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Influence of target clustering on exchange effects in internuclear interaction

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Nucleon-exchange effects are examined in the case where one of the nuclei involved exhibits properties of considerable clustering. The results, obtained by analyzing the exponential parts of the exchange-normalization kernel function in a three-cluster resonating-group formulation, indicate that, as in the case of no target clustering, the one-exchange and core-exchange terms are still the terms which make the dominant contribution. The characteristics of the one-exchange term are found to be rather insensitive to the clustering nature of the target nucleus. The core-exchange term, on the other hand, may acquire a rather long range compared to the direct potential when the degree of clustering is appreciable.

I. INTRODUCTION

Detailed analyses[1,2] of resonating-group kernel functions have led to a basic understanding of the roles played by antisymmetrization on the essential features of the internuclear interaction. For a two-cluster system that consists of clusters A and B with nucleon numbers N_A and N_B $(N_A > N_B)$ and described, respectively, by translationally-invariant shell-model functions of the lowest configuration in harmonic-oscillator wells having widths parameters α_A and α_B, it has been found that the following general conclusions can be made (see Ref. 3 for the definition of the various terms).

(i) Among all class-A terms, the one-exchange terms are the most important. At relatively high energies, the type-1c or knockon exchange term has, in particular, a dominant influence.

(ii) Among all class-B terms, the core-exchange terms make the largest contribution. For systems in which absorption is strong, the core-exchange type-a and type-d terms are particularly significant.

(iii) The one-exchange terms are generally important in all scattering systems and over a wide energy range, but the core-exchange terms are generally important only when the nucleon-number difference $(N_A - N_B)$ of the interacting nuclei is rather small.

(iv) At energies higher than about 25 MeV/nucleon, the contribution to the cross section in the decreasing part of the V-shaped angular-distribution curve comes essentially from the direct term and the one-exchange terms (mainly the type-1c term), while the contribution to the cross section in the increasing part of this curve comes essentially from the core-exchange terms.

An especially important consequence of the above-mentioned findings is that, for a scattering system in which the interacting nuclei have a rather small nucleon-number difference, the real part of the macroscopic, optical potential must contain a significant amount of Majorana, or parity-dependent, component. In fact, even the parity Π of the Pauli-favored states (i.e., the states with the stronger interaction) has recently been determined. By studying the diagonal elements of the parity-projected GCM (generator-coordinate method) norm kernel, Baye[4] has shown that Π is given by $(-1)^{N_A}$ for collisions between two p-shell nuclei, and by $(-1)^{N_B}$ for collisions between two s-shell or two sd-shell nuclei.

Numerical studies in specific nuclear systems[3,5] fully support these findings. In addition, it is noted[6] that the findings remain essentially unchanged even when absorption is present and when a common oscillator width parameter is chosen to describe the clusters involved. Furthermore, one finds[1] that the investigation of antisymmetrization effects can be substantially simplified by studying the norm kernel alone, although additional useful details do emerge if the Hamiltonian, or energy, kernel function is also examined.[2,7]

In this investigation, we continue our study by considering situations in which one of the nuclei involved, say nucleus A, exhibits considerable clustering properties itself. In reality, these are situations that occur rather fre-

quently. For example, in the case of $\alpha+{}^6$Li or $\alpha+{}^{20}$Ne scattering, one must necessarily take into account the fact that the nuclei ^{6}Li and ^{20}Ne have predominantly $\alpha+\mathrm{d}$ and $\alpha+{}^{16}$O cluster structures, respectively, and hence, cannot be adequately described by simple shell-model functions of the lowest configuration. Thus, it is clear that, in such situations, the proper way to study exchange effects is to examine the kernel functions occurring in a three-cluster formulation. This is obviously a rather difficult problem; fortunately, based on our experience from two-cluster investigations, one can still carry out the study by adopting the simplifications of choosing a common harmonic-oscillator width parameter for each of the three clusters involved and by examining only the main features of the norm kernel.

The evaluation of the norm kernel is fairly complicated. However, with the use of the complex-generator-coordinate technique,[8] one can readily derive the general expressions for the exponential parts of this kernel in the three-cluster $(A_1+A_2)+B$ case, where the nucleus A is now considered to have an (A_1+A_2) cluster structure. As in the two-cluster problem, this information is in fact sufficient for the purposes of this investigation. That is, one can proceed with these exponential parts to determine the characteristic ranges and characteristic energies of the various nucleon-exchange terms and, thereby, find out whether or not target clustering has any significant influence on the effects of antisymmetrization in the properties of the intercluster interaction.

