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Importance of the deuteron quadrupole moment in ^{2}H(d,γ)^{4}He

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We present a phenomenological study of the ^{2}H(d,γ)^{4}He reaction that allows for the D-state component of the colliding deuterons. The low-energy cross section is determined by an interference between $E2$ transitions from an initial $^5S_2$ scattering state to either the D- or S-state components of ^{4}He, the latter via the internal quadrupole moment of the deuteron. From a fit to the data, we determine the asymptotic D/S ratio of ^{4}He to be $\rho \sim -0.40$.

I. INTRODUCTION

In the simplest shell model picture, ^{4}He is the smallest doubly magic nucleus: the four nucleons occupy the lowest $0s_{1/2}$ orbital with the protons and neutrons paired into spin-singlet states. However, more realistic microscopic calculations[1-3] suggest a D-state component of the ^{4}He ground state in which the four nucleons are coupled in an $S=2$ spin state. As the ^{4}He ground state is $J^{\pi}=0^+$, this provides clear evidence in favor of a nonspherical configuration (orbital angular momentum $L=2$). The magnitude of this D-state component remains uncertain, however, with theoretical estimates ranging from $P_D \sim 5\%$ to 13%.

Recent experimental data on the ^{2}H(d,γ)^{4}He reaction[4,5] have been interpreted in terms of a D-state component in the ground state of ^{4}He. Theoretical calculations that reproduce the high energy ($E_{\text{c.m.}} > 3$ MeV) cross section without invoking a D-state component underestimate the data at lower energies by almost three orders of magnitude. This enormous discrepancy has been seen as one of the clearest signatures of a D-state component. Measurements of tensor analyzing powers with a 9.7 MeV polarized deuteron beam have been interpreted as indicating a 4.8% D-state admixture, based on a pure $E2$ direct capture model.[6]

Arguments based on spin-isospin selection rules suggest that the capture process involves $E2$ radiation almost exclusively. The $^1D_2 \to {}^1S_0$ transition, which dominates at high energy, is strongly suppressed by the centrifugal barrier at lower energies, and $^5S_2 \to {}^5D_0$ thus becomes important as the energy decreases (we use the notation $^{2S+1}L_J$). This theoretical picture now enjoys solid experimental support. Gamma ray angular distributions,[4,7,8] measured at center of mass energies between 75 keV and 4.85 MeV, indicate a transition from the $\sin^2 2\theta$ behavior characteristic of the $^1D_2 \to {}^1S_0$ transition at high energies to near isotropy at lower energies, as in characteristic of capture from an S state.

All theoretical studies of the ^{2}H(d,γ)^{4}He reaction to date have assumed either point deuterons,[4,6,8,9] or deuteron wave functions with no D-state admixture.[10] Our goal in this work is to study, in a simple phenomenological model, the influence of the D-state component of the deuteron wave function on the low-energy capture cross section and the extent to which it must be allowed for in attempting to extract the D state of ^{4}He from the data.

It is easy to see that the D state of the deuteron can be significant in the capture process. An $E2$ transition from an initial $^5S_2$ scattering state can populate both the large ($^1S_0$) and small ($^5D_0$) components of ^{4}He. The "external" $E2$ operator, $(\mathbf{R}\times\mathbf{R})_{2\mu}$, written in terms of the relative coordinate, $\mathbf{R}$, of the d+d system, effects a transition to the D-state component of ^{4}He; this is what is accounted for in the point-deuteron approximation. However, the "internal" $E2$ operator, $(\mathbf{r}_{12}\times\mathbf{r}_{12})_{2\mu}+(\mathbf{r}_{34}\times\mathbf{r}_{34})_{2\mu}$, written in terms of internal deuteron coordinates, $\mathbf{r}_{12}$, $\mathbf{r}_{34}$, does not change the angular momentum of the relative dd wave function, and therefore populates the S-state component of ^{4}He through the internal quadrupole moment of the deuteron. As these two contributions arise from the same $^5S_2$ continuum state, they will have a similar energy dependence at low energies. Further, as the deuteron D-state probability ($\sim 6\%$) is of the same magnitude as current estimates of the ^{4}He D-state probability, we might expect the two amplitudes to be comparable. Indeed, by fitting the absolute normalization of the low-energy cross section, we find an interference between these two amplitudes that fixes the asymptotic D/S ration in ^{4}He.

II. THEORY

On the basis of the experimental evidence discussed above, we restrict our study to $E2$ transitions from $^1D_2$

and $^5S_2$ dd scattering states to the $^1S_0$ and $^5D_0$ components of the ^{4}He ground state. Further, in light of recent evidence of additional multipolarities in ^{2}H(d,γ)^{4}He at $E_{\rm c.m.}=4.85$ MeV,[11] we limit our study to c.m. energies below 3 MeV.

