Isospin splitting of the giant dipole resonance in the *s*-*d* shell and the interacting boson model

G. Maino and A. Ventura
Comitato Nazionale per l'Energia Nucleare e le Energie Alternative, I-40138 Bologna, Italy

L. Zuffi
Dipartimento di Fisica dell'Università and Istituto Nazionale di Fisica Nucleare, Sezione di Milano, I-20122 Milano, Italy
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A phenomenological algebraic approach to nuclear isovector excitations is discussed within the framework of the interacting boson model. Analytic formulae are obtained for isospin splittings and excitation strengths of the giant dipole resonance in deformed nuclei. An application of the formalism to ^{26}Mg is presented, where both schematic and realistic calculations have been performed.

I. INTRODUCTION

The interacting boson model[1] (IBM) for even-even nuclei has been successfully applied to the description of low-lying collective states of light nuclei, just like those in the 2*s*-1*d* shell,[2] by including in the formalism further degrees of freedom,[3] such as isospin, which consist of considering boson excitations formed by neutron-proton correlated pairs, in addition to the usual neutron-neutron and proton-proton boson pairs. On the other hand, an extension of the IBM for the treatment of high-lying collective excitations,[4] like giant resonances, has been proved to be adequate in reproducing[5] experimental data (photon absorption, elastic, and inelastic scattering cross sections).

Since it has long been recognized[6] that isospin plays a crucial role in determining the main characteristics of isovector giant dipole resonances (GDR), it seems worthwhile to combine the two above-mentioned extensions of the IBM in unified formalism in order to obtain a more general description of GDR and a stringent test of the algebraic model. Moreover, by exploiting the limit symmetries of IBM, it is possible to derive analytic expressions for the GDR fragmentation due to isospin, in addition to that arising from nuclear deformation,[4] with axial [SU(3) (Ref. 1)] and triaxial [SU*(3) (Ref. 7)] symmetries.

In this paper we present the general formalism for the treatment of GDR, with particular emphasis on the closed-form formulae obtained for deformed nuclei (Sec. II). Then, in Sec. III, an application to ^{26}Mg is discussed and results of both schematic and realistic calculations are presented in two IBM dynamical symmetries, SU(3) and O(6). Conclusions and perspectives for further applications of the model are summarized in Sec. IV.

II. FORMALISM

In IBM language, the isovector GDR is represented by a *p* boson with quantum numbers $J^{\pi}=1^{-}$, which describes a coherent superposition of particle-hole excitations across a major shell, and strongly interacts with the low-energy *s* and *d* bosons, mainly through a quadrupole-quadrupole force.[4] This coupling is responsible for the GDR fragmentation which produces two, or three components,[4] for axially symmetric and triaxial nuclei, respectively. Relevant calculations for nuclei with SU(3) symmetry, as in lanthanide and actinide regions, are in fair agreement with the experimental data for photon absorption, elastic, and inelastic scattering in the GDR energy domain.[8] As a matter of fact, in addition to the effects related to the coupling to low-energy levels, arising in particular from ground-state deformation, it is well known that the GDR of nuclei with isospin $T_0\neq 0$ is split into components characterized by different isospin values;[9] this energy splitting mainly originates from nuclear symmetry potential.[10] Thus, we introduce an isovector *p* boson which couples with *s*-*d* bosons in the frame of the IBM-3 version[2] of the model, including isospin. In IBM-3, the low-lying collective states are defined by product functions in both the (s,d) boson and the isospin space. The usual *s* and *d* bosons carry one isospin unit and have a third component equal to $+1$ (neutron-neutron pair), 0 (neutron-proton pair), and -1 (proton-proton pair). Since the *p*-boson collective excitation leading to GDR states is characterized by isospin $t=1$ and third component $t_z=0$, a natural choice for overall isospin group symmetry is U(3),[2] reduced to SU(3) under isospin conservation.

It has been proven[3] that an IBM-3 description of states, totally symmetric in both the boson and isospin spaces, can be reduced to an equivalent IBM-1 calculation at the cost of introducing parameters explicitly dependent on isospin in the IBM-1 Hamiltonian. This is the approach adopted in the present work.

