Total Gamow-Teller strength and effect of configuration mixing and proton-neutron correlation in the even-even *sd*-shell nuclei

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We have calculated the total Gamow-Teller strength $S_{\pm}$ in the even-even *sd*-shell nuclei making use of the sum rule technique. The ground state wave function is obtained from the shell model calculation in the full $(sd)^n$ space. It is shown that the proton-neutron correlation effect causes a considerable amount of reduction of the total Gamow-Teller strength. It is also shown that the reduction of the isospin component in S_- induced by the configuration mixing becomes more significant with increasing isospin.

At present, much data on the Gamow-Teller (GT) excitation strength has been accumulated by means of (p,n) and (n,p) reactions in a wide range of the periodic table. However, systematic experimental study of the GT excitation in the even-even *sd*-shell nuclei seems to be still in progress in the *sd*-shell nuclei.[1-6] Hence, it becomes significant to study systematically the GT excitation from the theoretical point of view. Some theoretical studies concerning the GT excitation in the sd-shell nuclei exist, e.g., by Wildenthal,[7] Bloom *et al.*,[8] and Knüpfer and Metsch.[9] Recently, the sum rule technique was applied by Macfarlane[10] to investigate the influence of ground state correlations upon the total GT strength in the doubly closed shell nucleus, ^{208}Pb, taking into account the ground state occupation number distribution.[11,12] The assumption adopted was that in the ground state the protons and neutrons each have angular momentum zero. Macfarlane pointed out that the approximation is likely to be good for doubly closed shell nuclei, dubious elsewhere.[10] In this paper we have studied the total GT strength, making use of the sum rule technique in the region of the doubly open shell nuclei, i.e., even-even *sd*-shell nuclei, and have investigated the effect of the configuration mixing and the proton-neutron correlation on the total GT strength. Furthermore, the effect of the configuration mixing on the isospin component of the total GT strength is investigated.

The total strength for the GT operator, $t_{\pm}\sigma$, can be given by

$$S_{\pm}=\sum B(\mathrm{GT})=\langle 0\,|\,(t_{\pm}\sigma)^{+}\cdot(t_{\pm}\sigma)\,|\,0\rangle$$

where $|\,0\rangle$ is the ground state of the target nucleus with the isospin T_0. The sum rule operator $\hat{S}_{\pm}\equiv(t_{\pm}\sigma)^{+}\cdot(t_{\pm}\sigma)$ is expressed as a sum of the term including the occupation number probability and the additional proton-neutron correlation term,

$$\hat{S}_{\pm}=\sum_{j,j'}|\,(j'\|\sigma\|j)\,|^{2}\hat{n}_{j}\binom{\mathrm{p}}{\mathrm{n}}\left[1-\hat{n}_{j'}\binom{\mathrm{n}}{\mathrm{p}}\right] - \sum_{j_1,j_2,j_1',j_2',k\neq 0}(-1)^{k}(2k+1)W(j_1j_1'j_2j_2';1k)(j_2'\|\sigma\|j_2)(j_1\|\sigma\|j_1')[u^{(k)}(j_2j_1)]_{\mathrm{p}}\cdot[u^{(k)}(j_1'j_2')]_{\mathrm{n}}\ , \quad (1)$$

where $\hat{n}_j\binom{\mathrm{p}}{\mathrm{n}}$ is the occupation number probability of the single-proton(neutron) orbital j and $u^{(k)}(j_2j_1)$ is the one-body unit tensor operator defined by

$$(j_2'\|u^{(k)}(j_2j_1)\|j_1')=\delta(j_1j_1')\delta(j_2j_2')\ .$$

The product term $\hat{n}_j\hat{n}_{j'}$ and the proton-neutron correlation term in Eq. (1) are common in both the operators $\hat{S}_+$ and $\hat{S}_-$, and we obtain the relation[13]

$$\hat{S}_{-}-\hat{S}_{+}=3\sum_{j}[\hat{N}_j(\mathrm{n})-\hat{N}_j(\mathrm{p})]$$

