Fermion calculations in the boson space using the Dyson boson mapping

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An algorithm for obtaining a recursive algebraic expression for the various expansion coefficients appearing in the physical boson basis states is presented, using the Dyson boson mapping. It is observed that only the first few terms of the physical boson basis states, expressed in the present form, contribute to the matrix element of the Hamiltonian. Explicit numerical calculations have been performed for Ni and Sn isotopes in the obtained physical boson basis states. The pairing Hamiltonian has been used as the model interaction. The results are more than 90% in agreement with exact shell model calculations.

I. INTRODUCTION

In recent years, many attempts have been made to perform the nuclear shell model calculations in the boson representation.[1-8] The main objective of these studies is to formulate a microscopic theory that is capable of relating the parameters of the interacting boson model (IBM) (Ref. 9) Hamiltonian to the nuclear single particle energies as known from the experiment and the effective shell model nucleon-nucleon interaction. These microscopic formulations also referred to as boson expansion theories (BET's), mainly involve the expansion of bifermion operators in terms of bosons. The expansion coefficients are determined through the requirement that the commutation properties are mapped correctly. The explicit mapping is required due to the fact that the bifermions do not obey the simple boson commutation relations. Both unitary and nonunitary mappings have been used. The unitary mappings are mainly infinite boson expansions. The nonunitary mapping of the bifermion operators is achieved through the generalized Dyson boson mapping (DBM) (Refs. 3 and 7) and leads to a finite number of terms in the boson space. This mapping has been employed in the present work.

Due to the finite nature of the DBM, any fermion operator written in terms of bifermions has a finite number of terms in the boson space. For carrying out the explicit calculations in the boson space one needs to construct the physical boson basis (PBB) states.[7] The PBB states are obtained by replacing each bifermion operator in the fermion basis by the corresponding boson image using the DBM. An iterative procedure for obtaining the PBB states, recently formulated by Li,[10] has been found to be inadequate.[11]

In the present work, a general procedure for constructing the PBB states is outlined. An explicit recursive algebraic expression for the various expansion coefficients appearing in the PBB state for the ground state has been obtained. This PBB state, presented in Sec. III, is expressed in the form such that only the first few terms contribute to the matrix element. Therefore, in the present form one needs to construct the expressions for only the first few coefficients appearing in the PBB states. The present formulation has been applied to Sn and Ni isotopes. The numerical results [ground state energies (E_0) and occupation probabilities (v_a^2)] are presented in Sec. IV. The results are more than 90% in agreement with the exact shell model calculations. For completeness, the DBM is briefly reviewed in Sec. II.

II. DYSON BOSON MAPPING

The Dyson boson mapping in terms of coupled operators is written as

$$A^{\dagger}_{JM}(ab)\rightarrow\bar{b}^{\,\dagger}_{JM}(ab)$$

$$\equiv b^{\dagger}_{JM}(ab)-\sum_{\substack{cdJ_1J_2\\J_3J_4}}\hat{J}_1\hat{J}_2\hat{J}_3\hat{J}_4\begin{Bmatrix} j_a & j_c & J_1\\ j_b & j_d & J_2\\ J & J_3 & J_4\end{Bmatrix}(-1)^{J_3+J_4+J}[(b^{\dagger}_{J_1}(ac)\times b^{\dagger}_{J_2}(bd))_{J_4}\times\tilde{b}_{J_3}(cd)]_{JM}\ , \tag{1a}$$

$$A_{JM}(ab)\rightarrow\bar{b}_{JM}(ab)\equiv b_{JM}(ab)\ , \tag{1b}$$

$$(C^{\dagger}_a\tilde{C}_b)\rightarrow\bar{E}_{JM}(ab)\equiv-\sum_{cJ_1J_2}\hat{J}_1\hat{J}_2(-1)^{j_a+j_c+J+J_2}\begin{Bmatrix} j_a & j_c & J_1\\ J_2 & J & j_b\end{Bmatrix}[b^{\dagger}_{J_1}(ac)\times\tilde{b}_{J_2}(bc)]_{JM}\ , \tag{1c}$$

where

$$\tilde{C}_a=(-1)^{j_a-m_a}C_{j_a-m_a}\ , \tag{2}$$

$$\hat{J}_1=\sqrt{2J_1+1}\ ,$$

$$A^{\dagger}_{JM}(ab)=(C^{\dagger}_aC^{\dagger}_b)_{JM}\ ,$$

$$b^{\dagger}_{JM}(ab)=\sum_{m_am_b}\begin{bmatrix} j_a & j_b & J\\ m_a & m_b & M\end{bmatrix} b^{\dagger}_{j_am_aj_bm_b}\ , \tag{3}$$