We write each of the dd scattering states, as well as the components of the ground state of ^{4}He, as products of internal deuteron wave functions and a function $f(R)$ of relative motion:

$$\Psi_{LS}^J=\frac{f_{LS}^J(R)}{R}[Y_L(\hat{\mathbf{R}});(\Phi_d^{S_1^{=1}}\Phi_d^{S_2^{=1}})_s]_J \ . \tag{1}$$

Here L is the orbital angular momentum, S the channel spin, and J the total angular momentum. A knowledge of the exact form of the deuteron wave function, $\Phi_d^{S=1}$, is not essential; it suffices to know that $\Phi_d^{S=1}$ contains a D-state component, which leads to a nonzero quadrupole moment for the deuteron.

In the case of the ground state of ^{4}He,

$$\Psi_\alpha=\cos\omega\Psi_{L=S=0}^{J=0}+\sin\omega\Psi_{L=S=2}^{J=0} \ , \tag{2}$$

where the radial wave functions are normalized to one. We generate these from uncoupled radial Schrödinger equations involving phenomenological Woods-Saxon potentials

$$V(R)=\frac{V_0}{1+\exp[(R-R_0)/a]} \tag{3}$$

chosen to fit the experimental dd breakup energy of 23.84 MeV. The mixing angle ω parametrizes the D-state amplitude in ^{4}He. The scattering states

$$\Psi(^{2S+1}L_J)=i^L\left[\frac{4\pi(2L+1)}{v}\right]^{1/2}\frac{1}{p}\Psi_{LS}^J \ , \tag{4}$$

where v and p are the relative velocity and momentum of the colliding deuterons, are normalized to unit flux. We generate them from radial Schrödinger equations in the presence of nuclear and Coulomb potentials.

In the long wavelength approximation valid here, the $E2$ electromagnetic transition operator is

$$Q_{2\mu}^E=e\sum_{i=1}^{4}r_i^2Y_{2\mu}(\hat{\mathbf{r}}_i)\frac{1-\tau_{zi}}{2} \ , \tag{5}$$

where $\mathbf{r}_i$ is the coordinate of the $i^{\rm th}$ particle relative to the center-of-mass and $e(1-\tau_{zi})/2$ is its charge.

If we now introduce the internal coordinate of the deuterons $\mathbf{r}_{12},\mathbf{r}_{34}$, and their relative separation $\mathbf{R}$ through the equations

$$\mathbf{r}_{1,2}=\tfrac{1}{2}(\mathbf{R}\pm\mathbf{r}_{12}); \quad \mathbf{r}_{3,4}=\tfrac{1}{2}(-\mathbf{R}\pm\mathbf{r}_{34}) \ , \tag{6}$$

we can write the relevant (isoscalar) part of the $E2$ operator as

$$Q_{2\mu}^E=\frac{e}{2}[R^2Y_{2\mu}(\hat{\mathbf{R}})+\tfrac{1}{2}r_{12}^2Y_{2\mu}(\hat{\mathbf{r}}_{12}) +\tfrac{1}{2}r_{34}^2Y_{2\mu}(\hat{\mathbf{r}}_{34})+\text{“cross terms”}] \ , \tag{7}$$

where the cross terms are linear in $\mathbf{R}$ and therefore do not contribute to the processes of interest by the parity selection rule. The part of the $E2$ operator that depends on the internal coordinates of the two deuterons has been neglected in all previous phenomenological calculations. It is this part of the operator, together with the explicit D-state admixture in the deuteron wave functions, that leads to a new and important contribution to the ^{2}H(d,γ)^{4}He capture rate. To explicitly note its effect we calculate the contribution from this internal part of the $E2$ operator to the low-energy cross section. The reduced matrix element is given by

$$\begin{aligned}M&=\frac{e}{4}\langle\Psi_\alpha\|r_{12}^2Y_2(\hat{\mathbf{r}}_{12})+r_{34}^2Y_2(\hat{\mathbf{r}}_{34})\|\Psi_{^5S_2}\rangle\\&=\sqrt{50}\left[\frac{e^2}{\hbar v}\right]^{1/2}\frac{\cos\omega}{p}Q_{\rm d}\int_0^\infty dR\, f_s^\alpha(R)f_{^5S_2}(R) \ ,\end{aligned} \tag{8}$$

where

$$Q_{\rm d}=(8\pi/25)^{1/2}\langle\Phi_{\rm d}^{S=1}\|\left[\frac{r}{2}\right]^2Y_2(\hat{\mathbf{r}})\|\Phi_{\rm d}^{S=1}\rangle =0.2860\ {\rm fm}^2$$

is the experimentally measured quadrupole moment of the deuteron,[12] and f_S^g is the radial wave function of the $^1S_0$ component of ^{4}He. Note that if the $^5S_2$ and $^1S_0$ potentials were identical, the radial integral would vanish by orthogonality.