where $\hat{N}_j$ is the number operator of the single-particle orbital j, $\hat{N}_j\equiv(2j+1)\hat{n}_j$. It can be seen from Eq. (1) that if the ground state is composed only from the states

$$|\,J_{\mathrm{p}}=0^{+}\times J_{\mathrm{n}}=0^{+}\rangle\ ,$$

only the first term with the occupation number probability contributes to the total strength because the unit tensor operators for protons and neutrons included in the second term are not the scalar ($k\neq 0$) in the ordinary space. Therefore, the total strengths $S_{\pm}$ are affected through the second term by the states $|\,J_{\mathrm{p}}\times J_{\mathrm{n}}\rangle$ with $J\neq 0^{+}$ which are admixed into the ground state mainly by the proton-neutron correlation. For this reason, we call the second term in Eq. (1) the proton-neutron correlation term. This term, which Macfarlane neglected in his study of the total GT strength in the doubly closed shell nuclei,[10] becomes significant in the doubly open shell nuclei. In this study it is investigated as to what extent the configuration mixing within $(sd)^n$ shell model space, which contains the proton-neutron correlation, affects the total GT strength in the even-even *sd*-shell nuclei which include the nuclei with $N=Z$, $Z+2$, and $Z+4$. Only the wave functions of the ground state $|0\rangle$ are required to obtain the total GT strength, but the complicated wave functions of the excited 1^+ states are not necessary in our treatment. The ground state wave functions are obtained by the shell model calculation in the full $(0d_{5/2}, 1s_{1/2}, 0d_{3/2})^{A-16}$ configuration space and we adopt the effective interaction of Wildenthal.[7]

The calculated results for the total strength $S_\pm$ are displayed in Table I. As the lowest approximation, we consider the lowest configuration limit (LC). The ground state is assumed to be in the lowest configuration state $|\mathrm{LC}\rangle$ with the seniority 0, e.g., for ^{32}Si,

$$|\mathrm{LC}\rangle = |\pi(d_{5/2})^6 J_\mathrm{p}=0 \times \nu(d_{5/2})^6(s_{1/2})^2(d_{3/2})^2 J_\mathrm{n}=0\rangle ,$$

and we get the total strength as an expectation value of $\hat{S}_\pm$ in this state,

$$S_\pm^{\mathrm{LC}} = \sum_{j,j'} |(j'\|\sigma\|j)|^2 n_j^{\mathrm{LC}}(^{\mathrm{p}}_{\mathrm{n}})[1-n_{j'}^{\mathrm{LC}}(^{\mathrm{n}}_{\mathrm{p}})] \tag{2}$$

where n_j^{LC} is the occupation number probability in the $|\mathrm{LC}\rangle$ state. Only the first term in Eq. (1) contributes to the $S_\pm^{\mathrm{LC}}$ because the ground state $|\mathrm{LC}\rangle$ consists of only the states $|J_\mathrm{p}=0^+ \times J_\mathrm{n}=0^+\rangle$, as is described in the previous paragraph. The values for the LC limit are shown in the third and sixth columns in Table I where the contribution of the spin-flip transition, $d_{5/2}\to d_{3/2}$, for S_- is given in parentheses. The non-spin-flip transitions ($d_{5/2}\to d_{5/2}$, $s_{1/2}\to s_{1/2}$, and $d_{3/2}\to d_{3/2}$) also exist in $S_\pm^{\mathrm{LC}}$ and this is the different point from the total $M1$ strength where the non-spin-flip transitions are forbidden in the LC limit.[14-16]