The basic structure of the exchange-normalization kernel is discussed in Sec. II. Here one sees that, in contrast to the two-cluster case, many nucleon-exchange modes are now present. Among these, one exchange mode, to be called mode $\tilde{\alpha}$, turns out to be especially important. The properties of this particular mode will be described in detail in Sec. III. Numerical studies have been performed for $\alpha+{}^6$Li and $\alpha+{}^{20}$Ne scattering in mode $\tilde{\alpha}$, and the findings are also discussed in this section. These particular systems are chosen for illustration, because the nucleon-number difference of the interacting nuclei is small in the former case, but large in the latter case. Finally, in Sec. IV, we summarize the results and make some concluding remarks.

II. EXCHANGE NORMALIZATION KERNEL

In this section, we study the properties of the exchange normalization kernel $\tilde{N}_E$ for the general case of $A+B$ scattering, where the nucleus A is considered to have a cluster structure of A_1+A_2. For simplicity in notation, we shall conveniently label clusters A_1, A_2, and B as clusters 1, 2, and 3, respectively. The nucleon numbers of these clusters are equal to N_1, N_2, and N_3, with $N_1+N_2=N_A$ and $N_3=N_B$. For the construction of the cluster internal wave functions, we shall use, as mentioned in the preceding section, harmonic-oscillator wells having a common width parameter α.

In the single-channel resonating-group formulation for $A+B$ scattering, the wave function ψ of the system is written as

$$\psi=\mathcal{A}\{[\phi_1\phi_2G(\boldsymbol{\rho})]\phi_3F(\mathbf{R})Z(\mathbf{R}_{\mathrm{c.m.}})\}\ , \tag{1}$$

where $\mathcal{A}$ is an antisymmetrization operator, $Z(\mathbf{R}_{\mathrm{c.m.}})$ is any normalizable function describing the total center-of-mass motion, and

$$\begin{aligned}&\boldsymbol{\rho}=\mathbf{R}_1-\mathbf{R}_2\ ,\\ &\mathbf{R}=\mathbf{R}_3-(N_1\mathbf{R}_1+N_2\mathbf{R}_2)/(N_1+N_2)\ ,\end{aligned} \tag{2}$$

with $\mathbf{R}_1$, $\mathbf{R}_2$, and $\mathbf{R}_3$ being, respectively, the c.m. coordinates of clusters 1, 2, and 3. The functions ϕ_1, ϕ_2, and ϕ_3 represent the internal structures of the clusters; they are chosen to be translationally invariant products of single-particle functions of the lowest configuration in harmonic-oscillator wells having a common width parameter α. The function $F(\mathbf{R})$ describes the relative motion between clusters A and B; it is obtained by solving a projection equation, as discussed in detail in Refs. 8 and 9.

The relative-motion function $G(\boldsymbol{\rho})$ between clusters 1 and 2 is assumed to have the form

$$G(\boldsymbol{\rho})=h(\boldsymbol{\rho})\exp(-\tfrac{1}{2}\mu_{12}\gamma\rho^2)\ , \tag{3}$$

where $h(\boldsymbol{\rho})$ is a polynomial in ρ_x, ρ_y, and ρ_z, and μ_{12} is a reduced nucleon number given by

$$\mu_{12}=N_1N_2/(N_1+N_2)\ . \tag{4}$$

To describe the presence of appreciable clustering in nucleus A, we choose the parameter γ to be substantially smaller than α.[10] For example, in the case of ^{6}Li, an appropriate value[11] for the clustering parameter y, defined as

$$y=\gamma/\alpha\ , \tag{5}$$

is around 0.5. On the other hand, it should be noted that, when $y=1$ (i.e., $\gamma=\alpha$), the wave function for cluster A reduces to a translationally invariant shell-model function of the lowest allowable configuration in a harmonic-oscillator well with width parameter α and, consequently, we get back the results reported previously in Refs. 1–3 for the two-cluster limiting case.