$$b_{JM}(ab)=[b^{\dagger}_{JM}(ab)]^{\dagger}\ ,$$

and the symbols { } and [ ] denote a $6j$ and $3j$ coupling coefficient, respectively. The operators $C^{\dagger}_a$ (C_a) are the fermion single particle creation (annihilation) operators. The operators $b^{\dagger}_{JM}(ab)$ $[b^{\dagger}_{J'M'}(a'b')]$ are the coupled boson creation (annihilation) operators and satisfy

$$\begin{aligned}[b_{JM}(ab),b^{\dagger}_{J'M'}(a'b')] &=\delta_{JJ'}\delta_{MM'}(\delta_{aa'}\delta_{bb'}-(-1)^{j_a+j_b+J}\delta_{ab'}\delta_{ba'})\ ,\\ [b^{\dagger}_{JM}(ab),b^{\dagger}_{J'M'}(a'b')]&=[b_{JM}(ab),b_{J'M'}(a'b')]=0\ .\end{aligned} \tag{4}$$

In order to have a one to one correspondence with the phenomenological bosons, the collective operators are defined as

$$b^{\dagger\mu}_{JM}=\tfrac{1}{2}\sum_{a\le b}X^{\mu}_{JM}(ab)b^{\dagger}_{JM}(ab)\ . \tag{5}$$

These collective bosons satisfy the following commutation relations:

$$\begin{aligned}&[b_{\mu},b^{\dagger}_{\nu}]=\delta_{\mu\nu}\ ,\\ &[b^{\dagger}_{\mu},b^{\dagger}_{\nu}]=[b_{\mu},b_{\nu}]=0\ ,\end{aligned} \tag{6}$$

where the collective coefficients X are constrained to satisfy the orthogonality relation,

$$\sum_{ab}X^{\mu}_{JM}(ab)X^{\nu}_{JM}(ab)=2\delta_{\mu\nu}\ . \tag{7}$$

The quantum numbers $\nu,\rho,\ldots$ denote the eigenstates for a particular J value. In the present work the ground state boson is denoted as $s^{\dagger}_0(b^{\dagger 0}_{00})$.

Using the collective operators defined above, the DBM [Eqs. (1a)–(1c)] in the collective representation is

$$\bar{b}^{\dagger\mu}_{JM}\equiv b^{\dagger\mu}_{JM}-\sum_{\substack{J_1J_2J_3J_4\\ \nu\rho\sigma}}\tau^{J_1^{\nu}J_2^{\rho}(J_4)}_{J^{\mu}J_3^{\sigma}(J_4)}[(b^{\dagger\nu}_{J_1}\times b^{\dagger\rho}_{J_2})J_4\times\tilde{\bar{b}}^{\sigma}_{J_3}]_{JM}\ , \tag{8a}$$

$$\bar{b}^{\mu}_{JM}\equiv b^{\mu}_{JM}\ , \tag{8b}$$

$$\bar{E}_{JM}(ab)\equiv-\sum_{\substack{J_1J_2c\\ \mu\nu}}\hat{J}_1\hat{J}_2(-1)^{j_a+j_c+J+J_2}X^{\mu}_{J_1}(ac)X^{\nu}_{J_2}(bc)\begin{Bmatrix} j_a & j_c & J_1\\ J_2 & J & j_b\end{Bmatrix}[b^{\dagger\mu}_{J_1}\times\tilde{\bar{b}}^{\nu}_{J_2}]_{JM}\ , \tag{8c}$$

where

$$\tau^{J_1^{\nu}J_2^{\rho}(J_4)}_{J^{\mu}J_3^{\sigma}(J_4)}=\tfrac{1}{2}\sum_{abcd}\hat{J}_1\hat{J}_2\hat{J}_3\hat{J}_4\begin{Bmatrix} j_a & j_b & J_1\\ j_d & j_c & J_2\\ J_3 & J & J_4\end{Bmatrix}X^{\nu}_{J_1}(ab)X^{\rho}_{J_2}(dc)X^{\sigma *}_{J_3}(ad)X^{\mu *}_{J}(bc) \tag{9}$$

$$\begin{aligned}&=(-1)^{J_2+J_1+J_4}\tau^{J_2^{\rho}J_1^{\nu}(J_4)}_{J^{\mu}J_3^{\sigma}(J_4)}=\frac{\hat{J}_3}{\hat{J}}(-1)^{J_3+J_4+J}\tau^{J_1^{\nu}J_2^{\rho}(J_4)}_{J_3^{\sigma}J^{\mu}(J_4)}\\ &=\frac{\hat{J}_1}{\hat{J}}(-1)^{J_1+J_2+J_3+J}\tau^{J^{\mu}J_3^{\sigma}(J_4)}_{J_1^{\nu}J_3^{\rho}(J_4)}\ .\end{aligned} \tag{10}$$