Following Rowe and Iachello,[4] we consider the SU(3) symmetry in the (s,d) boson space, corresponding to the rotational pattern in deformed axially-symmetric nuclei. In particular, the $(\lambda,0)$ irreducible representation (irrep) of SU(3) in Elliott's notation[11] labels the ground-state rotational band, where $\lambda=2N$, with N the effective boson number. The *p* boson transforms like a first-rank tensor under SU(3). High-energy (GDR) states are then obtained by coupling the irreps that describe the low-lying states to the (1,0) irrep of the dipole collective excitation. Since the boson state functions must be completely symmetrized, isospin SU(3) irreps are represented by Young diagrams having the same antisymmetrized parts as the corresponding boson SU(3) irreps to which they are coupled. Therefore, the dipole excitation has isospin label (1,0) with $T_z=0$, while the ground-state isospin

$$T_0=T_{z0}=(A-2Z)/2 , \quad (1)$$

where A and Z are the mass and atomic numbers, respectively, belongs to the $(N,0)$ irrep,[2] where the allowed values of T are $N,N-2,\ldots,1$ or 0 depending on whether N is odd or even. The group decomposition chain to be investigated is thus the following:

$$\mathrm{SU}_B^{(sd)}(3)\otimes \mathrm{SU}_T^{(sd)}(3)\otimes SU_B^{(p)}(3)\otimes \mathrm{SU}_T^{(p)}(3) \supset \mathrm{SU}_B^{(sdp)}(3)\otimes \mathrm{SU}_T^{(sdp)}(3) . \quad (2)$$

Here, the indexes B and T label the boson and isospin symmetries, respectively, and s,d and p refer to the low-energy $L^\pi=0^+,2^+$ bosons and the high-energy $L^\pi=1^-$ boson, respectively.

Under the assumption that s-, d-, and p-boson isospins couple at the $\mathrm{SU}_T(3)$ level (the validity of the assumption will be discussed later), in complete analogy with the calculation of Rowe and Iachello[4] for $\mathrm{SU}_B(3)$, one has to deal with the following $\mathrm{SU}_T(3)$ product representation and its decomposition:

$$(N,0)\otimes(1,0)=(N+1,0)\oplus(N-1,1) . \quad (3)$$

The $(N+1,0)$ irrep contains $T=N+1,N-1,\ldots,T_0+1,\ldots,$ while the nonsymmetric $(N-1,1)$ irrep contains $T=N,N-1,\ldots,T_0+1,T_0,\ldots.$ Therefore, IBM-3 predicts a splitting of each $\mathrm{SU}_B(3)$ GDR state into three components $|(\lambda,\mu),T,T_z\rangle$, namely:

$$\begin{aligned} |1_1^-\rangle &= |(N+1,0),T_0+1,T_0\rangle ,\\ |1_2^-\rangle &= |(N-1,1),T_0+1,T_0\rangle , \quad (4)\\ |1_3^-\rangle &= |(N-1,1),T_0,T_0\rangle . \end{aligned}$$

Thus the total number of GDR states is six for axially symmetric nuclei.

The boson Hamiltonian, which includes a quadrupole-quadrupole interaction between high- and low-energy bosons and is responsible for the GDR fragmentation due to nuclear ground-state deformations, has been discussed in Ref. 4. Obviously, if isospin is conserved, the total Hamiltonian commutes with the total isospin operator (and—in any case—with its third component), but the boson interaction terms depend on the relevant isospin values. From a computational point of view in realistic cases different from an exact $\mathrm{SU}_B(3)$ symmetry, this corresponds to performing various IBM calculations, as in Ref. 5, for each isospin channel with a suitable choice of the relevant coupling.

A similar approach has been applied to the study of GDR fragmentation in medium-mass nuclei[12] within the framework of the dynamic collective model.[13] In the $\mathrm{SU}_T(3)$ case it is not necessary to write the explicit form of the isospin-dependent interaction, since it is not essential for GDR splitting considerations. However, a leading-order Hamiltonian term, linear in T_0, such as $\hat{H}'_T=k(\mathbf{t}\cdot\mathbf{T}_0)$, can be easily defined as a function of the $\mathrm{SU}_T(2)$ [$\approx \mathrm{SO}_T(3)$] Casimir operators in isospace.