The exchange normalization kernel $\tilde{N}_E$ is given by

$$\begin{aligned}\tilde{N}_E(\mathbf{R}',\mathbf{R}'')=\langle\,[\phi_1\phi_2G(\boldsymbol{\rho})]\phi_3\delta(\mathbf{R}-\mathbf{R}')Z\mid\\ \mathcal{A}''\{\mathcal{A}_{12}[\hat{\phi}_1\hat{\phi}_2G(\boldsymbol{\rho})]\hat{\phi}_3\delta(\mathbf{R}-\mathbf{R}'')Z\}\,\rangle,\end{aligned} \tag{6}$$

where $\hat{\phi}_1$, $\hat{\phi}_2$, and $\hat{\phi}_3$ are antisymmetric functions obtained by antisymmetrizing the cluster internal functions ϕ_1, ϕ_2, and ϕ_3, $\mathcal{A}_{12}$ is an intercluster antisymmetrization operator for clusters 1 and 2, and the operator $\mathcal{A}''$ is defined by the equation

$$\mathcal{A}''=\mathcal{A}'-1\ , \tag{7}$$

with $\mathcal{A}'$ being an antisymmetrization operator that interchanges nucleons in clusters A and B. By employing the complex-generator-coordinate technique,[8] one finds straightforwardly that $\tilde{N}_E$ has the form

$$\tilde{N}_E(\mathbf{R}',\mathbf{R}'')=\sum_x\sum_\nu [P_{x\nu}\exp(-A_{x\nu}\mathbf{R}'^2-C_{x\nu}\mathbf{R}'\cdot\mathbf{R}''-B_{x\nu}\mathbf{R}''^2)+\text{H.c.}] , \quad (8)$$

where $P_{x\nu}$ is a polynomial in $\mathbf{R}'^2$, $\mathbf{R}'\cdot\mathbf{R}''$, and $\mathbf{R}''^2$, and the quantity x represents the number of nucleons interchanged between A and B. The index ν is used to denote, for a fixed value of x, the many modes of nucleon exchanges among the three clusters; it is defined by the set of values for x_{21}, x_{31}, x_{32}, x_{231}, and x_{312}, where x_{ij} represents the number of interchanges for nucleons with the same spin and isospin in clusters i and j, and x_{ijk} has a similar meaning except that nucleons of the same spin and isospin in all three clusters are now involved in the interchange process. Clearly, one can see that the value of x is given by

$$x=x_{31}+x_{32}+x_{231}+x_{312} . \quad (9)$$

The general expressions for the coefficients $A_{x\nu}$, $B_{x\nu}$, and $C_{x\nu}$ can be derived in terms of N_1, N_2, N_3, α, and γ for each exchange mode. The results are

$$A_{x\nu}=\tfrac{1}{2}\mu_0\alpha-\Gamma(be^2+aj^2-cej)-\Gamma'(b'd'^2+a'f'^2-c'd'f') , \quad (10)$$

$$B_{x\nu}=-\tfrac{1}{2}\mu_0\alpha-\Gamma(bg^2+am^2-cgm)-\Gamma'(a'g'^2+b'e'^2-c'e'g') , \quad (11)$$

$$C_{x\nu}=-\Gamma[2beg+2ajm-c(gj+em)]-\Gamma'[2a'f'g'+2b'd'e'-c'(d'g'+e'f')] , \quad (12)$$

where

$$\mu_0=N_AN_B/(N_A+N_B) , \quad (13)$$

$$a=\tfrac{1}{4}\alpha(N_1+N_1^2/N_3-A_3^2/N_3-A_2^2/N_2-A_1^2/N_1) , \quad (14)$$

$$b=\tfrac{1}{4}\alpha(N_2+N_2^2/N_3-B_3^2/N_3-B_2^2/N_2-B_1^2/N_1) , \quad (15)$$

$$c=\tfrac{1}{2}\alpha(N_1N_2/N_3-A_3B_3/N_3-A_2B_2/N_2-A_1B_1/N_1) , \quad (16)$$

and

$$a'=\tfrac{1}{2}\mu_{12}(\alpha+\gamma)-\Gamma(bd^2+ah^2-cdh) , \quad (17)$$

$$b'=\tfrac{1}{2}\mu_{12}(\gamma-\alpha)-\Gamma(bf^2+al^2-cfl) , \quad (18)$$

$$c'=-\Gamma[2bdf+2ahl-c(dl+fh)] , \quad (19)$$

$$d'=-\Gamma[2bde+2ahj-c(dj+eh)] , \quad (20)$$

$$e'=-\Gamma[2bdg+2ahm-c(dm+gh)] , \quad (21)$$

$$f'=-\Gamma[2bef+2ajl-c(fj+el)] , \quad (22)$$

$$g'=-\Gamma[2bfg+2alm-c(fm+gl)] . \quad (23)$$

In addition, we have defined the following quantities in Eqs. (10)–(23):

