Reply to "Comment on 'Interpretation of relativistic dynamical effects in proton-nucleus scattering' "

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At issue is the identification of the physical mechanism that causes the distinction between the nucleon-nucleus scattering observables predicted by Dirac and Schrödinger implementations of the impulse approximation for the optical potential. We clarify the decisive role of an early variant of the Dirac approach in addressing this issue. Using this basis, recent conjectures concerning the physical mechanism are easily seen to be misleading. The elimination of apparent formal mathematical difficulties at high momenta, which arise from the noncompactness of local form factors, is also straightforward within this approach using well-known techniques in scattering theory.

In most treatments which have compared the relativistic (REL) and nonrelativistic (NR) approaches to nucleon-nucleus scattering, the on-shell input to the two approaches is effectively identical by construction; this is not generally the case for the corresponding off-shell behaviors. Because the off-shell extensions utilized in the two approaches can be quite different and are often highly model dependent, the essential physical distinction between the two approaches is obscured. Investigations into the sources of differences in the scattering observables generated from the full REL and from the NR treatments must then contend simultaneously with two distinct issues: (1) the different off-shell dependences in the particle-only space, and (2) the enlargement of the Hilbert space provided by the Dirac equation. We are in agreement with the authors of the preceding Comment that such off-shell differences must be treated with care. Similarly, the trivial arithmetical properties of Feynman propagators and their projections on the positive- and negative-energy Dirac subspaces are not at issue. However, in any theory the character of the interactions is not disconnected from that of the propagators. It is the interplay between the two which is required of numerical input and which results in qualitative predictions.

The central topic addressed in Refs. 1–4 is the identification of the dominant physical mechanism that causes the Dirac and Schrödinger implementations of the impulse approximation for the optical potential to yield different observables. In both Refs. 1 and 2, valid mathematical considerations related to the REL approach are discussed. We have not taken issue with the purely mathematical manipulations in either work. However, in both cases properties of propagators are developed in relative isolation from the associated interactions. Upon identifying mathematically valid characteristics associated with the REL propagators, both works then jump to the conclusion that these characteristics represent the root source or explanation of the REL successes. These unfounded presumptions are fostered by the neglect of the full behavior of interactions and by the presence of the off-shell ambiguity. After all, specific properties of propagators can be effectively emphasized, suppressed, or negated depending upon the corresponding behavior of the associated interaction. Thus, it is the inferences concerning the mechanistic source or explanation of the Dirac success which are drawn from the mathematical manipulations of Refs. 1 and 2 with which we strongly disagree.[3] In fact, the incorrectness of these inferences was already immediate from the developments of Refs. 4, which formed the basis for the discussion in Ref. 3. In Ref. 3 we analyzed the manner in which the physical conclusions from Refs. 1 and 2 are contrary to the studies we have made[4] and also contrary to several physical arguments. Here we attempt to clarify the analysis further.

The advantage of the REL treatment of Refs. 4 is that the off-shell ambiguity between the REL and NR approaches *does not exist.* The standard NR impulse approximation is employed for the construction of the optical potential in the NR case. For the REL case, the scalar-vector Dirac optical potential is *defined* in such a way that[4] when negative-energy intermediate states are excluded, that is when the theory is restricted to the positive-energy Dirac subspace; the preceding NR theory is recovered *identically.* This is achieved by defining the REL optical potential such that the interplay between this interaction and the positive-energy projection of the REL propagator simply reproduces the interplay between the NR interaction and propagator. One of the original motivations for the approach of Refs. 4 was to remove the ambiguity due to item (1) above in order to be able to investigate item (2) in as clean an environment as possible. This same consideration makes the approach of Refs. 4 ideally suited for assessing the merit of the inferences which were put forth in Refs. 1 and 2. Although the formulation of Refs. 4 adopts a slightly different perspective from other contemporary Dirac-equation approaches in order to achieve its objectives, it is representative of the characteristic successes and features generically common to such approaches. In what follows we adhere strictly to the framework of Refs. 4.

In Ref. 1 an inference is drawn which attributes the source of the Dirac successes to an accidental cancellation of "pathological" high-momentum behavior by the enlargement of the Hilbert space to include the negative-energy Dirac subspace. That this is not the case follows directly from Refs. 4. The source of the "undesirable" p^2 terms isolated by the manipulations of Ref. 1 is the Dirac spinors found in the spectral resolution of the positive-energy Dirac propagator. Reference 1 finds a resultant interaction[3] of the form $\boldsymbol{\sigma}\cdot\mathbf{p}W\boldsymbol{\sigma}\cdot\mathbf{p}$. However, in the analysis of Refs. 4 the REL optical potential sandwiched between these spinors (together with the rest of the REL propagator) simply reproduces the NR optical potential (and propagator) consisting of the usual central and spin-orbit forces. Because there is no "pathological" behavior in this standard NR case (as discussed shortly; in fact, only nearly on-shell intermediate momenta are numerically important for NR predictions[3]), none is implicit in the combination of the REL interaction and the positive-energy propagator. The "undesirable" high-momentum behavior apparently isolated in Ref. 1 from the positive-energy Dirac propagator is suppressed[5] upon combination with the REL optical potential. Therefore, *there is no "pathology"* in the positive-energy sector of the Dirac theory to be canceled: the explanation of the Dirac success cannot be that inferred in Ref. 1.