If one simply assumes that the isospin coupling interaction is expressed, without loss of generality by the relevant Casimir operators, it is found that the energy splitting of the first two states (4), which belong to different $\mathrm{SU}_T(3)$ irreps and have the same isospin, T_0+1, is given by

$$\begin{aligned} \Delta E_{21} &= E[(N+1,0),T_0+1]-E[(N-1,1),T_0+1]\\ &= a\,\Delta\mathcal{C}_2[\mathrm{SU}_T(3)] , \quad (5) \end{aligned}$$

where $\Delta\mathcal{C}_2$ is the difference between the quadratic Casimir operators for $\mathrm{SU}_T(3)$, evaluated for the relevant irreps and defined as usual

$$\mathcal{C}_2[\mathrm{SU}(3)]=\tfrac{2}{3}[\lambda^2+\mu^2+\lambda\mu+3(\lambda+\mu)] . \quad (6)$$

From Eqs. (5) and (6) one obtains

$$\Delta E_{21}=2a(N+1) . \quad (7)$$

In the same way, the energy difference of the second and the third state (4), with isospin T_0+1 and T_0, respectively, both in the $(N-1,1)$ irrep is simply

$$\begin{aligned} \Delta E_{23} &= E[(N-1,1),T_0+1]-E[(N-1,1),T_0]\\ &= b\,\Delta\mathcal{C}_2[\mathrm{SU}_T(2)] , \quad (8) \end{aligned}$$

where

$$\mathcal{C}_2[\mathrm{SU}_T(2)]=2L(L+1) . \quad (9)$$

From Eqs. (8) and (9) one obtains

$$\Delta E_{23}=4b(T_0+1) . \quad (10)$$

The constants a of Eq. (7) and b of Eq. (10) should be determined on the basis of available experimental data. Summing up, if the GDR coupling to the low-energy levels shows $\mathrm{SU}_T(3)$ symmetry, each GDR component, arising from fragmentation due to nuclear deformation, is split again into three components separated in energy in accordance with Eqs. (7) and (10). In particular, the coefficient b must be positive since the GDR states with $T=T_0+1$ are shifted to higher energy with respect to the $T=T_0$ state.

While there is no definite experimental evidence for an energy splitting of type (7), formula (10) can be related to the overall experimental information[14] on GDR isospin splitting and to the first theoretical estimate,[10] based on schematic considerations about symmetry potential energy and particle-hole interactions for spherical nuclei.

According to Ref. 10, one can obtain the following rough estimate for b in Eq. (10):

$$4b=V_1/A-2k\langle x^2\rangle_{n\,\mathrm{exc}}\cong 60/A\ (\mathrm{MeV}) , \quad (11)$$

where A is the mass number, V_1 an exchange term in the Lane potential,[15] and the term proportional to k results from the reduced dipole transition strength in the particle-hole interaction for mode $T=T_0+1$, $\langle x^2\rangle_{\rm n\,exc}$ being an average value of the neutron excess.

Once the energy displacements of isospin GDR states are established, it is possible to derive an expression for the dipole transition strengths. In the present approach, the total isovector $E1$ transition operator, $\hat{D}^{(1)}$, can be factorized in two parts which act on the boson and isospin space, $\hat{D}_B^{(1)}$ and $\hat{D}_T^{(1)}$, respectively.