$$\Gamma=(4ab-c^2)^{-1} , \quad (24)$$

$$\Gamma'=(4a'b'-c'^2)^{-1} , \quad (25)$$

$$A_1=(N_1-x_1)-(x_{31}+x_{312})N_1/N_3 , \quad (26)$$

$$A_2=(x_{21}+x_{312})-(x_{32}+x_{231})N_1/N_3 , \quad (27)$$

$$A_3=(x_{31}+x_{231})-(N_3-x_3)N_1/N_3 , \quad (28)$$

$$B_1=(x_{21}+x_{231})-(x_{31}+x_{312})N_2/N_3 , \quad (29)$$

$$B_2=(N_2-x_2)-(x_{32}+x_{231})N_2/N_3 , \quad (30)$$

$$B_3=(x_{32}+x_{312})-(N_3-x_3)N_2/N_3 , \quad (31)$$

$$d=i\alpha(N_1A_2-N_2A_1)/(N_1+N_2) , \quad (32)$$

$$e=i\alpha[(N_1+N_2)A_3-N_3(A_1+A_2)]/N , \quad (33)$$

$$f=i\mu_{12}\alpha , \quad (34)$$

$$g=iN_1\alpha , \quad (35)$$

$$h=i\alpha(N_1B_2-N_2B_1)/(N_1+N_2) , \quad (36)$$

$$j=i\alpha[(N_1+N_2)B_3-N_3(B_1+B_2)]/N , \quad (37)$$

$$l=-i\mu_{12}\alpha , \quad (38)$$

$$m=iN_2\alpha , \quad (39)$$

with i being the imaginary unit number,

$$x_1=x_{21}+x_{31}+x_{231}+x_{312} , \quad (40)$$

$$x_2=x_{21}+x_{32}+x_{231}+x_{312} , \quad (41)$$

$$x_3=x , \quad (42)$$

and

$$N=N_1+N_2+N_3 . \quad (43)$$

By adopting the Born-approximation procedure[12] used in previous analyses,[1,2] we can construct equivalent exchange potentials between clusters A and B which are characterized by characteristic wave numbers $k_{x\nu}$ and characteristic ranges $R_{x\nu}$, given by

$$k_{x\nu}=\left[\frac{4A_{x\nu}B_{x\nu}-C_{x\nu}^2}{A_{x\nu}+B_{x\nu}-|C_{x\nu}|}\right]^{1/2} , \quad (44)$$

$$R_{x\nu}=\left[\frac{2|C_{x\nu}|}{4A_{x\nu}B_{x\nu}-C_{x\nu}^2}\right]^{1/2} . \quad (45)$$

These equivalent potentials have either a Wigner or a Majorana character, depending upon whether $C_{x\nu}$ is smaller or larger than zero. Also, as has been explained previously,[3] it is useful to define two alternative characteristic quantities, i.e., the characteristic energy $E_{x\nu}$ and the characteristic weight $\zeta_{x\nu}$. These quantities are defined as

$$E_{x\nu}=\frac{\hbar^2}{2M\mu_0}k_{x\nu}^2 , \quad (46)$$

with M being the nucleon mass, and

$$\zeta_{xv}=(k_{xv}R_{xv})^2 \ . \tag{47}$$

This latter parameter (i.e., ζ_{xv}) may be considered as providing a qualitative measure of the relative importance of the corresponding nucleon-exchange term; that is, for exchange terms belonging to the same class, the one with the largest value of ζ_{xv} will generally be more important.

Because of the complicated structure of A_{xv}, B_{xv}, and C_{xv}, an analytical study of the characteristic quantities would be difficult. Hence, the procedure that we adopt to learn the systematic features of these quantities is to perform a number of numerical investigations involving many different choices of N_1, N_2, N_3, α, and γ. In Table I, we show, as an example, the values of R_{xv}, k_{xv}, E_{xv}, and ζ_{xv} obtained in the $\alpha+{}^6$Li case for a selected set of exchange modes and with $(N_1,N_2,N_3)=(4,2,4)$, $\alpha=0.367$ fm^{-2}, and $\gamma=0.18$ fm^{-2}. For comparison, we also list the values of these quantities (in parentheses) for the two-cluster case with $\gamma=\alpha=0.367$ fm^{-2}. In addition, it should be noted that, in this particular case, exchange terms having $x=1$ and 2 belong to class A with $C_{xv}<0$, while those having $x=3$ and 4 belong to class B with $C_{xv}>0$.