It is to be noted that the DBM is (a) finite and (b) nonunitary. Due to (a), any fermion operator written in terms of bifermion operators will have a finite number of terms in the boson space.

For the evaluation of matrix elements one needs a suitable set of basis states. The construction and the use of the PBB states are discussed in the following sections.

III. BASIS STATES

The physical boson basis states are obtained by replacing each bifermion operator appearing in the fermion basis states by its corresponding boson image using Eqs. (8a)–(8c). The procedure for constructing the PBB states is outlined and is illustrated for the ground state, $(\bar{s}^{\dagger}_0)^p\,|\,0)$. First we write $\bar{s}^{\dagger}_0$ in the following form:

$$
\begin{aligned}
\bar{s}_0^\dagger \equiv{}& s_0^\dagger - \tau_0^0 (s_0^\dagger)^2 \tilde{s}_0 - 2\sum_{\nu\neq 0} \tau_0^\nu s_0^\dagger s_\nu^\dagger \tilde{s}_0 - \sum_{\nu\rho\neq 0} \tau^{\nu\rho} s_\nu^\dagger s_\rho^\dagger \tilde{s}_0 - \sum_{\sigma\neq 0} \tau_{0\sigma}^0 (s_0^\dagger)^2 \tilde{s}_\sigma \\
&-2\sum_{\nu\sigma\neq 0} \tau_\sigma^\nu s_\nu^\dagger s_0^\dagger \tilde{s}_\sigma - \sum_{\nu\rho\sigma\neq 0} \tau_{0\sigma}^{\nu\rho} s_\nu^\dagger s_\rho^\dagger \tilde{s}_\sigma - \sum_{J_1\neq 0} \tau^{J_1} (b_{J_1}^{\dagger 0} \times b_{J_1}^{\dagger 0})_0 \tilde{s}_0 - 2\sum_{\nu J_1\neq 0} \tau^{J_1^\nu} (b_{J_1}^{\dagger\nu} \times b_{J_1}^{\dagger 0})_0 \tilde{s}_0 \\
&- \sum_{\nu\rho J_1\neq 0} \tau^{J_1^\nu J_1^\rho} (b_{J_1}^{\dagger\nu} \times b_{J_1}^{\dagger\rho})_0 \tilde{s}_0 - \sum_{J_1\sigma\neq 0} \tau^{J_1 J_1} (b_{J_1}^{\dagger 0} \times b_{J_1}^{\dagger 0})_0 \tilde{s}_\sigma - 2\sum_{\nu\rho J_1\neq 0} \tau_{0\sigma}^{J_1^\nu J_1^\rho} (b_{J_1}^{\dagger\nu} \times b_{J_1}^{\dagger 0})_0 \tilde{s}_\sigma \\
&- \sum_{\nu\rho\sigma J_1\neq 0} \tau_{0\sigma}^{J_1^\nu J_1^\rho} (b_{J_1}^{\dagger\nu} \times b_{J_1}^{\dagger\rho})_0 \tilde{s}_\sigma - 2\sum_{J_3\neq 0} \tau_{0J_3}^{0J_3} [(s_0^\dagger \times b_{J_3}^{\dagger 0})_{J_3} \times \tilde{b}_{J_3}^0]_0 - 2\sum_{\nu J_3\neq 0} \tau_{0J_3}^{\nu J_3} [(s_\nu^\dagger \times b_{J_3}^{\dagger 0})_{J_3} \times \tilde{b}_{J_3}^0]_0 \\
&-2\sum_{\nu\rho J_3\neq 0} \tau_{0J_3}^{\nu J_3^\rho} [(s_\nu^\dagger b_{J_3}^{\dagger\rho})_{J_3} \times \tilde{b}_{J_3}^0] - 2\sum_{J_3\sigma\neq 0} \tau_{0J_3^\sigma}^{0J_3} [(s_0^\dagger \times b_{J_3}^{\dagger 0})_{J_3} \times \tilde{b}_{J_3}^\sigma]_0 \\
&-2\sum_{\nu\sigma J_3\neq 0} \tau_{0J_3^\sigma}^{\nu J_3} [(s_\nu^\dagger \times b_{J_3}^{\dagger 0})_{J_3} \times \tilde{b}_{J_3}^\sigma]_0 - 2\sum_{\nu\rho\sigma J_3\neq 0} \tau_{0J_3^\sigma}^{\nu J_3^\sigma} [(s_\nu^\dagger \times b_{J_3}^{\dagger\rho})_{J_3} \times \tilde{b}_{J_3}^0]_0 - \sum_{\nu\rho\sigma J_1 J_2\neq 0} \sum \tau_{0\,0J_3^\sigma(J_3)}^{J_1^\nu J_2^\rho(J_3)} [(b_{J_1}^\dagger \times b_{J_2}^{\dagger\rho})_{J_3} \times \tilde{b}_{J_3}^\sigma]_0 ,
\end{aligned}
$$