The boson matrix elements have been derived in Ref. 4 in adiabatic approximation, where the energy spacing in the ground-state band is negligible compared with the GDR deformation splitting, and are simply proportional to the reduced Wigner coefficients[16] of the $SU_B(3)\supset SO_B(3)$ decomposition connecting the $L=0$ ground state in the $(2N,0)$ irrep to the two GDR states belonging to the $(2N+1,0)$ and $(2N-1,1)$ irreps, for an axially symmetric nucleus:

$$M_1=\langle (2N+1,0)\,||\,\hat{D}_B^{(1)}\,||\,(2N,0)\rangle \times\langle (2N,0),0;(1,0),1\,||\,(2N+1,0),1\rangle \,, \quad (12a)$$

$$M_2=\langle (2N-1,1)\,||\,\hat{D}_B^{(1)}\,||\,(2N,0)\rangle \times\langle (2N,0),0;(1,0),1\,||\,(2N-1,1),1\rangle \,. \quad (12b)$$

Since the reduced matrix elements of the $\hat{D}_B^{(1)}$ operator between the $SU_B(3)$ irreps in Eqs. (12a) and (12b) are equal, they amount to the same proportionality constant.

If we introduce the $SU_T(3)$ symmetry, each amplitude (12a) and (12b) splits into three through the multiplication by the following amplitudes for isospin transition:

$$N_1=\langle (N+1,0)\,||\,\hat{D}_T^{(1)}\,||\,(N,0)\rangle\langle (N,0),T_0;(1,0),1\,||\,(N+1,0),T_0+1\rangle\langle T_0,T_0;1,0\,|\,T_0+1,T_0\rangle \,, \quad (13a)$$

$$N_2=\langle (N-1,1)\,||\,\hat{D}_T^{(1)}\,||\,(N,0)\rangle\langle (N,0),T_0;(1,0),1\,||\,(N-1,1),T_0+1\rangle\langle T_0,T_0;1,0\,|\,T_0+1,T_0\rangle \,, \quad (13b)$$

$$N_3=\langle (N-1,1)\,||\,\hat{D}_T^{(1)}\,||\,(N,0)\rangle\langle (N,0),T_0;(1,0),1\,||\,(N-1,1),T_0\rangle\langle T_0,T_0;1,0\,|\,T_0,T_0\rangle \,. \quad (13c)$$

Here again the reduced $\hat{D}_T^{(1)}$ matrix elements have the same value and can be merged with the common coefficient of Eqs. (12a) and (12b) into only one constant, D_0. The ratio of transition strengths for the two different GDR isospin components is simply

$$S(T=T_0+1)/S(T=T_0) =(|N_1|^2+|N_2|^2)/|N_3|^2=1/T_0 \,. \quad (14)$$

A necessary condition for the Hamiltonian with GDR coupling to be diagonalized in the coupled $SU_T(3)$ basis (3) in adiabatic approximation is, however, that the energy spacing of the states with different T contained in the $(N,0)$ irrep, be small with respect to the energy splitting induced by the GDR coupling to above mentioned states. The latter can be estimated by means of formulae (10) and (11), the former is easily connected with the symmetry term in the Weiszäcker mass formula:[17]

$$\Delta E(T,T_0)\simeq\left[\frac{134-238A^{-1/3}}{A}\right] \times[T(T+1)-T_0(T_0+1)]\ \text{MeV} \,. \quad (15)$$

In the case where the above mentioned condition does not hold, the GDR isospin coupling has to be described at the $SU_T(2)$ level, which predicts two isospin components with $T=T_0$ and T_0+1, whose energy difference is given by Eqs. (10) and (11).

The ratio of transition strengths is simply given in terms of isospin Clebsch-Gordan coefficients:[18]

$$\frac{S(T=T_0+1)}{S(T=T_0)}=\left|\frac{\langle T_0,T_0;1,0\,|\,T_0+1,T_0\rangle}{\langle T_0,T_0;1,0\,|\,T_0,T_0\rangle}\right|^2=\frac{1}{T_0}\,, \quad (16)$$

which would be equivalent to Eq. (14) if the two $SU_T(3)$ states with $T=T_0+1$ were degenerate. If the boson Hamiltonian does not show a $SU_B(3)$ symmetry, the boson amplitudes cannot be obtained through the analytic formulae [(12a) and (12b)], but have to be evaluated numerically, as in Ref. 5, and multiplied by the $SU_T(3)$ or $SU_T(2)$ transition amplitudes, depending on which isospin symmetry is valid.

