rangle$ near midrapidity; if the *shape* of $\langle p^x(y)\rangle$ changes with energy, then $\langle p^x\rangle$ at forward rapidities need not scale in the same way. Overall, it is not clear that the balance of experimental evidence supports the view[2] that there is a softening of the EOS at the higher densities associated with beam energies at and above 1 GeV/nucleon.

Figure 4 also shows $\langle p^x(y)\rangle$ for Ar + BaI_2 in three M' intervals. Here, the VUU predictions show the same qualitative multiplicity trend as the experimental data, with the directed flow effect reaching a maximum at intermediate multiplicity, as expected. The extent of the agreement between the model and experiment is not affected by changing the definition of M' (i.e., changing

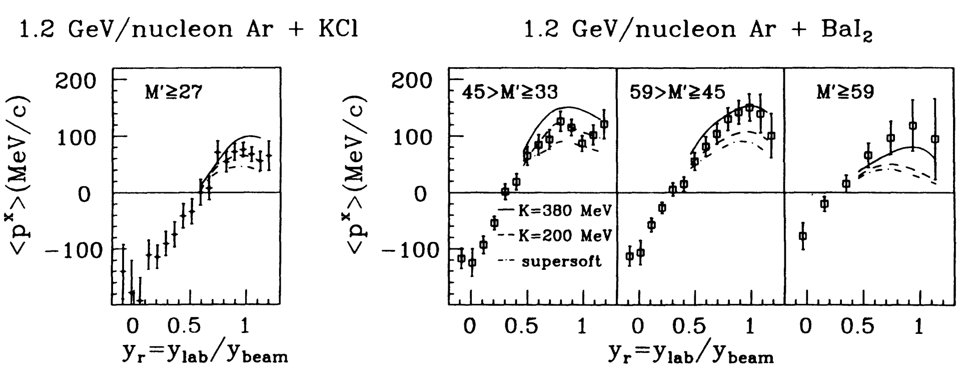

FIG. 4. Mean transverse momentum/nucleon in the reaction plane, as a function of rapidity. The VUU predictions are shown only over the rapidity region where there is useful sensitivity to the incompressibility K (see text).

the cuts). Over most of the M' spectrum, K values in the medium to stiff range are again favored. However, the predicted $\langle p^x \rangle$ drops off faster towards the highest multiplicities than indicated by experiment. (The last multiplicity interval, $M' \geq 59$, corresponds to the uppermost 5% of the inelastic multiplicity spectrum for Ar + BaI_2.) It is possible that the differing multiplicity dependence is associated with the fact that MDI (Refs. 16–18) effects are neglected in the VUU model. At the very least, there are theoretical indications that a model without MDI can lead to overestimates of the incompressibility,[17,18] with the consequence that the present work may yield only upper limits on the true stiffness of the EOS.

We emphasize that while appropriate cuts can partly circumvent the need to simulate detector distortions and inefficiencies when comparing a model with experiment, there is no simple substitute for correct simulation of the impact parameter averaging associated with multiplicity and/or trigger selected subsamples. In order to illustrate this effect, we have taken VUU events for $K=380$ MeV and plotted $\langle p^x(y) \rangle_{max}$ as a function of both impact parameter b and participant multiplicity M'. Taking the peak of these plots, we define the ratio

$$P_{bM} = \langle p^x(y,b) \rangle_{max} / \langle p^x(y,M') \rangle_{max} \ .$$

For 1.2 GeV/nucleon Ar + KCl, we find $P_{bM} \sim 1.24$; for Ar + BaI_2 at the same energy, we find $P_{bM} \sim 1.16$. With the possible exception of the very heaviest systems, it is evident that nontrivial uncertainties arise if is assumed[2] that $P_{bM} \sim 1$.

In summary, charged particle exclusive streamer chamber data for Ar + KCl and Ar + BaI_2 at 1.2 GeV/nucleon are presented with cuts to facilitate model comparisons. Both inclusive and exclusive parameters are compared with VUU model predictions based on three different density-dependent mean field potentials. VUU rapidity and transverse momentum spectra for high multiplicity events are not sensitive to the mean field and are in good agreement with experiment, as are the multiplicity distributions over the region under study. Rapidity spectra show evidence of being useful in determining whether the model uses the correct cross sections for binary collisions in the nuclear medium, and whether momentum-dependent interactions are correctly incorporated. Sideward flow parameters do not favor the same nuclear incompressibility at all multiplicities, and there are indications that the present model may provide only an upper limit on the true stiffness of the equation of state. Questions relating to impact parameter averaging, and the energy dependence of transverse flow are also addressed.

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[24]This value allows for the difference in event selection between the two samples.

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