$$\tau^0 = \tau_{00\,00(0)}^{00\,00(0)} , \tag{11}$$

$$\tau^\nu \equiv \tau_{00(0)}^{00(0)} ,$$

and

$$\tau_\sigma^{J_1 J_2} = \tau_{0\sigma\,00(0)}^{J_1^0 J_2^0(0)} . \tag{12}$$

The above form of the $\bar{s}_0^\dagger$ is very useful. For example, in determining the terms of $(\bar{s}_0^\dagger)^p$ with all $b^\dagger$'s coupled to $J=0$, only the first six terms of Eq. (11) contribute. The PBB state for $p=1$ (two particles) is

$$\bar{s}_0^\dagger |0) = s_0^\dagger |0) , \tag{13}$$

for $p=2$,

$$(\bar{s}_0^\dagger)^2 |0) = \bar{s}_0^\dagger \bar{s}_0^\dagger |0) = \bar{s}_0^\dagger s_0^\dagger |0) .$$

Using the expansion of $\bar{s}_0^\dagger$ [Eq. (11)], we obtain

$$
\begin{aligned}
(\bar{s}_0^\dagger)^2 |0) ={}& N_0^2 (s_0^\dagger)^2 |0) + \sum_{\nu\neq 0} N_\nu^2 s_\nu^\dagger s_0^\dagger |0) \\
&+ \sum_{\nu\rho\neq 0} N_{\nu\rho}^2 s_\nu^\dagger s_\rho^\dagger |0) + (\text{terms with } b_{J\neq 0}) ,
\end{aligned} \tag{14}
$$

where

$$
\begin{aligned}
N_0^2 &= (1-\tau^0) , \\
N_\nu^2 &= -2\tau_0^\nu ,
\end{aligned} \tag{15}
$$

and

$$N_{\nu\rho}^2 = -\tau^{\nu\rho} .$$

Following as in Eq. (14), the PBB state for p pairs of identical valence nucleons is

$$
\begin{aligned}
(\bar{s}_0^\dagger)^p |0) ={}& N_0^p (s_0^\dagger)^p |0) + \sum_{\nu\neq 0} N_\nu^p s_\nu^\dagger (s_0^\dagger)^{p-1} |0) \\
&+ \sum_{\nu\rho\neq 0} N_{\nu\rho}^p s_\nu^\dagger s_\rho^\dagger (s_0^\dagger)^{p-2} |0) + \cdots ,
\end{aligned} \tag{16}
$$