III. NUMERICAL APPLICATIONS

It is well known[11] that most nuclei in the $2s$-$1d$ shell exhibit collective features in low-energy spectra. The IBM yields a reasonable description of nuclear properties

TABLE I. Fragmentation of the giant dipole resonance in ^{26}Mg.

	Excitation energy (MeV)		Dipole strength[a]	
Isospin T	Expt. (Ref. 21)	Calc. $SU_B(3)$	Expt. (Ref. 21)	Calc. $SU_B(3)$
1	11.17–18.20	17.5	1.00[a]	1.00[a]
2	18.45–22.30	22.1	0.77–1.00	1.00
1	23.50–27.00	25.0	1.80–2.03	1.54
2		29.6		1.54

[a]Arbitrary normalization to unit.

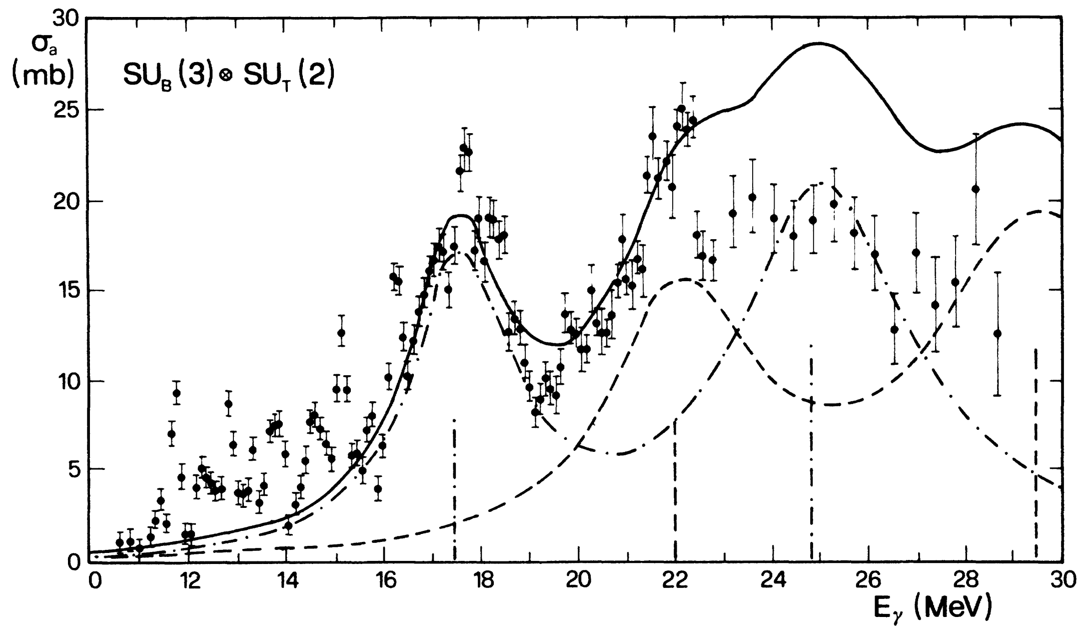

FIG. 1. Experimental (Ref. 21) and calculated [$SU_B(3)\otimes SU_T(2)$, solid line] photoabsorption cross section of ^{26}Mg. Dotted-dashed line, $T=1$ component; dashed line, $T=2$ component; bars at the bottom represent calculated dipole strengths (in arbitrary units).

in this mass region.[2,3,19] Moreover a large amount of experimental data is available regarding different GDR fragmentation mechanisms, mainly deformation, isospin, and configurational splitting.[18]

In particular, GDR in ^{26}Mg has been investigated by means of (e,e'),[20] (γ,xn),[21] (γ,p),[22,23] (γ,α),[22] and $(\gamma,x\gamma')$ (Ref. 24) reactions up to an excitation energy $E^*=30$ MeV. Different decay modes of GDR states, excited by the various probes, give reliable determination of the isospin values of observed states. In this way, it is possible to identify a large fragmentation pattern in ^{26}Mg, probably arising from both deformation and isospin effects.