From Table I, one notes the following features.

(i) For a given value of x, the values of the characteristic weight ζ_{xv} are not only almost the same for all exchange modes, but also nearly equal to the value in the two-cluster limiting case with $y=1$. This indicates that, even when a strong degree of target clustering is present, the one-exchange term is dominant among class-A exchange terms, and the core-exchange term ($x=N_B$) is dominant among class-B exchange terms. Additionally, one can conclude from this observation that, in a qualitative sense, the overall importance of the effects of antisymmetrization seems to be rather insensitive to the clustering properties of the nuclei involved.

(ii) For each value of x, the characteristic ranges R_{xv} of various exchange modes vary over an appreciable range. In particular, it is seen that the variation is especially large in the core-exchange case. Also, one should note that, even though γ is appreciably smaller than α in this calculation, there do exist exchange modes for which the values of R_{xv} in the three-cluster formulation are quite close to the value of R_{xv} in the two-cluster limiting case.[13]

(iii) In the present calculation, where γ is realistically chosen to be smaller than α, the value of k_{xv} for each exchange mode is smaller than that in the two-cluster limiting case. This is of course related to the fact that, as γ becomes smaller than α, there is less probability for the

TABLE I. Values of characteristic quantities in the $\alpha+{}^6$Li case.

	Exchange mode v					R_{xv}	k_{xv}	E_{xv}	
x	x_{31}	x_{32}	x_{21}	x_{231}	x_{312}	(fm)	(fm^{-1})	(MeV)	ζ_{xv}
1	0	1	0	0	0	2.219	1.729	25.8	14.71
	0	0	1	1	0	2.111	1.755	26.6	13.73
	0	0	0	1	0	2.088	1.739	26.1	13.18
	1	0	0	0	0	2.060	1.805	28.1	13.82
			(two-cluster case; $y=1$)			(2.004)	(1.829)	(28.9)	(13.44)
2	0	2	0	0	0	1.179	0.935	7.6	1.21
	0	0	0	2	0	1.027	0.983	8.3	1.02
	0	1	0	1	0	0.994	0.993	8.5	0.97
	2	0	0	0	0	0.975	1.069	9.9	1.09
	1	0	1	0	1	0.921	1.080	10.1	0.99
	1	0	0	0	1	0.903	1.073	10.0	0.94
	1	1	0	0	0	0.901	1.099	10.4	0.98
	1	1	1	0	0	0.891	1.096	10.4	0.96
			(two-cluster case; $y=1$)			(0.882)	(1.110)	(10.7)	(0.96)
3	1	2	0	0	0	1.217	1.073	10.0	1.71
	2	0	1	1	0	1.171	1.138	11.2	1.78
	2	0	0	1	0	1.170	1.140	11.2	1.52
	3	0	0	0	0	1.167	1.044	9.4	1.49
	2	1	0	0	0	1.101	1.212	12.7	1.78
			(two-cluster case; $y=1$)			(1.100)	(1.212)	(12.7)	(1.78)
4	4	0	0	0	0	3.333	1.470	18.7	24.00
	2	0	0	2	0	2.965	1.652	23.6	24.00
	3	0	0	1	0	2.898	1.690	24.7	24.00
	2	2	0	0	0	2.543	1.926	32.1	24.00
	2	1	0	1	0	2.465	1.987	34.1	24.00
	3	1	1	0	0	2.393	2.059	36.6	24.28
	3	1	0	0	0	2.379	2.048	36.2	23.73
			(two-cluster case; $y=1$)			(2.334)	(2.099)	(38.1)	(24.00)

existence of high-momentum components in the A_1+A_2 relative motion.

Also, it is noted from Table I that the characteristic ranges for $x=1$ are much larger than those for $x=2$, and the characteristic ranges for $x=4$ are much larger than those for $x=3$. This indicates that, in the case where absorption is strong, a complete omission of two- and three-exchange terms should have only minor effects on the differential cross-section results over the whole angular region. Indeed, a three-cluster resonating-group study[14] of the $\alpha+{}^6$Li system did show that, with moderately strong absorption, this expectation turned out to be entirely valid.

The features mentioned above [i.e., items (i)–(iii)] have been found not only in the $\alpha+{}^6$Li example, but also in all other cases examined. Hence, even though we have investigated only a limited number of cases, we do believe that these features are general and can be considered as representing the essential properties of nucleon-exchange terms in a three-cluster formulation.

