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Comments

Comments are short papers which comment on papers of other authors previously published in **Physical Review C**. *Each Comment should state clearly to which paper it refers and must be accompanied by a brief abstract.*

Comment on "Interpretation of relativistic dynamical effects in proton-nucleus scattering"

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We show that the criticism of the work of Thies and of Cooper and Jennings is based on a misunderstanding of the relation between the relativistic and nonrelativistic models, as well as the relation between coordinate space and momentum space calculations. The presence of structure in the particle-only propagator is shown by direct calculation.

It is claimed in a paper by Picklesimer and Tandy[1] that the particle-only part of the full Feynman propagator corresponds to the nonrelativistic (NR) limit. This is generally *not* true. For the NR model to correspond to the particle-only part of the propagator it is necessary for the T-matrix elements to correspond both on and off shell. All that is guaranteed by the fitting of the free nucleon-nucleon scattering data is that they agree on shell. The presence of explicit momenta in the lower component of the free Dirac spinor generates a stronger momentum dependence in the off-shell behavior for the relativistic case than is customarily assumed in the nonrelativistic case. To be specific, the positive-energy-only T matrix is parametrized as

$$t_D^{(+)}(k,k')=a(q)+\sigma_1\cdot k b(q)\sigma_2\cdot k'+\cdots \,, \tag{1}$$

where the neglected terms do not contribute to the optical potential in lowest order (except through exchange terms). The Schrödinger parametrization is usually taken to be

$$t_S(k,k')=f(q)+g(q)(\sigma_1+\sigma_2)\cdot L+\cdots \,, \tag{2}$$

where k and k' are the initial and final c.m. momenta, $q=k-k'$, and L is the total angular momentum operator ($q\times\nabla_q$). In the Dirac case it is the stronger off-shell momentum dependence [the explicit k and k' in Eq. (1)] that is responsible for most of the short-range structure in the particle-only propagation. This misunderstanding of the relationship between the particle-only and the nonrelativistic approach contributes to the confusion in the rest of Ref. 1.

We now come to the question of short-range structure in the particle-only part of the propagator. In Fig. 1 we show the full Feynman propagator and the particle-only propagator in coordinate space for a 200 MeV nucleon. The short-range structure in the particle-only propagator is immediately obvious (a similar figure is shown in Ref. 2). This is *precisely* the short-range structure that was discussed by Thies,[3] whose use of nonrelativistic kinematics resulted in its being approximated by a delta function. Contrary to the claims of Ref. 1 it is "existent." In view of Fig. 1, any claims that there is no short-range structure must be regarded as inoperative. We might note that this structure is a property of the propagator and not of the potentials.

The claim that the high-momentum components are suppressed by the nuclear form factor, while true, is not relevant. First we note that a coordinate-space delta function contributes equally for all momenta and not just for high momenta. However, when dealing with arguments based on ranges it is best to work in coordinate space. (It is always possible to Fourier transform from momentum to coordinate space and back again.) Consider the double scattering term

$$T^{(2)}=\int d^3x\, d^3x'\rho(x)tG(x-x')\rho(x')t \tag{3}$$

assuming that we have a zero-range t (this assumption can be relaxed). In order to get a momentum-space form factor from the density $\rho(x)$, it is necessary to integrate over all space. The suppression of high momentum mentioned in Ref. 1 comes about *only* because they have integrated over all ranges of $(x-x')$. Once that has been done any information concerning the values of $(x-x')$ which are important in double scattering has been lost. To get that information it is necessary to work in coordinate space (see Refs. 2 and 3 or even Ref. 4). It is not possible to simultaneously talk about coordinate-space ranges and momentum-space nuclear form factors. The