All other contributions to the $E2$ transition matrix element can be evaluated in a similar fashion, and we simply display our results for the capture cross section:[10,13]

$$\sigma(E_{\rm c.m.})=\sum_{S=0,2}\frac{4\pi}{75\hbar}\frac{2S+1}{45}\left[\frac{E_\gamma}{\hbar c}\right]^5\frac{1}{p^2}\frac{e^2}{\hbar v}\,|A_{^{2S+1}L_J}|^2 \ , \tag{9}$$

where $E_\gamma=E_{\rm c.m.}+23.84$ MeV is the energy of the emitted photon,

$$A_{^1D_2}=-\tfrac{5}{2}\cos\omega\int_0^\infty R^2dR\ f_S^g(R)f_{^1D_2}(R) \tag{10a}$$

is the continuum $^1D_2$ component of the amplitude, and

$$\begin{aligned}A_{^5S_2}=&\sqrt{50}Q_{\rm d}\cos\omega\int_0^\infty dR\ f_S^g(R)f_{^5S_2}(R)\\&+\tfrac{1}{2}\sin\omega\int_0^\infty R^2dR\ f_D^g(R)f_{^5S_2}(R)\end{aligned} \tag{10b}$$

is the $^5S_2$ amplitude. As mentioned above, we will adjust ω to reproduce the absolute normalization of the low energy experimental cross section.

III. RESULTS

To specify the radial wave functions of the $^1S_0$ and $^5D_0$ components of ^{4}He and the $^1D_2$ and $^5S_2$ continuum states, we have constructed a singlet ($S=0$) Woods-Saxon potential (parameters $V_0=-74.00$ MeV, $R_0=1.70$ fm, $a=0.90$ fm) that simultaneously reproduces the dd breakup energy of ^{4}He and the experimental capture cross section at $\sim$2 MeV. The quintet ($S=2$) potential (parameters $V_0=+60.00$ MeV, $R_0=1.70$ fm, $a=0.90$ fm) was constrained to reproduce the low energy, 25–175 keV, $^5S_2$ dd elastic phase shifts calculated by Meier and Glöckle[14]

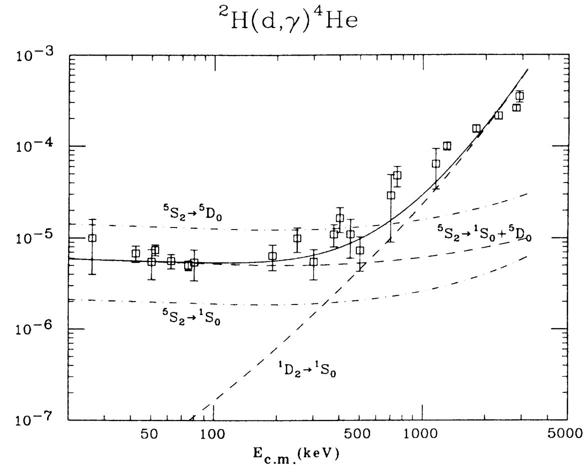

FIG. 1. Shown are the contributions from the $^1D_2 \to {}^1S_0$ transition (dashed line), the $^5S_2 \to {}^5D_0$ and $^5S_2 \to {}^1S_0$ transitions (dashed-dot-lines), the coherent sum of these latter two (dashed line), and the total calculated S factor (solid line). From the fit to the data points shown (Refs. 4, 5, and 15-17) we determine an asymptotic D/S ratio of $\rho \sim -0.40$.

using a resonating group method. The repulsive character of the potential, required to fit the phase shifts, will clearly not bind the two deuterons in the $L = S = 2$ channel. We have therefore generated f_D^α from a third potential (parameters $V_0 = -191.50$ MeV, $R_0 = 1.70$ fm, $a = 0.90$ fm) that binds the d + d system in the $^5D_0$ channel with the correct energy. Tensor coupling between the components of the wave function has been neglected throughout. We note that the $^1D_2$ and $^5S_2$ scattering wave functions have no nodes, reflecting the absence of bound states with these quantum numbers. Furthermore, the smooth energy dependence of the phase shifts indicates no resonant structure in these channels. Both of these facts are supported by experiment.