To gain further understanding of three-cluster exchange effects, we shall concentrate on a particularly important exchange mode, to be called mode $\tilde{\alpha}$, which is characterized by the fact that all nucleon-exchanges take place entirely between, say, clusters 1 and 3 (for simplicity, the discussion below will be restricted to the interesting case where $N_1 \geq N_3$). We single out this particular mode for a detailed consideration, simply because numerical studies in many systems have shown that, for the core-exchange case,[15] the corresponding equivalent potential acquires the longest characteristic range among all core-exchange modes. For example, consider the core-exchange terms in the $\alpha+{}^6$Li case described above. As is seen from Table I, the characteristic range for mode $\tilde{\alpha}$ is equal to 3.33 fm. On the other hand, for the mode where two nucleons are interchanged between clusters 1 and 3, and two other nucleons are interchanged between clusters 2 and 3, the characteristic range has an appreciably smaller value equal to 2.54 fm.

III. PROPERTIES OF EXCHANGE MODE $\tilde{\alpha}$

For the exchange mode $\tilde{\alpha}$, the coefficients $A_{x\alpha}$ and $C_{x\alpha}$ ($B_{x\alpha}=A_{x\alpha}$ for this mode) have much simpler expressions and are obtained by solving the following simple equations:

$$2A_{x\alpha}-C_{x\alpha}=\mu_0\alpha\frac{(2\mu_{13}-x)+(1-\xi)yx}{\xi x}\ , \tag{48}$$

$$2A_{x\alpha}+C_{x\alpha}=\mu_0\alpha\frac{\xi yx}{(2\mu_{13}-x)y+(1-\xi)x}\ , \tag{49}$$

where

$$\xi=\mu_{13}/\mu_0\ , \tag{50}$$

with

$$\mu_{13}=N_1N_3/(N_1+N_3)\ . \tag{51}$$

Using the resulting expression for $C_{x\alpha}$, one can then find the quantity x_α for which $C_{x\alpha}$ is equal to zero. The result is

$$x_\alpha=\mu_0\{(1-y)^2-\xi(1+y^2)+[(1-\xi)^2(1-y^2)^2+4\xi^2y^2]^{1/2}\}\left[\frac{1}{\xi}(1-\xi)(1-y)^2\right]^{-1}\ , \tag{52}$$

which depends rather complicatedly on μ_0, y, and ξ. However, in practical cases of interest where y is between about 0.5 and 1, one can show that x_α is close to μ_0. This is illustrated in the $\alpha+{}^6$Li case with $(N_1,N_2,N_3)=(4,2,4)$ and $\alpha+{}^{20}$Ne case with $(N_1,N_2,N_3)=(4,16,4)$ in Table II. Here one sees that, for $0.5\leq y\leq 1$, x_α/μ_0 differs from 1 by an insignificant amount.

The expressions of the characteristic range $R_{x\alpha}$ and the characteristic wave number $k_{x\alpha}$ turn out to be comparatively simple for this exchange mode and, consequently, an analytical analysis can be made. The results shows that for $x<x_\alpha$, $R_{x\alpha}$, and $k_{x\alpha}$ have largest values when $x=1$, and for $x>x_\alpha$, $R_{x\alpha}$, and $k_{x\alpha}$ have largest values when $x=N_B$. Therefore, even with target clustering taken into account, one can conclude that, among all exchange terms of mode $\tilde{\alpha}$, the one-exchange term has the largest influence for $x<x_\alpha$ and the core-exchange term has the largest influence for $x>x_\alpha$.

For the core-exchange mode $\tilde{\alpha}$ in the special case where $N_1=N_3$, there is one particularly interesting finding. This finding is that the characteristic weight $\zeta_{c\alpha}$ is independent of the clustering parameter y and is given by

$$\zeta_{c\alpha}=4N_AN_B/(N_A-N_B)^2\ . \tag{53}$$

TABLE II. Values of x_α/μ_0.