An estimate of the GDR isospin splitting in ^{26}Mg can be immediately obtained from Eqs. (10) and (11) with $T_0=1$ and $A=26$, leading to a value

$$\Delta E = E(T=2)-E(T=1)\cong 4.6 \text{ MeV} .$$

On the other hand, the states contained in the (5,0) irrep have $T=1$, 3, and 5. Formula (15) yields, for the excitation energies of the lowest $T=3$ and $T=5$ states, 20.6 and 57.8 MeV, respectively. Therefore, the adiabatic approximation does not hold, and the total Hamiltonian cannot be diagonalized in the coupled $SU_T(3)$ basis (3). One is then forced to abandon $SU_T(3)$ for the description of the GDR isospin coupling and to resort to the simpler $SU_T(2)$ symmetry.

As for the GDR deformation splitting, an order-of-magnitude estimate is obtained by assuming a $SU_B(3)$ symmetry, although it does not allow a realistic description of low-energy states. From the analysis of ^{24}Mg where the GDR strength, as observed in the photoabsorption cross section,[21] is concentrated in two energy regions around $E_1=18.5$ and $E_2=25.0$ MeV, if the splitting is mainly due to the nuclear deformation, the

TABLE II. IBM parameters (Ref. 5) for ^{26}Mg.

Parameter	Value
N	5
a_0 (MeV)	−0.310
a_1 (MeV)	0.133
a_2 (MeV)	−0.211
χ	−0.0008
ϵ_p (MeV)	21.9
b_2 (MeV)	2.60
D_0 (fm)	1.70

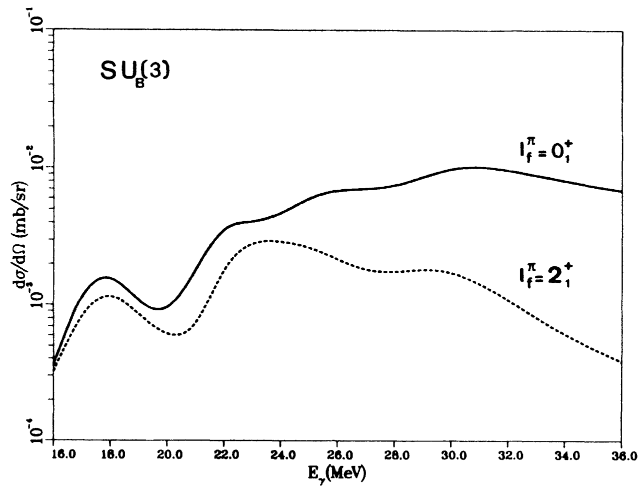

FIG. 2. Photon elastic (solid line) and inelastic (dashed line) scattering cross sections for ^{26}Mg, assuming $SU_B(3)\otimes SU_T(2)$ symmetry, at scattering angle $\vartheta=130°$.