In the next step, we consider the occupation number (ON) approximation in which the $\hat{n}_j$ is replaced by the expectation value $\langle \hat{n}_j\rangle$ in the fully diagonalized ground state $|0\rangle$, and the proton-neutron correlation term is neglected although there exist the states $|J_\mathrm{p}\times J_\mathrm{n}\rangle$ with $J\neq 0$ in the ground state $|0\rangle$. The total strength in the ON approximation is accordingly given by

$$S_\pm^{\mathrm{ON}} = \sum_{j,j'} |(j'\|\sigma\|j)|^2 \langle n_j(^{\mathrm{p}}_{\mathrm{n}})\rangle[1-\langle n_{j'}(^{\mathrm{n}}_{\mathrm{p}})\rangle] . \tag{3}$$

The ON approximation corresponds to Macfarlane's approximation of the GT sum rule in Ref. 10. By the ON approximation, the effect of the configuration mixing is partly taken into account through the change of the distribution of the occupation probability of the single particle orbitals. $S_\pm^{\mathrm{ON}}$ is shown in the fourth and seventh columns in Table I and again the contributions of the $d_{5/2}\to d_{3/2}$ transition are in the parentheses. As is seen in Table I, LC and ON gives not so different total strengths from each other except for ^{30}Si, ^{28}Mg, and ^{32}Si, in which the $s_{1/2}\to s_{1/2}$ transition contributes to the LC limit to a considerable extent. Consequently, the ON approximation does not reduce the LC value although the occupation number distribution is considerably changed by the configuration mixing, which can be seen from the reduction of the $d_{5/2}\to d_{3/2}$ contribution in the S_- going from LC to ON, e.g.,

$$S_-^{\mathrm{ON}}(d_{5/2}\to d_{3/2}) \simeq 0.64 S_-^{\mathrm{LC}}(d_{5/2}\to d_{3/2})$$

on the average in the $N=Z$ nuclei. On the other hand, the strengths of the transitions other than $d_{5/2}\to d_{3/2}$ are enhanced, filling up the reduction of the strength of the $d_{5/2}\to d_{3/2}$ transition and therefore $S_\pm^{\mathrm{ON}}$ have nearly the same values with $S_\pm^{\mathrm{LC}}$.

The full results, $S_\pm^{\mathrm{full}} \equiv \langle 0|\hat{S}_\pm|0\rangle$, which is the expectation value of the full expression of $\hat{S}_\pm$ in the fully diagonalized ground state $|0\rangle$, are shown in the fifth and eighth columns in Table I. The contribution of the

TABLE I. Total Gamow-Teller strength $S_\pm$ for the lowest configuration limit (LC), occupation number approximation (ON), and full calculation result (Full). The contributions of the transition $d_{5/2}\to d_{3/2}$ are also shown in the parentheses for S_-.

			S_-			S_+	
T_0	Nuclei	LC	ON	Full	LC	ON	Full
0	^{20}Ne	5.07 (3.20)	4.99 (1.80)	0.55 (1.81)			
	^{24}Mg	8.27 (6.40)	8.22 (4.11)	2.33 (4.14)			
	^{28}Si	9.60 (9.60)	9.71 (6.15)	3.89 (6.32)			
	^{32}S	9.60 (9.60)	8.89 (6.16)	4.00 (6.44)			
	^{36}Ar	5.40 (4.80)	5.25 (3.43)	2.10 (3.44)			
1	^{18}O	6.00 (3.20)	6.00 (2.53)	6.00 (2.53)	0.0	0.0	0.0
	^{22}Ne	10.13 (6.40)	10.23 (4.85)	6.54 (4.86)	4.13	4.23	0.54
	^{26}Mg	12.40 (9.60)	12.40 (6.84)	7.78 (7.00)	6.40	6.40	1.78
	^{30}Si	15.60 (9.60)	13.09 (7.76)	8.52 (7.97)	9.60	7.09	2.52
	^{34}S	10.80 (9.60)	10.56 (7.53)	7.59 (7.77)	4.80	4.56	1.59
	^{38}Ar	6.00 (4.80)	6.00 (4.51)	6.00 (4.51)	0.0	0.0	0.0
2	^{20}O	12.00 (6.40)	12.00 (5.57)	12.00 (5.57)	0.0	0.0	0.0
	^{24}Ne	15.20 (9.60)	15.27 (7.81)	12.51 (7.89)	3.20	3.27	0.51
	^{28}Mg	18.40 (9.60)	16.60 (8.33)	12.63 (8.38)	6.40	4.60	0.63
	^{32}Si	16.80 (9.60)	15.56 (8.67)	12.75 (8.73)	4.80	3.56	0.75
	^{36}S	12.00 (9.60)	12.00 (9.03)	12.00 (9.03)	0.0	0.0	0.0