	System	
y	$\alpha+{}^6$Li	$\alpha+{}^{20}$Ne
0.2	1.118	1.099
0.4	1.035	1.041
0.5	1.020	1.025
0.6	1.011	1.014
0.8	1.002	1.003
1.0	1.000	1.000

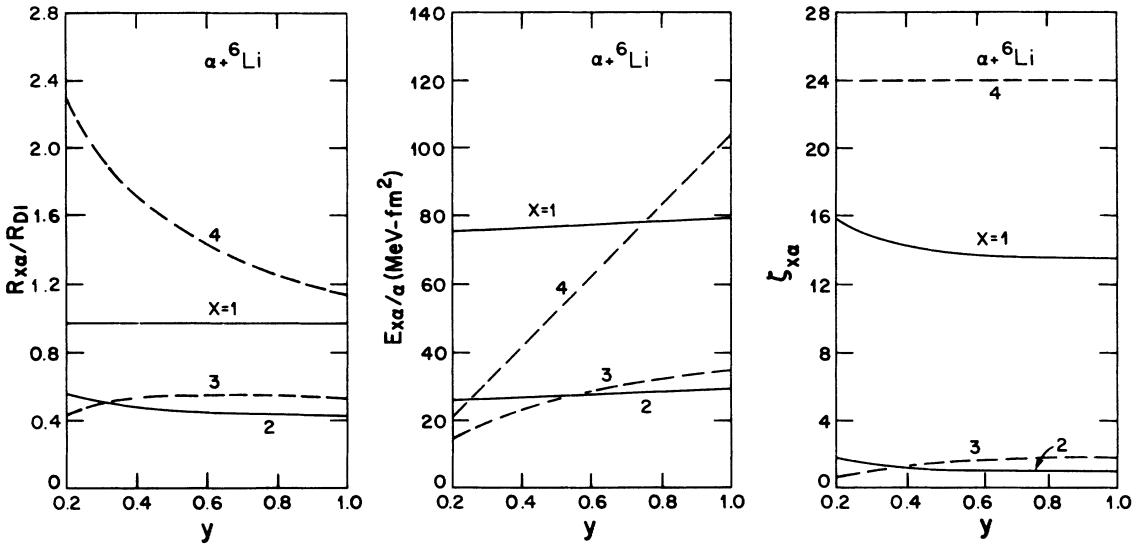

FIG. 1. $R_{x\alpha}/R_{D1}$, $E_{x\alpha}/\alpha$, and $\zeta_{x\alpha}$ as a function of the clustering parameter y in the $\alpha+{}^6$Li case. The solid and dashed curves represent results for class-A and class-B terms, respectively.

Quite obviously, this simple relation can be considered to support the viewpoint, expressed in Sec. II, concerning the overall importance of antisymmetrization effects with regard to the degree of target clustering.[16]

As numerical examples, we consider mode-$\tilde{\alpha}$ in the $\alpha+{}^6$Li case with $(N_1N_2,N_3)=(4,2,4)$ and $\alpha+{}^{20}$Ne case with $(N_1,N_2,N_3)=(4,16,4)$. The results are shown in Figs. 1 and 2, where the solid (for class-A terms) and dashed (for class-B terms) curves represent the values of $R_{x\alpha}/R_{D1}$, $E_{x\alpha}/\alpha$, and $\zeta_{x\alpha}$ as functions of the clustering parameter y for various values of x. In these figures, it is noted that the characteristic range $R_{x\alpha}$ is compared with the range parameter R_{D1} of the direct interaction, obtained by the use of a zero-range nucleon-nucleon potential interacting between nucleons in clusters 1 and 3; its expression is

$$R_{D1}=\left[\frac{N_2N_3+y(N_1+N_2)(2N_1N_3-N_1-N_3)}{y(N_1+N_2)N_1N_3\alpha}\right]^{1/2}. \tag{54}$$

We choose to show the behavior of $R_{x\alpha}/R_{D1}$ rather than the behavior of $R_{x\alpha}$ itself, simply because, as y becomes smaller, R_{D1} becomes larger and, thus, it is more interesting to know whether the equivalent exchange potential extends beyond the direct potential or not.

The interesting features contained in Figs. 1 and 2 are as follows.