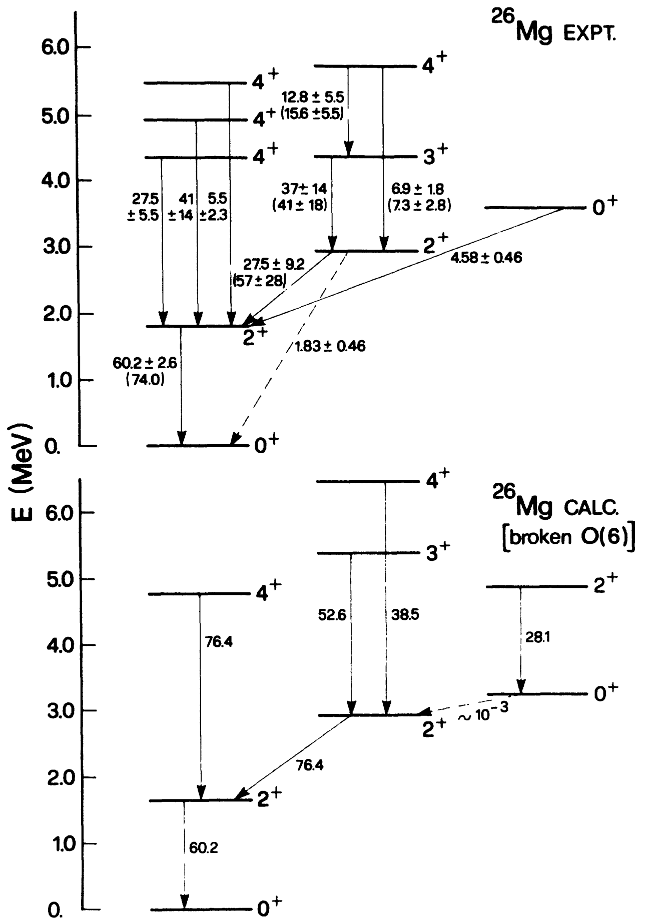

FIG. 3. Experimental (Refs. 25 and 26) and calculated level schemes of ^{26}Mg.

ground-state isospin being $T_0=0$, we derive a quadrupole-quadrupole coupling constant 0.830 MeV and a p-boson energy $\varepsilon_p=22.35$ MeV. The resulting ratio of dipole strengths in the two regions is $S(E_2)/S(E_1)=4N/(2N+3)=1.45$, where $N=4$ is the number of effective bosons in ^{24}Mg, to be compared with the experimental integrated value[21] $S(E_2)/S(E_1)\simeq 1.37$.

We assume the same parameters for ^{26}Mg, by varying simply ϵ_p according to the $A^{-1/3}$ law. Moreover, a value $b=60/(4A)=0.58$ MeV is adopted for the GDR isospin splitting in $SU_T(2)$.

By applying Eqs. (10) and (16), one obtains the results of Table I, to be compared with the experimental integrated values.[21] By associating to each GDR component, at energy E, a width $\Gamma(E)$ obeying the phenomenological power law

$$\Gamma(E)=0.035E^{3/2}\text{ MeV },\qquad(17)$$

with E expressed in MeV, one obtains the photoabsorption cross section compared with experiment[21] in Fig. 1 and photon scattering cross sections of Fig. 2. As expected, the agreement with experimental data is rather poor, with the exception of the main peak at $E_1\cong 17.5$ MeV. In particular, too much strength is concentrated in the region above 22 MeV. The "pigmy" resonance below the GDR is, of course, not reproduced because it cannot be described by the present collective model.

Since the low-energy spectrum is rather close to that of a γ-soft nucleus, we have also performed a more realistic IBM calculation based on a broken $O_B(6)$ symmetry.[1]

The IBM parameters of Table II are obtained by a least-squares fitting procedure; the calculated levels and $B(E2)$ transition intensities are compared with the experi-