$d_{5/2} \rightarrow d_{3/2}$ transition is again listed in the parentheses. We can see a large amount of reduction compared with the ON approximation. The reduction factors are given by $S_-^{\rm full}/S_-^{\rm ON} = 0.33$, 0.66, and 0.80 for the nuclei with $N=Z$, $Z+2$, and $Z+4$, respectively, and $S_+^{\rm full}/S_+^{\rm ON} = 0.28$ and 0.17 for those with $N=Z+2$ and $Z+4$ on the average. In these averages, the equivalent particle systems in the model space $(sd)^n$, i.e., ^{18,20}O, ^{38}Ar, and ^{36}S, are not included, where the ground state is evidently $|J_{\rm p}=0^+ \times J_{\rm n}=0^+\rangle$ and consequently $S_\pm^{\rm ON}$ is equivalent to $S_\pm^{\rm full}$. It is noted that the total strength of S_+ is more reduced than that of S_- because S_- is larger than S_+ generally in the nuclei with $N>Z$ and the amount of the reduction is common to the both strengths S_+ and S_- due to the symmetry $S_- - S_+ = 3(N-Z)$. In the full case, the contribution of the transition $j \rightarrow j'$, i.e., the term with $|(j'\|\sigma\|j)|^2$, is included not only in the first term in Eq. (1) but also in the second term with $j_1=j_2=j$ and $j'_1=j'_2=j'$. It can be seen in Table I that the contribution of the $d_{5/2} \rightarrow d_{3/2}$ transition to $S_-^{\rm full}$ is almost equal to or slightly larger than those to $S_-^{\rm ON}$. Therefore, the diagonal contribution, i.e., $|(j'\|\sigma\|j)|^2$, can be well approximated by $S_-^{\rm ON}$. On the other hand, the interference terms of the different single particle transitions $j_1 \rightarrow j'_1$ and $j_2 \rightarrow j'_2$, i.e., the terms with

$$(j'_2\|\sigma\|j_2)\cdot(j_1\|\sigma\|j'_1) \quad (j_1 \neq j_2 \text{ or } j'_1 \neq j'_2) ,$$

contribute to the total GT strength in a destructive way through the admixture of the proton-neutron correlated states, $|J_{\rm p} \times J_{\rm n}\rangle$, with $J \neq 0$ into the ground state, e.g., $|J_{\rm p}=2^+ \times J_{\rm n}=2^+\rangle$ which is considered to be most strongly admixed to the ground state by the quadrupole-type correlation between protons and neutrons.

So far, the effect of the configuration mixing, particularly the proton-neutron correlation effect on the total GT strength $S_\pm$, has been investigated by comparing the LC, ON, and full calculations. Here the isospin components of the total strength S_- are estimated and the effect of the configuration mixing, including proton-neutron correlation on these components, is investigated. The total GT strength S_- is composed of the isospin components $S_-(T)$ with $T=T_0-1$, T_0, and T_0+1:

$$S_- = S_-(T_0-1) + S_-(T_0) + S_-(T_0+1) .$$

These isospin components are defined by

$$S_-(T) \equiv \sum_f |\langle f;T\|t_-\sigma\|0\rangle|^2 \qquad (T=T_0-1,\ T_0,\ T_0+1) ,$$

and they are expressed as linear combinations of $S_\pm$ and S_0,[14,17,18] e.g., for $N=Z+2$ nuclei,