where

$$
\begin{aligned}
N^p_{\substack{\nu_1\nu_2\cdots \\ \sum_i \nu_i = m}} ={}& \{1-[p-(m+1)]\tau^0\} N^{p-1}_{\substack{\nu_1\nu_2\cdots \\ \sum_i \nu_i = m}} - 2(p-m)\tau^{\sigma_m} N^{p-1}_{\substack{\nu_1\nu_2\cdots \\ \sum_i \nu_i = m-1}} \\
&+ [p-(m-1)]\tau^{\nu_{m-1}\nu_m} N^{p-1}_{\substack{\nu_1\nu_2\cdots \\ \sum_i \nu_i = m-2}} - 2\sum_{\sigma\neq 0} \tau_\sigma^{\nu_m} \mathbf{P}_m(\nu_1\nu_2 \ldots \nu_{m-1},\sigma) N^{p-1}_{\substack{\nu_1\nu_2\cdots\sigma \\ \sum_i \nu_i = m-1}} \\
&- \sum_{\sigma\neq 0} \tau_\sigma^{\nu_{m-1}\nu_m} \mathbf{P}_{m-1}(\nu_1\nu_2 \ldots \nu_{m-1},\sigma) N^{p-1}_{\substack{\nu_1\nu_2\cdots\sigma \\ \sum_i \nu_i = m-2}} - \sum_{\sigma\neq 0} \tau_\sigma^0 \mathbf{P}_{m+1}(\nu_1\nu_2 \ldots \nu_m,\sigma) N^{p-1}_{\substack{\nu_1\nu_2\cdots\sigma \\ \sum_i \nu_i = m}} ,
\end{aligned} \tag{17}
$$

with

$$n' > m' \geq 0 , \tag{18}$$

where n' is the superscript and m' the number of coordinates appearing in the various terms of Eq. (17). Here,

$$\mathbf{P}_m(\nu_1\nu_2 \ldots \nu_m,\sigma) = \sum_{i=1}^m \nu_i \leftrightarrow \sigma = 0, \quad \text{for } m=0 . \tag{19}$$

Equation (17) is a closed algebraic expression for the various expansion coefficients appearing in Eq. (16). For example, $m=0$ in Eq. (17) corresponds to the first term of Eq. (16), i.e., N_0^p, $m=1$ corresponds to N_ν^p, etc.

Using the procedure outlined above, a similar recursive algebraic expression can be obtained for the terms in Eq. (16) with $b_{J\neq 0}^\dagger$ and for higher seniority (v) states. In fact, where the Hamiltonian admits only a single boson associated with each J, the PBB state for $v=2$ has been reported in Ref. 7.

It is observed that only the first few terms of Eq. (16) contribute to the matrix element of the two body interaction. This is the main advantage in expressing the PBB states in the form of Eq. (16). As will be shown in Sec. IV, only the first three terms, i.e., N_0, N_ν^p, and $N_{\nu_\rho}^p$, contribute to the matrix element of the pairing Hamiltonian.

IV. APPLICATION TO THE PAIRING HAMILTONIAN

The pairing Hamiltonian between the identical nucleons is written as

$$H_F=-\sum_a \varepsilon_a \hat{j}_a (C_a^\dagger \tilde{C}_a)_0 - \frac{G}{4}\sum_{ab}\hat{j}_a\hat{j}_b A_{00}^\dagger(aa)A_{00}(bb) , \tag{20}$$

where ε_a denotes the single particle energies and G having the dimensions of energy is quoted as the strength of the pairing interaction. Using the mapping [Eqs. (8a)–(8c)], the pairing Hamiltonian takes the following form in the boson space:

$$H_B=\sum_{\substack{abJM\\ \mu\nu}} \varepsilon_a X_{JM}^{\mu}(ab)X_{JM}(ab)b_{JM}^{\dagger\mu}b_{JM}^{\nu}-\frac{G}{4}\sum_{ab\mu}\hat{j}_a\hat{j}_b X_{00}^{\mu}(aa)X_{00}^{\mu}(bb)\left\{s_\mu^\dagger-\sum_{\substack{J_1J_2J_3\\ \nu_1\rho_1\sigma_1}}\tau_{0^\mu J_3^{\sigma_1}(J_3)}^{J_1 J_2^{\rho}(J_3)}[(b_{J_1}^{\dagger\rho_1}\times b_{J_2}^{\dagger\rho_1})_{J_3}\times\tilde{b}_{J_3}^{\sigma_1}]_{00}\right\}s_\nu . \tag{21}$$

The ground state wave function for p pairs of identical valence nucleons is given by

$$|\phi_0)=N_R(\bar{s}_0^\dagger)^p|0) . \tag{22}$$

The bar state obtained using Eq. (8b) is

$$(\phi_0|=N_L(0|(s_0)^p . \tag{23}$$

As is clear from Eqs. (22) and (23) we have

$$[\,|\phi_0)]^\dagger\neq(\phi_0|\ . \tag{24}$$

Due to this nonunitary character of the basis states, it is required to use the biorthonormal basis states.[6] The following normalization prescription is adopted[8,10] for the biorthonormal basis states:

$$N_L=\frac{1}{[(0|s_0^p(s_0^\dagger)^p|0)]^{1/2}} ,\qquad N_R=\frac{1}{N_L(0|s_0^p(\bar{s}_0^\dagger)^p|0)} . \tag{25}$$