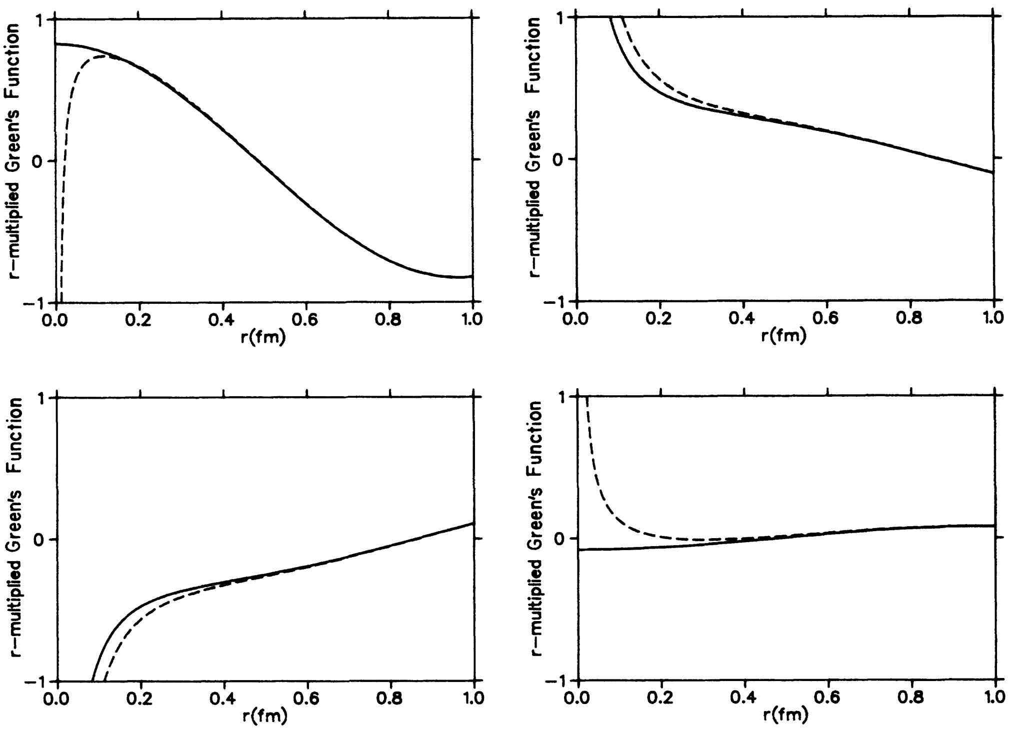

FIG. 1. The product of $r = |x - x'|$ times the Green's function, i.e., $rG(r)$, for 200 MeV protons. The solid line is the full propagator while the dashed line is the positive frequency only part of the propagator. The four graphs correspond to the four "block" components (Ref. 2) of the propagator.

criticsm of Thies[3] in Ref. 1 is thus based on a misunderstanding of coordinate-space calculations and their relation to momentum-space calculations.

A rather bizarre example is provided by Eq. (5) of Ref. 1 where it is argued that by starting from a well-behaved potential in momentum space $V(p,p')$, one can "pull out" a factor of p,p' and this can generate an illusion of a delta function. However, if $V(p,p')$ is well behaved and finite at $p = p' = 0$, then their $\hat{V}(p,p')$ will be singular at low values of p, and we have hardly improved the situation by "smoothing." The delta function referred to by Thies,[3] however, has the property that once it is removed there are no pathologies left behind. When working in momentum space this is not always obvious, which emphasizes the importance of using coordinate space when discussing short *distance* behaviors.

The claim that G_0^- cancels short-ranged contributions in G_0^+ is not a truism. Everyone seems to agree that G_0^- is short ranged. Given that, it then follows that either G_0^+ also has short-range structure that cancels that from G_0^- or else the full propagator, S_F, must have short-range structure. Reference 1 implies, from their discussion following Eq. (5), that there is no (spurious) short distance structure arising when just the positive energy Dirac equation is used, which implies that G_0^+ is smooth. The authors have, however, rearranged their equation so that the nucleon is no longer free between scatterings (it has a position-dependent effective mass) and thus contact with the multiple scattering series is lost. Our statement (see also Ref. 2), verified by Fig. 1, is that the structure is in G_0^+ and not S_F. (The other possibility is not eliminated *a priori*.) This means that G_0^- does not add additional structure but eliminates it. This seems to us a most significant point. We also emphasize again that this is a discussion of the propagator and will hold for a wide range of potentials and processes. The numerical calculation of Ref. 1 actually strongly supports our conclusion. They say that eliminating the cuts in G_0^+ gives a smooth result. This is precisely what we showed (Ref. 2). They also say that removing the short-range structure does not effect the final result if the full propagator is used. Again this is just what we saw analytically without the need for numerical calculations. Their numerical calculations show very clearly that G_0^- does indeed eliminate the short-range structure in G_0^+. For the authors to then conclude that "thus, SR details are insignificant and do not effect the role of G_0^- (Ref. 1) seems very strange since G_0^- is itself a short-ranged object, and its removal causes a significant change to the calculations.

The discussion of the role of short-range structure is not new with Dirac phenomenology but rather was taken from pion-nucleus scattering.[3] In the context of pion-nucleus scattering the origin and supression of delta functions has been discussed in *great* detail (see Ref. 4 and references therein). For example, the role of nucleon form factors and their interplay with short-range correlations was discussed in Ref. 5, and the possibility of other effects looking like the suppression of delta functions was discussed in Ref. 6. We recommend that anyone interested in how momentum dependent terms give rise to delta functions and how they can be suppressed study that literature. There is no need for the same mistakes to be made twice.

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