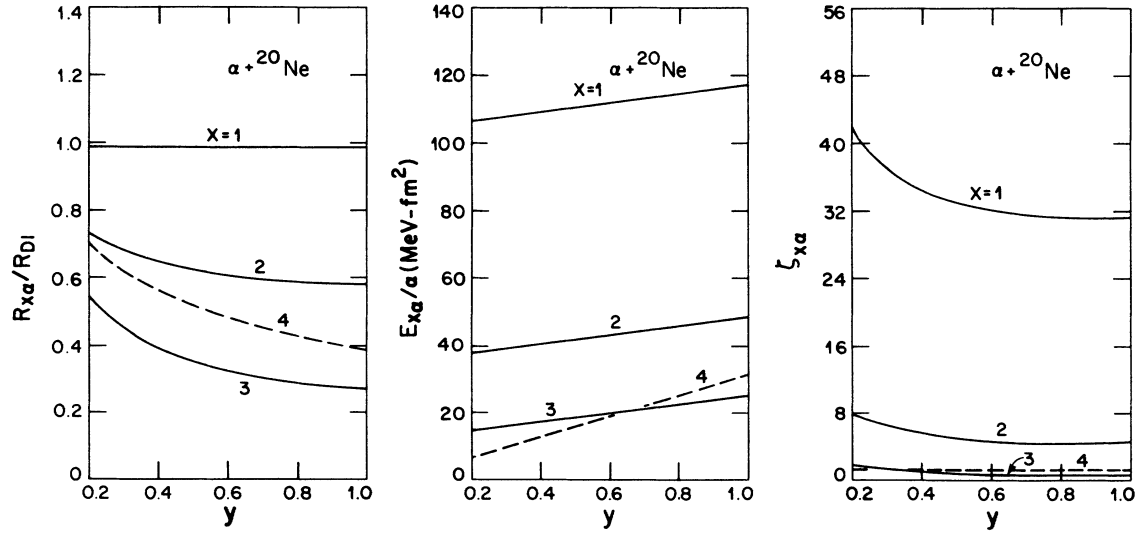

FIG. 2. Same as Fig. 1, except that the case under consideration is $\alpha+{}^{20}$Ne.

(i) For $x=1$, $R_{x\alpha}/R_{D1}$ is smaller than 1 and nearly independent of y. In contrast to this, it is noted that, for the core-exchange case ($x=4$), $R_{x\alpha}/R_{D1}$ increases rapidly as y decreases. In the $\alpha+{}^6$Li case where core exchange has important effects, the value of $R_{x\alpha}/R_{D1}$ at the realistic value of $y=0.5$ is equal to 1.55 which is substantially larger than the value of 1.12 in the two-cluster limiting case ($y=1$).

(ii) In the core-exchange case, $E_{x\alpha}/\alpha$ decreases rapidly as y decreases. Together with finding (i) above, this means that when there is a large degree of target clustering, the core-exchange contribution may be quite important even when strong absorption is present, but this contribution may rapidly diminish when the energy becomes high.

(iii) In the relevant range of y between 0.5 and 1, the characteristic weight $\zeta_{x\alpha}$ depends very insensitively upon y. In the $\alpha+{}^{20}$Ne case, for example, one sees that the three- and four-exchange terms have very small characteristic weight, for any value of y within this range. This is an indication that, in this case, one may simplify the resonating-group calculation by omitting these exchange terms and yet obtain a reasonable description of the behavior of this system.

IV. CONCLUSION

In this investigation, we examine the effects of antisymmetrization in the case where one of the nuclei involved exhibits properties of considerable clustering. The results, obtained by analyzing the exponential parts of the exchange-normalization kernel function in a three-cluster formulation, indicate that the overall importance of exchange effects seems to be only weakly affected by the degree of target clustering, and the one-exchange and core-exchange terms remain to be the terms which make the dominant contribution. For the one-exchange term, it is additionally found that its characteristics are rather insensitive to the clustering nature of the target nucleus. On the other hand, for the core-exchange term, there is an indication that its range may become quite large when the degree of clustering is appreciable, but its influence may decrease fairly rapidly with increasing energy.

The important task now is to examine the systematic features of experimental scattering data in order to verify the findings of our series of investigations. Unfortunately, this cannot be carried out at present. As is evident from our discussion, exchange effects are best studied by examining data which are performed at relatively high energies between about 25 and 50 MeV/nucleon, and which cover a wide angular range. At present, experimental results that meet these criteria exist only in very light systems, such as ^{3}He$+\alpha$ scattering where resonating-group calculations have been found to be highly successful.[17] For systems involving heavier nuclei, there are almost no data that are suitable for our purposes. In this respect, it would indeed be desirable to perform many careful measurements in the above-mentioned energy range, as such measurements would certainly help us to better comprehend the effects of nucleon exchange and, consequently, achieve an eventual understanding of the many intricate phenomena exhibited by nuclear systems.

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[16]It is important to remember that, in the core-exchange case, the characteristic range in mode $\tilde{\alpha}$ is substantially longer than that in any other mode. Therefore, especially in systems where absorption is strong, mode $\tilde{\alpha}$ may be considered as the dominant one among all core-exchange modes.
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