The ground state energy is given by

$$E_0(p)=(\phi_0|H_B|\phi_0) . \tag{26}$$

Using Eqs. (14), (21), and (25) in (26) we obtain

$$E_0(p)=\sum_{ab}\left[\left[\varepsilon_a\delta_{ab}-\frac{G}{4}\hat{j}_a\hat{j}_b\right]\left[pX_aX_b+2X_aP_1^p(b)-\sum_c X_cX_bX_cP_1^p(c)\right]\right.$$
$$+\frac{G}{2}\frac{\hat{j}_b}{\hat{j}_a}X_a^2\left\{p(p-1)X_aX_b+(p-1)X_aP_1^p(b)+(p-1)X_bP_1^p(a)+4P_2^p(ba)\right.$$
$$\left.\left.+\sum_c X_c\left[-(p-1)X_bX_aP_1^p(c)-2X_aP_2^p(bd)-2X_bP_2^p(ca)+\sum_d X_aX_bX_dP_2^b(cd)\right]\right\}\right] , \tag{27}$$

and the occupation probability

$$v_a^2=\frac{(\phi_0|(-)\bar{E}_{00}(aa)|\phi_0)}{\hat{j}_a}=\frac{pX_a^2}{\hat{j}_a^{\,2}}+\frac{1}{N_0^p\hat{j}_a^{\,2}}\left[2X_aP_1^p(a)-\sum_b X_a^2X_bP_1^p(b)\right] . \tag{28}$$

In Eqs. (27) and (28), the following notation has been used:

$$X_a=X_{00}^0(aa) ,$$
$$P_1^p(a)=\sum_\nu X_{00}^\nu(aa)N^p , \tag{29}$$
$$P_2^p(ab)=\sum_{\nu\rho}X_{00}^\nu(aa)X_{00}^\rho(bb)N_{\nu\rho}^p .$$

From Eqs. (27) and (28), the following points can be noted: (a) the coefficients of one the first three terms of Eq. (16), i.e., N_0^p, N_ν^p, and $N_{\nu\rho}^p$ appear in Eqs. (27) and (28); (b) the ground state energy [Eq. (27)] depends only on the collective coefficients X_a even though the ground state wave function depends on all X's; (c) for $p=2$, the expressions are exactly the same as those of Li.[10]

The ground state parameters X_a can be obtained in the fermion space[12] as well as in the boson space. In the boson space these can be obtained through the variational procedure

$$\frac{\partial}{\partial X_a}E_0(p)=0 , \tag{30}$$

subject to the constraint

$$\sum_a X_a^2 = 2 \ . \tag{31}$$

The minimization condition, Eq. (30), results in a set of nonlinear coupled equations.[6] The Newton-Rampson method has been employed for solving these equations.

Numerical calculations have been carried out for Sn and Ni isotopes. In the case of Sn isotopes the single particle energies used are $2d_{5/2}=0.0$, $1g_{7/2}=0.22$, $3s_{1/2}=1.90$, $2d_{3/2}=2.20$, and $1h_{11/2}=2.80$ MeV. The strength $G=0.187$ MeV has been used. In the case of Ni isotopes the input parameters used are $2p_{3/2}=0.0$, $1f_{5/2}=0.78$, $2p_{1/2}=1.56$, $1g_{9/2}=4.52$ MeV, and $G=0.331$ MeV. The results of calculations are shown in Tables I and II for Sn and Ni isotopes, respectively. The results labeled PW are obtained in the present work using Eq. (30). The results obtained by Li[10] and by exact shell model calculations are designated by Li and ESM, respectively. As is clear from the Table I, the occupation probabilities for $2d_{5/2}$ and $1g_{7/2}$ exceed unity[11] in the case of Li which is a gross violation of the Pauli principle. This violation occurs because Li has employed an approximate PBB state. In fact, Li has taken only the first two terms of the PBB state into consideration. As mentioned before, for $p=2$ our expressions for the ground state energy, Eq. (27), and the occupation probability, Eq. (28), are exactly identical to Li's corresponding expressions. This is also evident from the numerical results presented in Tables I and II. The results of the present work (PW) have been carried out with the full PBB states. Therefore, the present work can be considered as an improvement of the work of Li. It is evident from Tables I and II that the results PW are more than 90% in agreement with the ESM and there is no violation of the Pauli principle.