Similarly, in Ref. 2 the source of the REL successes is attributed to the cancellation of "spurious" short-range behavior in the positive-energy propagator by the negative-energy propagator. Although the assertion in Ref. 2 that the role of the negative-energy propagator is its cancellation of "undesirable" behavior of the positive-energy propagator is true by definition,[3] the further inference drawn in Ref. 2 that this is the dominant new physical mechanism introduced by the Dirac approach is merely a speculation and requires proof. For example, the two projections of the propagator couple differently to the interaction and their high-momentum components are not equally effective in the scattering problem. In fact, the approach of Refs. 4 renders the actuality completely transparent. Any such "undesirable" short-range behavior in the positive-energy propagator is canceled by the REL optical potential since the two together simply reproduce the NR theory. The role of the negative-energy subspace in producing the Dirac successes cannot be to cancel "anomalous" positive-energy propagator structure, since that has already been removed. It is in this regard that "anomalous" short-range structure in the positive-energy sector is nonexistent. Any numerically significant structure in the propagator is suppressed upon combination with the interaction. This is clear from the absence of such short-range structure in the NR case (as discussed in Ref. 3). Moreover, the simple numerical experiments of Ref. 3 provide concrete examples of the insensitivity to the details of the short-range behavior of the positive- and negative-energy Dirac propagators. One can only conclude that it does not matter whether short-range structure in the particle-only propagator is smoothed, removed, or ignored. At any rate, it is not physically appropriate to characterize the sought-after dominant relativistic mechanism in such terms.

Finally, the importance of treating the interaction (V) and the propagator $[G_0(E+i\varepsilon)]$ as a whole is also emphasized by the mathematical considerations needed to show the applicability of Fredholm (finite rank numerical) methods in, e.g., the NR theory. The key entity in such an analysis is the full scattering kernel $K=VG_0(E+i\varepsilon)$ and a sufficient (but *not* necessary) condition for the applicability of discrete numerical methods is that the L^2 norm of K, $\tau=\mathrm{Tr}(KK^\dagger)$, be finite.[6] Because for local potentials $\langle\mathbf{k}'|V|\mathbf{k}\rangle=V(q)$, $\mathbf{q}=\mathbf{k}'-\mathbf{k}$, it is useful in considering the trace τ to employ the momentum variables $\mathbf{q}$ and $\mathbf{k}$ rather than $\mathbf{k}'$ and $\mathbf{k}$. For local potentials, an apparent problem occurs due to the divergence of τ for physical scattering energies $(E+i0)$. However, this problem, which arises from a contour pinch, can be removed by a similarity transformation.[7] Additionally, despite the fact that $V(q)$ does not fall off at large $\mathbf{k}$ for fixed $\mathbf{q}$, there is no divergence of τ due to the behavior of the kernel at large k because the Green's functions provide the necessary falloff as $k\to\infty$. Thus τ is finite and Fredholm methods are applicable.[7] However, consider the standard (nonlocal) NR spin-orbit force where

$$V(\mathbf{k}',\mathbf{k})=W(q)\boldsymbol{\sigma}\cdot\mathbf{q}\times\mathbf{k}\ . \tag{1}$$

Even with the contour pinch handled by the a similarity transformation, one now finds that τ diverges due to a failure of the integrand to fall off as $k\to\infty$: the falloff due to the Green's functions is not sufficient to overcome the linearity in k found in Eq. (1). This is the essence of the high-momentum question raised in Ref. 1.[8] The root cause of this difficulty is simply that W does not decay in the momentum-space direction orthogonal to $\mathbf{q}$, the same property that causes local potentials to be noncompact operators.[9] This difficulty does not occur for nonlocal W with finite range in all directions in the (k',k) space. The fact that τ diverges impedes our establishing the applicability of finite matrix methods. Fortunately, however, the weaker condition that the nth power of the kernel has a finite L^2 norm is sufficient to establish convergence of finite rank representations of an equation in which only the first power of the kernel appears.[6] In the present circumstance, the case $n=2$ is sufficient to tame the divergence as $k\to\infty$. Thus, when the relevant interplay between propagation and interaction is properly taken into account, the high-momentum components of the spin-orbit operator are effectively suppressed by the structure of the scattering theory. Were that not the case, matrix methods would not converge. Apart from numerical verification, this forms the basis for our statement in Ref. 3 that only nearly on-shell intermediate momenta are important for our REL and NR predictions. High-momentum problems are only apparent and are of no real consequence either for the validity of the theory or for numerical predictions.

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