$$S_-(T=0) = S_- - \tfrac{1}{2}S_0 + \tfrac{1}{3}S_+ , \tag{4}$$

$$S_-(T=1) = \tfrac{1}{2}S_0 - \tfrac{1}{2}S_+ , \tag{5}$$

$$S_-(T=2) = \tfrac{1}{6}S_+ . \tag{6}$$

Here S_0 is the total strength of the $M1$-type operator, $\tau_0\sigma$, and it is expressed similarly to Eq. (1),

$$\begin{aligned}\hat{S}_0 = &\sum_{j,j',a={\rm p,n}} |(j'\|\sigma\|j)|^2 \hat{n}_j(a)[1-\hat{n}_{j'}(a)] \\ &- \sum_{j_1,j_2,j'_1,j'_2,k\neq 0,a={\rm p,n}} (-1)^k(2k+1)(j'_2\|\sigma\|j_2)(j_1\|\sigma\|j'_1)W(j_1j'_1j_2j'_2;1k)[u^{(k)}(j_2j_1)]_a\cdot[u^{(k)}(j'_1j'_2)]_a \\ &-2\sum_{j_1,j_2,j'_1,j'_2} (j_2\|\sigma\|j'_2)(j'_1\|\sigma\|j_1)[u^{(1)}(j_2j'_2)]_{\rm p}\cdot[u^{(1)}(j'_1j_1)]_{\rm n} .\end{aligned} \tag{7}$$

The contribution of the first term corresponds to the ON approximation, $S_0^{\rm ON}$, as in the case of $S_\pm^{\rm ON}$. Of the remainder, the third term represents the proton-neutron correlation term. This term does not contribute significantly to S_0. On the other hand, although the second term does not correspond to the proton-neutron correlation, it is significantly influenced by the configuration mixing and has a large negative value. The calculated results for the isospin components with $T=0$, 1, and 2 for the $N=Z+2$ nuclei with $T_0=1$, are shown in Fig. 1. It can be seen that the full results of $S_-(T=0)$ component are not so different from those of the ON approximation, while the full values for $S_-(T=1)$ are largely reduced from the values of the ON approximation. Moreover, $S_-(T=2)$ is seen to be more reduced than $S_-(T=1)$ components, although the magnitude of $S_-(T=2)$ is so small. The reduction factors, $S_-(T)^{\rm full}/S_-(T)^{\rm ON}$, are 0.88, 0.50, and 0.28 for $T=0$, 1, and 2 components, respectively, on the average over ^{22}Ne, ^{26}Mg, ^{30}Si, and ^{34}S.

This isospin dependence of the reduction has been recently pointed out by Madey *et al.*[19] in the study of the GT strength distribution deduced from the reaction ^{26}Mg(p,n)^{26}Al. From the shell model calculation of the GT strength distribution, they pointed out that the reduction of the GT strength brought about by the effect other than the configuration mixing within the $(sd)^n$ model space is more reliably investigated by examining the $T=T_0-1$ strength, because of the smallness of the configuration mixing effect in the $T=T_0-1$ strength. In our study with the sum rule technique, we can qualitatively understand this isospin dependence of the configuration mixing effect, defining Δ_ν by $S_\nu^{\rm full} = S_\nu^{\rm ON} + \Delta_\nu (\nu = -, 0, +)$, where Δ_ν denotes the effect of the