V. CONCLUSIONS

The major obstacle in carrying out the explicit calculations in the boson space using the Dyson boson mapping is the construction of the basis states. There are essentially two ways of performing the calculations. One is to employ the boson basis states and the other is to use the physical boson basis states. The boson basis states are overcomplete and therefore contain the spurious states. The physical boson basis (PBB) states are obtained by replacing each bifermion operator appearing in the fermion basis by the corresponding boson image using the Dyson boson mapping. These PBB states have a very complex structure,[7,10] thereby nullifying any advantage in working in the boson representation.

Many attempts have been made to obtain the PBB states in an approximate way.[10,11] These various approximate methods have been shown to be inadequate.[11] In these investigations, the occupation probabilities exceed unity. In the present work an algorithm has been developed for obtaining the explicit expressions of the PBB states. The algorithm is illustrated for the ground state wave function.

It is observed that only the first few terms of the PBB state, expressed in the present form, contribute to the matrix element. In fact, in the case of the pairing interaction only the first three terms of the PBB state [Eq. (14)] contribute to the matrix element. Therefore, one needs to derive the algebraic expressions of only the first few expansion coefficients appearing in the PBB state.

The present formulation has been applied to Sn and Ni isotopes with the pairing Hamiltonian as the model interaction. The results of the present investigations have been compared with those of Li.[10] It is shown in Table I that for $p=8$ (16 particles), the occupation probability exceeds unity in the case of Li. This is a gross violation of the Pauli principle. The results of the present work do not suffer from this drawback. The present formulation can be straightaway used to obtain the higher seniority PBB states. This work is now underway.

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TABLE I. Calculated ground state energies E_0 and occupation probabilities for the tin isotopes. The strength G of the pairing Hamiltonian and single particle energies used are $G=0.187$ MeV and $\varepsilon_a=0.22$, 1.90, 2.20, and 2.80 MeV for $1d_{5/2}$, $1g_{7/2}$, $3s_{1/2}$, $2d_{3/2}$, and $1h_{11/2}$, respectively.

p	E_0	$v^2_{2d_{5/2}}$	$v^2_{1g_{7/2}}$	$v^2_{3s_{1/2}}$	$v^2_{2d_{3/2}}$	$v^2_{1h_{11/2}}$	Source
2	−2.623	0.325	0.214	0.029	0.023	0.016	PW
	−2.623	0.325	0.214	0.029	0.023	0.016	Li
	−2.624	0.325	0.214	0.029	0.023	0.016	ESM
5	−3.080	0.714	0.608	0.078	0.060	0.038	PW
	−3.085	0.712	0.609	0.078	0.060	0.038	Li
	−3.084	0.715	0.607	0.078	0.060	0.038	ESM
7	−0.673	0.935	0.910	0.119	0.083	0.046	PW
	−0.741	0.913	0.912	0.127	0.090	0.051	Li
	−0.700	0.936	0.909	0.120	0.085	0.048	ESM
8	2.180	0.932	0.919	0.380	0.240	0.103	PW
	1.506	1.030	1.010	0.208	0.135	0.067	Li
	2.150	0.931	0.914	0.370	0.249	0.115	ESM

TABLE II. Calculated ground state energies E_0 and occupation probabilities for the nickel isotopes. The parameters used for the pairing Hamiltonian are the following: $G=0.331$ MeV, and $\varepsilon_a=0.0$, 0.78, 1.56, and 4.52 MeV for $2p_{3/2}$, $1f_{5/2}$, $2p_{1/2}$, and $1g_{9/2}$, respectively.

p	E_0	$v^2_{2p_{1/2}}$	$v^2_{1f_{5/2}}$	$v^2_{2p_{1/2}}$	$v^2_{1g_{9/2}}$	Source
2	−2.090	0.624	0.201	0.081	0.013	PW
	−2.090	0.624	0.201	0.081	0.013	Li
	−2.100	0.629	0.198	0.081	0.013	ESM
3	−1.745	0.762	0.407	0.155	0.021	PW
	−1.770	0.801	0.383	0.148	0.020	Li
	−1.750	0.764	0.404	0.153	0.021	ESM
5	1.723	0.932	0.855	0.413	0.031	PW
	1.720	0.925	0.866	0.395	0.031	Li
	1.700	0.934	0.856	0.408	0.031	ESM

APPENDIX

Here, the relevant commutation relations used in the present work are listed.