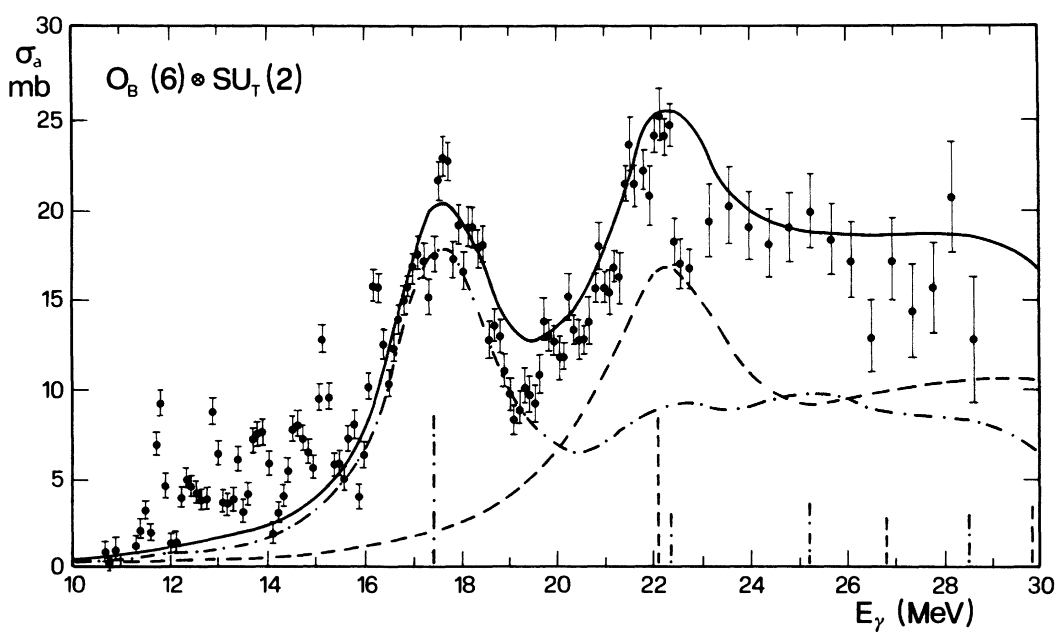

FIG. 4. Experimental (Ref. 21) and calculated [broken $O_B(6)\otimes SU_T(2)$, solid line] photoabsorption cross section of ^{26}Mg. Dotted-dashed line, $T=1$ component; dashed line, $T=2$ component; bars at the bottom represent calculated dipole strengths (in arbitrary units).

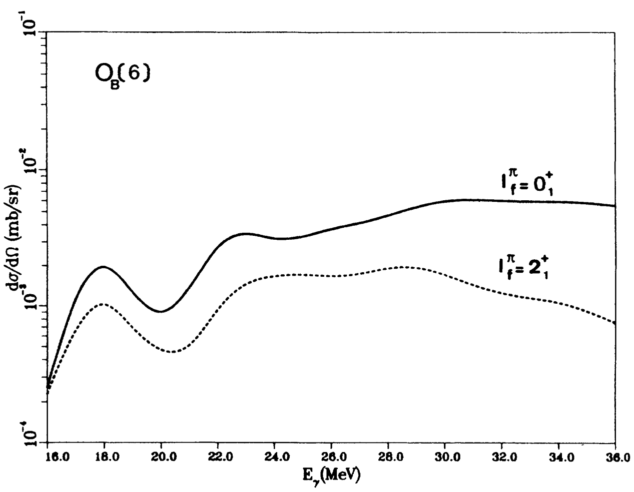

FIG. 5. Photon elastic (solid line) and inelastic (dashed line) scattering cross sections for ^{26}Mg, assuming a broken $O_B(6) \otimes SU_T(2)$ symmetry, at $\vartheta = 130°$.

mental ones in Fig. 3. With a suitable choice of the quadrupole-quadrupole interaction strength among *s*, *d*, and *p* bosons, assuming a ratio of 1.0 between the strengths in the $T=1$ and $T=2$ channels [according to formula (16)], we obtain the total photoabsorption cross section shown in Fig. 4. Here again, the simple power law (17) has been adopted for the GDR widths. Obviously, the fine structure in the cross section, in particular below GDR excitation, cannot be reproduced by the present model; its complete description would require detailed shell-model calculations.

The different fragmentation of dipole strengths in the schematic $SU_B(3)$ model and in the realistic $O_B(6)$ limit is worth noting: while the former deformation splitting of each GDR state into two components can be obtained in other GDR models, the latter, showing a dominant state and a series of smaller equivalent strengths, is rather peculiar to the IBM model and in much better agreement with the experiment.

Figure 5 shows the photon elastic and inelastic (to the 2_1^+ state) scattering cross sections. Such a measurement on ^{26}Mg would be highly desirable, since the (γ,γ') reaction, which is essentially nonstatistical, yields a useful probe to investigate GDR components with different isospin[12] and, therefore, a good test of the model.

IV. CONCLUSIONS

The $SU_B(6) \otimes SU_T(2)$ approach, proposed in Sec. II, constitutes a simple yet accurate tool to deal with isospin effects in the GDR region for light even-even nuclei. In addition, thanks to the particularly flexible IBM structure, it allows realistic calculations, as that shown for ^{26}Mg, with little computational effort.

Of course, more experimental information about photon scattering is necessary in order to test the model. Moreover, it is to be pointed out that the present procedure can be applied with slight modifications to the study of reactions other than photon absorption and scattering, proceeding through GDR excitation like charge-exchange processes, induced, for instance, by low-energy pions, and muon nuclear capture.

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