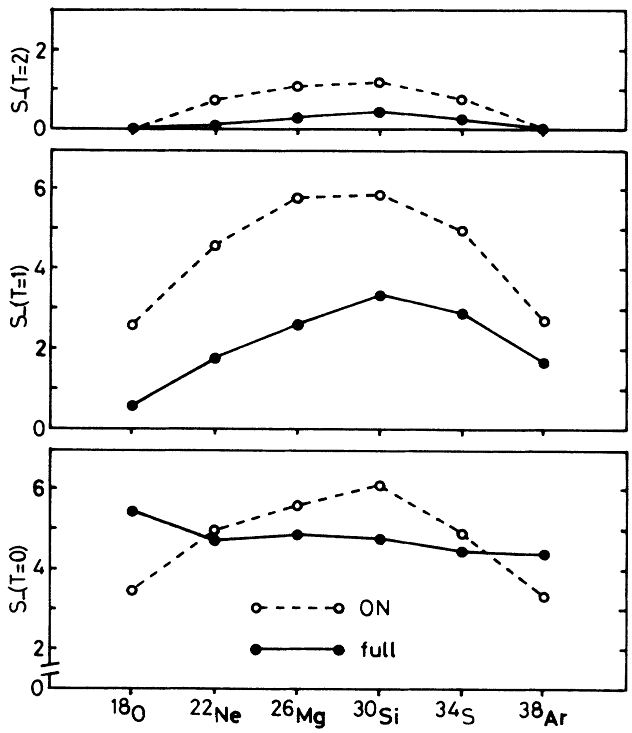

FIG. 1. Mass number dependence of the isospin components $S_{-}(T)$ ($T=0,1,2$) for $N=Z+2$ nuclei. The ON (open circles) and the full values (solid circles) are connected by broken and solid lines, respectively.

configuration mixing which cannot be expressed by means of the change of the occupation probability. The amount of the configuration mixing effect for $S_{-}(T=0)$, $S_{-}(T=1)$, and $S_{-}(T=2)$ is expressed as $\Delta_{-}-\frac{1}{2}\Delta_0+\frac{1}{3}\Delta_{+}$, $\frac{1}{2}\Delta_0-\frac{1}{2}\Delta_{+}$, and $\frac{1}{6}\Delta_{+}$, respectively. In the expression $\Delta_{-}-\frac{1}{2}\Delta_0+\frac{1}{3}\Delta_{+}$ for the component $S_{-}(T=0)$, three terms almost cancel out each other. On the other hand, for $S_{-}(T=1)$, the cancellation between $\frac{1}{2}\Delta_0$ and $-\frac{1}{2}\Delta_{+}$ is not so remarkable, and there does not exist any cancellation for $S_{-}(T=2)$. These cancellations are typically illustrated as, e.g., for ^{22}Ne, $-0.74+0.94-0.25=-0.05$, $-1.02+0.40=-0.62$, and -0.87, normalized by $S_{-}^{\mathrm{ON}}(T)$ for $T=0$, 1, and 2 components, respectively. For this reason, the effect of the configuration mixing increases as the isospin increases. In other words, the reduction factor due to the configuration mixing decreases with decreasing isospin. This tendency of the isospin dependence also holds in the $N=Z+4$ nuclei with $T_0=2$, i.e., $S_{-}(T)^{\mathrm{full}}/S_{-}(T)^{\mathrm{ON}}=0.89$, 0.53, and 0.17 for $T=1$, 2, and 3, respectively, on the average over ^{24}Ne, ^{28}Mg, and ^{32}Si, and the reason for this tendency is the same as for the $N=Z+2$ nuclei. Hence, from the above discussion, the isospin-dependent property of the reduction factor of the component $S_{-}(T)$ is found to be caused by the configuration mixing which cannot be expressed by means of the change of the occupation probability.

In conclusion, we have calculated the total GT strength in the even-even *sd*-shell nuclei with $N=Z$, $Z+2$, and $Z+4$. It has been shown that the ON approximation for $S_{\pm}$ does not reduce the LC limit values although the effect of the configuration mixing is partly taken into account in the ON approximation through the change of the occupation probabilities of the single particle orbitals. On the other hand, in the full calculation, the proton-neutron correlation has been found to cause the admixture of the $|J_p \times J_n\rangle$ component with $J \neq 0$ into the ground state, resulting in the significant reduction of the total GT strength. It has also been found that the amount of the reduction by the configuration mixing becomes more significant with increasing isospin.

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