$$[s_0^m, s_0^{\dagger n}] = \frac{n!}{(n-m)!} s_0^{\dagger n-m} \quad \text{for } n \geq m \ , \tag{A1}$$

$$\left[s_\mu, \prod_{i=1}^{k} s^\dagger_{\nu_i}\right] = \sum_{r=1}^{k} \delta_{\mu\nu_r} \prod_{\substack{i=1 \\ i\neq r}}^{k} s^\dagger_{\nu_i} \ , \tag{A2}$$

$$[b_{JM}(ab), \bar{b}^\dagger_{J'M'}(cd)] = \mathbf{P}_J(cd) \Bigg| \delta_{JJ'}\delta_{MM'}\delta_{ac}\delta_{bd} - \mathbf{P}_J(ab)\delta_{cb} \times \sum_{J''} \hat{J}\hat{J}'(-1)^{J'+J''+M} \begin{Bmatrix} J & j_a & j_b \\ j_d & J' & J'' \end{Bmatrix} \begin{bmatrix} J' & J & J'' \\ M' & -M & M-M' \end{bmatrix} \bar{E}_{J''M-M'} \Bigg| \ , \tag{A3}$$

$$[E^{(ab)}_{JM}, (s_0^\dagger)^n] = -2n\hat{j}_b^{-1}\chi_b b^\dagger_{JM}(ab)(s_0^\dagger)^{n-1} \ , \tag{A4}$$

$$[s_0^n, \bar{b}^\dagger_{JM}(cd)] = 2n[\delta_{J0}\delta_{M0}\delta_{cd}\chi_c s_0^{n-1} + \mathbf{P}_J(cd)\hat{j}_d^{-1}\chi_d \bar{E}_{JM}(cd)s_0^{n-1} - (n-1)(-1)^{J-M}\hat{j}_c^{-1}\hat{j}_d^{-1}\chi_c\chi_d b_{J-M}(cd)s_0^{n-2}] \ , \tag{A5}$$

where

$$\mathbf{P}_J(ab) = 1 - (-1)^{j_a+j_b+J} a \leftrightarrow b \ .$$

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[1] D. Bonatsos and A. Klein, Ann. Phys. (N.Y.) **169**, 61 (1986); D. Bonatsos, A. Klein, and Q. Y. Zhang, Phys. Rev. C **34**, 686 (1986).

[2] T. Tamura, Phys. Rev. C **28**, 2840 (1983), and references therein.

[3] H. B. Geyer and S. Y. Lee, Phys. Rev. C **26**, 642 (1982); H. B. Geyer, Phys. Rev. C **34**, 2373 (1986); H. B. Geyer, C. A. Engelbrecht, and F. J. W. Hahne, *ibid.* **33**, 1041 (1986); C. T. Li, Phys. Lett. **120B**, 251 (1983); P. Ring and P. Schuck, *The Nuclear Many-Body Problem* (Springer, New York, 1980), Chap. 9.

[4] Y. K. Gambhir, P. Ring, and P. Schuck, Phys. Rev. C **25**, 2858 (1982), S. Pittel, P. D. Dual, and B. R. Barrett, Ann. Phys. (N.Y.) **144**, 168 (1982); A. Van Egmond and K. Allaart, Nucl. Phys. **A394**, 173 (1983).

[5] O. Scholten, Phys. Rev. C **28**, 1783 (1983), and references therein.

[6] J. A. Sheikh, Ph.D. thesis, Indian Institute of Technology, Bombay, 1986 (unpublished).

[7] Y. K. Gambhir, J. A. Sheikh, P. Ring, and P. Schuck, Phys. Rev. C **31**, 1519 (1985).

[8] Y. K. Gambhir, R. S. Nikam, C. R. Sarma, and J. A. Sheikh, J. Math. Phys. **26**, 2067 (1985); J. A. Sheikh, J. Sita, and C. R. Sarma, J. Math. Phys. **28**, 751 (1987).

[9] A. Arima and F. Iachello, in *Advances in Nuclear Physics*, edited by J. W. Negele and E. Vogt (Plenum, New York, 1984), Vol. 13, p. 139.

[10] C. T. Li, Phys. Rev. C **29**, 2309 (1984).

[11] J. A. Sheikh and Y. K. Gambhir, Phys. Rev. C **24**, 2344 (1986).

[12] Y. K. Gambhir and J. A. Sheikh, Phys. Rev. C **33**, 2188 (1986), and references therein.