As mentioned above, our primary goal here is to assess the importance of the D-state component of the deuteron in the capture process and its use in determining the D state of ^{4}He. A model-independent determination of the latter would be possible only if the radial integrals in (10a and 10b) are dominated by the nuclear exterior, and therefore depend only on the asymptotic form of the wave functions. Unfortunately, this is not the case, even at low energies. The large binding energy of ^{4}He makes the bound-state wave functions decay rapidly, and so the dominant contribution to the integrals comes from a region between 3 and 4 fm; our results are sensitive to the details of the nuclear interior. To quantify this sensitivity, we have performed calculations with two sets of potentials: the "single geometry" set described above (set I), and also one in which the quintet geometry was changed to $R_0 = 2.00$ fm, $a = 1.00$ fm (set II). The strengths of these potentials, $V_0 = +28.00$ MeV and $V_0 = -156.50$ MeV, were readjusted to fit the elastic phase shifts and the binding energy of the D state, respectively.

In Fig. 1 we present the experimental data[4,5,15–17] for the ^{2}H(d,γ)^{4}He capture reaction, together with our theoretical calculations for the single geometry potential set. At low energies, it is most convenient to present the data in terms of the astrophysical S factor, defined by

$$S(E_{\text{c.m.}}) = \sigma(E_{\text{c.m.}}) E_{\text{c.m.}} \exp(2\pi\eta) , \tag{11}$$

where $\eta = e^2/\hbar v$ is the Sommerfeld parameter. We can well reproduce the energy dependence of the S factor if we choose $\tan\omega = -0.41$ (-0.31) for potential set I (II), leading to a ^{4}He D-state probability of $P_D \sim 14\%$ (9%) and to an asymptotic D/S ratio of $\rho \equiv (N_D/N_S)\tan\omega \sim -0.40$ (-0.46). Here, N_S (N_D) is the asymptotic normalization of the $^1S_0$ ($^5D_0$) component of the ground state of ^{4}He,

$$\begin{aligned} f_S^\alpha(R) &\to N_S \exp(-\kappa R) , \\ f_D^\alpha(R) &\to N_D \left[1 + \frac{3}{\kappa R} + \frac{3}{(\kappa R)^2} \right] \exp(-\kappa R) , \end{aligned} \tag{12}$$

and $\kappa = 1.072$ fm^{-1} is the dd threshold wave number. The uncertainty in our results is a direct consequence of the strong model dependence of all phenomenological analyses.

These values of the ^{4}He D-state admixture are larger than that of a recent calculation of the ^{2}H(d,γ)^{4}He, which gave $P_D \sim$ 5–7 %.[10] They tend to favor the 13% result of a theoretical calculation with soft-core potentials[1] over the 5.4% value obtained with the Paris potential.[2] Our results agree with the value of $-0.5 < \rho < -0.4$ obtained in the analysis of tensor analyzing powers at $E_{\text{c.m.}} = 4.85$ MeV,[9] but are lower than a more recent calculation which gave $\rho = -0.2 \pm 0.05$ (Ref. 8) from the study of gamma ray angular distributions at lower energies.

It is important to note that our calculated capture cross section arises from a delicate destructive interference between the $^5S_2 \to {}^1S_0$ and $^5S_2 \to {}^5D_0$ amplitudes. If we had neglected the first process, and had attempted to fit the cross sections, we would have obtained $|\rho| = 0.24$ (0.26) and $P_D \sim 6\%$ (2.8%) for the two potential sets. We also note that our results quoted above correspond to only one of two possible mixing angles, both of which fit the data for a given potential set. These solutions correspond to $P_D \sim 1\%$ and $\rho > 0$; we have discarded these *a priori* as unphysical.

IV. CONCLUSIONS

We have tried to use a simple phenomenological model to estimate the effect of the internal structure of the deuteron in the determination of the ^{2}H(d,γ)^{4}He capture rate. We have found substantial changes in the extracted D-state probability of ^{4}He as compared with a similar calculation that neglects the internal structure of the deuteron. Despite the difficulty that phenomenological models may have in predicting the D/S state ratio in ^{4}He, we feel that the importance of the D-state component of the deuteron, in the determination of the capture rate, has been established.

In our model, we have ignored the rearrangement (^{3}He+n or ^{3}H+p) components of the continuum wave function. Although these positive Q-value channels are strongly populated in d+d collisions, we suspect that they are not important in the ^{2}H(d,γ)^{4}He reaction. In a simple cluster model, the channel spin of the 3+1 system is either $S=0$ or $S=1$. Therefore, $E2$ capture proceeds from an $L=2$, $S=0$ scattering state, which is suppressed at lower energies due to the centrifugal barrier. Furthermore, a semimicroscopic calculation[10] of the ^{2}H(d,γ)^{4}He reaction that neglects rearrangement yields the correct normalization of the high-energy cross section, indicating that rearrangement effects are probably small. Of course, a definite quantitative statement requires a detailed calculation.

It is clear that the ^{2}H(d,γ)^{4}He reaction can be an important tool in investigating the *D*-state component of ^{4}He. However, a more fundamental calculation of this reaction, incorporating both the internal structure of the colliding deuterons and the tensor components in the NN force, is essential for a reliable and unambiguous analysis.

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